



## INTERNAL AND TRANSITIONAL BALLISTIC SOLUTION FOR SPHERICAL AND PERFORATED PROPELLANTS AND VERIFICATION WITH EXPERIMENTAL RESULTS

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**Abstract:** The solutions of internal ballistics in addition with the transitional ballistic equations are very important for gun design. During the design phase of a gun, some of the main parameters decided on are the selection of the propellant chemical structure and the grain geometry which may be spherical, tubular or multi perforated tubular. In this study, the Résal equation, which is one of the basic internal ballistic equations, is solved by means of Runge Kutta method for spherical and perforated propellants. The propellant chemical structure, propellant geometry, projectile mass properties and barrel geometry are the main inputs for a typical gun design. The pressure distribution and the linear and tangential velocity of the projectile in the barrel are predicted depending on time along the barrel length. The friction force between barrel and projectile, resistance force created by the rifling against the motion of projectile are taken into consideration. Thermodynamic and heat transfer parameters required in order to solve heat transfer problems of barrel such as cook-off problem are also obtained. The calculation of erosion at the inner surface of barrel is another problem solved for the life prediction of the barrel. To design a gun, solving internal ballistics is not enough, transitional ballistic of the barrel after the shot ejection has to be also solved. The computer code of “Internal Ballistic AKÇAY” is generated for the solution of universal gun design problems. Experiments are carried out with test barrel of 7.62x51 mm M80 ammunition at MKE ballistic test facilities. The theoretical results are compared with the experimental results. The agreements between them are quite satisfactory.

**Keywords:** Gun barrels, interior ballistics, propellants, thermodynamics.

## KÜRESEL VE ÇOK DELİKLİ BARUTLAR İÇİN İÇ BALİSTİK VE GEÇİŞ BALİSTİĞİ PROBLEMLERİNİN ÇÖZÜMÜ VE DENEYSEL SONUÇLAR İLE KARŞILAŞTIRILMASI

**Özet:** Silah tasarımında iç balistik problem çözümünün yanında geçiş balistiği probleminin de çözümü gerekmektedir. Silahın tasarım safhasında öncelikle seçilerek karar verilmesi gereken parametreler barutun kimyasal yapısı ve dane geometrisidir. Barutun dane geometrisi, genel olarak; küresel, silindirik veya çok delikli silindirik olabilmektedir. Bu çalışmada, temel iç balistik denklemi oluşturan Résal eşitliği küresel ve tek delikli barut dane geometrisi için Runge Kutta metodu kullanılarak çözülmüştür. Tipik bir silahın tasarımında temel girdiler; barut kimyasal yapısı, barut dane geometrisi, mermi kütleli değerleri ve namlu geometrisinden oluşmaktadır. Namlu boyunca, namlu içinde basınç dağılımı, merminin lineer ve teğetsel hız değişimi zamana bağlı olarak hesaplanmaktadır. Bu hesaplamalarda, mermi ve namlu arasındaki sürtünme kuvveti ile namlu içinde merminin hareketine karşı yiv ve setlerin yarattığı direnç kuvveti de göz önüne alınmaktadır. Hesaplamalar sonucunda, namluda ısı transferi ve barutun kendi kendine tutuşması gibi problemlerin çözümü için gerekli termodinamik ve ısı transferi parametreleri de elde edilmektedir. Namluların ömür probleminin çözümü için atış sırasında namlu iç yüzeyinde meydana gelen aşınmanın da hesabı gerekmektedir. Oluşturulan modelde namlu aşınması hesaplanabilmektedir. Namlunun tasarımının yapılabilmesi için iç balistik problemin çözülmesi yeterli olmamakta, mermi namluyu terk ettikten sonra oluşan geçiş balistiği probleminde çözümü gerekmektedir. Universal silah tasarım problemlerinin çözümü için “AKÇAY İç Balistik” bilgisayar kodu geliştirilmiştir. Elde edilen teorik sonuçlar, MKE Atış Poligonunda 7.62x51 mm M80 fişekleri ile yapılan test sonuçları ve literatürde mevcut test sonuçları ile karşılaştırılmıştır. Teorik ve deneysel sonuçlar arasındaki uyum gayet tatminkârdır.

**Anahtar Kelimeler:** Silah namluları, iç balistik, barutlar, termodinamik

### NOMENCLATURE

$A$	Inner cross sectional of the barrel	$m_M$	projectile mass
$A_P$	surface area of a grain	$m_B$	propellant mass
$A_{P_0}$	initial surface area of a grain	$m^*$	effective projectile mass
$a$	perforation diameter	$m_{B_x}$	mass of the burned propellant
$b_z$	width of the rifling grooves	$n$	burning rate pressure index of the propellant
$B_a$	burning coefficient	$n_z$	number of rifling grooves
$D$	caliber of the barrel	$N$	number of perforation of a grain
$L_B$	length of a tubular grain	$P$	pressure in the barrel
		$P_a$	atmospheric pressure
		$R$	radius of a tubular propellant grain

$\mathfrak{R}$	universal gas constant
$R_G$	gas constant
$Q_B$	specific heat of propellant
$Q_K$	heat loss
$t$	time
$t_z$	depth of the rifling grooves
$T$	temperature
$T_0$	initial temperature
$U$	projectile velocity
$W(x)$	unit energy lost due to friction
$W_d$	energy used for the rotation of the projectile
$x$	location of the projectile in the barrel

$z$	fraction of burned propellant mass to initial propellant mass
$V$	volume behind the projectile
$V_B$	combustion chamber volume

#### Greek symbols

$\varepsilon$	web thickness
$\gamma$	ratio of specific heats
$\gamma'$	measured ratio of specific heat
$\varphi(z)$	form function of the propellant
$\eta$	covolume of the combustion gases
$\rho$	propellant density
$\eta_{piezo}$	piezometric efficiency
$\eta_b$	ballistic efficiency

