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MODELING OF TWO-DIMENSIONAL SOLIDIFICATION OF A FINITE CYLINDER

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Abstract: Two-dimensional solidification problem of a finite cylinder, in which the liquid phase is initially at the fusion temperature, is solved by using a front fixing approach. The external surfaces of the cylinder are subjected to a temporally or spatially varying temperature below freezing. The method employed is based on one used for the solution of a solidification problem in Cartesian domain. A coordinate transformation is applied in both radial and axial directions to obtain a square computational domain. This transformation results in a computationally intensive grid generation for every time step of solution. Finite difference form of the transformed energy equation is solved for the temperature distribution in the solid phase and the solid-liquid interface energy balance is integrated for the new position of the moving solidification front. The effect of the aspect ratio and spatially varying boundary temperatures on solidification is studied.
Keywords: solidification, phase change, moving boundary problem, coordinate transformation, front fixing

SONLU BİR SİLİNDİRDEKİ DONMANIN İKİ BOYUTLU MODELLENMESİ

Özet: Silindirik geometride sıvı fazın ergime sıcaklığında bulunduğu donma problemi katı – sıvı sınırını sabitleme yöntemi kullanılarak çözülmüştür. Dış yüzeyde ergime sıcaklığının altında olmak üzere zamana veya pozisyona göre değişken olan sıcaklık sınır koşulu tanımlanmıştır. Daha önce kartezyen koordinat sisteminde kullanılmış olan koordinat dönüşümü tekniği radyal ve eksenel yönde uygulanarak çözüm bölgesi olarak sabit bir kare elde edilmiştir. Bu dönüşüm yöntemi her çözüm adımı için yeniden çözüm ağı oluşturmayı gerektirmektedir. Koordinat dönüşümü ile elde edilen enerji denklemi sonlu farklar yöntemi ile katı fazdaki sıcaklık dağılımı ve katı – sıvı sınırının ilerlemesini belirlemek üzere çözülmüş ve yükseklik/yarıçap oranı ve dış yüzeydeki konuma göre değişen sınır koşullarının etkileri incelenmiştir.
Anahtar Kelimeler: donma, faz değişimi, hareketli sınır problemi, koordinat dönüşümü, sınır sabitleme

LIST OF SYMBOLS

Ar	Aspect ratio A/R
A	Cylinder height [m]
C	Specific heat [J/kg K]
J	Jacobian
k	Thermal conductivity [W/m K]
L	Latent heat of fusion [J/kg]
n	Normal direction
n	Unit normal vector
R	Cylinder radius [m]
r	Dimensionless radial coordinate
St	Stefan number, $\mathrm{St} = C(T_f^* - T_{ref}^*)/L$
T	Dimensionless temperature
t	Time [s]
V	Interface velocity [m/s]
z	Dimensionless axial coordinate

Subscript

f	Fusion
n	Differentiation with respect to the outward normal direction
nor	Normal direction
n-r	Component of normal in r – direction
n-z	Component of normal in z – direction
r	Differentiation with respect to r – direction
ref	Reference
sur	Surface
z	Differentiation with respect to z – direction
λ	Differentiation with respect to λ – direction
ξ	Differentiation with respect to ξ – direction

Superscript

*	Dimensional quantity

Greek letters

α	Thermal diffusivity
δ	Interface position
λ	Dimensionless transformed coordinate
ρ	Density
τ	Dimensionless transformed time
ξ	Dimensionless transformed coordinate

INTRODUCTION

Solidification problems are a subclass in a larger class of moving boundary problems. Due to the presence of a moving boundary there are inherent mathematical difficulties in the solution of these problems. To overcome these difficulties immobilization techniques have been employed to fix the moving boundary *via* a coordinate transformation. The drawback of this approach is that the energy equation is rendered complex, introducing additional terms due to the coordinate transformation. The cases, when a transformation in more than one of the coordinates is needed are particularly problematic.

