



## PRESSURE-DRIVEN LAMINAR PULSATING FLOW IN A PIPE: EFFECT OF THE AMPLITUDE

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**Abstract:** In this study, pressure-driven laminar pulsating flow in a pipe is examined numerically. Both the hydrodynamically developing flow and fully developed conditions are considered. A commercially available CFD package, FLUENT, is used in the simulations. A constant value of the time-averaged Reynolds number is considered,  $Re=1000$ . Six different values of the dimensionless frequency ( $F=0.01, 0.1, 1, 10, 100, 1000$ ) and three different values of the dimensionless amplitude ( $A=0.1, 0.5$  and  $0.95$ ) are considered. Both the hydrodynamically developing and fully developed flows are analyzed. Effects of the amplitude and frequency of the pulsating inlet velocity on the friction coefficient as well as velocity field are predicted.

**Keywords:** Hydrodynamics, Pulsating flow, Internal flow, Frequency, amplitude, friction coefficient

## BİR BORU İÇERİSİNDE BASINÇ-KAYNAKLI LAMİNER ATIMLI AKIŞ İÇİN GENLİK ETKİSİ

**Özet:** Bu çalışmada, bir boru içerisindeki basınç kaynaklı laminer atımlı akış sayısal olarak incelenmiştir. Hidrodinamik olarak gelişmekte olan akış ve tam gelişmiş akış koşullarının her ikisi de göz önüne alınmıştır. Simülasyonlarda bir ticari CFD paket programı olan FLUENT kullanılmaktadır. Zaman-ortalama Reynolds sayısı için sabit bir değer dikkate alınmıştır ( $Re=1000$ ). Boyutsuz frekansın altı farklı değeri ( $F=0.01, 0.1, 1, 10, 100, 1000$ ) ve boyutsuz genliğin üç farklı değeri ( $A=0.1, 0.5$  and  $0.95$ ) incelenmiştir. Hem hidrodinamik olarak gelişmekte olan akış hem de tam gelişmiş akış analiz edilmiştir. Atımlı giriş hızı genlik ve frekansının, hız alanının yanı sıra sürtünme katsayısına etkisi belirlenmiştir.

**Anahtar kelimeler:** Hidrodinamik, Atımlı akış, İç akış, Frekans, Genlik, Sürtünme katsayısı

### NOMENCLATURE

$u, u^*$	dimensional [m/s] and dimensionless axial velocity component, $u/u_m$
$u_A$	dimensional velocity fluctuation amplitude [m/s]
$v$	radial velocity component [m/s]
$wt$	dimensional phase
Greek letters	
$\rho$	density
$\mu$	dynamic viscosity
$\nu$	kinematic viscosity
$\omega$	angular frequency, $2\pi f$
Subscripts	
$A$	amplitude
$av$	average
$m$	mean
$0$	base or initial
$ta$	time-averaged
$w$	wall
$A$	dimensionless amplitude, $u_A/u_m$
$C_f$	friction coefficient
$D$	pipe diameter[m]
$f, F$	dimensional [Hz] and dimensionless frequency, $R^2 f/\nu$
$F\tau$	dimensionless phase
$L$	length of the pipe [m]
$L_p$	length of the piston joint [m]
$P$	pressure [Pa]
$r, r^*$	dimensional [m] and dimensionless radial coordinate, $r/R$
$R$	radius of the pipe [m]
$Re$	Reynolds number, $u_{av,ta} D/\nu$
$x$	axial coordinate [m]
$t, \tau$	dimensional [s] and dimensionless time (period), $vt/R^2$

## INTRODUCTION

Pulsating flow is specific type of unsteady flow, in which an periodic oscillating flow component is superimposed on a steady flow. Pulsating flows received a great deal of research interest because of their large application areas spanning from biological systems (e.g. blood flow in arteries and veins) to engineering systems (e.g. intake and exhaust flows of internal combustion engines, nuclear reactors, etc.). Some other applications areas can be listed as follows: heat transfer augmentation, enhancement of cleaning, fluid mixing, mass transport in porous media, thermoacoustic devices, reciprocating pumps and turbines and biofluids engineering (Nabayi et. al., 2010)

According to the nature of the pulse, pulsating flows can be categorized into two groups: pressure-driven and boundary-driven. As it is inferred from its name, a pressure driven flow can be produced creating a pulsating pressure in the entrance. Such a flow can be generated using piston-cylinder mechanism, Scotch-Yoke mechanism, servo valves, mass flow rate controller (Carpinlioglu et. al.,2013) etc. In boundary-driven pulsating flows, additional momentum is supplied to the main flow with the pulsation of the walls.