## INTRODUCTION

The gun design has been the main interest for defence people since the gun powder had been discovered centuries ago. So any of the development on this matter still has a great importance. The main point in this study is to find out relationships among the interior ballistic parameters and to optimise a new gun barrel for the new weapon systems under development. Interior ballistics is a science to be studied on to the development of a gun which throws a useful material from one point to another one. In classical weapons, chemical energy of the propellants is converted to the kinetic energy in order to overcome the resistance forces against the movement of the projectile in the barrel and to accelerate the projectile up to desired level of velocity at the muzzle of the barrel. The main goals for the solution of internal ballistic problems are to obtain the desired muzzle velocity without damaging the weapon from excess internal barrel pressure and to deliver the same type of projectiles to a definite target with the successive propellant charges for heavy weapons or with the cartridge case for light weapons with high accuracy and dispersion. Some of the studies on internal ballistics are focused on chemical and mechanical structure of the propellants which are the main parameters for internal ballistics; most of the other studies have been carried out for the solution of the internal ballistic equations (Résal 1864, Corner 1950, AMCP 1964). Practical internal ballistic applications such as the method of Vallier-Heydenreich have also been well studied during the last fifty years (Öztürk 1981, Akçay 1992). Some of the basic approaches like Lagrange Gradient method and the general considerations in internal ballistics are summarized in the literature (Carlucci et al., 2007). The role of exothermic gas phase reactions behind the projectile, the complications of the combustion processes of the nitrocellulose and nitramine based propellants, ignition problems of the solid propellants are examined in detail (Summerfeld et al., 1986). The internal ballistics is mainly based on both thermodynamics and fluid dynamics and some models based on thermodynamics are developed in order to predict pressure time history in the combustion chamber and velocity of projectile in the barrel for small caliber weapons (Celens, 1986). The similar model was also developed for the prediction of pressure and velocity in the barrel for small and large caliber weapons (Akçay, 1981).

The design of a gun and its barrel require the time dependent pressure and temperature history in the barrel not only during the internal ballistic period but also during the transitional

ballistic period. The transitional ballistics covers the events between the internal ballistic period and the external ballistic period. It starts at the time when the projectile leaves the muzzle of the barrel and is over when the pressure in the barrel becomes equal to the atmospheric pressure. The transitional ballistics has been extensively studied in order to reduce the muzzle flush, blast effect and to predict its effects on the initial velocity of the projectile (Moore, 1974). It is found that the increase in the projectile velocity during intermediate ballistic period is in the order of per thousand of the muzzle velocity that might be neglected in comparison with the velocity changes occurred due to the other projectile constructive tolerances (Trebinski et al., 2015). Any of the above studies mentions the importance of transitional ballistics for gun design.

The computational efforts and developments of numerical simulation methods have been increased during the last two decades and the more complex solutions in internal ballistics are performed such as the two phase flow problems in gun barrels which are modeled with the ready to use fluid dynamics programs (Bougamra et al., 2014). Some theoretical and experimental studies have been carried out on internal ballistics. They ignored some of the parameters like co-volume of the propellant and friction force between the barrel and projectile and they could not discriminate the combustion chamber pressure and the base pressure at the projectile moving along the barrel. They compared their results with the semiempirical methods which require the maximum gas pressure and muzzle velocity in advance (Cronemberger et al., 2015). In fact, the main goals of the internal ballistic solution are the correct prediction of complete internal ballistic parameters including base pressure and velocity of the projectile along the barrel depending on time without neglecting any of the input parameters of the barrel, projectile, igniter and propellant. The interior ballistic parameters like combustion gas temperature and pressure have been used to perform barrel stress analysis (Şenturk A. et al., 2016). The mathematical modeling of internal ballistics is still very important due to the high costs, long durations and technical difficulties in new gun and ammunition prototype constructions and experimental setups for their testings.

In this paper, the Code of Internal Ballistic AKÇAY being generated since 1977 has been enlarged to cover the intermediate ballistic period which has been necessary to be investigated not only to see its effects on muzzle flush, blast effect and muzzle velocity but also the contribution to the recoil force to design recoil cylinders and recuperator for large

caliber weapons and motion mechanism for small caliber weapons. Our interest in transitional ballistics in this study is the prediction of the thermodynamic properties like pressure, density and temperature versus time along the barrel during the intermediate ballistic period. This interest is rather different from the information given above literature about transitional ballistics. Here the theoretical solutions that have been carried out for spherical and multi tubular propellant grain geometries have given pressure time history, projectile position and velocity along the barrel length. Complete thermodynamic parameters in the barrel for internal and transitional ballistic periods such as density, pressure, temperature, specific heat coefficients, convective heat transfer coefficient of the combustion gases are also predicted (Akçay 2014, White 2003). The dimensions and angle of rifling of the barrel and connection force between cartridge case and projectile, engraving force, the friction force between barrel and projectile are considered in these calculations. In this way, calculation of the rotational motion of the projectile in the barrel becomes possible also.

## STATEMENT OF THE PROBLEM AND GOVERNING EQUATIONS

The theoretical principles of the internal ballistic problem based on energy balance were first established by Résal in 1864. During the interior ballistic cycle in a barrel, the amount of heat generated by the combustion of the propellant was consumed as follows;

- Energy of linear motion of the projectile,
- Energy of rotational motion of the projectile,
- Energy of recoiling parts of the weapon,
- Energy of driving automatic weapons,
- Energy of the flow of the burned propellant gases,
- Internal energy of the burned propellant gases,
- Heating the barrel, projectile and cartridge case,
- Energy to overcome friction resistance forces,
- Work done against the extractor resistance for cartridge,
- Work done in engraving the rotating band for spin stabilized projectiles.

The energies mentioned above how to be taken into consideration during theoretical predictions are explained below.

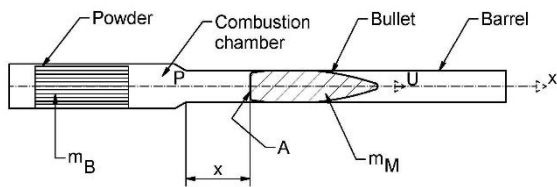


Figure 1. The geometrical interpretation of a small caliber gun barrel.

