



## ADOMIAN DECOMPOSITION METHOD FOR HEAT CONDUCTION IN AN ANNULAR FIN OF HYPERBOLIC PROFILE WITH TEMPERATURE DEPENDENT THERMAL CONDUCTIVITY

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**Abstract:** The Adomian decomposition method (ADM) is applied to analyze the thermal performance of an annular fin of hyperbolic profile with temperature dependent thermal conductivity. ADM provides the solution in an infinite series with easily computable components. It has been observed that the variation of thermo-geometric fin parameter and the thermal conductivity parameter have a significant effect on the temperature distribution in the fin and its efficiency. For engineering analysis and design, a regression equation is also proposed for fin efficiency as a function of fin radii ratio and thermo-geometric fin parameter. Obtained results are in a good agreement with numerical results of finite difference method (FDM) and exact solution with the constant thermal conductivity.

**Keywords:** Hyperbolic profile annular fin, Fin efficiency, Variable conductivity, Adomian decomposition method.

## ADOMIAN AYRIŞTIRMA METODU İLE ISIL İLETKENLİĞİ SICAKLIKLA DEĞİŞEN HİPERBOLİK PROFİLLİ DAİRESEL KANATTA ISI İLETİMİ

**Özet:** Bu çalışmada, Adomian ayrıştırma metodu (ADM) ile ısı iletkenliği sıcaklıkla değişen hiperbolik profilli dairesel bir kanadın ısı performansı incelenmiştir. ADM terimleri kolayca hesaplanabilen sonsuz seri formunda bir çözüm sunar. Yapılan analizde, termo-geometrik kanat parametresi ve ısı iletkenlik parametresi değişimlerinin kanat boyunca sıcaklık dağılımına ve kanat verimine önemli bir etkisi olduğu görülmüştür. Ayrıca, kanat verimi için mühendislik analizi ve tasarımında kullanılmak amacı ile kanat yarıçap oranı ve termo-geometrik kanat parametresinin fonksiyonu olan bir korelasyon denklemi elde edilmiştir. Elde edilen sonuçların sonlu farklar yönteminden ve ısı iletkenliği sabit durum için analitik çözümden elde edilen sayısal sonuçlarla uyum içinde olduğu görülmüştür.

**Anahtar Kelimeler:** Dairesel kanat, Kanat verimi, Değişken ısı iletkenlik, Adomian ayrıştırma yöntemi.

### NOMENCLATURE

$A_c$	cross-sectional area of the fin [ $m^2$ ]
$A_k$	Adomian's polynomials
$dA_s$	elemental surface area [ $m^2$ ]
$h$	heat transfer coefficient [ $W/(m^2K)$ ]
$k$	thermal conductivity of the fin material [ $W/(mK)$ ]
$k_a$	thermal conductivity at the ambient fluid temperature [ $W/(mK)$ ]
$L$	the higher order derivative
$L^{-1}$	inverse operator of $L$
$N$	nonlinear operator
$r$	radial coordinate [m]
$r_1$	inner radius of the annular fin [m]
$r_2$	outer radius of the annular fin [m]
$T$	temperature [K]
$FDM$	Finite difference method
$ADM$	Adomian decomposition method
$Q$	heat transfer rate [W]

### Greek symbols

$\alpha$	integral constant
$\beta$	dimensionless thermal conductivity parameter [ $=\lambda(T_b-T_a)$ ]
$\eta$	fin efficiency
$\xi$	dimensionless coordinate [ $=(r-r_1)/r_1$ ]
$\lambda$	slope of the thermal conductivity-temperature curve [1/K]
$\psi$	thermo-geometric fin parameter [ $=2hr_1^2/k_a\delta_1$ ] <sup>1/2</sup>
$\theta$	dimensionless temperature [ $=(T-T_a)/(T_b-T_a)$ ]
$\delta_1$	fin base thickness [m]
$\delta_2$	fin tip thickness [m]
$\gamma$	the radii ratio [ $=r_2/r_1$ ]

