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# HEAT TRANSFER ANALYSIS FROM ROTATING POROUS PLATE WITH TEMPERATURE-DEPENDENT THERMAL CONDUCTIVITY

**Mohsen TORABI and Hessameddin YAGHOOBI**
Young Researchers Club, Central Tehran Branch, Islamic Azad University, Tehran, IRAN
torabi.mech@gmail.com, yaghoobi.hessam@gmail.com

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**Abstract:** In the present study, an analytical solution for the nonlinear energy equation due to temperature-dependent thermal conductivity is presented. The effects of variable thermal conductivity and other parameters on heat transfer are investigated for a hydromagnetic flow of an incompressible viscous electrically conducting fluid past a rotating porous plate. The plate rotates with a uniform angular velocity about an axis normal to the plate and the fluid at infinity rotates with the same angular velocity about a non-coincident parallel axis. The governing equations are solved analytically using the variational iteration method (VIM), and numerically using the Richardson extrapolation method. The temperature distribution and heat transfer of various suction parameters, Brinkman numbers, magnetic parameters and Prandtl numbers are presented. It is demonstrated that for large values of Brinkman number and magnetic parameter, a significant amount of heat is generated near the plate.
**Keywords:** Temperature-dependent thermal conductivity, Hydromagnetic, Viscous fluid, Rotating porous plate, Variational iteration method (VIM).

## ISI İLETİM KATSAYISI SICAKLIĞA BAĞLI DÖNEN GÖZENEKLİ LEVHADAN ISI TRANSFERİNİN ANALİZİ

**Özet:** Bu çalışmada, ısı iletim katsayısının sıcaklığa bağlı olması nedeniyle non-lineer olan enerji denkleminin analitik çözümü sunulmuştur. Dönen gözenekli levha üzerinden akan, elektrik iletken, sıkıştırılamaz ve viskoz hidromanyetik akışkanın, değişken ısı iletim katsayısının ve diğer parametrelerin ısı transferine etkileri araştırılmıştır. Levha, sabit bir açısal hızla levhaya dik bir eksen etrafında dönmektedir ve akışkan, levhadan uzakta levha eksenine paralel farklı bir eksen etrafında aynı hızla dönmektedir. Denklemler, değişken iterasyon metodu (VIM) kullanılarak analitik olarak ve Richardason ekstrapolasyon metodu kullanılarak sayısal olarak çözülmüştür. Değişik parametreler, Brinkman sayısı, manyetik parametreler ve Prandtl sayısı için sıcaklık dağılımı ve ısı transferi sonuçları sunulmuştur. Brinkman sayısı ve manyetik parametrelerin yüksek değerlerinde, levhanın yakınında önemli ısı üretimi olduğu görülmüştür.
**Anahtar Kelimeler:** Sıcaklığa bağlı ısıl iletkenlik, Hidromanyetik, Viskoz akış, Dönen gözenekli plaka, Varyasyonel iterasyon metodu (VIM)

**Nomenclature**

| | |
|---|---|
| $B$ | Magnetic field |
| $Br$ | Brinkman number |
| $b$ | Temperature coefficient of thermal conductivity |
| $c_p$ | Specific heat of fluid [ $kJ / kg \cdot K$ ] |
| $Ec$ | Eckert number |
| $J$ | Current density |
| $k(T)$ | Thermal conductivity [ $W / m \cdot K$ ] |
| $L$ | Linear operator |
| $\ell$ | Distance between two axes [ $m$ ] |
| $M$ | Dimensionless magnetic parameter |
| $N$ | Nonlinear operator |
| Pr | Prandtl number |
| $p$ | Fluid pressure |
| $q$ | Heat flux [ $W / m^2$ ] |
| $q^*$ | Dimensionless heat flux |
| $S$ | Dimensionless suction parameter |
| $T$ | Temperature [ $K$ ] |
| $w_0$ | Suction/blowing velocity [ $m / s$ ] |
| $u,\ v$ | Velocity components in $x$ and $y$ directions [ $m / s$ ] |

**Greek symbols**

| | |
|---|---|
| $\zeta$ | Dimensionless distance from plate |
| $\theta$ | Dimensionless temperature |
| $\mu$ | Absolute viscosity of fluid [ $kg / m \cdot s$ ] |
| $\nu$ | Kinematic viscosity of fluid [ $m^2 / s$ ] |
| $\lambda$ | Lagrange multiplier |
| $\rho$ | Fluid density [ $kg / m^3$ ] |
| $\sigma$ | Fluid thermal conductivity [ $W / m \cdot K$ ] |
| $\Omega$ | Angular velocity [ $rad / s$ ] |