### Governing equations

The balance equations for mass, momentum, energy, propellant burning and flow of combustion gases in the barrel are given as follows;

Equation of motion

$$m^* \frac{d^2x}{dt^2} = PA \quad (1)$$

Combustion equation

$$\frac{dz}{dt} = B_a \varphi(z) \left( \frac{P}{P_a} \right)^n \quad (2)$$

Energy equation

$$m_{B_x} Q_B = \frac{1}{2} m^* \left( \frac{dx}{dt} \right)^2 + \frac{PV}{(\gamma-1)} + Q_K + \int_0^x W(x) dx + W_d \quad (3)$$

where  $m^* = m_M + \frac{1}{2} m_B$ ,  $m^*$  is the effective projectile mass,

$m_M$  is the projectile mass,  $m_B$  is the propellant mass (it is assumed that half of the propellant mass is converted to gas molecules moving with the projectile along the barrel),

$A = \frac{\pi D^2}{4} + n_z b_z t_z$  is the inner cross sectional area of barrel

including the geometrical effects of grooving,  $x$  is the location of the projectile in the barrel,  $z$  is the fraction of burned propellant mass to initial propellant mass,  $P$  is the pressure in the barrel,  $P_a$  is the atmospheric pressure,  $B_a$  is the burning coefficient,  $\varphi(z)$  is the form function of the propellant,  $n$  is the burning rate pressure index of the propellant,  $m_{B_x}$  is the mass of the burned propellant at time  $t$ ,  $Q_B$  is the specific heat of propellant,  $\gamma$  is the ratio of specific heats,  $Q_K$  is the heat loss through barrel and projectile,  $W(x)$  is the unit energy lost due to friction along the barrel and  $W_d$  is the energy used for the rotation of the projectile and the recoil of the gun. The percentage of the total energy used for the rotation of the projectile and the recoil is in the order of 1% so that it can be neglected. Energy loss due to friction may be considered within the heat loss. Total energy loss through heat transfer to the barrel, cartridge and projectile during firing is taken into account by considering polytropic expansion of the combustion gases in the barrel. For this reason, the measured ratio of specific heat  $\gamma'$  value is used instead of theoretical value of  $\gamma$  and  $Q_K$  the heat loss parameter can be neglected (Corner, 1950). Energy Eq. (3) expressing the energy balance during interior ballistic cycle proposed by Résal is rearranged;

$$m_{B_x} Q_B = \frac{1}{2} m^* \left( \frac{dx}{dt} \right)^2 + \frac{PV}{(\gamma'-1)} \quad (4)$$

At a certain time  $t$ , during firing, percentage  $z$  of the propellant converted from solid phase to the gas phase and released energy is equal to the internal energy of combustion gases and the kinetic energy of the effective projectile mass.

$$m_{B_x} = m_B z \quad (5)$$

$V$  is the volume behind the projectile in which the combustion of the propellant occurs at time  $t$ ;

$$V = V_B + Ax - m_B z \eta - \frac{(1-z)m_B}{\rho} \quad (6)$$

where  $V_B$  is the initial volume used for propellant storage,  $x$  is the location of the projectile in the barrel at time  $t$ ,  $\eta$  is the covolume of the combustion gases and  $\rho$  is the propellant density. When equation (1) inserted into

equation (2), the new combustion equation is obtained as a function of projectile acceleration.

$$\frac{dz}{dt} = B_a \varphi(z) \left( \frac{m^* \ddot{x}}{AP_a} \right)^n \quad (7)$$

Equation (1) and Equation (4) can be combined as;

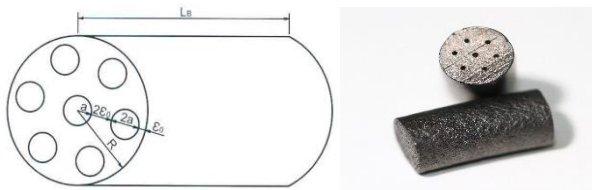
$$\frac{d^2x}{dt^2} = \frac{A(\gamma' - 1)}{\left( V_B + Ax - m_B z \eta - \frac{(1-z)m_B}{\rho} \right)} \cdot \left[ \frac{m_B Q_B z}{m^*} - \frac{1}{2} \left( \frac{dx}{dt} \right)^2 \right] \quad (8)$$

In order to obtain ballistic parameters of a specific gun, equations (7) and (8) have to be solved simultaneously. For the solution of this set of equation, form function of propellant grain  $\varphi(z)$  has to be known. Form function is expressed as follows;

$$\varphi(z) = \frac{A_p}{A_{p_0}} \quad (9)$$

here  $A_{p_0}$  is the initial surface area of the propellant grain;  $A_p$  is the surface area of the grain at any time during combustion. Form function has a value of between one and zero. Propellant grains have wide variety of geometrical shapes, like cubical, spherical, strip, solid cylinder, tubular or multi perforated. The value of the form function for tubular propellant grains is equal to one. The form functions of different grain geometries and their effects on burning rate can be found in the literature (Corner 1950, AMCP 1964, Akçay, 1992). Form factor is a very important parameter which effects the burning rate, burning duration, combustion pressure and velocity of the projectile in the barrel. A geometrical representation of a multi perforated tubular propellant grain is shown in Fig.2. At any time  $t$  during burning there is a relationship among the parameters of a grain;

$$3a + 4\varepsilon = R \quad (10)$$



**Figure 2.** Geometry of multi perforated tubular propellant grain (courtesy of Mr. Osman Başak).

Considering  $N$  as the number of perforations and  $\varepsilon$  as the web thickness, the mass fraction of the burned propellant is

$$z = \frac{\left[ 2(N-1)\varepsilon^3 + [(N-1)L_B - 4(R+Na)]\varepsilon^2 + 2[(R+Na)L_B + (R^2 - Na^2)]\varepsilon \right]}{(R^2 - Na^2)L_B} \quad (11)$$

Here, the form function for the multi perforated propellant grains is given as follows;

$$\varphi(z) = \frac{\left[ -3(N-1)\varepsilon^2 + [(N-1)L_B - 4(R+Na)]\varepsilon + [(R+Na)L_B + (R^2 - Na^2)] \right]}{[(R+Na)L_B + (R^2 - Na^2)]} \quad (12)$$

If Eq.7, Eq.11 and Eq.12 are rearranged, the linear burning rate for multi perforated propellant grains is obtained as;

$$\frac{d\varepsilon}{dt} = \frac{(R^2 - Na^2)L_B}{2[(R+Na)L_B - (R^2 - Na^2)]} B_a \left[ \left( \frac{m^*}{P_a A} \right) \frac{d^2x}{dt^2} \right]^n \quad (13)$$