### Subscripts

$s$	surface
$a$	ambient fluid
$b$	base
$f$	fin

## INTRODUCTION

Fins are extended surfaces that are used to enhance heat transfer between a primary surface and the surrounding fluid. Finned surfaces are widely seen in liquid-gas heat exchangers, air-cooled internal combustion engines, the cooling of electronic equipment and many other engineering applications. For this reason, numerous studies have been performed on different fin configuration with constant and variable thermal properties. Kraus et al. (2001) provided a very detailed review on this subject devoted to various aspects of extended surface convective heat transfer. Mokheimer (2002) investigated the performance of annular fins with different profiles subject to variable heat transfer coefficient. The performance of the fin expressed in terms of fin efficiency as a function of the ambient and fin geometry parameters. Zubair et al. (1996) presented the optimal dimensions of convective-radiating circular fins. A correlation for the optimal dimensions of a constant and variable profile fins is presented in terms of reduced heat-transfer rate. Arauzo et al. (2005) investigated the heat transfer characteristics of annular fins of hyperbolic profile with the power series method. They applied an elementary analytic procedure for the approximate solution of the quasi-one-dimensional heat conduction equation (a generalized Bessel equation) that governs the temperature variation in annular fins of hyperbolic profile. Campo and Cui (2008) examined temperature and heat analysis of annular fins of hyperbolic profile relying on the simple theory for straight fins of uniform profile. Their technical brief addresses an elementary analytic procedure for solving approximately the quasi-1D heat conduction equation (a generalized Airy equation) governing the annular fin of hyperbolic profile. Razelos and Imre (1980) obtained the optimum dimensions of circular fins with a profile of constant slope, including the effects of a linear variation of the thermal conductivity and a heat transfer coefficient that is assumed to vary according to a power law with distance from the bore.

An extensive literature is seen on the thermal analysis of fins with variable thermal conductivity. Chiu and Chen (2002) used a decomposition method for solving the convective longitudinal fins with variable thermal conductivity. In their paper the Adomian decomposition method is used to evaluate the efficiency and the optimum length of a convective rectangular fin with variable thermal conductivity and to determine the temperature distribution within the fin. Arslanturk (2005) made an analysis for the efficiency of convective straight fins with temperature-dependent thermal conductivity by using the Adomian decomposition method. The fin efficiency of the straight fins has been obtained as a function of thermo-geometric fin parameter and the thermal conductivity parameter. Yu and Chen (1998) applied the differential transformation method to optimize the rectangular fin with variable thermal parameters. Chiu and Chen (2002) used decomposition method for the thermal stresses in

isotropic circular fins with temperature-dependent thermal conductivity. Arslanturk (2009) obtained correlation equations for optimum design of annular fins with temperature dependent thermal conductivity. Nonlinear fin equation is solved by Adomian decomposition method. Chang (2005) applied Adomian decomposition method to investigate a straight fin governed by a power-law-type temperature dependent heat transfer coefficient. Yang et al. (2010) used a double decomposition method for solving the annular hyperbolic profile fins with variable thermal conductivity. The double decomposition method uses the same operator as Adomian decomposition method, but decomposes the first undefined parameters. They compare the results with the exact solution in the case of constant thermal conductivity.

In this study, Adomian decomposition method is applied to determine the temperature distribution within the annular fin of hyperbolic profile with temperature dependent thermal conductivity. The effects of thermo-geometric fin parameter and the thermal conductivity parameter variations on the temperature distribution are also investigated. In addition, fin efficiency obtained from temperature distribution within the fin is determined. Results from ADM are compared with numerical results of finite difference method (FDM) and exact solution with the constant thermal conductivity. Adomian decomposition method provides an easy and direct way to find an analytic solution without any linearization in the form of an infinite series.

## PROBLEM DESCRIPTION

For the radial fin of hyperbolic profile displayed with its terminology and coordinate system in Fig. 1. A generalized differential equation and boundary conditions for radial fin of hyperbolic profile function can be written as following forms,

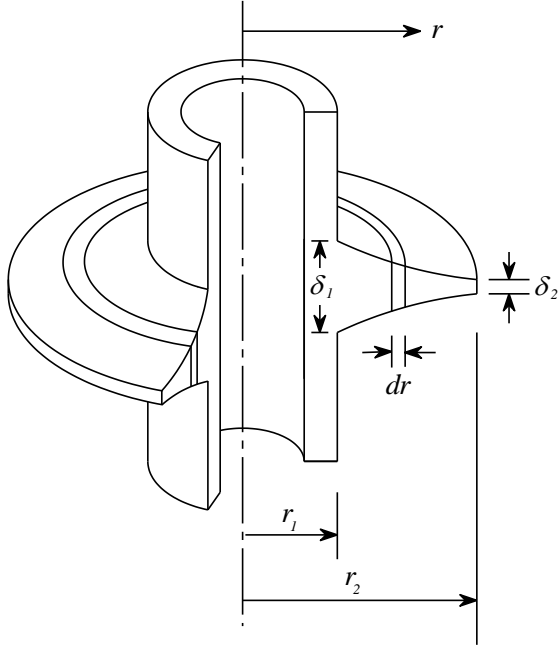
$$\frac{d}{dr} \left( k A_c \frac{dT}{dr} \right) - \frac{h dA_s}{dr} (T - T_a) = 0 \quad (1)$$