A number of researchers have studied the phase change problem in the cylindrical geometry. The problems, where the original medium is at the fusion temperature, are single-phase problems, since only the temperature distribution in the newly formed phase is to be solved. When the original medium is at a different temperature than solidification, the solution involves the computation of temperatures in both phases, hence a two-phase problem. Duda et al., 1975; solved a two-dimensional ($r-z$ plane) two-phase problem of a finite cylinder with convective boundary condition, in which the penetration of the interface in the axial direction is used for immobilization. Perturbation method was applied by Huang and Shih, 1975; for inward and outward solidification of saturated liquid in spherical and cylindrical containers. The solution for the inward solidification in cylindrical geometry was for a cylinder with infinite length, resulting in a one-dimensional formulation of the problem. The original phase was at the fusion temperature; hence the method was applied to single phase problems. Saitoh, 1976; reported experiments for the solidification of water in various geometries, including cylindrical containers. The solidification process resulted in a one-dimensional motion of the interface, in the radial direction. A temporal variation of boundary temperature was applied, including linearly decreasing and sinusoidally varying boundary temperatures. Coordinate transformation method generally used in one-dimensional problems was extended by Saitoh, 1978; to the multi-dimensional problem by using an independent variable, which takes constant values at the physical boundaries and the freezing front. In this method, both the phase and fixed boundaries could be selected arbitrarily and the functions defining the boundaries were used for the definitions of the transformed independent variables. Calculations were performed for single-phase freezing problems with constant physical properties in regular squares, triangles and ellipses. In this study, problems with variable and constant surface temperature were analyzed. Voller and Cross, 1981 solved the one-dimensional, radial solidification of an infinitely long cylinder, where the liquid is initially at the melting temperature, and reported the total solidification time. The enthalpy method in conjunction with a finite difference formulation was used and the results were compared to an existing approximate analytical solution.

The inward one-dimensional, radial solidification in a liquid cylinder initially at the fusion temperature was given by Hill and Dewynne, 1986. In this problem a front fixing method was applied, where the radial position of the boundary was fixed through a logarithmic transformation and an analytic series solution was introduced for the transformed energy equation. The complete solidification time was approximated using the first three terms of the series, where the boundary conditions are of the constant temperature and convection type. Saitou and Hirata, 1993; considered the effect of buoyancy in the freezing of a finite cylinder. The problem involved the solution of the conduction on the solid side and the buoyancy induced convection on the liquid side. Kharche and Howarth, 2000; solved the single-phase solidification of a finite cylinder, where the solidification occurs along the radial and axial directions, subjected to a spatial variation of boundary temperatures. A perturbation solution was obtained for large Stefan numbers. The temperature profiles and interface locations were reported in the form of infinite series.

Dursunkaya and Odabaşı, 2003; used a coordinate transformation technique to immobilize the moving interface in the two-dimensional solidification of an infinite square prism. In this case the phase change interface was assumed to have a complex shape and a coordinate transformation was applied where both of the original x and y coordinates are transformed. This was applied in conjunction with a boundary-fitted mesh, with constant arc length along a constant coordinate line. The predictions were compared to experimental results having constant and temporally varying boundary temperatures, and new results for spatially varying boundary temperatures were presented.

Phase change materials (PCM) encapsulated in containers are used as thermal storage systems which take the advantage of latent heat removal capacity. Bilir and İlken, 2005; studied total solidification time for the inward solidification for cylindrical and spherical containers. The phase change in radial direction was modeled by enthalpy method, where a convection boundary condition was defined for the external boundary. A parametric study was conducted and total solidification time was correlated to Stefan and Biot numbers and the difference between the melting and ambient temperatures.

Rattanadecho and Wongwises, 2008; applied a front fixing method on a structured grid in a rectangular cavity to study freezing of a porous material, where constant boundary condition was defined for a small portion of the upper side of the domain and the rest of the boundaries were assumed to be insulated. The transformed form of the energy equation in both solid and liquid regions were solved by finite difference method for the interface position as a function of time and compared to experimental results.

A phase change material along horizontal tube, which was used a cooling medium, was analyzed by Kamal *et al.*, 2014; using solid-liquid interface boundary fixing method. Landau transformation was used to fix the boundary and the energy equation in the $r-z$ plane was solved by finite volume method. The analysis was validated by the experimental results for the, interface velocity. To optimize the thermal storage system, multiple tubes with different diameters served as heat sink array were placed inside a phase change material contained in a cylinder. Melting of the PCM around the heat sinks was modeled by enthalpy method using commercial code FLUENT. The temperature distribution and phase change interface motion were obtained and the effect of the number and diameter of the heat sink combinations was analyzed for total melt time by Huawei *et al.*, 2014.