More comprehensive information about the form function of multi perforated propellant grains and its use in propellant burning has been given in Akçay (1992). The pressure of the combustion gases at any point of the barrel can be calculated as

$$P = \frac{A(\gamma' - 1)}{\left( V_B + Ax - m_B z \eta - \frac{(1-z)m_B}{\rho} \right)} \left[ m_B z Q_B - \frac{1}{2} m^* \left( \frac{dx}{dt} \right)^2 \right] \quad (14)$$

The temperature of the combustion gases is given as follows;

$$T = \frac{P \left[ V_B - m_B \left( z \eta + \frac{1-z}{\rho} \right) + Ax \right]}{(n_{mol} m_B z \mathfrak{R})} \quad (15)$$

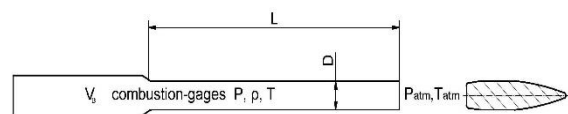
and the density of the combustion gases is given as follows

$$\rho = \frac{m_B z}{\left[ V_B - m_B \left( z \eta + \frac{1-z}{\rho} \right) + Ax \right]} \quad (16)$$

### Transitional Ballistics

When the projectile in the barrel, the parameters like gas pressure, gas temperature and density of the gases have been examined in the context of internal ballistics. As soon as the projectile leaves the barrel, the combustion gases inside the barrel starts to move towards atmosphere. The evacuation of gases takes rather longer time than the projectile movement duration in the barrel. The evacuation of combustion gases from the barrel is examined under the name of “transitional ballistics”. During the gas evacuation period, barrel starts to function like a rocket motor and it moves the gun in the reverse direction. This occurs up to the time when the value of the gas pressure in the barrel decreases to the value of the atmospheric pressure. In fact, transitional ballistics covers the events between the internal ballistic period and the external ballistic period.

In this study, during the transitional ballistic period, combustion chamber or cartridge case is considered like a gas container as shown in Fig.3. The physical problem to be studied can be treated as the evacuation of a gas from a container through a pipe. In this study the flow is considered as a kind of blowdown process (White, 2003). In this case;



**Figure 3.** Evacuation of the combustion gases from barrel to the atmosphere

Continuity equation:

$$\frac{d}{dt}(\rho_o V) = -\dot{m} \quad (17)$$

$$\frac{d}{dt}\left(\frac{P_o}{R_G T_o} V\right) = -\gamma^{\frac{1}{2}} \left(\frac{2}{\gamma+1}\right)^{\frac{1}{2}\left(\frac{\gamma+1}{\gamma-1}\right)} \frac{A P_o}{(R_G T_o)^{\frac{1}{2}}} \quad (18)$$

Here,

$$V = V_B + AL \quad (19)$$

Energy equation:

$$\frac{dQ}{dt} + \frac{dW}{dt} = 0 = \frac{d}{dt}\left(\frac{P_o}{R_G T_o} V C_V T_o\right) + \dot{m} C_p T_o \quad (20)$$

By combining Eq.17 and Eq.19 and defining:  
Considering

$$B = \gamma^{\frac{1}{2}} \left(\frac{2}{\gamma+1}\right)^{\frac{1}{2}\left(\frac{\gamma+1}{\gamma-1}\right)} \quad (21)$$

The pressure change in the barrel becomes

$$\frac{dP_o}{P_o} = -B \frac{A}{\sqrt{R_G T_o}} \frac{R_G T_o}{V} dt \quad (22)$$

$$\int_{P_o}^P \frac{dP_o}{P_o} = -B \frac{A}{\sqrt{R_G T_o}} \frac{R_G T_o}{V} \int_0^t dt \quad (23)$$

So that at any time  $t$ , the pressure in the barrel can be given as follows;

$$P(t) = P_o e^{-B \frac{A \sqrt{R_G T_o}}{V} t} \quad (24)$$

The duration of gas evacuation period lasts until the pressure in the barrel becomes equal to the atmospheric pressure. If this time period considered as  $\Delta t$ , from Eq.23

$$\Delta t = \frac{\ln \frac{P_a}{P_o}}{-B \frac{A}{V} \sqrt{R_G T_o}} \quad (25)$$

obtained. Here  $A$  is the cross sectional area of the barrel and  $V$  is the total volume including the combustion chamber volume or the cartridge case volume and the barrel volume given as in Eq.19.

The temperature of the combustion gases at any time  $t$  during evacuation period becomes

$$T_o(t) = \left[ \frac{1}{T_o(0)^{\frac{1}{2}}} + \frac{1}{2} \frac{B}{V} (\gamma-1) A \sqrt{R_G} t \right]^{-2} \quad (26)$$

The relationship between the pressure and the temperature during the evacuation period is

$$\frac{P_o(t)}{P_o(0)} = \left[ \frac{T_o(t)}{T_o(0)} \right]^{\frac{\gamma}{\gamma-1}} = \left[ 1 + \frac{1}{2} \frac{B}{V} (\gamma-1) A \sqrt{R_G} T_o(0)^{\frac{1}{2}} t \right]^{-\frac{2\gamma}{\gamma-1}} \quad (27)$$

The density of the combustion gases at any time  $t$  during evacuation period can be given as follows

$$\frac{\rho_o(t)}{\rho_o(0)} = \left[ 1 + \frac{1}{2} \frac{B}{V} (\gamma-1) A \sqrt{R_G} T_o(0)^{\frac{1}{2}} t \right]^{-\frac{2}{\gamma-1}} \quad (28)$$

### Efficiency problem for internal ballistics

The two types of efficiencies are described in internal ballistics. One is the piezometric efficiency  $\eta_{piezo} = P_{average}/P_{max}$  reflects the shape of the pressure curve  $P(x)$ . In our case the low piezometric efficiency means that thinner barrel wall thickness, less recoil, lighter and more mobile weapon, low muzzle pressure, less flash and less dispersion. The second one is the ballistic efficiency  $\eta_b = \frac{1}{2} m U^2 / m_B Q_B$  which shows the ratio between the kinetic energy of the projectile at muzzle of the barrel and the total energy stored in the propellant.