$$T = T_b \quad \text{at } r = r_1 \quad (2)$$

$$\frac{dT}{dr} = 0 \quad \text{at } r = r_2 \quad (3)$$

where,  $k$  is the thermal conductivity,  $A_c$  is the cross-sectional area,  $h$  is the heat transfer coefficient,  $dA_s$  is the elemental surface area,  $T_a$  is the ambient temperature. The profile function for this fin is considered as  $f = C/r$ , where  $C$  is a constant. Moreover,  $f(r_1) = \delta_1/2$  and the constant becomes  $C = \delta_1 r_1 / 2$ . Cross-sectional area and elemental surface area of the fin will be  $A_c = 4\pi C$  and

$dA_s = 4\pi r ds$ . Here, the surface element length  $ds$  approximates to the incremental radius  $dr$ .



**Figure 1.** Annular fin of hyperbolic profile.

Thermal conductivity of the fin is assumed to be a linear function of temperature according to

$$k(T) = k_a(1 + \lambda(T - T_a)) \quad (4)$$

where,  $k_a$  is the thermal conductivity of the fin at ambient temperature  $T_a$ ,  $\lambda$  is the parameter describing the variation of thermal conductivity.

Substituting the following dimensionless variables,

$$\theta = \frac{T - T_a}{T_b - T_a}, \quad \xi = \frac{r - r_1}{r_1}, \quad \beta = \lambda(T_b - T_a) \quad (5)$$

$$\gamma = \frac{r_2}{r_1}, \quad \psi = \left( \frac{2hr_1^2}{k_a \delta_1} \right)^{1/2}$$

the governing differential equation and its boundary conditions reduces to

$$\frac{d^2\theta}{d\xi^2} + \beta\theta \frac{d^2\theta}{d\xi^2} + \beta \left( \frac{d\theta}{d\xi} \right)^2 - \psi^2(1 + \xi)\theta = 0 \quad (6)$$

$$\theta = 1 \text{ at } \xi = 0 \quad (7)$$

$$\frac{d\theta}{dr} = 0 \text{ at } \xi = \gamma - 1 \quad (8)$$

## THE ADOMIAN DECOMPOSITION METHOD

The Adomian decomposition method (Adomian, 1994) has been applied to obtain analytical solutions in terms of convergent power series to a wide of problems involving algebraic, differential, integro-differential, and partial differential equations. The convergence of the decomposition series has been investigated by several researchers (Cherruault, 1989; Cherruault and Adomian, 1993; Abbaoui and Cherruault, 1994).

Consider the differential equation in an operator form:

$$Lu + Ru + Nu = g \quad (9)$$

where  $L$  is the highest-order derivative which is assumed to be invertible,  $R$  is a linear differential operator of order less than  $L$ ,  $Nu$  represents the nonlinear terms, and  $g$  is the source term. Because  $L$  is invertible, applying the inverse operator  $L^{-1}$  to both sides of Eq. (9) and using the given conditions, we obtain

$$u = \phi + L^{-1}(g) - L^{-1}(Ru) - L^{-1}(Nu) \quad (10)$$

where  $\phi$  satisfies the  $L\phi = 0$ . The constant of integrations can be found from the given boundary or initial conditions.