Bourdillon *et al.*, 2014; implemented the enthalpy method on the Open FOAM code to analyze two-dimensional freezing of water in rectangular and cylindrical cavities with constant temperature boundary condition. Energy and momentum equations in both solid and liquid regions were solved and mushy zone was also considered to include the complete physical phenomena.

Huang and Wu, 2014; applied the lattice Boltzmann method to model two dimensional melting in a cylinder with constant temperature boundary condition. Immersed boundary method was adapted to the model to capture the velocity and temperature at the solid-liquid interface. The model was validated by analytical and numerical solution of solidification in a semi-infinite domain and a rectangular cavity. The temperature and velocity distributions were solved for the cylindrical geometry, where the solid phase position was defined as either fixed or free, where the latter case reduced the melting time.

Phase change of semi-transparent material was analyzed by Piotr and Piotr, 2012; by a front tracking algorithm. The production of glass, oxide crystals or ceramics, melting of ice due to sunlight were some examples of phase change for such materials, where in modeling this phenomena the effect of radiation was included. Fluid flow and energy equations were solved in a fixed grid using finite volume method, which was applied for the solidification of a semitransparent material in an axisymmetric rectangular and cylindrical cavity and the results for the interface movements were obtained.

In the present study the two-dimensional solidification in the $r-z$ plane of a finite cylinder is solved. A front-fixing approach similar to Dursunkaya and Odabaşı, 2003; was used, where a coordinate transformation involving both the radial and axial directions was applied. Computations were performed for only half of the physical domain due to symmetry, but in this domain the boundary assumed a circular shape, i.e. it was a double-valued function of one of the coordinates, which required the application of the above mentioned front fixing scheme. Finite difference forms of the governing equations were solved in the solid phase only, since the liquid phase was at the fusion temperature throughout the computation. Therefore, the present study concentrates on a single-phase, two-dimensional solidification problem. Due to the changing shape of the solidified region, a new solution grid was generated by solving a new set of nonlinear algebraic equations iteratively at every time step.

PROBLEM FORMULATION

The problem studied is the two-dimensional freezing of an initially liquid cylinder of height A and radius R. Liquid phase is initially at the fusion temperature, and is assumed to be at that temperature throughout solidification, so the temperature distribution only in the solid phase is unknown. This system is exposed to the surface temperature T^*_{sur}, which is, in general, temporally and spatially changing, and is below the fusion temperature. The physical properties are assumed to be constant. Energy equation for the solid phase can be written as,

$$T^*_{t^*} = \alpha\left(T^*_{r^*r^*} + \frac{T^*_{r^*}}{r^*} + T^*_{z^*z^*} \right) \tag{1}$$

where initial and boundary conditions are,

$$\begin{aligned}
T^*(r^*, z^*, 0) &= T^*_f \\
T^*_{r^*}(0, z^*, 0) &= 0 \\
T^*(R, z^*, t^*) &= T^*_{sur}(z^*, t^*) \\
T^*(r^*, 0, t^*) &= T^*_{sur}(r^*, t^*) \\
T^*(\delta^*, t^*) &= T^*_f \\
T^*(r^*, A, t^*) &= T^*_{sur}(r^*, t^*)
\end{aligned} \tag{2}$$

Equations (1) and (2) are nondimensionalized using the following definitions,

$$T = \frac{T^* - T^*_{ref}}{T^*_f - T^*_{ref}},$$
$$r = \frac{r^*}{R}, \qquad z = \frac{z^*}{R},$$
$$t = \frac{\alpha\ t^*}{R^2}, \qquad \delta = \frac{\delta^*}{R} \tag{3}$$

For problems with a constant surface temperature, the reference temperature T^*_{ref} is equal to the surface temperature T^*_{sur}; for variable surface temperature boundary conditions, it is a suitably defined reference temperature. The dimensionless energy equation becomes,

$$T_t = T_{rr} + \frac{T_r}{r} + T_{zz}, \tag{4}$$

subject to the initial and boundary conditions,

$$\begin{aligned}
&T(r,z,0) = 1, && T(\delta,t) = 1, \\
&T_r(0,z,t) = 0, && T(1,z,t) = T(z,t), \\
&T(r,0,t) = T(r,t), && T(r,Ar,t) = T(r,t)
\end{aligned} \tag{5}$$

