



## NUMERICAL ANALYSIS OF COMBINED NATURAL CONVECTION AND RADIATION IN A SQUARE ENCLOSURE PARTIALLY HEATED VERTICAL WALL

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**Abstract:** Combined natural convection and thermal radiation heat transfer in air-filled square gray enclosure are numerically investigated for three different cases having the same constant temperature difference. The partially heated patch of the left wall and the cold right wall are maintained at constant temperatures while the other walls are insulated. The surface radiation heat transfer is considered from the heat sources (patch) to the other surfaces, and the medium is assumed to be nonparticipating. The emissivity ratio of the walls are kept the same. The transport equations along with Boussinesq approximation as well as the energy equation are solved using FLUENT software. As the wall emissivities were changed, the steady state mean Nusselt numbers over the cold surface were computed for each case as a function of the Rayleigh number. In this study, the Rayleigh number was ranged from  $10^5$  to  $10^7$  while the surface emissivity ratio was changed from  $\varepsilon=0$  to 1. The heat transfer and fluid flow characteristics were investigated with respect to the effect of the conduction-radiation parameter, Rayleigh number and the wall emissivity.

**Keywords:** Natural convection, Surface radiation, Laminar flow, Partially heated square enclosure, Heat transfer

## KİSMİ OLARAK DİKEY DUVARINDAN ISITILMIŞ KAPALI KARE GEOMETRİDE BİRLEŞİK DOĞAL TAŞINIM VE IŞINIMIN SAYISAL İNCELENMESİ

**Abstract:** Gri duvarlı içi hava dolu kare geometride doğal taşınım ve ısı ışınım ısı transferi sabit sıcaklık farkına sahip üç farklı durum için sayısal olarak incelenmiştir. Kısmi olarak ısıtılmış sol duvar ve soğuk sağ duvar sabit sıcaklıkta muhafaza edilmiş diğer duvarlar yalıtılmıştır. Isı kaynağından diğer yüzeylere ışınım ile ısı transferi olduğu göz önüne alınmış ve ortamın katılımcı olmadığı kabul edilmiştir. Duvarların ışınım yayma oranlarının aynı olduğu düşünülmüştür. Boussinesq yaklaşımı ile beraber taşınım denklemleri ve enerji denklemi FLUENT yazılımı ile çözülmüştür. Sürekli rejimde, yüzey ışınım yayma oranı değiştirilirken, her durum için soğuk plaka yüzey alanı üzerinden hesaplanan ortalama Nusselt sayısı Rayleigh sayısının fonksiyonu olarak hesaplanmıştır. Bu çalışmada, Rayleigh sayısı  $10^5$  ile  $10^7$  arasında değiştirilirken, yüzey ışınım yayma oranları  $\varepsilon=0$  ile 1 arasında değiştirilmiştir. Isı transferi ve akışkan akış karakteristikleri iletim-ışınım parametresi, Rayleigh sayısı ve duvar ışınım yayma oranı etkisi açısından incelenmiştir.

**Anahtar Kelimeler:** Doğal Taşınım, Yüzey Işınımı, Laminar Akış, Kısmi olarak ısıtılmış kare geometri, Isı transferi

### NOMENCLATURE

$g$	Earth's gravitational acceleration [ $=m/s^2$ ]	Ra	Rayleigh number [ $= g\beta L^3\Delta T/\alpha\nu$ ]
$I_k$	Dimensionless irradiation of the $k$ th element	$q$	Heat flux
$k$	Conductivity of fluid [ $=W/mK$ ]	$T$	Temperature [K]
$L$	Length of enclosure [m]	$T_0$	Referance temperate [ $=(T_h + T_c)/2$ ]
$N$	Total number of radiative surfaces	$u, v$	Fluid velocity components [m/s]
Nu	Nusselt number	$U, V$	Dimensionless velocity components [ $= uL/\alpha, =vL/\alpha$ ]
$N_{rc}$	Conduction-radiation number [ $= \sigma T_h^4 L/k\Delta T$ ]	$x, y$	Coordinate axes [m]
$p$	Pressure [Pa]	$X, Y$	Dimensionless coordinates [ $= x/L, =y/L$ ]
$P$	Dimensionless pressure [ $= pL^2/\rho\alpha^2$ ]	<i>Special characters</i>	
Pr	Prandtl number [ $= \nu/\alpha$ ]	$\alpha$	Thermal diffusivity [ $=m^2/s$ ]
$R$	Radiosity	$\beta$	Volumetric thermal expansion coefficient [1/K]
		$\Delta T$	Temperature difference [ $= T_h - T_c$ ]
		$\varepsilon$	Surface emissivity