The nonlinear operator  $Nu = F(u)$  is usually represented by an infinite series,

$$F(u) = \sum_{k=0}^{\infty} A_k \quad (11)$$

where  $A_k$  are special polynomials obtained for the particular nonlinearity. The  $A_k$  can be found from the formula,

$$A_k(u_0, u_1, \dots, u_k) = \frac{1}{k!} \frac{d^k}{d\lambda^k} \left[ F \left[ \sum_{i=0}^{\infty} \lambda^i u_i \right] \right]_{\lambda=0} \quad (12)$$

Adomian's decomposition method defines the solution  $u(x)$  by the infinite series

$$u = \sum_{k=0}^{\infty} u_k \quad (13)$$

Substituting the Eq. (11) and Eq. (13) in Eq. (10) results

$$\sum_{k=0}^{\infty} u_k = \phi + L^{-1}g - L^{-1}(Ru) - L^{-1} \sum_{k=0}^{\infty} A_k \quad (14)$$

here the components  $u_0, u_1, u_2, \dots$  are usually determined recursively by:

$$\begin{aligned} u_0 &= \phi + L^{-1}g, \\ u_{k+1} &= -L^{-1}(Ru_k) - L^{-1}(A_k), \quad k \geq 0 \end{aligned} \quad (15)$$

where Adomian's polynomials from Eq. (12) gives

$$\begin{aligned} A_0 &= F(u_0) \\ A_1 &= u_1 F'(u_0) \\ A_2 &= u_2 F'(u_0) + \left( \frac{u_1^2}{2!} \right) F''(u_0) \\ A_3 &= u_3 F'(u_0) + u_1 u_2 F''(u_0) + \left( \frac{u_1^3}{3!} \right) F'''(u_0) \\ A_4 &= u_4 F'(u_0) + \left( \frac{1}{2!} u_2^2 + u_1 u_3 \right) F''(u_0) \\ &\quad + \frac{1}{2!} u_1^2 u_2 F'''(u_0) + \frac{1}{4!} F^{(iv)}(u_0) \\ &\dots \\ &\dots \end{aligned} \quad (16)$$

The solution for the n-term approximation is defined as

$$\theta_n = \sum_{k=0}^{n-1} u_k$$

## THE FIN TEMPERATURE DISTRIBUTION

By defining the differential operator  $L = d^2 / d\xi^2$ , Eq. (6) can be written as

$$\begin{aligned} L\theta &= -\beta\theta \frac{d^2\theta}{d\xi^2} - \beta \left( \frac{d\theta}{d\xi} \right)^2 + \psi^2(1+\xi)\theta \\ &= -\beta NA - \beta NB + \psi^2(1+\xi)\theta \end{aligned} \quad (17)$$

where  $NA$  and  $NB$  are nonlinear terms defined as,

$$\theta = \sum_{k=0}^{\infty} \theta_k \quad (18a)$$

$$NA = \theta \frac{d^2\theta}{d\xi^2} = \sum_{k=0}^{\infty} A_k \quad (18b)$$

$$NB = \left( \frac{d\theta}{d\xi} \right)^2 = \sum_{k=0}^{\infty} B_k \quad (18c)$$

Adomian's polynomials can be written by using Eq. (12) or Eqs. (16a-16d) as

$$\begin{aligned} A_0 &= \theta_0 \frac{d^2\theta_0}{d\xi^2} \\ A_1 &= \theta_1 \frac{d^2\theta_0}{d\xi^2} + \theta_0 \frac{d^2\theta_1}{d\xi^2} \\ A_2 &= \theta_2 \frac{d^2\theta_0}{d\xi^2} + \theta_1 \frac{d^2\theta_1}{d\xi^2} + \theta_0 \frac{d^2\theta_2}{d\xi^2} \\ A_3 &= \theta_3 \frac{d^2\theta_0}{d\xi^2} + \theta_2 \frac{d^2\theta_1}{d\xi^2} + \theta_1 \frac{d^2\theta_2}{d\xi^2} + \theta_0 \frac{d^2\theta_3}{d\xi^2} \\ A_4 &= \theta_4 \frac{d^2\theta_0}{d\xi^2} + \theta_3 \frac{d^2\theta_1}{d\xi^2} + \theta_2 \frac{d^2\theta_2}{d\xi^2} + \theta_1 \frac{d^2\theta_3}{d\xi^2} \\ &\quad + \theta_0 \frac{d^2\theta_4}{d\xi^2} \\ &\dots \\ &\dots \end{aligned} \quad (19)$$

and

$$\begin{aligned} B_0 &= \left( \frac{d\theta_0}{d\xi} \right)^2 \\ B_1 &= 2 \frac{d\theta_0}{d\xi} \frac{d\theta_1}{d\xi} \\ B_2 &= \left( \frac{d\theta_1}{d\xi} \right)^2 + 2 \frac{d\theta_0}{d\xi} \frac{d\theta_2}{d\xi} \\ B_3 &= 2 \frac{d\theta_1}{d\xi} \frac{d\theta_2}{d\xi} + 2 \frac{d\theta_0}{d\xi} \frac{d\theta_3}{d\xi} \\ B_4 &= \left( \frac{d\theta_2}{d\xi} \right)^2 + 2 \frac{d\theta_1}{d\xi} \frac{d\theta_3}{d\xi} + 2 \frac{d\theta_0}{d\xi} \frac{d\theta_4}{d\xi} \\ &\dots \\ &\dots \end{aligned} \quad (20)$$