Figure 1 shows the original and transformed computational domains. This transformation is accomplished as follows: The curved interface is transformed to the straight line $\lambda = 1$ and the external boundary *i.e.*, the top, the sides, and the bottom surfaces are transformed to the straight line $\lambda = 0$. The upper –above the interface– and lower –below the interface– side of the center line become the last two boundaries, where the former is transformed to the straight line, $\xi = 0$ and the latter becomes $\xi = 1$. The interior points are obtained by solving a system of nonlinear equations, which is formed by equating the physical distance between two successive grid points along constant λ and ξ lines.

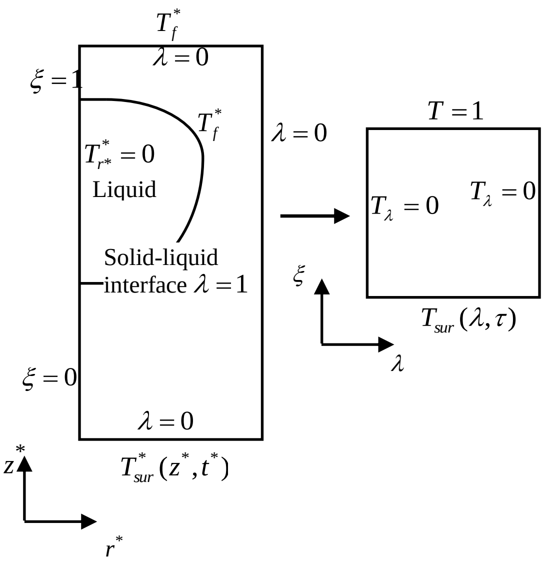

Figure 1. The physical domain, transformed computational domain with boundary conditions

In order to transform the physical domain into the computational domain, new independent variables λ, ξ in space and τ in time replace the original set of r, z and t, as,

$$r = r(\lambda, \xi, \tau), \qquad z = z(\lambda, \xi, \tau), \qquad t = t(\tau). \qquad (6)$$

With this definition, the final form of the transformed dimensionless energy equation is obtained as,

$$\begin{aligned} T_\tau = {} & \frac{T_{\lambda\lambda}}{J^2}\left[z_\xi^2 + r_\xi^2\right] + \frac{T_{\xi\xi}}{J^2}\left[z_\lambda^2 + r_\lambda^2\right] - 2\frac{T_{\lambda\xi}}{J^2}\left[z_\lambda z_\xi + r_\lambda r_\xi\right] \\ & + \frac{T_\lambda}{J^3}\Big[J_\xi\left(z_\lambda z_\xi + r_\lambda r_\xi\right) - J_\lambda\left(z_\xi^2 + r_\xi^2\right) + \\ & \quad \frac{1}{J^2}\left(z_{\lambda\xi} z_\xi + r_{\lambda\xi} r_\xi - z_{\xi\xi} z_\lambda - r_{\xi\xi} r_\lambda\right) + \frac{z_\xi}{rJ} + \\ & \quad \frac{1}{J}\left(r_\tau z_\xi - z_\tau r_\xi\right)\Big] \\ & + \frac{T_\xi}{J^3}\Big[J_\lambda\left(z_\lambda z_\xi + r_\lambda r_\xi\right) - J_\xi\left(z_\lambda^2 + r_\lambda^2\right) + \\ & \quad \frac{1}{J^2}\left(z_{\lambda\xi} z_\lambda + r_{\lambda\xi} r_\lambda - z_{\lambda\lambda} z_\xi - r_{\lambda\lambda} r_\xi\right) - \frac{z_\lambda}{rJ} + \\ & \quad \frac{1}{J}\left(z_\tau r_\lambda - r_\tau z_\lambda\right)\Big] \end{aligned} \qquad (7)$$

where, J is the Jacobian given by,

$$J = r_\lambda z_\xi - r_\xi z_\lambda. \qquad (8)$$

Initial and boundary conditions in transformed variables become,

$$\begin{aligned} & T(\lambda, \xi, 0) = 1, && T(1, \xi, \tau) = 1, \\ & T(0, \xi, \tau) = T(\xi, \tau), && T_\lambda(\lambda, 0, \tau) = 0, \\ & T_\lambda(\lambda, 1, \tau) = 0. && \end{aligned} \qquad (9)$$