$\Theta$	Dimensionless temperature ratio [= $T_k / T_h$ ]
$\theta$	Dimensionless temperature [= $(T - T_c) / \Delta T$ ]
$\rho$	Fluid density [= $kg/m^3$ ]
$\sigma$	Stefan-Boltzmann constant [= $5.67 \times 10^{-8} W/m^2 K^4$ ]
$\nu$	Kinematic viscosity [= $m/s^2$ ]
<i>Subscripts</i>	
<i>c</i>	Cold
<i>cd</i>	Conduction
<i>cv</i>	Convection
<i>h</i>	Hot
<i>r</i>	Radiation
<i>t</i>	Total

## 1. INTRODUCTION

Natural convection heat transfer has received considerable attention due to their importance in thermal engineering applications such as thermal energy storage, design of solar collectors, cooling of electronic equipment, and many other technological processes. In literature, many studies dealing with natural convection in enclosure neglect the effect of radiation. The influence of radiation on natural convection is generally stronger than that on forced convection due to the inherent coupling between the temperature and flow fields in natural convection.

The first numerical study of the coupled heat transfer problem involving both convection and radiation in a rectangular cavity seems to be that of Larson and Viskanta (1997). They found that radiation heats up the cavity surface and thus considerably modifies the flow pattern and the corresponding convection process. Chang et al. (1983) considered square enclosures with partitions filled with air, carbon dioxide, or ammonia gas. Radiative heat transfer was analyzed by the radial flux method. Lauriat (1982) treated the radiation part of the problem by using the  $P_1$  differential approximation. Yücel et al. (1989) studied coupled natural convection and radiation in a rectangular participating medium using the finite differences and the two-dimensional discrete ordinates ( $S_4$  and  $S_8$ ) method. Tan and Howell (1991) studied combined radiation and natural convection in a two-dimensional participating square medium using finite differences and the product-integral method.

Most of the numerical studies reported in literature have considered one of the vertical walls heated, and the other wall cooled with horizontal insulated walls. Balaji and Venkateskan (1993), by using air as a working fluid, showed that the surface radiation leads to reduction of the convection component which decreases monotonically with the emissivity. However, in general, this reduction was largely compensated by the radiative transfer between the walls of the cavity. Balaji and Venkateskan (1994) proposed general correlations for convection and radiation Nusselt numbers for different emissivities of the walls. A good agreement between the

numerical data and the correlations was observed. Sen and Sarkar (1995) studied the interaction of variable property convection and surface radiation in a differentially heated square cavity. Akiyama and Chong (1997) studied the interaction of natural convection with thermal radiation by gray surfaces in a square enclosure filled with air. Their results showed that the surface radiation has significant influence on the temperature distribution and flow patterns in the enclosure, especially at high Rayleigh numbers. The mean convection Nusselt number increased with the increase of Ra, but only limited variations were observed when the emissivity was varied. However, the mean radiative Nusselt number rises quickly as the emissivity increases. A similar study was performed by Wang et al. (2006). Their analysis showed that the net radiative heat flux is linear with  $\Delta T$  if  $\Delta T \ll T_0$ , which is the case at low Rayleigh number, and that radiative Nusselt number is a linear function of cavity height. Mezrhab and Behir (1999), using the finite volume method (FVM), studied the heat transfer by the radiation and the natural convection in an air-filled square enclosure with a vertical partition of finite thickness and varying height. Mahapatra et al. (1999), reported a finite-element solution of the interaction of surface radiation and variable property laminar natural convection in a differentially heated square cavity. Colomer et al. (2004) studied natural convection with radiation for participating and nonparticipating media in a three-dimensional differentially heated cavity. The problem was solved for a transparent medium showing that the radiation contributes to increase significantly the heat transfer in the cavity. Kumar and Eswaran (2010) analyzed a numerical simulation of combined radiation and natural convection for participating and nonparticipating media in a three-dimensional differentially heated rectangular cavity. They found that the wall emissivity had a strong influence on the heat transfer but the scattering albedo did not.

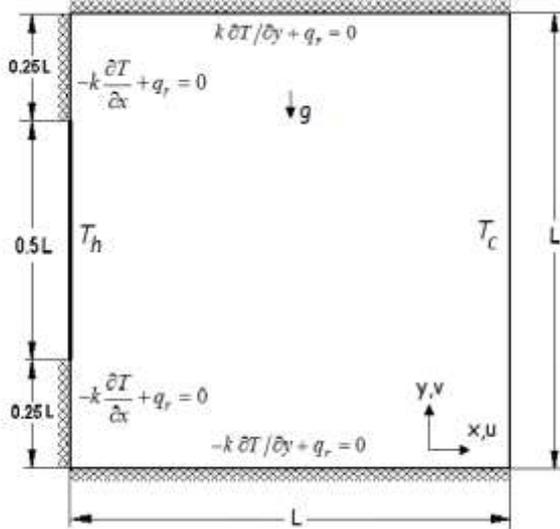
The latter is the famous case of Rayleigh Bernard problem with hot bottom wall and cold top wall with adiabatic vertical walls. Ridouane et al. (2004, 2006a, 2006b) found the surface radiation that significantly alters the existence ranges of the solutions. For each solution, the convective and radiative contributions to the global heat transfer are also quantified for various Ra and  $\varepsilon$ . Cheikl et al. (2007) studied a small and a large source corresponding to 20% and 80% of total length of the bottom wall, respectively. Gad and Balaji (2010) investigated the combined natural convection and surface radiation in enclosures for Rayleigh Bernard configuration of air by using FLUENT 6.3. They carried out an investigation to determine the onset of convection and to propose correlations for convection and radiation Nusselt numbers based on a detailed parametric study. Several studies dealing with the combined effect of the natural convection and surface radiation have considered in differentially heated inclined cavities (Sharma et al. (2008) and Vivek et al. (2012)).