Application of the inverse operator  $L^{-1}$  on both sides of Eq. (17), we get

$$L^{-1}L\theta = -\beta L^{-1}NA - \beta L^{-1}NB + \psi^2 L^{-1}[(1+\xi)\theta] \quad (21)$$

The inverse operator  $L^{-1}$  can be taken as two-fold definite integral defined as

$$L^{-1} = \int_0^\xi \int_0^\xi \cdot d\xi d\xi \quad (22)$$

$$L^{-1}L\theta = \theta - \theta(0) - \xi \frac{d\theta(0)}{d\xi} \quad (23)$$

and we define

$$\theta_0 = \theta(0) + \xi \frac{d\theta(0)}{d\xi} \quad (24)$$

Then

$$L^{-1}L = \theta - \theta_0, \quad (25)$$

Hence, Eq. (21) can be rewritten as

$$\theta = \theta_0 - \beta L^{-1}NA - \beta L^{-1}NB + \psi^2 L^{-1}[(I + \xi)\theta] \quad (26)$$

The term  $d\theta(0)/d\xi$  in Eq. (24) represents the temperature gradient at the fin base and can be evaluated from the boundary condition given in Eq. (8). If we equalize that  $d\theta(0)/d\xi = \alpha$ , Eq. (24) can be rewritten as

$$\theta_0 = I + \alpha\xi \quad (27)$$

The next iterates are determined recursively as:

$$\theta_{k+1} = -\beta L^{-1}A_k - \beta L^{-1}B_k + \psi^2 L^{-1}[(I + \xi)\theta_k] \quad (28)$$

First three iterations are expressed as:

$$\begin{aligned} \theta_1 = & \frac{1}{2}(-\alpha^2\beta + \psi^2)\xi^2 + \frac{1}{6}(\alpha + 1)\psi^2\xi^3 \\ & + \frac{1}{12}\alpha\psi^2\xi^4 \end{aligned} \quad (29a)$$

$$\begin{aligned} \theta_2 = & \frac{1}{2}\beta(\alpha^2\beta + \psi^2)\xi^2 + \frac{1}{6}\beta(3\alpha^3\beta - 4\alpha\psi^2 \\ & - \psi^2)\xi^3 + \frac{1}{24}(-5\alpha^2\beta - 6\alpha\beta + \psi^2)\psi^2\xi^4 \\ & + \frac{1}{120}(-13\alpha^2\beta + \alpha\psi^2 + 4\psi^2)\psi^2\xi^5 \\ & + \frac{1}{360}(3\alpha + 2)\psi^4\xi^6 + \frac{1}{504}\alpha\psi^4\xi^7 \end{aligned} \quad (29b)$$

$$\begin{aligned} \theta_3 = & \frac{1}{2}\beta^2(-\alpha^2\beta + \psi^2)\xi^2 + \frac{1}{6}\beta^2(-6\alpha^3\beta + \psi^2 \\ & + 7\alpha\psi^2)\xi^3 + \frac{1}{24}\beta(-15\alpha^4\beta^2 + 28\alpha^2\beta\psi^2 \\ & + 10\alpha\beta\psi^2 - 5\psi^4)\xi^4 + \frac{1}{60}\beta(19\alpha^3\beta \\ & + 28\alpha^2\beta - 10\alpha\psi^2 - 9\psi^2)\psi^2\xi^5 + \frac{1}{720} \\ & (120\alpha^3\beta^2 - 21\alpha^2\beta\psi^2 - 102\alpha\beta\psi^2 \\ & - 18\beta\psi^2 + \psi^4)\psi^2\xi^6 + \frac{1}{5040}(-150\alpha^2\beta \\ & - 138\alpha\beta + \alpha\psi^2 + 9\psi^2)\psi^4\xi^7 + \frac{1}{20160} \\ & (-149\alpha^2\beta + 6\alpha\psi^2 + 14\psi^2)\psi^4\xi^8 \\ & + \frac{1}{90720}(13\alpha + 7)\psi^6\xi^9 + \frac{1}{45360}\alpha\psi^6\xi^{10} \end{aligned} \quad (29c)$$