### NUMERICAL METHOD OF SOLUTION

In order to solve the set of the differential equations through Eq.1 to Eq.8, fourth order modified Runge Kutta method (Kopchenova et al., 1975) is used in the computer program written in visual Fortran. The original code was developed in 1977 during the development of 155 mm gun barrel, recoil cylinders and recuperator (Akçay, 1981). The total internal ballistic cycle is completed around one mili second for the small caliber guns, so that the time step is set to 0.00000001 second. Boundary conditions are

For internal ballistics;

$$\begin{array}{lll} \text{Initial condition} & t=0 \quad x=0, & P = P_a \\ \text{Final condition} & t=t_1 \quad x=L, & P = P_e \end{array}$$

For transitional ballistics;

$$\begin{array}{lll} \text{Initial condition} & t=t_1, & P = P_e \\ \text{Final condition} & t=t_2, & P = P_a \end{array}$$

Here  $t_1$  is the calculated time,  $P_e$  is the calculated pressure of combustion gases at the time that the projectile leaves the barrel muzzle. These two values are the initial conditions for transitional ballistic calculations. The calculations are over when the gas pressure in the barrel becomes equal to the atmospheric pressure  $P_a$ .

### EXPERIMENTS

The interior and the transitional ballistic tests have been carried out for 7.62x51 mm M80 and 9x19 mm Parabellum cartridges by using the test barrels at MKE internal ballistic test facilities. Length of the barrel in which projectile moves along is 508.8 mm. According to NATO standard,

ammunitions to be used for internal ballistic tests have to be conditioned at 21°C for two hours for normal conditioning. All the ammunitions are conditioned at the specified temperature and duration before the tests. The pressure inside the barrel is measured at two points on the barrel, first one just at the head of cartridge case, second one at 32.7 cm ahead of this point. Piezoelectric probes are used for the measurements of the pressure. The timing and the velocity of projectiles are measured at 23.7 m ahead of the barrel muzzle. The air temperature was around 20°C and air pressure was 0.898 Bars during all tests. At least twenty ammunitions have to be used to statistical evaluation of velocity and pressure test results. Not only the mean values of velocity and pressure are considered during test evaluation, standard deviation, that means homogeneity of the samples tested is also important parameter for internal ballistic tests. With the technical information given about 7.62x51 mm M80 cartridge by Cronemberger et al., the internal ballistic equations are resolved with the current method in order to compare the experimental results given in Cronemberger's study and the results obtained by this method. Although MKE cartridges have spherical propellant grains, the cartridges used in Cronemberger's study has single perforated tubular propellant grains.

## RESULTS AND DISCUSSION

In this section, the results obtained from theoretical calculations and experimental results are presented and compared with the experimental results for both internal and transitional ballistics. During the tests performed at MKE facilities, M80 cartridges including 0.275 g double base propellants having spherical geometrical grain are used. The results of these tests are given in Figs.4-11. There is a time lag between the values of measured and calculated time depending on the position of the projectile along the barrel. This correction is applied to the measured time values. The motion of projectile up to the muzzle of the barrel took 1.112 milliseconds. The maximum projectile base pressure reached a value of 3478 Bars at 0.229 milliseconds and the exit pressure was 550 Bars at 1.057 milliseconds. The results of the prediction showed that the maximum pressure was calculated as 3526 Bars at 0.29 milliseconds and the exit pressure was 551 Bars at 1.109 milliseconds. Theoretical maximum pressure occurred at 1.79 cm ahead of the cartridge case which was in very good agreement with the experimental result shown in Fig.4a. The general agreement with the theoretical and experimental results of the base pressure values along the barrel length is quite good. The second experimental results are given by Cronemberger et.al is also evaluated in this study for the confirmation of the current theoretical approach. M80 cartridge case with 2.6762 g tubular grains is used in their tests. The maximum pressure obtained during their tests was 3260 Bars at 0.250 milliseconds, their experimentally correlated exit pressure was not obtained but the pressure around 700 Bars at 0.9 milliseconds as shown in Fig.4b. The theoretical internal ballistic cycle for Cronemberger test was around 1.213 milliseconds. The results of the prediction show that the maximum pressure was calculated 3197 Bars at 0.24 milliseconds and the exit pressure was 598 Bars at 1.19 milliseconds.

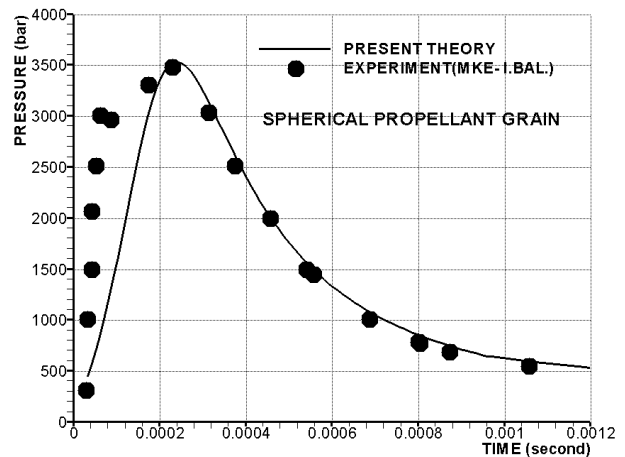


Figure 4a. Pressure time history during internal ballistic phase in MKE tests.

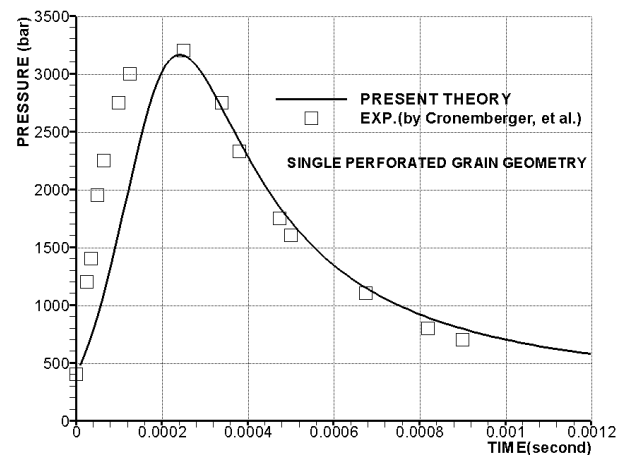


Figure 4b. Pressure time history during internal ballistic phase in Cronemberger's tests.