Since the temperature in the liquid is constant, the energy balance at the interface can be obtained by equating the amount of heat released due to solidification to the heat conducted to the solid at the interface as,

$$\rho L V_{nor}^* = k\, T_{n^*}^* \cdot \qquad (10)$$

This equation can be nondimensionalized using the following definitions,

$$T = \frac{T^* - T_{ref}^*}{T_f^* - T_{ref}^*}, \qquad V_n = \frac{V_n^* R}{\alpha}, \qquad n = \frac{n^*}{R} \qquad (11)$$

and takes the dimensionless form,

$$V_{nor} = T_n \,\mathrm{St}. \qquad (12)$$

Using the definition of the surface normal vector, $\mathbf{n} = \nabla T / |\nabla T|$, the components of the normal velocity in r and z directions can be written as follows.

$$V_{n-r} = T_r \,\mathrm{St}, \qquad V_{n-z} = T_z \,\mathrm{St} \qquad (13)$$

SOLUTION METHODOLOGY

The original physical, and the transformed computational domains, and the corresponding coordinates are given in Figure 1. A general transformation of the form

$$\lambda = \lambda(r, z, t), \quad \xi = \xi(r, z, t) \qquad (14)$$

is used for the mapping. The coordinate transformation and grid generation are obtained simultaneously. The external boundaries are divided into equal arc length segments, and equations are written for the locations of the interior points by enforcing equal arc length grid spacing, in the physical domain along constant λ and ξ lines. This results in a set of nonlinear equations for the interior points where the temperatures are to be calculated. This set is solved using an iterative, multi-dimensional Newton-Raphson scheme. The details of the grid generation to facilitate equal arc length are given in Dursunkaya and Odabaşı, 2003.

Once the grid is generated, the finite difference form of the energy equation is solved for the temperatures, the derivatives of which are used in the computation of radial and axial components of the interface velocity using Equation (13). A numerical integration of Equation (13) in time gives the new position of the phase change interface.

RESULTS

The results in the cylindrical geometry were validated using experimental data available in the literature. These cases involved a temporal variation of boundary temperatures. The method was then applied to problems where the boundary temperatures have a spatial variation.

Saitoh, 1976; carried out experiments for the freezing of water in various geometries. For the cylindrical solidification problem a cylinder with an aspect ratio of 5 was used. The temperature on the external surface of the cylinder was altered temporally and temperature measurements were taken at various radial locations, along the lateral mid-plane of the cylinder. The results were similar to a one-dimensional solidification in an infinitely long cylinder, since no coolant was applied to the top and bottom surfaces of the cylinder. Although, the exact nature of the boundary condition at the top and bottom of the cylinder was unclear, it was reasonable to assume insulated conditions to prevail at the top and bottom, which render this solidification problem one-dimensional, changes occurring only in the radial direction. To validate the present method, a temporally changing boundary temperature, identical to the experiments, was specified on all surfaces and the interface motion on the lateral mid-plane of the cylinder was compared to the findings of Saitoh, 1976. Since the aspect ratio is large ($Ar = 5$), it can be argued that the motion of the phase change interface along the lateral mid-plane assumes that of an infinitely long cylinder. This assumption was later justified by observing the shape of the interface contours. To enable a comparison, the graphical results given in Saitoh, 1976; were digitized and plotted. The physical properties for ice were as follows: $\alpha = 1.2358\times10^{-6}\,\text{m}^2/\text{sec}$, $\alpha / R^2 = 8.7904\times10^{-4}\,\text{sec}$, $L = 333.4\,\text{kJ/kg}$ and $C = 1.93\,\text{kJ/kg}^o\text{C}$. It should be noted that the simulations showed the results to be sensitive to thermal properties, the differences existing in literature for the values of these lead to deviation from the experimental results. Figure 2 shows the experimental results by Saitoh, 1976; and the results of the current study. In this set, the boundary temperature of the cylinder is reduced linearly in time. The three curves represent three different cooling rates as given in the legend. In all three cases there is a good match between the experimental results and the predictions of the current study.