By summing those iterations, the n-term approximation is defined as

$$\Phi_{n+1} = \sum_{k=0}^n \theta_k = \theta_0 + \theta_1 + \theta_2 + \dots + \theta_n \quad (30)$$

## FIN EFFICIENCY

The heat transfer rate can be written by applying Newton's law of cooling,

$$Q_f = \int_s h(T - T_a) dA_s \quad (31)$$

The fin efficiency is defined as the ratio of actual heat transfer rate to the maximum possible heat transfer rate which would be achieved if the entire fin were at the base temperature,  $T_b$ . Then, fin efficiency in terms of dimensionless parameters in Eq. (5) can be written as

$$\eta = \frac{Q_f}{Q_{max}} = \frac{4\pi h \int_{r_1}^{r_2} (T - T_a) r dr}{2\pi h (r_2^2 - r_1^2) (T_b - T_a)} = \frac{2 \int_0^{\gamma-1} (I + \xi) \alpha d\xi}{\gamma^2 - 1} \quad (32)$$

## RESULTS AND DISCUSSION

The dimensionless temperature distribution along the fin was calculated by taking the fifteen terms from the series solution. In order to compare the accuracy of the results, problem is also solved numerically by using the MATLAB bvp4c finite difference code. The function bvp4c solves two-point boundary value problems for ordinary differential equations. Obtained results from the numerical and ADM solution are given in Table 1. A comparison is also made with the analytical solution in the case of constant thermal conductivity ( $\beta = 0$ ) in the same table. A very good agreement was obtained which represents the validity of the ADM. ADM results are identical with the numerical and analytical results for constant thermal conductivity.

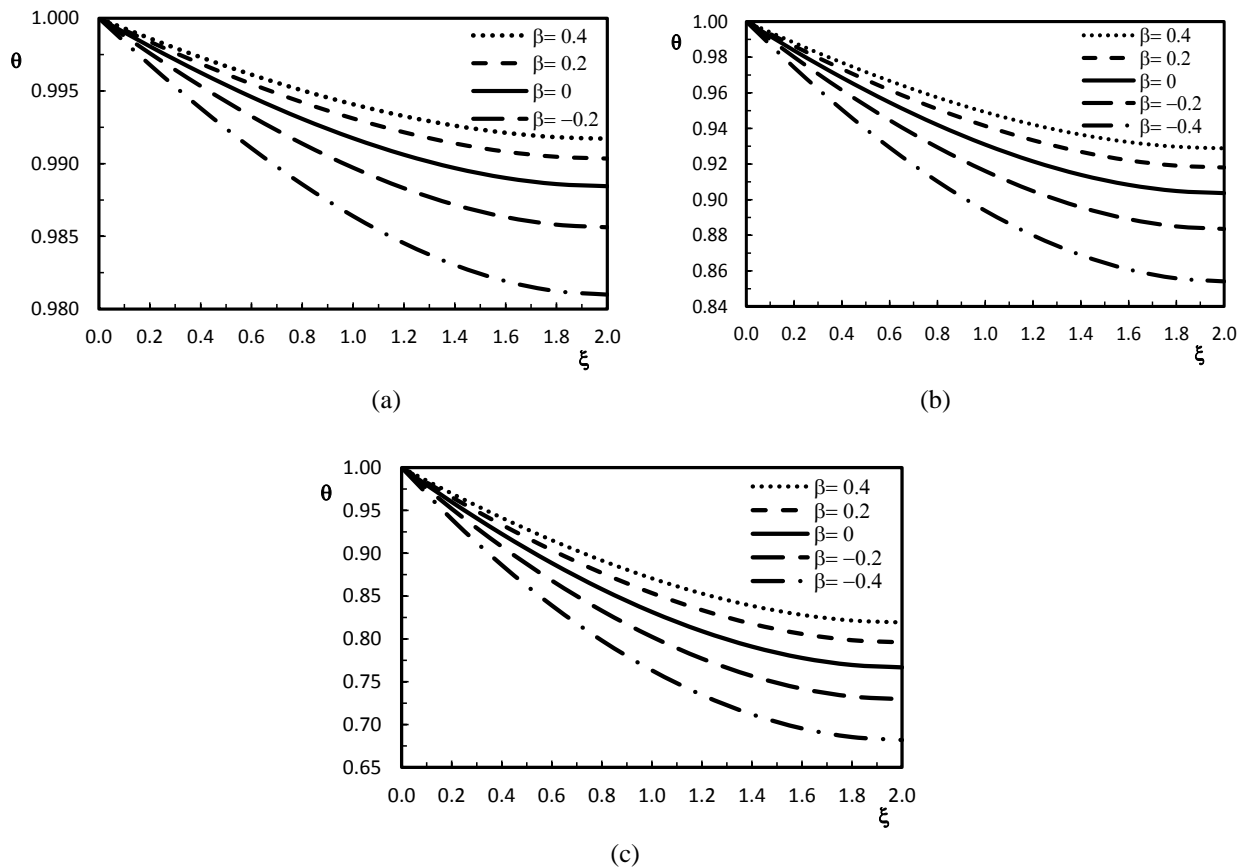
Dimensionless temperature distribution along the fin with  $\beta$  varying from -0.4 to 0.4 are displayed in Fig. 2, (a)-(c) for the case of  $\gamma = 3$  and different values of thermo-geometric fin parameter  $\psi = 0.05, 0.15$  and  $0.25$ , respectively. It will be observed that dimensionless temperature gradient along the fin decreases monotonically from the base to the tip. If the thermal conductivity of the fin's material increases with the temperature ( $\beta > 0$ ), the dimensionless temperature gradient along the fin increases. Oppositely, if the thermal conductivity decreases with temperature ( $\beta < 0$ ), the temperature distribution decreases. As a result, this is a consequence of the nonlinearity due to temperature dependent thermal conductivity. From figures, it can also mentioned that dimensionless