The percentage of the burned propellant for spherical and tubular grain geometries is shown in Fig.5. These are the calculated values from MKE and Cronemberger's tests. At the end of internal ballistic cycle 79% of the propellant was burned during MKE tests and 87% of the propellant was burned during Cronemberger's tests. The spherical propellant grains are consumed faster than the tubular propellant grains up to the time of 0.8 milliseconds and gives higher maximum pressure value.

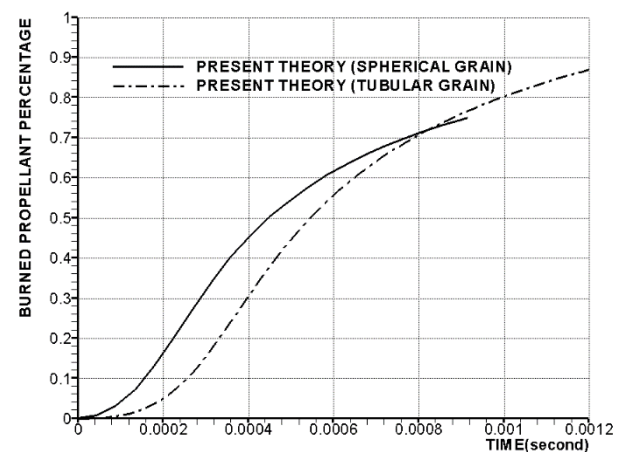
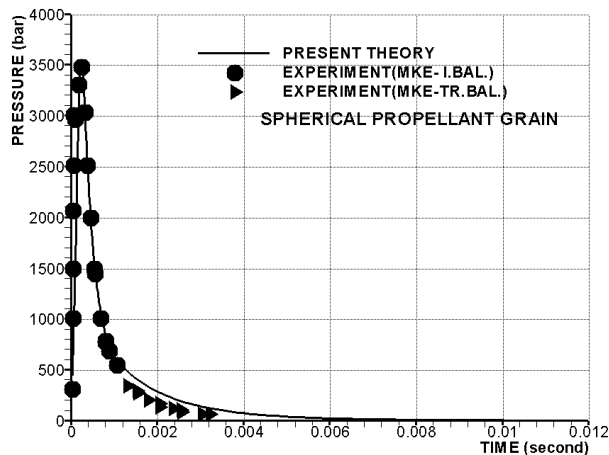
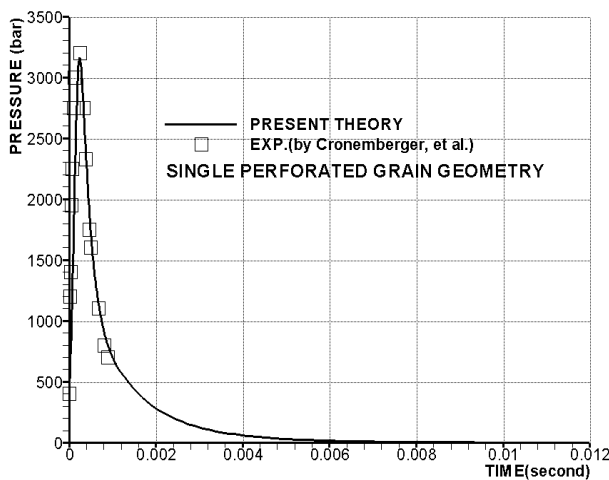


Figure 5. The percentage of the burned propellant by the time during internal ballistic phase.



**Figure 6a.** Pressure time history during internal and transitional ballistic phases in MKE tests.



**Figure 6b.** Pressure time history during internal and transitional ballistic phases in Cronmberger's tests.

One of the most important parameters for gun design is the pressure time function for a considered barrel during the evacuation of combustion gases after the projectile leaves the barrel. It is necessary to be known for design of recoil and recuperator mechanisms. The gas evacuation goes on until the gas pressure in the barrel becomes equal to the atmospheric pressure where the test is carried out. Here, the pressure measurement of the gases was carried out depending on time at MKE facilities. The calculated time during the intermediate ballistic starts at 1.109 milliseconds and lasts up to 14.27278 milliseconds. The duration of intermediate ballistic phase is thirteen times greater than the internal ballistic phase. The experimental measurements cover the pressure values up to 3.22 milliseconds. At this point, the pressure inside the barrel drops from 551 Bars to 69 Bars. The agreement between the theoretically obtained values and the experimental values are quite good for both internal and transitional ballistic phases as shown in Fig.6a. The results of the theoretical and the experimental values of the gas pressure versus time history for Cronemberger's case are shown in Fig.6b.

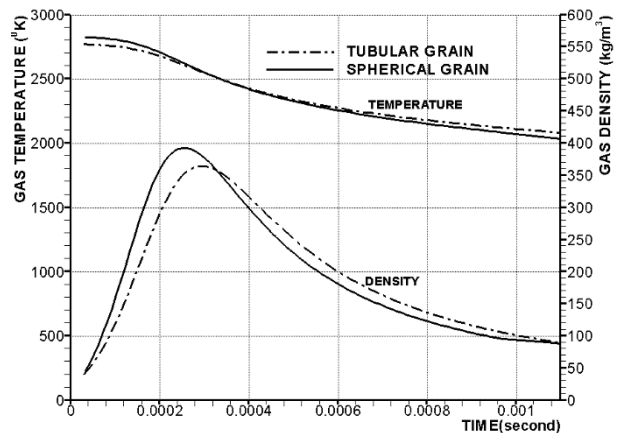
In Cronenberg test's the intermediate ballistic cycle was not examined so that no comparison was possible. The calculated time during the intermediate ballistics for Cronemberger's case starts at 1.213 milliseconds and lasts to 14.40304 milliseconds. The motion of a flying projectile in front of the

barrel in the discharging gases from barrel muzzle during intermediate ballistic phase is shown in Fig.7.