Very limited experimental work has been conducted for combined natural convection and radiation in differentially heated rectangular enclosures. Ramesh and Venkateskan (1999), reported in an experimental study involving two cases corresponding to highly emissive ( $\varepsilon=0.85$ ) and highly polished ( $\varepsilon=0.05$ ) walls. Ramesh et al. (1999), using the same experimental method, analyzed the effect of boundary conditions on natural convection in a square enclosure with the surface radiation. Results of the experiments were compared with those obtained numerically with FVM, and reasonable agreement was observed.

The objective of this study is to investigate the natural convection and the thermal radiation heat transfer from air-filled square gray enclosures for three different temperature cases preserving the temperature difference between the left wall patch and right wall.

## 2. MATHEMATICAL FORMULATION

A two-dimensional configuration under investigation, with the system of coordinates, is depicted in Figure 1. A patch on the left wall is (partially) heated, and the right wall of the enclosure is uniformly cooled with constant temperature, respectively. The remaining walls are considered to be adiabatic. Three different cases having the same temperature difference are examined. All the inner surfaces in contact with the studied fluid are assumed to be gray, diffuse emitters and reflectors of radiation. The fluid properties are evaluated at a mean temperature, and the buoyancy-driven air flow is conceived to be laminar and incompressible. The Rayleigh numbers considered in this study ranged from  $10^5$  to  $10^7$ .



**Figure 1.** Schematic of the problem and the coordinate system.

The continuity, momentum and energy equations for two-dimensional cartesian coordinate system are solved using the Boussinesq approximation. The governing equations are defined as follows:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \quad (1)$$

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = -\frac{1}{\rho} \frac{\partial p}{\partial x} + \nu \left( \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} \right) \quad (2)$$

$$u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} = -\frac{1}{\rho} \frac{\partial p}{\partial y} + \nu \left( \frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} \right) + g\beta(T - T_c) \quad (3)$$

$$u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \alpha \left( \frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} \right) \quad (4)$$

where  $\beta$  is the volumetric thermal expansion coefficient,  $\alpha$  is the thermal diffusivity,  $\nu$  is the kinematic viscosity,  $g$  is the gravitational acceleration,  $p$  is the pressure,  $\rho$  is the fluid density,  $T$  is the temperature. All walls are impermeable no-slip boundaries. The boundary conditions;

$$u = v = 0, \quad T = T_h, \quad 0.25L \leq y \leq 0.75L \quad \text{and} \quad x = 0$$

$$u = v = 0, \quad T = T_c, \quad 0 \leq y \leq L \quad \text{and} \quad x = L$$

$$u = v = 0, \quad -k \frac{\partial T}{\partial x} + q_r = 0, \quad x = 0, \quad 0 \leq y \leq 0.25L \quad \text{and} \quad 0.75L \leq y \leq L \quad (5)$$

$$u = v = 0, \quad -k \frac{\partial T}{\partial y} + q_r = 0, \quad y = 0 \quad \text{and} \quad 0 \leq x \leq L$$

$$u = v = 0, \quad k \frac{\partial T}{\partial y} + q_r = 0, \quad y = L \quad \text{and} \quad 0 \leq x \leq L$$

Introducing the following dimensional variables,

$$X = \frac{x}{L}, \quad Y = \frac{y}{L}, \quad U = \frac{u}{\alpha/L}, \quad V = \frac{v}{\alpha/L}, \quad (6)$$

$$P = \frac{p}{\rho \alpha^2 / L^2}, \quad \theta = \frac{T - T_c}{T_h - T_c}$$

The governing Eqs. (1)-(4) can be written in dimensionless form as

$$\frac{\partial U}{\partial X} + \frac{\partial V}{\partial Y} = 0 \quad (7)$$

$$U \frac{\partial U}{\partial X} + V \frac{\partial U}{\partial Y} = -\frac{\partial P}{\partial X} + \text{Pr} \left( \frac{\partial^2 U}{\partial X^2} + \frac{\partial^2 U}{\partial Y^2} \right) \quad (8)$$