The interest of this study is limited to the pulsating flows in pipes. On this topic, a large number of studies have existed in the literature, most of which are on the hydrodynamics aspects. For an extensive review of the literature, readers are referred to see review articles by Gundogdu and Carpinlioglu (1999,1999) and Carpinlioglu and Gundogdu (2001). Uchida (1956) and Atabek and Chang (1961) are the pioneering studies on the topic. Uchida (1956) analytically studied pulsating flow in a pipe for an arbitrary, time-varying, axial pressure gradient. Atabek and Chang (1961) obtained an analytical solution for unsteady laminar incompressible oscillatory flow near the entry of a circular tube. Krijger et al. (1991) numerically investigated sinusoidally pulsating channel flow. Zhao and Cheng (1996) studied fully developed laminar reciprocating pipe flow analytically and experimentally. They showed that the oscillation amplitude affected the friction coefficient considerably.

Haslam and Zamir (1998) analytically treated fully developed laminar pulsatile flow in tubes of elliptic cross sections. Yakhot et al. (1999) numerically investigated fully developed, laminar pulsating flow in a rectangular duct. They analyzed influence of the aspect ratio of the rectangular duct and the pulsating pressure gradient frequency on the phase lag, the amplitude of the induced oscillating velocity, and the wall shear. Yu and Zhao (1999) experimentally investigated flow characteristics in straight tubes with and without a lateral circular protrusion. Recently, a group from LSTM-Erlangen (Institute of Fluid Mechanics, Friedrich-Alexander University, Erlangen, Nurnberg,

Germany) contributed a lot into the existing knowledge on pulsating flows in pipes. Their articles present a good summary of the efforts made in the open literature. Ray and Durst (2004) studied laminar, fully developed pressure-driven flow through any arbitrary-shaped duct by using a semi-analytical method. Similar to the circular pipes or parallel plate channels, they defined three different flow regimes: quasi-steady, intermediate and inertia-dominated. Unsal et al. (2005) and Ray et al. (2005) investigated sinusoidal mass-flow controlled, pulsating, laminar and fully developed pipe flow analytically and experimentally. They showed the dependence of the ratio dimensionless amplitudes of the mass flow rate and the phase lag as a function of the dimensionless pulsation frequency. Haddad et al. (2010) analytically investigated pulsating laminar incompressible pipe and channel flows. Ray et al. (2012) carried out extensive numerical calculations to determine development length of sinusoidally pulsating laminar pipe flow in moderate and high Reynolds number regimes.

Chan et al. (2002) numerically studied the effects of pulsating frequency and amplitude of oscillations on the flow behavior for a pressure-driven developing pulsating flow in a pipe. McGinty et al. (2009) obtained general analytical solutions of flows in cylindrical and annular pipes subject to an arbitrary time-dependent pressure gradient and arbitrary steady initial flow of both Newtonian and non-Newtonian fluids. Trip et al. (2012) investigated the transitional regime of a sinusoidal pulsatile flow in a pipe using the particle image velocimeter. Chang (2012) studied pressure-driven laminar pulsating flow both in circular pipes and parallel-plate channels and analyzed the existence of the phase-lag between pressure gradient and flow rate. Recently, Çarpınlioğlu and her co-workers (2012,2013,2003) studied transition onset of pulsatile pipe flows.

Aygun and Aydin studied pulsating flow in a pipe experimentally and numerically both for the cases hydrodynamically developing and fully developed flow, focusing on the effect of the frequency. Aydin and Aygun (2014) numerically studied the hydrodynamical entrance length for pulsating flow in a pipe through a scale analysis.

The effect of the frequency has been well documented in the literature. The aim of the present study is to numerically investigate the effect of the amplitude for pulsating flows in pipes for a range of the corresponding parameters, mainly the frequency of the pulsating flow.