**Subscripts**

| | |
|---|---|
| $w$ | Wall |
| 0 | Initial guess |
| $\infty$ | Freestream condition |

## INTRODUCTION

Investigation about convective flow through porous media in rotating system is one of the important problems for engineers and continuing interest due to their applications in many industrial and technological applications. This study is noticeable in the design of turbines and turbo mechanics, in estimating the flight path of rotating wheels and spin-stabilized missiles. Also rotating heat exchangers are extensively used by the chemical and automobile industries.

The study of magnetohydrodynamic (MHD) boundary layers under the influence of viscous forces is of immense importance and continuing interest due to their applications in many industrial, geothermal, geophysical, technological and engineering applications such as those that occur at the core-mantle interface of the earth. Since the pioneering work of Berker (1693), there has been a research undertaken on the flow of a viscous incompressible fluid between two parallel plates rotating non-coaxially but with the same angular velocity. Coirier (1972) studied the flow due to a disk and fluid at infinity which are rotating non-coaxially at slightly different angular velocity. Abbott and Walters (1970) studied the hydrodynamic flow between two disks, rotating with the same angular velocity about non-coincident axes. The MHDs flow between two disks, rotating with the same angular velocity about two different axes has been studied by Mohanty (1972) on neglecting the induced magnetic field.

An extension of this problem for micropolar fluid has been made by Rao and Kasiviswanathan (1987). Other extensions of this type of flow to an Oldroyd-B fluid were studied by Rajagopal (1996) and in case of electrically conducting Oldroyd-B fluid by Ersoy (1999). Erdogan (1976) investigated the flow of a viscous fluid past a porous plate which rotates with uniform angular velocity about an axis normal to the plate, while the fluid at infinity rotates with the same angular velocity about a non-coincident parallel axis. Also, he studied the unsteady hydrodynamic viscous flow between eccentric rotating disks (Erdogan, 1995). Rao and Kasiviswanathan (1987) considered the flow of an incompressible viscous fluid between two eccentric rotating disks for unsteady cases. Lai *et al.* (1984) discussed the three-dimensional flow between two parallel plates which are rotating about a common axis or about distinct axes. Knight (1980) investigated the inertia effects of the non-Newtonian flow between eccentric disks rotating at different speeds. Erdogan (2000) obtained the exact solution of the time-dependent Navier–Stokes equations for the flow due to non-coaxial rotations of a disk. Ersoy (2003) found the velocity field and the shear stress components on the disks exactly by a Fourier series solution. Hayat *et al.* (2001) derived an exact solution of the unsteady three-dimensional Navier-Stokes equations for the case of flow due to non-coaxial rotations of a porous disk and a fluid at infinity in the presence of a uniform transverse magnetic field. Hayat *et al.*(2004) investigated the exact analytic solutions of two problems of a second order fluid in presence of a uniform transverse magnetic field by perturbation method. Chakraborti *et al.* (2005) investigated the hydromagnetic flow of an incompressible viscous electrically conducting fluid past a porous plate when the plate rotates with a uniform angular velocity about an axis normal to the plate and the fluid at infinity rotates with the same angular velocity about a non-coincident parallel axis. Guria *et al.* (2007a, 2007b, 2007c, 2007d) have studied the non-coaxial rotations of two porous disks or the rotations of porous disk and a fluid at infinity under different environments.

Such work seems to be important and useful for gaining our basic understanding of such flow and partly for possible applications to geophysical and astrophysical problems. Recently, Guria *et al.* (2008) studied hydromagnetic flow between two porous disks rotating with same angular velocity about two non-coincident axes in the presence of a uniform transverse magnetic field. Singh *et al.* (2009) investigated hydromagnetic convective flow of an incompressible homogeneous viscous liquid over an accelerated porous plate with suction/injection using Laplace transform technique.

The pursuit of analytical solutions for the nonlinear equation arising in heat transfer from rotating porous plate is of intrinsic scientific interest. The primary purpose of the present paper is to investigate an analytical solution on heat transfer for an electrically conducting incompressible viscous fluid past a porous plate in the presence of a uniform transverse magnetic field with variable thermal conductivity. Analytical expressions for the temperature profile and the heat transfer from the plate with Dirichlet condition on the surface are determined using the variational iteration method (VIM) given by He (1999). It may be pointed out that, later, the VIM has been successfully used in a series of literature (Seadodin *et al.*, 2011; Yaghoobi and Torabi, 2012, 2013) dealing with many engineering problems.