$$U \frac{\partial V}{\partial X} + V \frac{\partial V}{\partial Y} = -\frac{\partial P}{\partial Y} + \text{Pr} \left( \frac{\partial^2 V}{\partial X^2} + \frac{\partial^2 V}{\partial Y^2} \right) + \text{Ra Pr} \theta \quad (9)$$

$$U \frac{\partial \theta}{\partial X} + V \frac{\partial \theta}{\partial Y} = \frac{\partial^2 \theta}{\partial X^2} + \frac{\partial^2 \theta}{\partial Y^2} \quad (10)$$

with the corresponding boundary conditions

$$U = V = 0, \quad \theta = 1, \quad 0.25 \leq Y \leq 0.75 \quad \text{and} \quad X = 0$$

$$U = V = 0, \quad \theta = 0, \quad 0 \leq Y \leq 1 \quad \text{and} \quad X = 1$$

$$U = V = 0, \quad \frac{\partial \theta}{\partial X} = N_{rc} Q_r, \quad X = 0, \quad 0 \leq Y \leq 0.25 \quad \text{and} \quad 0.75 \leq Y \leq 1 \quad (11)$$

$$U = V = 0, \quad \frac{\partial \theta}{\partial Y} = N_{rc} Q_r, \quad Y = 0 \quad \text{and} \quad 0 \leq X \leq 1$$

$$U = V = 0, \quad \frac{\partial \theta}{\partial Y} = -N_{rc} Q_r, \quad Y = 1 \quad \text{and} \quad 0 \leq X \leq 1$$

The dimensionless parameter appearing in the Eqs. (7)–(11) are defined as

$$\text{Ra} = \frac{g\beta L^3 \Delta T}{\nu\alpha}, \quad \text{Pr} = \frac{\nu}{\alpha}, \quad N_{rc} = \frac{\sigma T_h^4 L}{k\Delta T}, \quad Q_r = \frac{q_r}{\sigma T_h^4} \quad (12)$$

The walls of the cavity are divided into  $N$  radiative surface elements, which coincide with the control volume fluid–solid interfaces. Therefore, the net radiative flux of the  $k$ th element is expressed as

$$q_{r,k} = q_{o,k} - q_{i,k} \quad k = 1, 2, \dots, N \quad (13)$$

where the subscripts  $o$  and  $i$  refer respectively the outgoing (radiosity) and incoming (irradiation) components. They are defined as

$$q_{o,k} - (1 - \varepsilon_k) \sum_{j=1}^N F_{kj} q_{o,j} = \varepsilon_k \sigma T_k^4 \quad k = 1, 2, \dots, N \quad (14)$$

$$q_{i,k} = \sum_{j=1}^N F_{kj} q_{o,j} \quad k = 1, 2, \dots, N \quad (15)$$

where  $\varepsilon_k$  is the emissivity of the  $k$ th element and  $F_{kj}$  is the view factor from the  $k$ th element to the  $j$ th element of the radiative surface elements. Using the definitions  $Q_{r,k} = q_{r,k} / \sigma T_h^4$ ,  $R_k = q_{o,k} / \sigma T_h^4$ ,  $I_k = q_{i,k} / \sigma T_h^4$  and  $\Theta_k = T_k / T_h$ , we obtain the dimensionless form of net radiative flux, radiosity and irradiation respectively,

$$Q_{r,k} = I_k - R_k \quad k = 1, 2, \dots, N \quad (16)$$

$$R_k - (1 - \varepsilon_k) \sum_{j=1}^N F_{kj} R_j = \varepsilon_k \Theta_k^4 \quad k = 1, 2, \dots, N \quad (17)$$

$$I_k = \sum_{j=1}^N F_{kj} R_j \quad k = 1, 2, \dots, N \quad (18)$$

In order to determine the total heat transfer rate, we need to define the local convective and radiative heat transfer rates along the right wall by the local

convective and radiative Nusselt numbers, respectively, as follows

$$\text{Nu}_{l,cv} = \frac{q_{cv}}{q_{cd}} = \frac{k \left. \frac{\partial T}{\partial x} \right|_{x=L}}{k \frac{\Delta T}{L}} = \left. \frac{\partial \theta}{\partial X} \right|_{X=1} \quad (19)$$

$$\text{Nu}_{l,r} = \frac{q_r}{q_{cd}} = \frac{q_r}{k \frac{\Delta T}{L}} = -N_{rc} Q_r \Big|_{X=1} \quad (20)$$

Thus, the mean Nusselt number along the right wall is defined by summing the mean convective and radiative Nusselt numbers:

$$\text{Nu}_t = \text{Nu}_{cv} + \text{Nu}_r = \int_0^1 \left. \frac{\partial \theta}{\partial X} \right|_{X=1} dY - \int_0^1 N_{rc} Q_r \Big|_{X=1} dY \quad (21)$$

The governing Eqs. (7)–(10) and the radiative transfer equation are discretized using the FVM, and the resulting equations are solved in an iterative procedure using the standard implicit, second order upwind solver with velocity and pressure coupling achieved by the SIMPLE algorithm. The surface-to-surface heat transfer model is used. The computations are carried out using FLUENT 6.3 (2005). Segregated, implicit, 2-D, laminar steady incompressible flow solver is employed for the numerical study.