## NUMERICAL STUDY

### Governing Equations

Figure 1 shows the schematic of the problem. For this simple geometry, the axially symmetric, unsteady, 2-D, incompressible and hydrodynamically developing

laminar flow with constant thermo-physical properties is considered. With the regarding simplifications, the corresponding governing equations in the cylindrical coordinate system are reduced as:



**Figure 1.** Schematic of the problem geometry.

Continuity equation:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial r} + \frac{v}{r} = 0 \quad (1)$$

x- Momentum equation:

$$\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial r} = -\frac{1}{\rho} \frac{\partial P}{\partial x} + \nu \left( \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial r^2} + \frac{1}{r} \frac{\partial u}{\partial r} \right) \quad (2)$$

r- Momentum equation:

$$\frac{\partial v}{\partial t} + u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial r} = -\frac{1}{\rho} \frac{\partial P}{\partial r} + \nu \left( \frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial r^2} + \frac{1}{r} \frac{\partial v}{\partial r} - \frac{v}{r^2} \right) \quad (3)$$

The regarding boundary conditions of the problem are as follows:

$$\frac{\partial u(0, t)}{\partial r} = 0 \quad \text{at } r=0 \quad (4)$$

$$u(R, t) = 0 \quad \text{at } r=R \quad (5)$$

At the pipe inlet,

$$u(r, t) = u_0(r, t), \quad v = 0 \quad \text{at } x=0 \quad (6)$$

where  $u_0(r, t)$  is constant for a uniform inlet flow, i.e.

$$u_0(t) = u_m \quad (7)$$

For a pressure-driven flow, the axial pressure gradient is given as

$$\frac{dP}{dx} = \left( \frac{dP}{dx} \right)_0 + \left( \frac{dP}{dx} \right)_A \sin(\omega t) \quad (8)$$

The corresponding inlet velocity profile for a pressure-driven flow is defined as

$$u(r, t) = u_0 + u_A \sin(\omega t) \quad (9)$$

At the solid pipe walls, the usual no-slip conditions are applied:

$$u = v = 0 \quad (10)$$

At the exit, the second derivative of the regarding velocities are set to be zero:

$$\frac{\partial^2 u}{\partial x^2} = 0 \quad (11)$$

The Reynolds number is defined to be based on the cross-sectional-time averaged velocity, which is

$$Re = \frac{u_{av,ta} D}{\nu} \quad (12)$$

where  $u_{av,ta}$  is the cross-sectional-time averaged velocity.

The dimensionless frequency and the dimensionless time are defined as:

$$F = \frac{R^2 f}{\nu} \quad (13)$$

$$\tau = \frac{\nu t}{R^2} \quad (14)$$

The friction coefficient is defined as

$$C_f = \frac{|\tau_w|}{\frac{1}{2} \rho u_{av}^2} \quad (15)$$

where  $\tau_w$  is the wall shear stress defined as

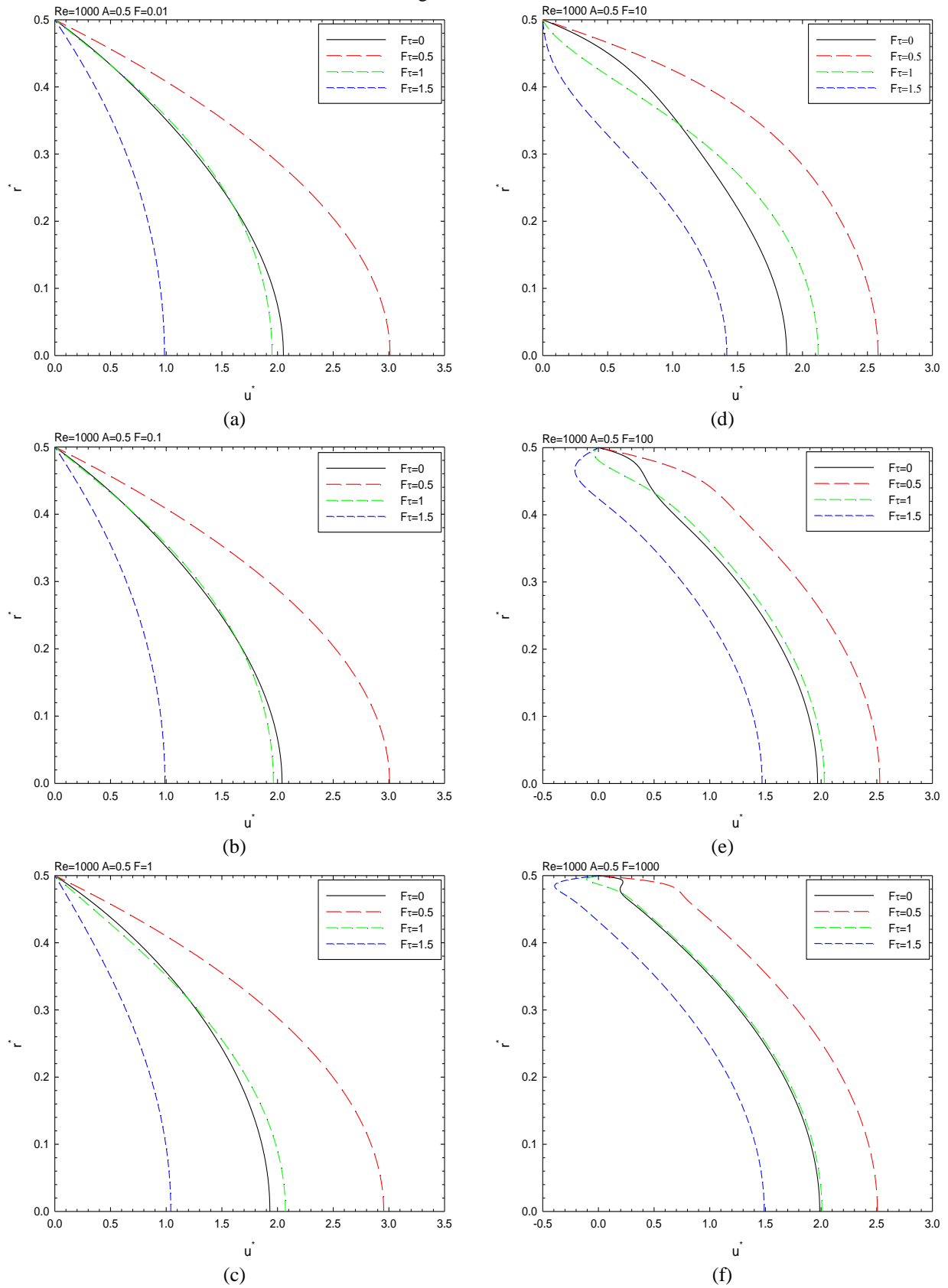
$$\tau_w = -\mu \left. \frac{\partial u}{\partial r} \right|_{r=R} \quad (16)$$