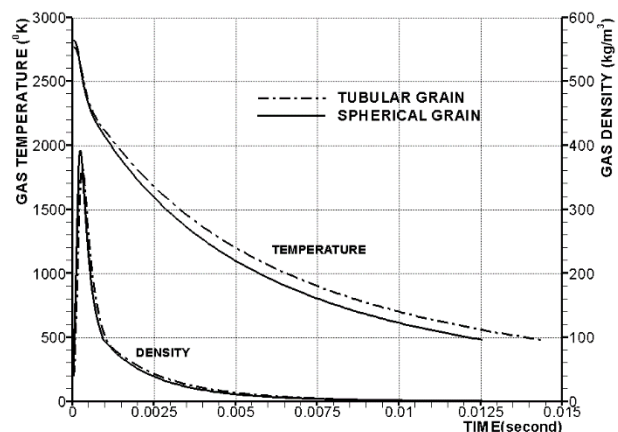


**Figure 7.** The motion of a 7,62 mm M80 bullet during intermediate ballistic phase (courtesy of Mr. Osman Başak).

Another two important parameters for internal and transitional ballistic periods are the combustion gas temperature and the combustion gas density. The results of the theoretical predictions for both of these parameters during the internal ballistic period are seen in Fig.8a. Up to the 0.3 milliseconds, the gas temperature and the gas density for propellant having spherical grain geometry is higher than that of the propellant having tubular grain geometry. At the time greater than 0.3 milliseconds, these two parameters mentioned above have smaller values for spherical grains than that of the tubular grains.



**Figure 8a.** Gas temperature and gas density time history during internal ballistic phases.



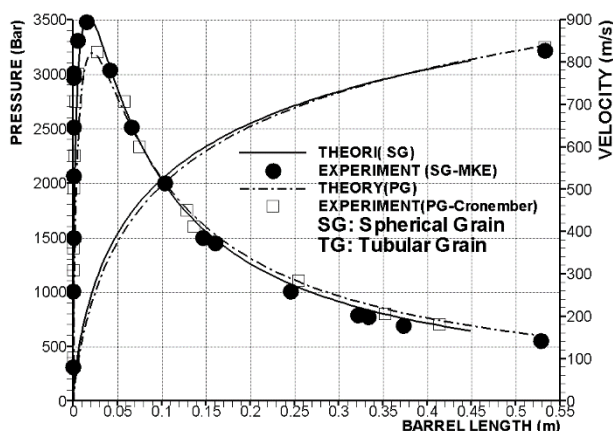
**Figure 8b.** Gas temperature and gas density time history during internal and transitional ballistic phases.

The gas temperature drops from the maximum value of 2822 K to 2050 K and the gas density drops from a maximum value of 392kg/m<sup>3</sup> to 85kg/m<sup>3</sup> at the time when the projectile leaves the muzzle. The change of the gas temperature and the gas density with the time during transitional ballistic period are shown in Fig.8b. The gas temperature in the gun barrel drops

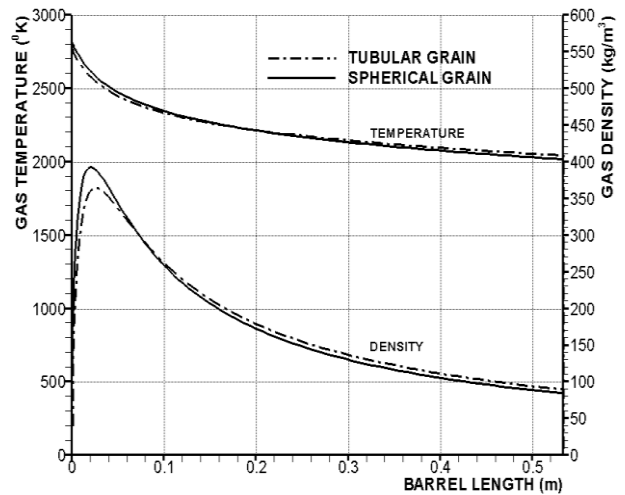
from 2050 K to 477 K, the gas density drops from 85kg/m<sup>3</sup> to 0.66kg/m<sup>3</sup> during transitional ballistic period.

During the design period of a gun, the pressure distribution and the velocity of the projectile along the barrel length have to be known to calculate the barrel thickness and barrel vibration parameters which are of vital importance for accuracy and dispersion of the projectile at the target. Theoretical and experimental pressure distribution and velocity of projectile in the barrel are shown in Fig.9a. The position at which the maximum pressure occurs is very important for the prediction of the barrel thickness in terms of strength of the barrel. The maximum pressure is measured as 3478 Bars and calculated as 3526 Bars at 1.79 cm ahead of the cartridge case for MKE tests although measured as 3260 Bars and calculated as 3197 Bars at 2.20 cm ahead of the cartridge case for Cronenberg's tests. This means that the barrel root must be 0.46 cm longer for the guns in which the tubular grain propellant used. Experimental exit pressure at the barrel muzzle is 550 Bars, theoretical muzzle pressure is 551 Bars for MKE tests, and calculated exit pressure value is 598 Bars and no available measured value for Cronenberg's test results. There is a very good agreement between the predicted and measured pressure values for both MKE and Cronenberg's tests. The velocity of projectile in the barrel increases very rapidly up to 20 cm ahead of cartridge case and starts steadily increasing for the rest of the length of the barrel. The calculated muzzle velocities are 833.7 m/s for MKE tests and 839.4 m/s for Cronenberg' test. The velocities of the projectiles are measured at 23.7 m ahead of the muzzle and obtained as 828 m/s for MKE test and 834 m/s at 7 m ahead of the muzzle for Cronenberg's tests. The data reduction applied to the measured velocity data for the two cases was applied with the External Ballistic Code AKÇAY and the experimental muzzle velocities are corrected as 835 m/s and 839 m/s.

The change of the gas temperature and the gas density along the barrel length are seen in Fig.9b. The maximum gas temperature of 2822 K occurs at the head of the cartridge case and drops to 2050 K, the maximum gas density of 393 kg/m<sup>3</sup> occurs at 0.0205 m ahead of the grooving drops to 83.6 kg/m<sup>3</sup> at the muzzle of the barrel.