The current study was also validated using the case of a sinusoidal variation of the boundary temperature in time. Figure 3 shows the predictions of the current study and experimental results of Saitoh, 1976. In the original study by Saitoh, 1976; time was nondimensionalized using the frequency of the sinusoidal variation of the boundary temperature. For this case, the same nondimensionalization was used in the present study in order to enable a comparison; in addition a reference temperature $T^*_{ref} = -10^o\text{C}$ was used in the computations of Stefan numbers and dimensionless temperatures. The three different set of data points depict the results of three experiments given in Saitoh, 1976. It can be seen that there is a good match between the predictions of the current study and the experimental results.

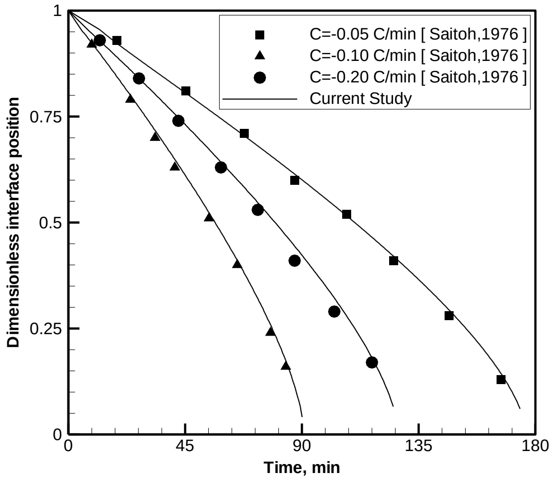

Figure 2. Radial movement of solidification front in a cylinder subject to a linearly varying boundary temperature in time

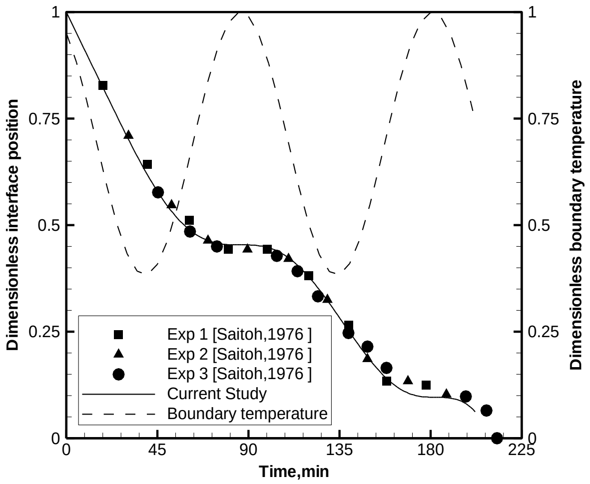

Figure 3. Radial movement of solidification front in a cylinder, subject to a sinusoidal variation of the boundary temperature.

The computational mesh consisted of 17 base mesh points in the λ direction with 2 mesh subdivisions near the interface resulting in 19 mesh points, and 85 mesh points in the ξ direction, giving a mesh with 19×85 nodal points. Since the coordinates of the mesh points on the boundaries including the points on the moving interface are calculated using equidistant node intervals, at every time step only the locations of the interior points should be recalculated (Dursunkaya and Odabaşı, 2003). The determination of the *r* and *z* coordinates of all interior nodal points requires the solution of a set of simultaneous nonlinear equations, iteratively, at every time step.

Figure 4 shows the motion of the phase change interface for the sinusoidally varying boundary temperature. The plot is given for half the domain, since the problem in this case is symmetric with respect to the lateral plane passing through the geometric center. An examination of this figure reveals that after about 150 minutes, the angle between the phase change interface lines and the cylinder axis deviate from perpendicular. This is due to the number of mesh points used in the λ direction, which in this case is 85. The effect of mesh size was studied extensively, and it was observed that this behavior starts earlier for coarser mesh. It was also observed that for cases, when the shape of the solidification front resembles a circle, a course mesh gives satisfactory results, which occurs when the aspect ratio is close to unity. For larger or smaller aspect ratios, or in case of an asymmetric spatial boundary temperature variation, the moving interface assumes a complex shape, necessitating the utilization of a finer mesh.