## Numerical Study

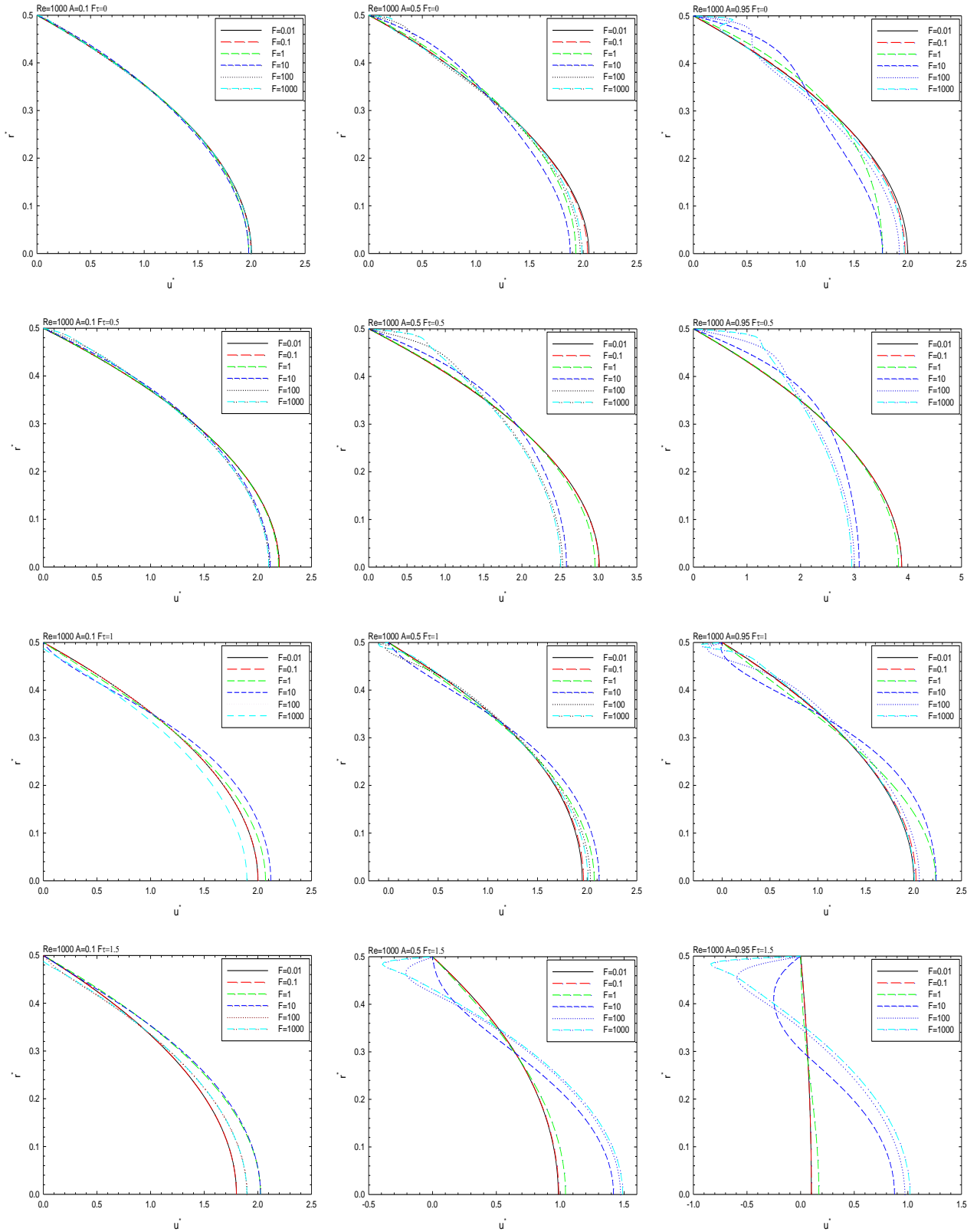
A commercial CFD software, FLUENT 6.1.22, is used in the numerical analysis. This package employs a finite volume method for the discretization of the continuity, momentum and energy equations. The SIMPLE algorithm is used to couple the pressure and velocity terms. Discretization of the momentum equations is performed by a second order upwind scheme and pressure interpolation is provided by PISO scheme (x). Convergence criterion considered as residuals is admitted  $10^{-6}$  for momentum and continuity equations. In order to define the pulsating (sinusoidal profile) inlet velocities, UDF (User-Defined Function) file is introduced to the FLUENT case file.