## BASIC IDEA OF VARIATIONAL ITERATION METHOD

To illustrate the basic concept of the technique, we consider the following general differential equation:

$$Lu + Nu = g(x) \tag{1}$$

where $L$ is a linear operator, $N$ a nonlinear operator, and $g(x)$ is the forcing term. According to variational iteration method, we can construct a correct functional as follows:

$$u_{n+1}(x) = u_n(x) + \int_0^x \lambda(Lu_n(t) + N\tilde{u}_n(t) - g(t))dt \tag{2}$$

where $\lambda$ is a Lagrange multiplier, which can be identified optimally via variational iteration method.

The subscripts $n$ denote the nth approximation, $\tilde{u}_n$ is considered as a restricted variation, that is, $\delta\tilde{u}_n = 0$; and (2) is called as a correct functional. The solution of the linear problems can be solved in a single iteration step due to the exact identification of the Lagrange multiplier. In this method, it is required first to determine the Lagrange multiplier λ optimally. The successive approximation $u_{n+1}$, $n \geq 0$ of the solution $u$ will be readily obtained upon using the determined Lagrange multiplier and any selective function $u_0$, consequently, the solution is given by

$$u = \lim_{n\to\infty} u_n \tag{3}$$

**PROBLEM STATEMENT**

Consider a porous plate coincident with the plane $z = 0$ and rotating about the $z$ axis, with uniform angular velocity $\Omega$, in an incompressible viscous electrically conducting fluid with thermal conductivity $\sigma$. The plate is assumed to be electrically non-conducting. The geometry of the problem is shown in Fig. 1. A uniform magnetic field $B$ is applied parallel to the $z$ axis, and the fluid is rotating about an axis parallel to the $z$ axis with the same angular velocity $\Omega$. The plate is maintained at a constant temperature $T_w$. The distance between both axes of rotation is $\ell$.

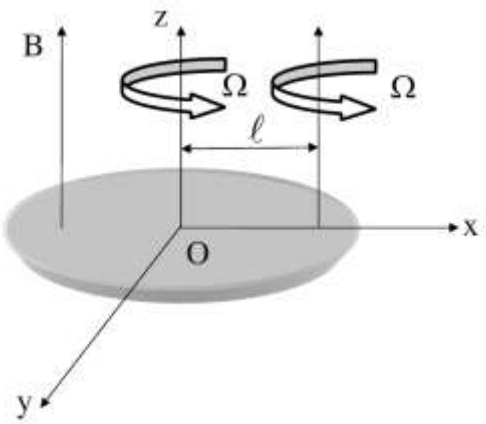


**Figure 1.** Geometry and coordinate system.

Let $(u, v)$ be the velocity components in the $x$ and $y$ directions, respectively. Following Chakraborti *et al.* (2005), the hydrodynamic boundary conditions can be written as

$$z = 0; \qquad u = -\Omega y \qquad v = \Omega x \qquad w = -w_0 \tag{4a}$$

$$z \to \infty; \qquad u \to -\Omega(y - \ell) \qquad v \to \Omega x \qquad w = 0 \tag{4b}$$

where $w_0$ is the suction/blowing velocity at the plate. Chakraborti *et al.* (2005) suggested the following velocity field for the plate:

$$u = -\Omega y + f(z); \qquad v = \Omega x + g(z) \tag{5}$$

where $f(z)$ and $g(z)$ are the components of the velocity field in the direction normal to the plane containing the axis of rotation and in the transverse direction parallel to the plane of the plate, respectively. Chakraborti *et al.* (2005) used the following equations of momentum along the $x$ and $y$ directions:

$$\nu f_1'' + w_0 f_1' - \frac{\sigma B^2}{\rho} f_1 = \frac{1}{\rho}\frac{\partial p}{\partial x} - \Omega^2 x - \Omega g \tag{6a}$$

$$\nu g'' + w_0 g' - \frac{\sigma B^2}{\rho} g = \frac{1}{\rho}\frac{\partial p}{\partial y} - \Omega^2 y - \Omega f_1 \tag{6b}$$

where the prime denotes differentiation with respect to $z$. Using mass and momentum equations, they obtained these components in dimensionless form for suction:

$$\frac{f_1(z)}{\Omega\ell} = 1 - e^{(-\zeta\alpha)}\cos(\zeta\beta) \tag{7a}$$

$$\frac{g(z)}{\Omega\ell} = e^{(-\zeta\alpha)}\sin(\zeta\beta) \tag{7b}$$

where

$$\zeta = \left(\frac{\Omega}{2\nu}\right)^{1/2} z \tag{8a}$$

$$\alpha = S + \left[\sqrt{1 + \frac{(S^2 + M)^2}{4}} + \frac{(S^2 + M)}{2}\right]^{1/2} \tag{8b}$$

$$\beta = \left[\sqrt{1 + \frac{(S^2 + M)^2}{4}} - \frac{(S^2 + M)}{2}\right]^{1/2} \tag{8c}$$

with the dimensionless suction parameter $S$ and the magnetic parameter $M$, respectively, as

$$S = \frac{w_0}{(2\nu\Omega)^{1/2}} \qquad M = \frac{2\sigma B^2}{\rho\Omega} \tag{9}$$

For blowing at the plate $(S < 0)$, the dimensionless velocity components are the same as given by Eq. (7). In this case, we replace $S$ with $S_1 > 0$ in Eqs. (8b) and (8c), so that, for blowing at the plate,

$$\alpha_1 = S_1 + \left[\sqrt{1 + \frac{(S_1^2 + M)^2}{4}} + \frac{(S_1^2 + M)}{2}\right]^{1/2} \tag{10a}$$

$$\beta_1 = \left[\sqrt{1 + \frac{(S_1^2 + M)^2}{4}} - \frac{(S_1^2 + M)}{2}\right]^{1/2} \tag{10b}$$

**HEAT TRANSFER**

The governing boundary-layer equation with viscous dissipation and Joule effect for the steady-state heat

transfer equation can be written as (Chakraborti *et al.*, 2005)

$$-\rho c_p w_0 \frac{dT}{dz} = \frac{d}{dz}\left[k(T)\frac{dT}{dz}\right] + \mu\left[\left(\frac{du}{dz}\right)^2 + \left(\frac{dv}{dz}\right)^2\right] + \frac{1}{\sigma}\left[J_x^2 + J_y^2\right] \quad (11)$$

where $c_p$ is the specific heat of the fluid, and $k(T)$ is the thermal conductivity of the fluid material that varies with temperature. When this variation in the range of practical interest is large, it is necessary to account for this variation to minimize the error in heat transfer. Accounting for the variation of the thermal conductivity with temperature makes the governing conduction equation nonlinear. The variation in thermal conductivity of a material with the temperature can be approximated in the following manner:

$$k(T) = k_a\left[1 + b(T - T_\infty)\right] \quad (12)$$

where $k_a$ is the thermal conductivity of the material at the reference temperature, and $b$ is the temperature coefficient of thermal conductivity. This temperature coefficient may be positive or negative, depending upon heating or cooling. The thermal boundary conditions are

$$T = T_w \qquad at \quad z = 0 \quad (13a)$$

$$T \to T_\infty \qquad as \quad z \to \infty \quad (13b)$$

$T_w$ and $T_\infty$ are constants, with $T_w > T_\infty$. For convenience in the subsequent analysis, the following dimensionless parameters are introduced in Eq. (6),
dimensionless temperature

$$\theta(\zeta) = \frac{T(z) - T_\infty}{T_w - T_\infty} \quad (14a)$$

dimensionless coordinate

$$\zeta = \left(\frac{\Omega}{2\nu}\right)^{1/2} z \quad (14b)$$

dimensionless slop of thermal conductivity-temperature curve

$$\varepsilon = b(T_w - T_\infty) \quad (14c)$$

Prandtl number

$$\mathrm{Pr} = \frac{\mu c_p}{k_a} \quad (14d)$$

Rotation parameter or Eckert number

$$Ec = \frac{\Omega^2 \ell^2}{c_p(T_w - T_\infty)} \quad (14e)$$

Brinkman number

$$Br = \mathrm{Pr}\, Ec \quad (14f)$$

and simplifying, we get the nonlinear energy equation,

$$\frac{d^2\theta}{d\zeta^2} + \varepsilon\theta\frac{d^2\theta}{d\zeta^2} + \varepsilon\left(\frac{d\theta}{d\zeta}\right)^2 + (2S\,\mathrm{Pr})\frac{d\theta}{d\zeta} + Bre^{-2\alpha\zeta}(\alpha^2 + \beta^2 + M) = 0 \quad (15)$$

with the dimensionless thermal boundary conditions:

$$\theta(0) = 1 \quad (16a)$$

$$\theta(\infty) = 0 \quad (16b)$$

It should be noted that Prandtl is the ratio of momentum diffusivity (kinematic viscosity) to thermal diffusivity, and Eckert number expresses the relationship between a flow's kinetic energy and enthalpy.