The numerical procedure was validated against the results of Wang et al. (2006). The comparisons of mean convection, radiation and total Nusselt numbers of this study and Wang et al. (2006) for differentially heated square cavity, which is heated and cooled from the lateral walls are given in Table 1. Good agreement is observed in the range of the Rayleigh numbers considered; the maximum difference is about 2.7%.

**Table 1.** Comparison of Nusselt numbers.

Ra		H. Wang et al.			Present Study		
Number	$\varepsilon$	$\text{Nu}_{cv}$	$\text{Nu}_r$	$\text{Nu}_t$	$\text{Nu}_{cv}$	$\text{Nu}_r$	$\text{Nu}_t$
$10^5$	0.0	4.540	0.000	4.540	4.524	0.000	4.524
$10^5$	0.2	4.411	1.073	5.484	4.404	1.073	5.477
$10^5$	0.8	4.247	5.137	9.384	4.241	5.136	9.377
$10^6$	0.0	8.852	0.000	8.852	8.849	0.000	8.849
$10^6$	0.2	8.417	2.319	10.736	8.391	2.318	10.709
$10^6$	0.8	7.930	11.150	19.078	7.930	11.148	19.078

The accuracy of numerical results was also checked through numerous detailed tests on grid size effects. For instance, in Table 2, the convergence of convective, radiative and total mean Nusselt numbers for increasing Rayleigh numbers and increasing number of grids are presented for  $\varepsilon = 0.9$  of Case2. The triangular non-uniform grid for results reported in this study was used. It was observed that the maximum difference between the results of 45473 nodes and those of 181089 nodes are of 0.6% for the mean convection Nusselt number, 2.7% for the mean radiation Nusselt number and 2.7% for the mean total Nusselt number.

### 3. RESULTS AND DISCUSSION

In this study, combined convection and radiation heat transfer analysis was carried out for three cases which the hot, and the cold wall temperatures are varied while keeping the temperature difference the same. These cases are

$$\text{Case1: } T_h = 300\text{K}, T_c = 290\text{K};$$

$$\text{Case2: } T_h = 320\text{K}, T_c = 310\text{K};$$

$$\text{Case3: } T_h = 340\text{K}, T_c = 330\text{K}$$

The thermophysical properties of air were evaluated at the reference temperature ( $T_0$ ). The thermophysical

properties of air for the reference temperature were given in Table 3. The numerical simulations were performed for  $10^5 \leq Ra \leq 10^7$  and  $0 \leq \varepsilon \leq 1$ . The surfaces of the enclosure were considered to be gray having the same emissivity value. The results were expressed in the form of the streamlines, and the isotherms contour plots and mean Nusselt number over the cold surface wall.

In Figure 2, for Case 2 the streamlines and the isotherms are shown for various Rayleigh numbers and the surface emissivities. The fluid near the hot wall rises up due to the convective heating and the fluid