A non-uniform mesh of 320 x 64 rectangular elements was used, for which solutions were ensured to be grid-

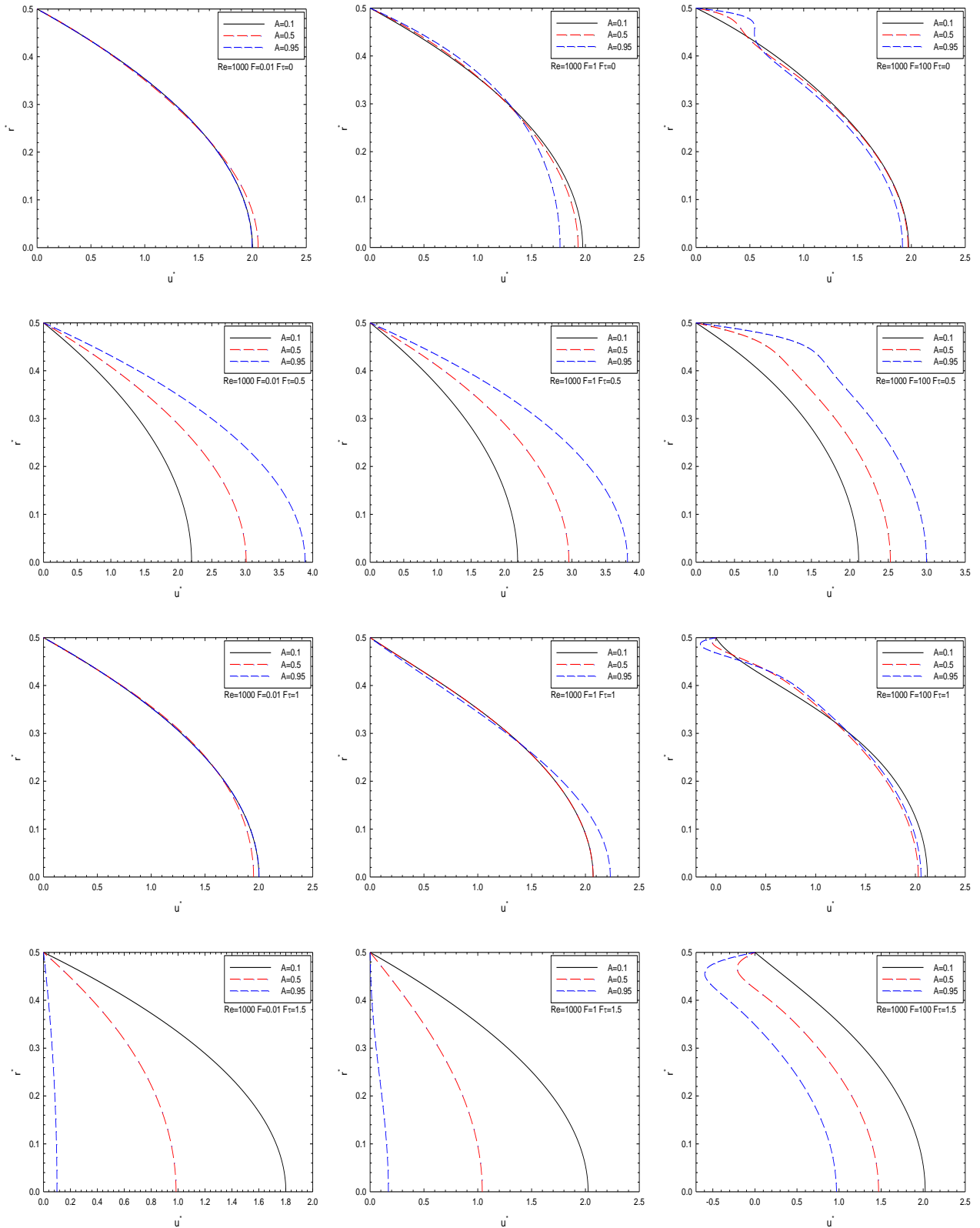
independent.