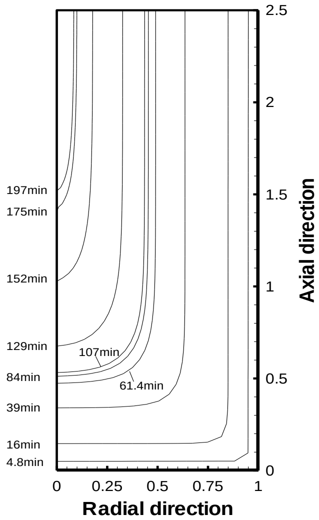

Figure 4. Transient interface motion for the sinusoidal boundary condition

New results will be presented for three different aspect ratios, $Ar = 0.5,\ 1$ and 3. Figure 5 shows the transient interface locations for an aspect ratio of 0.5. In this problem all the boundaries of the cylinder are suddenly exposed to a temperature less than melting and the Stefan number is 0.01, corresponding to approximately $2^{o}C$ temperature difference for ice. In this case the original mesh is 23×71 with 2 mesh subdivisions near the moving interface, resulting in a 25×71 grid system. The results show that solidification time τ for this problem is 2.4, approximately. By this time, the phase change interface has moved to approximately 45% of the distance in the axial direction both from the top and the bottom and 38% of the distance radially. This is expected, since more heat transfer occurs through the top and bottom surfaces in this geometry, resulting in a faster advance in the axial direction. An examination of the shape of the phase change interface reveals that, to resolve the sharp turn in the vicinity of $r \cong 0.62$ a finer mesh in the λ direction is needed. A parametric study of the mesh size revealed that, with a courser mesh, the interface shape assumes a discontinuous slope in earlier dimensionless times.

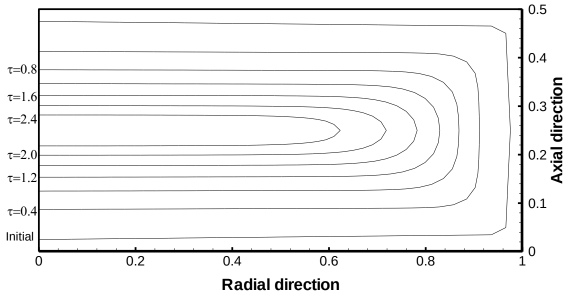

Figure 5. Transient interface motion for the case $Ar = 0.5,\ \mathrm{St} = 0.01$ with constant boundary temperatures

The results for an aspect ratio $Ar = 1$ with a constant temperature boundary condition are given in Figure 6. The solution was obtained using a 21×67 base mesh with 4 mesh subdivision at the interface, resulting in a 25×67 mesh. The figure shows that time for full solidification is about $\tau \approx 9.5$ compared to $\tau \approx 2.4$ for the case when aspect ratio was 0.5. The interface motion for the same aspect ratio but for a linearly varying boundary temperature along the axial direction is given in Figure 7. In this case the bottom of the cylinder is kept at zero dimensionless temperature ($T = 0$), the top at 0.75 ($T = 0.75$), and the external surface temperature varies linearly from the temperature at the bottom to the top. In casting, void and crack formation due to solidification is of interest, and such formations can be avoided by controlling the solidification process. Figure 7 shows that, using the linearly varying boundary temperature, the liquid core remains at the top of the cylinder. Full solidification time, however, is increased to $\tau \approx 17$ almost double the constant temperature case.

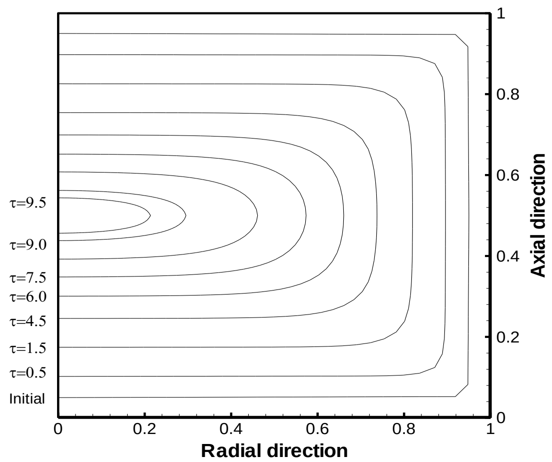

Figure 6. Transient interface motion for the case $Ar = 1,\ \mathrm{St} = 0.01$ with constant boundary temperatures

To assess the effect of the aspect ratio on solidification, the study was repeated for an aspect ratio of 3. Figures 8 and 9 show the interface motion for the case of constant boundary temperature and a spatially linearly varying boundary temperature, respectively. In the first case total solidification occurs at $\tau \approx 23$, whereas this increases to $\tau \approx 42$ for the spatially linearly changing boundary temperature. It can also be observed that the unsolidified region moves towards the top of the cylinder for the variable temperature boundary condition.