temperature gradient along the fin increases with increasing the thermo-geometric fin parameter. Increasing thermo-geometric fin parameter value means

that thermal conductivity of the fin decreases, hence internal thermal resistance of the fin increases due to conduction.

**Table 1.** Comparison of the results of FDM, ADM and exact solutions for  $\theta(\xi)$  (in the case of  $\psi=0.2$  and  $\gamma=4$ ).

$\xi$	$\beta = -0.3$		$\beta = 0$			$\beta = 0.3$	
	FDM	ADM	FDM	ADM	Exact	FDM	ADM
0.0	1.000000	1.000000	1.000000	1.000000	1.000000	1.000000	1.000000
0.2	0.944655	0.944653	0.957412	0.957412	0.957412	0.965157	0.965156
0.4	0.893036	0.893034	0.916661	0.916661	0.916661	0.931477	0.931475
0.6	0.845162	0.845159	0.877961	0.877961	0.877961	0.899170	0.899168
0.8	0.801036	0.801033	0.841507	0.841507	0.841507	0.868441	0.868439
1.0	0.760652	0.760648	0.807476	0.807476	0.807476	0.839482	0.839480
1.2	0.723997	0.723994	0.776027	0.776027	0.776027	0.812481	0.812478
1.4	0.691063	0.691059	0.747309	0.747309	0.747309	0.787616	0.787613
1.6	0.661842	0.661839	0.721460	0.721461	0.721461	0.765060	0.765057
1.8	0.636338	0.636335	0.698613	0.698613	0.698613	0.744981	0.744978
2.0	0.614565	0.614563	0.678895	0.678895	0.678895	0.727544	0.727540
2.2	0.596557	0.596555	0.662436	0.662436	0.662436	0.712909	0.712905
2.4	0.582366	0.582365	0.649370	0.649370	0.649370	0.701238	0.701235
2.6	0.572070	0.572070	0.639837	0.639837	0.639837	0.692694	0.692690
2.8	0.565780	0.565780	0.633990	0.633990	0.633990	0.687442	0.687438
3.0	0.563644	0.563644	0.632001	0.632001	0.632001	0.685652	0.685648