**Figure 2.** The fully developed radial velocity profile for various phase angles.



**Figure 3.** The effect of the dimensionless frequency,  $F$  on the radial velocity profile.



**Figure 4.** The effect of the dimensionless amplitude,  $A$  on the radial velocity profile.

## RESULTS AND DISCUSSION

Pressure-driven laminar pulsating flow in a pipe is studied numerically. A constant value of the Reynolds number,  $Re=1000$ , is considered. Six different values of the dimensionless frequency are considered, ( $F=0.01, 0.1, 1, 10, 100, 1000$ ). Tests are carried out for three

different values of the dimensionless amplitude ( $A=0.1, 0.5$  and  $0.95$ ). Both the hydrodynamically developing and fully developed flows are tested. The validity of the numerical procedure was previously tested in our earlier articles (Çarpinoglu and Ozahi, 2012, Ozahi and Carpinoglu, 2013, Carpinoglu, 2003) and fairly very

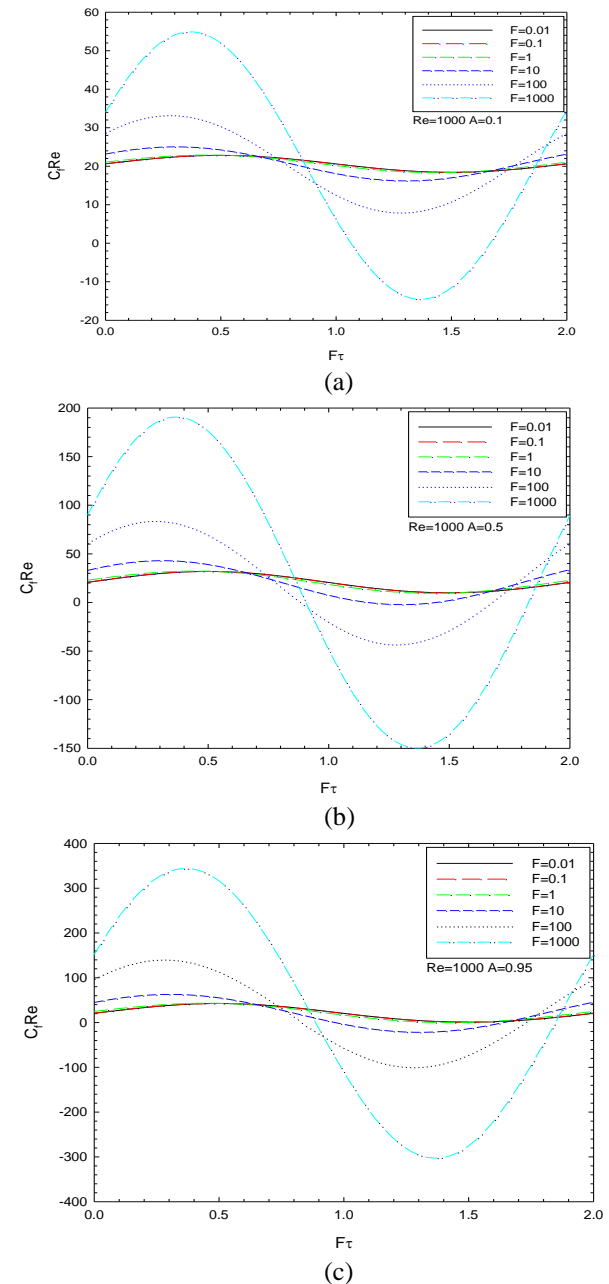
good agreements against the results in the literature for some specific cases have been observed.