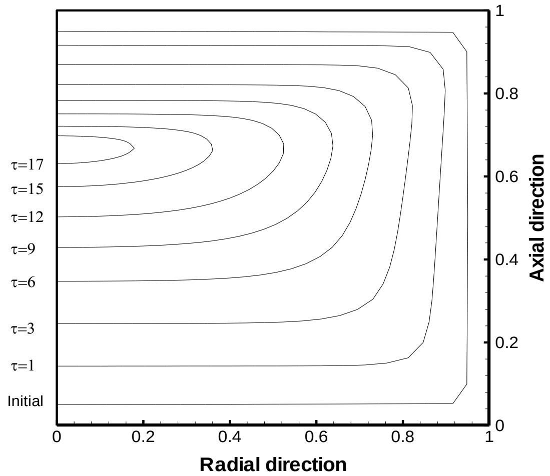

Figure 7. Transient interface motion for the case $Ar = 1,\ \mathrm{St} = 0.01$ with linearly varying boundary temperature in the axial direction

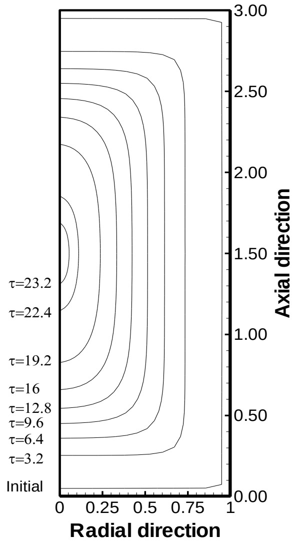

Figure 8. Transient interface motion for the case $Ar = 3,\ \mathrm{St} = 0.01$ with constant boundary temperatures

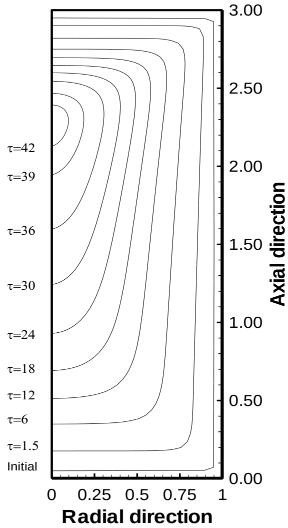

Figure 9. Transient interface motion for the case $Ar = 3,\ \mathrm{St} = 0.01$ with linearly varying boundary temperature in the axial direction

CONCLUSION

The problem of the inward solidification in a finite cylinder, subjected to temporally and spatially varying boundary temperatures was analyzed. A boundary fitted, equal arc length mesh was utilized in the solution. The interface position variation of the present study is compared to the experimental results of Saitoh, 1976 for the case, where the external boundary temperature varies linearly and sinusoidally with time and the match was found to be acceptable. The method is used to calculate the variation of the interface position in time for different aspect ratios. It is found that relatively course mesh can be utilized where interface position assumes a circular shape until total solidification is obtained. Total solidification time is compared for different boundary conditions and aspect ratios. It is shown that total solidification time increases with the aspect ratio when the external boundary set to a constant temperature, where the nondimensional solidification time is 2.4 for aspect ratio 0.5, 9.5 for aspect ratio 1.0 and 23 for aspect ratio 3.0. The effect of linearly varying external boundary temperature on total solidification time was found, when the nondimensional time increased from 9.5 to 17 for aspect ratio 1.0 and 23 to 42 for aspect ratio 3 with respect to constant temperature condition. Linearly varying boundary temperature causes unsolidified liquid region to move to the upper side of the cylinder. The results of the present study can be employed to determine total solidification time and possible void or crack position. By positioning the location of final solidifying region close to a boundary open to the atmosphere, through using different boundary temperature profiles, the formation of interior voids or cracks can be avoided.

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