Using Fourier's law, the heat flux from the plate to the fluid can be written as

$$q = -k(T)\left.\frac{dT}{dz}\right|_{z=0} \quad (17)$$

Accordingly, using Eqs. (12) and (14), heat flux in dimensionless form can be written as

$$q^* = \frac{q\sqrt{2\nu/\Omega}}{k_a(T_w - T_\infty)} = -(1 + \varepsilon\theta)\left.\frac{d\theta}{d\zeta}\right|_{\zeta=0} \quad (18)$$

## IMPLEMENTATION OF VIM

In order to solve Eq. (15) using VIM, we construct a correction functional, as follows

$$\theta_{n+1}(\zeta) = \theta_n(\zeta) + \int_0^\zeta \lambda\left\{\begin{array}{l}\dfrac{d^2\theta_n(t)}{dt^2} + (2S\,\mathrm{Pr})\dfrac{d\theta_n(t)}{dt} + \varepsilon\tilde{\theta}_n(t)\dfrac{d^2\tilde{\theta}_n(t)}{dt^2} \\ + \varepsilon\left(\dfrac{d\tilde{\theta}_n(t)}{dt}\right)^2 + Bre^{-2\alpha\zeta}(\alpha^2 + \beta^2 + M)\end{array}\right\}dt \quad (19)$$

Its stationary conditions can be obtained as follows:

$$\lambda''(t)\big|_{t=\zeta} - (2S\,\mathrm{Pr})\lambda'(t)\big|_{t=\zeta} = 0 \quad (20a)$$

$$1 - \lambda'(t)\big|_{t=\zeta} + (2S\,\mathrm{Pr})\lambda(t)\big|_{t=\zeta} = 0 \quad (20b)$$

$$\lambda(t)\big|_{t=\zeta} = 0 \quad (20c)$$

Thus the Lagrangian multiplier can therefore be identified as

$$\lambda = \frac{e^{2S\,\mathrm{Pr}(t-\zeta)} - 1}{2S\,\mathrm{Pr}} \quad (21)$$

As a result, we obtain the following iteration formula:

$$\theta_{n+1}(\zeta) = \theta_n(\zeta) + \int_0^\zeta \left(\frac{e^{2S\,\mathrm{Pr}(t-\zeta)} - 1}{2S\,\mathrm{Pr}}\right)\left\{\begin{array}{l}\dfrac{d^2\theta_n(t)}{dt^2} + (2S\,\mathrm{Pr})\dfrac{d\theta_n(t)}{dt} + \varepsilon\tilde{\theta}_n(t)\dfrac{d^2\tilde{\theta}_n(t)}{dt^2} \\ + \varepsilon\left(\dfrac{d\tilde{\theta}_n(t)}{dt}\right)^2 + Bre^{-2\alpha\zeta}(\alpha^2 + \beta^2 + M)\end{array}\right\}dt \quad (22)$$

From the boundary condition in Eq. (16a), that we have it in point $\zeta = 0$ an arbitrary initial approximation can be obtained

$$\theta_0(\zeta) = \frac{2S\,\mathrm{Pr} + C - Ce^{-2S\,\mathrm{Pr}\,t}}{2S\,\mathrm{Pr}} \tag{23}$$

where $C$ is constant, and we will calculate it with considering another boundary condition in Eq. (16b) in point $\zeta = \infty$.