Figure 2 illustrates the effect of the dimensionless frequency on the dimensionless radial velocity profiles at various axial locations downstream for constant values of the Reynolds number and the dimensionless amplitude ( $Re=1000$  and  $A=0.5$ ). As it can be seen, velocity increases or decreases depending on the phase angle. As expected, the velocity increases as a result of positive acceleration in the accelerating phase, while the opposite is true for the decelerating phase. For  $F \leq 0.1$ , the dimensionless radial velocity profile at  $F\tau=0$  presents higher values than those at  $F\tau=1$  while an opposite behavior is observed for  $F \geq 1$ . This can be explained in view of the flow physics in the accelerating and decelerating phases. For  $F \leq 0.1$ , the inertia forces in the accelerating phase ( $F\tau=0-0.5$ ) dominate over those in the decelerating phase ( $F\tau=0.5-1$ ). Thus, the radial velocity profile obtained at  $F\tau=0$  presents higher velocity values than those at  $F\tau=0.5$ . Similarly, for values of the dimensionless frequency of  $F \geq 1$ , the inertia forces becomes more dominated in the decelerating phase and, in follows, the radial velocity profile obtained at  $F\tau=0.5$  presents higher velocity values than at  $F\tau=0$ . This physical evidence is the main reason of the annular effect occurring comparatively at higher dimensionless frequencies.

The effect of the frequency on the fully developed dimensionless radial velocity profile for various values of the dimensionless amplitude is shown in Fig. 3. As seen from the figure, velocity profiles are nearly identical for low values of the dimensionless frequency ( $F \leq 0.1$ ). With an increase in  $F$  beyond this range, the variations in the dimensionless velocity increase. Figure 4 illustrates the effect of the amplitude on the fully developed dimensionless radial velocity profile more clearly. As seen, the variations are much more discernible at the crest and trough points ( $F\tau=0.5$  and  $1.5$ ) of the accelerating and decelerating phases, respectively. For the values of the dimensionless amplitude of  $A=0.5$  and  $0.95$ , flow reversals exist in the decelerating period ( $F\tau=1$  and  $1.5$ ) at higher values of the dimensionless frequency. These flow reversals become much clearer at the trough point ( $F\tau=1.5$ ), which is the end of the decelerating phase and, at the same time, the start of the accelerating phase. This effect is also called as the annular effect, which was detected by others (e.g. Zhao and Cheng, 1996, Yakhot et al., 1999, Pendyala et al., 2008).

From the practical interest, the friction coefficient is an important parameter to determine, which leads to pressure drops, and, in follows, to pumping/fan power required. For a constant value of the Reynolds number ( $Re=1000$ ) and for the hydrodynamically fully developed condition, the variation of the friction coefficient with the phase angle for a cycle at various values of the dimensional frequency and the dimensionless amplitude is depicted in Fig. 5. As seen

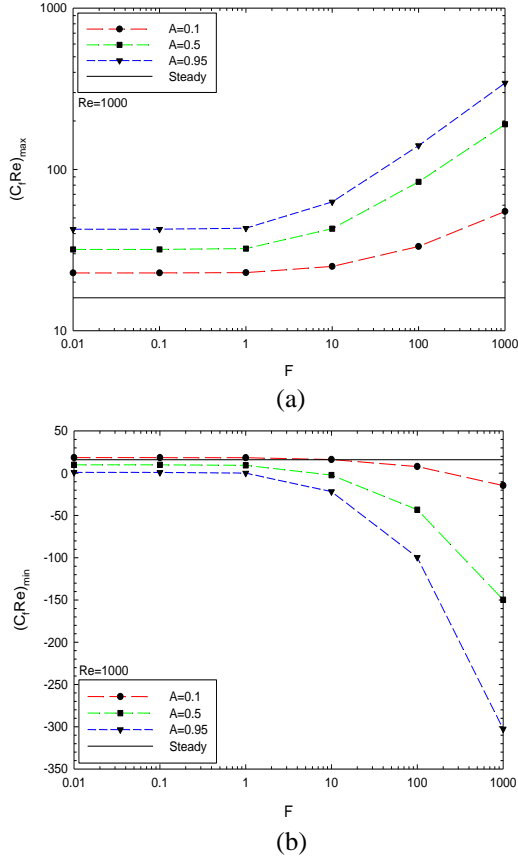
and expected, the amplitude of the friction coefficient increases with an increase in the frequency. In follows, its maximum and minimum values at the crest and trough points increase. These increases become much more considerable for  $F \geq 10$ .