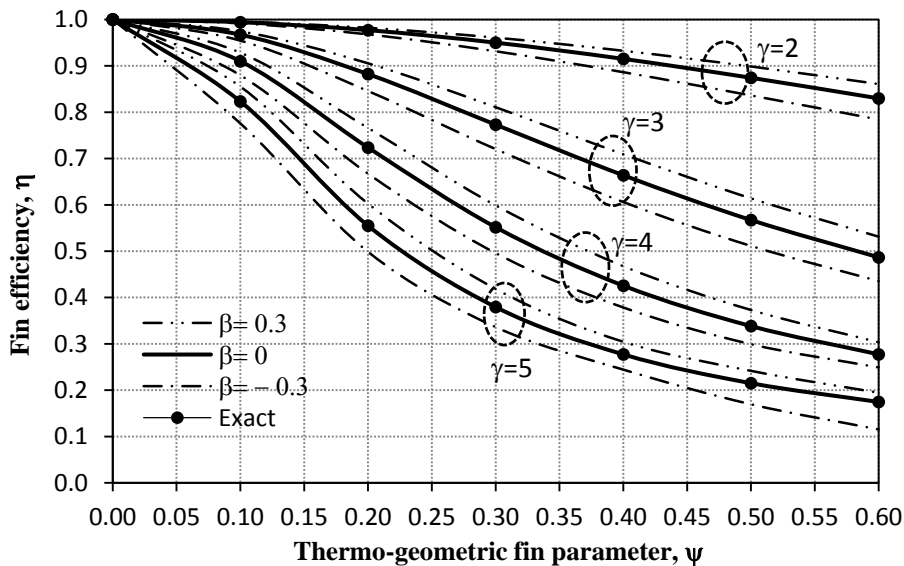
**Figure 2.** Dimensionless temperature distribution along the fin for  $\gamma=3$  and different values of  $\beta$  (a)  $\psi=0.05$ , (b)  $\psi=0.15$ , (c)  $\psi=0.25$

Variation of fin efficiency as a function of thermo-geometric fin parameter for different values of thermal conductivity parameter and radii ratios is given in Fig. 3. It can be observed that, fin efficiency increases with decreasing thermo-geometric fin parameter value. It can be found that fin efficiency increases with increasing radii ratio and thermal conductivity parameter for a given thermo-geometric fin parameter. From Fig.3, it can be seen that the results of exact solution match with the Adomian decomposition solution for the case of constant thermal conductivity,  $\beta = 0$ .

A non-linear regression equation for fin efficiency as a function of radii ratio ( $\gamma$ ) and thermo-geometric fin parameter ( $\psi$ ) is proposed in Eq. (33). The variable parameters in the regression equation are taken as in the range of  $0.1 \leq \psi \leq 0.6$  and  $2 \leq \gamma \leq 5$ .

$$\eta = \frac{a + b\gamma + c \exp(\psi)}{1 + d\gamma + f \exp(\gamma\psi)} \quad (33)$$

Coefficients in the regression equation are given in Table 2 for different values of thermal conductivity parameter ( $\beta$ ). The R-square value which is an indicator of how well the regression equation fits the data is also given for each specified case in the Table 2. As seen as, R-square values for each specified case are close to 1, hence the proposed regression equation can accurately be applied to calculate the fin efficiency for a given radii ratio and thermo-geometric fin parameter under specified cases given in Table 2.



**Figure 3.** Variation of fin efficiency with the thermo-geometric fin parameter ( $\psi$ ) for different values of thermal conductivity parameter ( $\beta$ ) and radii ratios ( $\gamma$ ).

**Table 2.** Coefficients in Eq. (33) for the fin efficiency.

$\beta$	a	b	c	d	f	R <sup>2</sup>
-0.3	0.098035	-0.239989	1.811962	-0.296460	0.993969	0.999
0	0.312944	-0.188082	1.258718	-0.226359	0.640906	0.998
0.3	0.503211	-0.167128	0.871628	-0.193759	0.428201	0.997

## CONCLUSION

The radial fin of hyperbolic profile with temperature dependent thermal conductivity has been analyzed using the Adomian decomposition method. This method provides an easy and direct way to find an analytic solution without any linearization in the form of an infinite series. Dimensionless temperature distribution and fin efficiency as a function of thermo-geometric fin parameter for different values of thermal conductivity parameter and radii ratios have been obtained. The results are expressed in terms of suitable dimensionless parameters and are presented in graphical forms. A

regression equation as a function of thermo-geometric fin parameter is also proposed for fin efficiency. It is found that the variation of thermo-geometric parameter and thermal conductivity parameter have a significant effect on the temperature distribution in the fin and its efficiency.

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