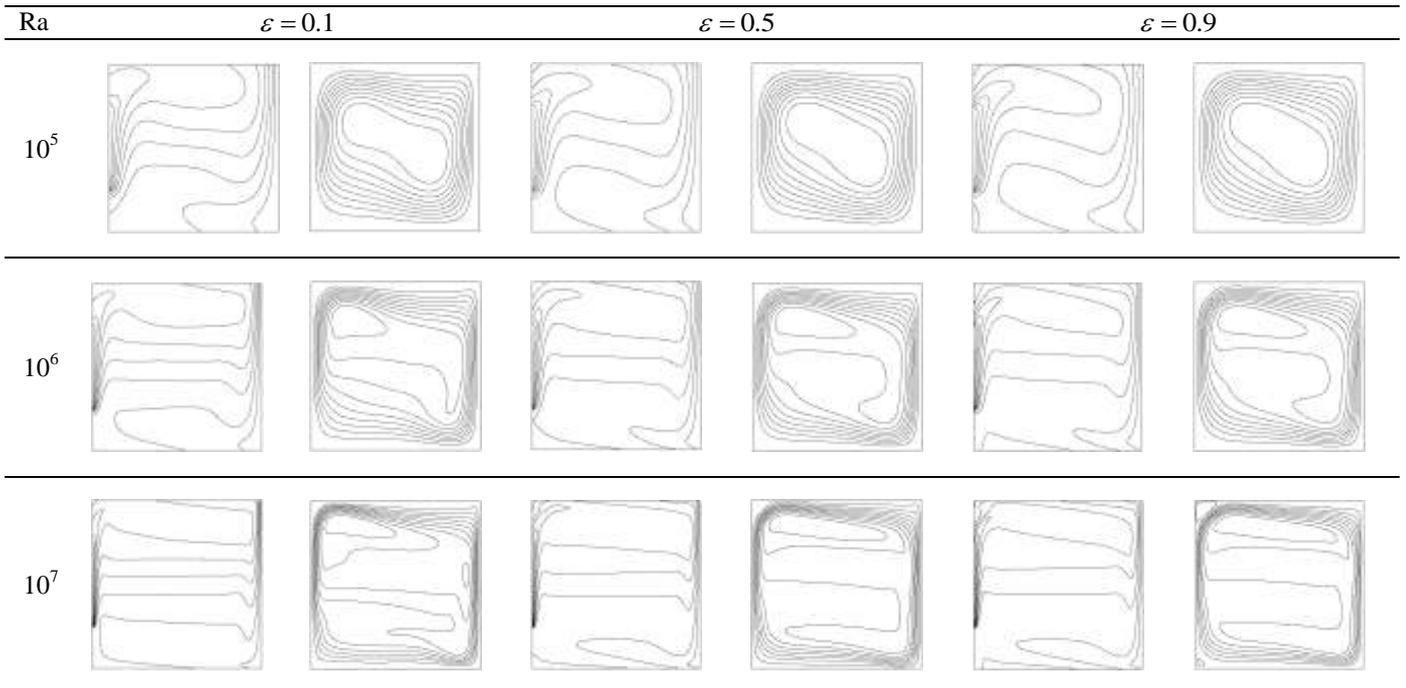
moves downwards near the cold wall; thus, the heated patch primarily facilitates a unicellular counter-clockwise circulation. As the Rayleigh number increases, this unicellular flow structure expands towards the enclosure walls thereby causing the isotherms and the streamlines to condense near the wall. This fluid and heat flow structure imply steepening flow and heat gradients with increasing Rayleigh number. It is noted that the streamlines more maintain the flow pattern while the isotherms depict the influence of the radiative heat transfer due to increased surface emissivity.

**Table 2.** Grid Sensitivity table for Case2 ( $\varepsilon = 0.9$ ).

Ra	11469 nodes			45473 nodes			181089 nodes		
	$Nu_{cv}$	$Nu_r$	$Nu_t$	$Nu_{cv}$	$Nu_r$	$Nu_t$	$Nu_{cv}$	$Nu_r$	$Nu_t$
$10^5$	3.024	5.916	8.940	3.034	5.932	8.966	3.038	5.941	8.979
$10^6$	5.388	12.490	17.878	5.397	12.518	17.916	5.403	12.535	17.938
$10^7$	9.677	26.558	36.235	9.604	26.576	36.180	9.603	26.603	36.207

**Table 3.** The thermophysical properties of air for the reference temperature.

$T_0$ (K)	$\rho$ ( $kg/m^3$ )	$c_p$ (j/kgK)	$k$ (W/mK)	$\mu$ (kg/ms)	$\beta$ (1/K)
295	1.1971	1006.2	0.025734	$1.8294 \times 10^{-5}$	$3.3898 \times 10^{-3}$
315	1.1209	1007	0.027211	$1.9233 \times 10^{-5}$	$3.1746 \times 10^{-3}$
335	1.0538	1008.2	0.028648	$2.0144 \times 10^{-5}$	$2.9851 \times 10^{-3}$



**Figure 2.** The variation of the streamlines (right) and the isotherms (left) with surface emissivity and Rayleigh number for Case 2.

The computed mean Nusselt values for the convective, radiative and total heat transfer in the enclosure are tabulated in Table 4 for  $\varepsilon=0.1, 0.5, 0.9$  and  $Ra=10^5, 10^6$  and  $10^7$ . For constant surface emissivity, as the temperature of the hot wall is increased, the mean convective Nusselt numbers slightly decrease, and the mean radiative Nusselt numbers increase as expected. Since the radiative heat transfer is proportional with the

fourth power of temperature and the wall emissivity, the increase in the mean radiative Nusselt number for higher surface emissivity is higher. The mean convective Nusselt numbers slightly decrease with increasing emissivity. For  $\varepsilon=0.1$ , the absolute decrease observed in  $Nu_{cv}$  in comparison of Case 1 to other cases is 0.03 and 0.05 for  $Ra=10^5$ , 0.06 and 0.11 for  $Ra=10^6$  and 0.11 and 0.22 for  $Ra=10^7$ . Similarly, for  $\varepsilon=0.5$ , the

absolute decrease in  $Nu_{cv}$  in comparison of Case 1 to other cases is 0.03 and 0.06 for  $Ra=10^5$ , 0.08 and 0.15 for  $Ra=10^6$  and 0.15 and 0.29 for  $Ra=10^7$ . On the other hand, for  $\varepsilon=0.9$ , the absolute decrease in  $Nu_{cv}$  in comparison of Case 1 to other cases is 0.03 and 0.06 for  $Ra=10^5$ , 0.07 and 0.14 for  $Ra=10^6$  and 0.14 and 0.26 for  $Ra=10^7$ . The radiative heat exchange increases with increasing wall emissivity in all cases due to radiative interactions between the walls. For  $\varepsilon=0.1$ , the absolute