**Figure 5.** The variation of the friction coefficient with the phase angle for various  $F$ .

Figure 6 illustrates the variation of the maximum and minimum values of the friction coefficient with the dimensionless frequency at the peak points, i.e. the crest and trough points ( $F\tau=0.5$  and  $1.5$ ). For  $F \leq 1$ , this effect is negligible but it increases with an increase of  $F$  beyond this limit. As seen, the friction coefficient increases with an increase in the dimensionless amplitude,  $A$ , too.

A clearer view of the effect of the dimensionless amplitude on the friction coefficient can be seen from Fig. 7. As it can be seen from the figure, increasing the dimensionless amplitude results in increases in the maximum and minimum values of the friction coefficient at the crest and trough points.



**Figure 6.** The variations of the max. and min. values of  $C_f$  with  $F$ .

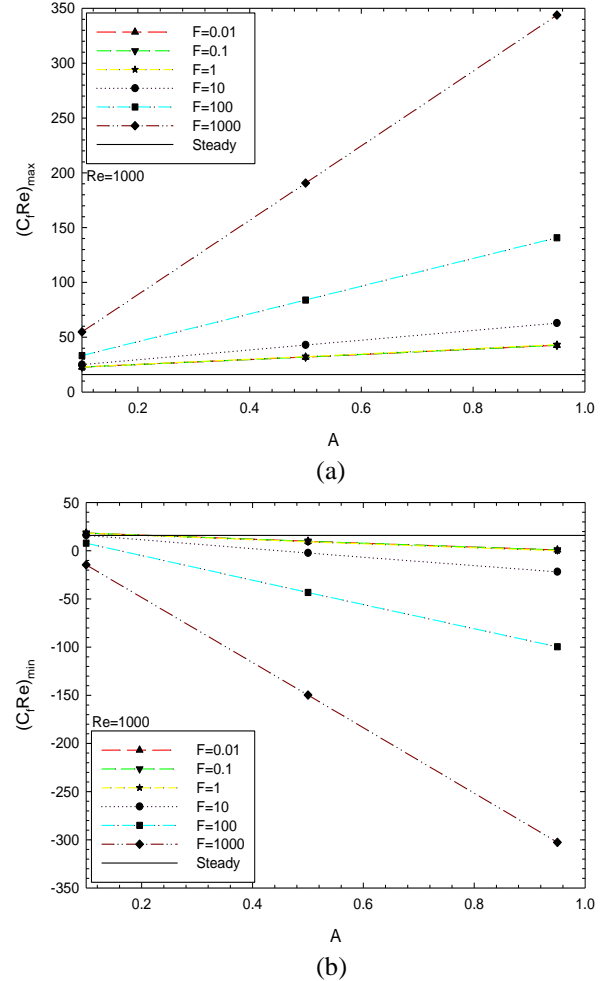
## CONCLUSIONS

In this study, we numerically analyzed both the hydrodynamically developing and fully developed pressure-driven laminar pulsating flow in a pipe via a commercial CFD package, FLUENT. The followings disclosures can be withdrawn from this study:

- The inertia forces in the accelerating phase ( $F\tau=0-0.5$ ) dominate over those in the decelerating phase ( $F\tau=0.5-1$ ) for  $F\leq 0.1$  while the opposite is true for  $F\geq 1$ .
- As expected, the nature of the radial velocity profiles has been found to critically depend on both the frequency, the amplitude and the phase angle.
- For higher values of the dimensionless frequency at the values of the dimensionless amplitude of  $A=0.5$  and  $0.95$ , flow reversals have been observed to arise in the decelerating period ( $F\tau=1$  and  $1.5$ ).
- The friction coefficient, and, in follows, its maximum and minimum values at the crest and trough points are shown to increase with an

increase in the frequency. These increases are found to become much more considerable for  $F\geq 10$ .

- Finally, it is also disclosed that the friction coefficient increases with an increase in the dimensionless amplitude,  $A$ , too.



**Figure 7.** The variations of the max. and min. values of  $C_f$  with  $A$ .

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