**Figure 9a.** Theoretical and experimental pressure velocity distribution along the barrel.

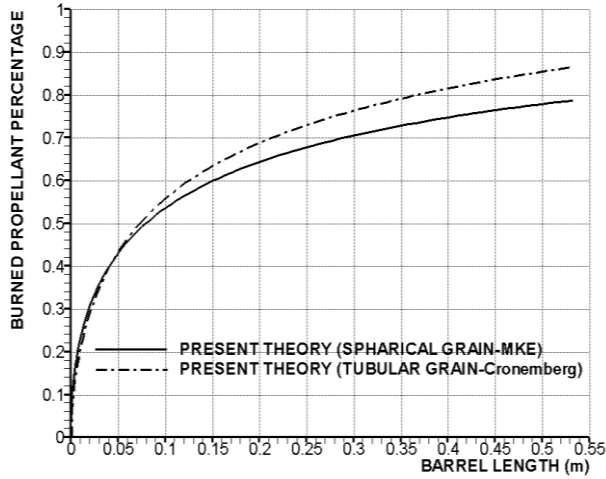


**Figure 9b.** Gas temperature and density distributions along the barrel.

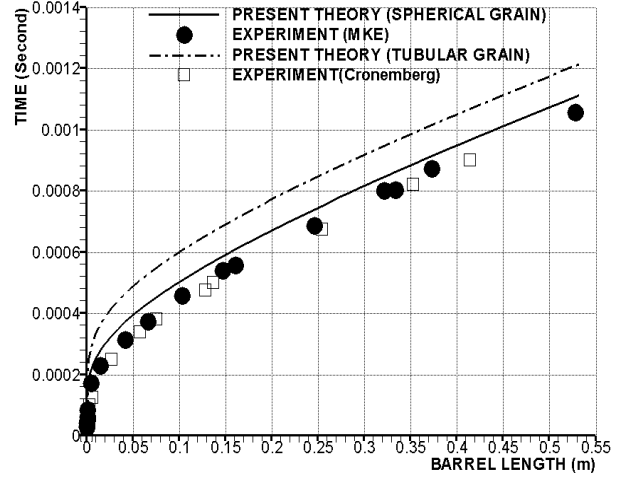
The propellant burning process for both spherical and tubular propellant grains is shown in Fig.10. The combustion of the propellant grains is steadily increasing along the barrel length and the consumption of the tubular propellant grain is 8.2% higher than the consumption of spherical propellant grain. Tests are conducted at MKE facilities to measure the unburned propellants of 9x19 mm Parabellum cartridges at three different propellant temperatures as shown in Table 1. To measure the unburned propellant, 60 shots for 21 °C, 105 shots for 52 °C and 105 shots for -54 °C are fired. The theoretical predictions were carried out with the Code of Internal Ballistics AKÇAY which is capable of considering the propellant temperature effects during the burning process. The experimental unburned propellant mass values are less than the predicted values, the mass difference between these values is due to the combustion process going on in the barrel and outside the barrel during the transitional period and seen as muzzle flashes. The theoretical predictions show that unburned propellant mass is greater for spherical grains than the tubular grains. No experimental results have been available yet for 7.62x51 mm cartridges.

The internal ballistic efficiencies of 7.62x51 mm M80 cartridges with two different grain geometries with test barrel are seen in Table 2. The theoretical and experimental ballistic efficiencies were about 30% and agree well with each other with a difference of 0.5-0.7% difference. The evaluated mean values of experimental base pressure are used to make a prediction for experimental piezometric efficiency. The theoretical and experimental piezometric efficiencies were around 37% for MKE tests and 40.5% for Cronenberg's tests. This means that the piezometric efficiency of tubular propellant is 3% higher than that of the spherical propellant. The comparison between the theoretical and experimental results of the projectile position in the barrel shown in Fig.11 shows that there is a discrepancy between these two values in the Cronenberg's studies.





**Figure 10.** The percentage of the burned propellant along the barrel length.



**Figure 11.** The projectile position by the time along the barrel length.

**Table 1** Unburned propellants for 7.62x51 mm M80 and 9x19 mm Parabellum Cartridges.

CARTRIDGE	GRAIN GEOMETRY	INITIAL PROPELLANT MASS g	TEMPERATURE C	NUMBER OF SHOT	UNBURNED PROPELLANT			
					THEORY		EXPERIMENT	
					MASS g	%	MASS g	%
7.62x51 mm M80	SPHERICAL	0.275	+21	1	0.057	20.88	-	-
	TUBULAR	0.268	+21	1	0.034	12.64	-	-
9x19 mm PARABELLUM	SPHERICAL	0.43	+21	60	4.98	19.30	3.6	14
			+52	105	8.80	19.50	2.9	6.4
			-54	105	8.49	18.80	5.8	12.8

**Table 2** Piezometric and ballistic efficiencies of 7.62x51 mm Cartridges with experimental barrel.

REFERENCE	$P_{MAX}$ (bar)		$P_{MEAN}$ (bar)		$V_O$ (m/s)		$\eta_{piezo}$		$\eta_{bal}$	
	TEORI	EXP.	TEORI	EXP.	TEORI	EXP.	TEORI	EXP.	TEORI	EXP.
MKE-AKÇAY	3526	3487	1326	1322	833.7	835	%36	%37.9	%29	%29.5
CRONEMBERGER	3197	3240	1338	1325	839.4	839	%40	%40.9	%31	%30.3

## CONCLUSION

The interior ballistic prediction method based on Résal equation is presented in this study. The geometrical structure of the propellant grains is considered as spherical and multi perforated. The method is also extended to cover the intermediate ballistic period of the considered propellant and gun combination. Intermediate ballistic period is very important for gun design. The calculations are carried out for 7.62x51 mm cartridge and 9x19 mm Parabellum cartridge with spherical and tubular grained propellants for both internal and transitional ballistic periods.

The interior ballistic tests of 7.62x51 mm M80 Cartridge and the unburned propellant tests for 9x19 mm Parabellum Cartridge with test barrels are also carried out at MKE Ballistic Test Facilities. The experimentally obtained results and the results in the literature are in good agreement with the theoretical results namely, the maximum pressure, pressure time history along the barrel, muzzle velocity, projectile exit time, percentage of unburned propellant, ballistic and piezometric efficiencies.

## ACKNOWLEDGMENT

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