increase observed in  $Nu_r$  in comparison of Case 1 to other cases is 0.10 and 0.22 for  $Ra=10^5$ , 0.22 and 0.46 for  $Ra=10^6$  and 0.46 and 0.98 for  $Ra=10^7$ . For  $\varepsilon=0.5$ , the absolute increase in  $Nu_r$  in comparison of Case 1 to other cases is 0.52 and 1.11 for  $Ra=10^5$ , 1.10 and 2.35 for  $Ra=10^6$  and 2.35 and 5.00 for  $Ra=10^7$ . Finally, for  $\varepsilon=0.9$ , the absolute increase in  $Nu_r$  in comparison of Case 1 to other cases is 1.01 and 2.15 for  $Ra=10^5$ , 2.14 and 4.54 for  $Ra=10^6$  and 4.54 and 9.68 for  $Ra=10^7$ .

**Table 4.** The variation of mean convective, radiative and total Nusselt numbers.

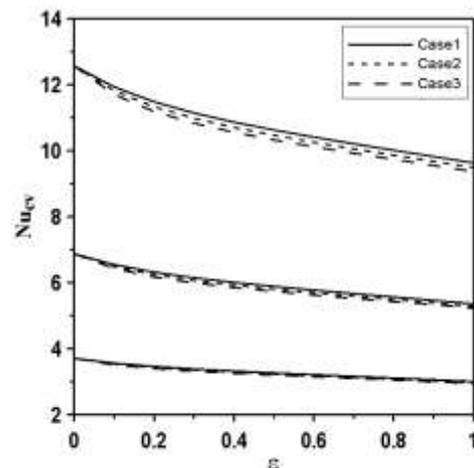
Case1	$\varepsilon = 0.1$			$\varepsilon = 0.5$			$\varepsilon = 0.9$		
Ra	$Nu_{cv}$	$Nu_r$	$Nu_t$	$Nu_{cv}$	$Nu_r$	$Nu_t$	$Nu_{cv}$	$Nu_r$	$Nu_t$
$10^5$	3.571	0.469	4.040	3.276	2.494	5.770	3.070	4.930	8.001
$10^6$	6.548	0.990	7.538	5.895	5.287	11.182	5.477	10.400	15.877
$10^7$	11.797	2.101	13.898	10.530	11.244	21.774	9.744	22.059	31.803
Case2	$\varepsilon = 0.1$			$\varepsilon = 0.5$			$\varepsilon = 0.9$		
$10^5$	3.546	0.570	4.116	3.242	3.014	6.256	3.038	5.941	8.979
$10^6$	6.492	1.205	7.697	5.816	6.390	12.207	5.403	12.535	17.938
$10^7$	11.687	2.559	14.246	10.376	13.596	23.972	9.603	26.603	36.207
Case3	$\varepsilon = 0.1$			$\varepsilon = 0.5$			$\varepsilon = 0.9$		
$10^5$	3.520	0.685	4.205	3.214	3.600	6.814	3.013	7.079	10.093
$10^6$	6.436	1.448	7.884	5.747	7.634	13.381	5.342	14.944	20.286
$10^7$	11.574	3.079	14.652	10.241	16.246	26.488	9.487	31.737	41.224

The variation of the mean convective Nusselt number as a function of the Rayleigh number and the surface emissivity are depicted in Figure 3. The mean convective Nusselt number decreases with increasing enclosure wall emissivity due to changes in the gradients of the isotherms caused by the radiative heat transfer within the medium. For  $Ra=10^5$ , since the circulation is weak, the flow pattern is less influenced by the radiative transfer; thus, the mean convective Nusselt number decreases only slightly with increasing emissivity of the enclosure walls. For  $Ra=10^6$  and  $Ra=10^7$ , since the circulation gains strength with increasing Rayleigh number, the effect of the radiative transfer becomes more pronounced. In this case, the mean convective Nusselt number increases with increasing Rayleigh number, but slightly decreases at  $Ra=10^5$  and moderately decreases at  $Ra=10^6$  and  $Ra=10^7$  with the increasing emissivity. The maximum absolute decrease in emissivity in comparison of Case 1 to other cases is 0.03 and 0.07 for  $Ra=10^5$ , 0.08 and 0.15 for  $Ra=10^6$  and 0.16 and 0.31 for  $Ra=10^7$ .

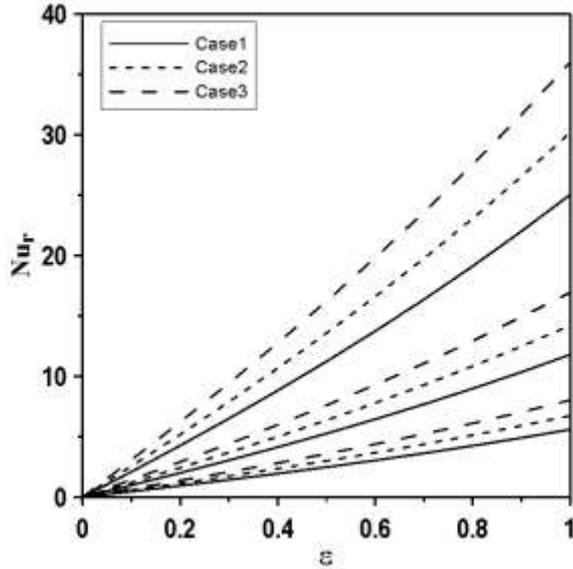
The variation of the mean radiative Nusselt number as a function of the Rayleigh number and the surface emissivity are depicted in Figure 4. The mean radiative Nusselt number increases sharply with increasing Rayleigh number and wall emissivity. Since the working fluid is air, especially for high wall emissivities, the surface radiation plays an important role in the heat transfer. As the wall emissivity changes, the temperature profiles (isotherms) near the walls,

especially near isolated walls, change. This change in the temperature field causes a change in the flow field. For  $\varepsilon=1.0$ , the absolute increase in the mean radiative Nusselt number in comparison of Case 1 to other cases is 1.14 and 2.43 for  $Ra=10^5$ , 2.41 and 5.14 for  $Ra=10^6$  and 5.14 and 10.96 for  $Ra=10^7$ .

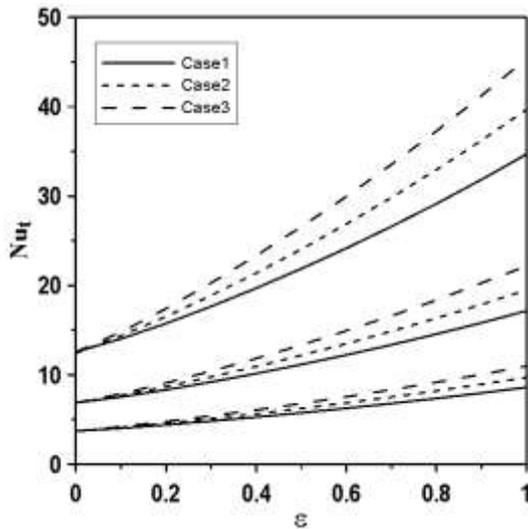
The variation of the mean total Nusselt number as a function of the Rayleigh number and the surface emissivity is depicted in Figure 5. The mean total Nusselt number increases with increasing Rayleigh number and wall emissivity, mainly due to the increase experienced in the radiative heat exchange.



**Figure 3.** Variation of the mean convective Nusselt number with emissivity and Rayleigh number.



**Figure 4.** Variation of the mean radiative Nusselt number with emissivity and Rayleigh number.



**Figure 5.** Variation of the mean total Nusselt number with emissivity and Rayleigh number.

#### Heat Transfer Correlations

In this study, the heat transfer, as the mean Nusselt numbers for various Rayleigh numbers and surface emissivities, is correlated by means of least-square regression as a function of  $Ra$ ,  $\varepsilon$  and  $N_{rc}$ . These correlations valid for  $10^5 \leq Ra \leq 10^7$  and  $0 < \varepsilon \leq 1$  ranges are given as:

$$Nu_{cv} = 0.209Ra^{0.267} (1+\varepsilon)^{-0.338} N_{rc}^{-0.04}, \quad r^2 = 0.999 \quad (22)$$

$$Nu_r = 0.0064Ra^{0.083} (1+\varepsilon)^{3.848} N_{rc}^{0.730}, \quad r^2 = 0.976 \quad (23)$$

$$Nu_t = 0.091Ra^{0.183} (1+\varepsilon)^{1.543} N_{rc}^{0.316}, \quad r^2 = 0.995 \quad (24)$$

#### 4. CONCLUSION

Convective and radiative heat transfer in a square enclosure bounded with gray surfaces were numerically investigated. The results indicate that the flow and the temperature field in the enclosure changed significantly due to the radiative heat transfer from bounding walls. When the surface walls do not emit, the mean convective Nusselt number increases with increasing Rayleigh number. When the surfaces emit, the mean convective Nusselt number decreases with increasing emissivity due to changes experienced in the temperature field because of the radiative heating. As the hot wall temperature is increased, although the temperature difference in each case is kept constant, the mean convective Nusselt number does not change much for increasing wall emissivity up to  $Ra=10^7$ . The mean radiative Nusselt number increases significantly with increasing surface emissivity and Rayleigh number as expected. This study indicates that especially for  $Ra > 10^5$  the radiative heat exchange cannot be neglected.

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