Using the variational Eq (22), we have

$$\theta_1(\zeta) = \theta_0(\zeta) + \int_0^{\zeta} \left( \frac{e^{2S\,\mathrm{Pr}(t-\zeta)} - 1}{2S\,\mathrm{Pr}} \right) \left\{ \begin{array}{l} \dfrac{d^2\theta_0(t)}{dt^2} + (2S\,\mathrm{Pr}) \dfrac{d\theta_0(t)}{dt} + \varepsilon\theta_0(t) \dfrac{d^2\theta_0(t)}{dt^2} \\ + \varepsilon \left( \dfrac{d\theta_0(t)}{dt} \right)^2 + Bre^{-2\alpha\zeta} (\alpha^2 + \beta^2 + M) \end{array} \right\} dt \tag{24}$$

Substituting Eq. (23) into Eq. (24) we have

$$\begin{aligned} \theta_1(\zeta) = & -\frac{1}{4S^2\,\mathrm{Pr}^2\,\alpha(S\,\mathrm{Pr} - \alpha)} (2S^2\,\mathrm{Pr}^2\,\varepsilon C\alpha + AS^2\,\mathrm{Pr}^2 \\ & + 2S\,\mathrm{Pr}\,\varepsilon C\alpha^2 - S\,\mathrm{Pr}\,\alpha A - 4e^{-2S\,\mathrm{Pr}\zeta}\alpha^2\varepsilon C\zeta S^2\,\mathrm{Pr}^2 \\ & - 2e^{-2S\,\mathrm{Pr}\zeta}\alpha^2\varepsilon C^2\zeta S\,\mathrm{Pr} - 2e^{-2S\,\mathrm{Pr}\zeta}\alpha^2\varepsilon CS\,\mathrm{Pr} + e^{-2S\,\mathrm{Pr}\zeta}\alpha^2\varepsilon C^2 \\ & + 4e^{-2S\,\mathrm{Pr}\zeta}\alpha\varepsilon C\zeta S^3\,\mathrm{Pr}^3 + 2e^{-2S\,\mathrm{Pr}\zeta}\alpha\varepsilon C^2\zeta S^2\,\mathrm{Pr}^2 \\ & + 2e^{-2S\,\mathrm{Pr}\zeta}\alpha\varepsilon C\zeta S^2\,\mathrm{Pr}^2 + e^{-2S\,\mathrm{Pr}\zeta}\alpha S\,\mathrm{Pr}\,A - e^{-2S\,\mathrm{Pr}\zeta}\alpha\varepsilon C^2 S\,\mathrm{Pr} \\ & - e^{-2\alpha\zeta}S^2\,\mathrm{Pr}^2\,A + e^{-4S\,\mathrm{Pr}\zeta}\alpha\varepsilon C^2 S\,\mathrm{Pr} - e^{-4S\,\mathrm{Pr}\zeta}\alpha^2\varepsilon C^2 - 4\alpha S^3\,\mathrm{Pr}^3 \\ & + 2e^{-2S\,\mathrm{Pr}\zeta}\alpha CS^2\,\mathrm{Pr}^2 - 2\alpha CS^2\,\mathrm{Pr}^2 + 4\alpha^2 S^2\,\mathrm{Pr}^2 \\ & - e^{-2S\,\mathrm{Pr}\zeta}\alpha^2 CS\,\mathrm{Pr} + 2\alpha^2 CS\,\mathrm{Pr}) \end{aligned} \tag{25}$$

where

$$A = Br(\alpha^2 + \beta^2 + M) \tag{26}$$

Accordingly, in the same manner the rest of the components of the iteration formula can be obtained.

## RESULTS AND DISCUSSION

In this section we present the results with the VIM, described in the previous section for solving Eq. (15). To test the validity and accuracy of the method used in this study, the temperature distribution $\theta$ for several values of $Br$, $M$, $S$ and Pr obtained by the VIM and well-established Richardson numerical solution (Richardson extrapolation) are displayed in Fig. 2. This figure shows very good agreement between the VIM and numerical solution. Moreover, this interesting agreement is tabulated in Table 1. In addition, it can be clearly seen from Fig. 2 that for sufficiently large values of $Br$ and $M$, a significant amount of heat is generated near the plate due to viscous dissipation as well as due to Joule heating arising from the flow of electric current in the fluid. Thus heat flows from the fluid to the plate even if the plate temperature is higher than the ambient temperature. Also, it can be seen from this figure that the temperature at a point decreases with an increase in $S$ or Pr. One can find the maximum temperature and its location by differentiating the temperature filed and solving the resultant equation for $\zeta$. For example, using $S = \mathrm{Pr} = Br = 1$, $M = 20$ and $\varepsilon = 0.1$ the maximum value of temperature is $\theta \approx 1.0698$, which occurs at $\zeta \approx 0.0693$. Assuming temperature independent thermal conductivity for the fluid, i.e. $\varepsilon = 0.0$, for the same parameters the maximum value of temperature is $\theta \approx 1.0754$, which occurs at $\zeta \approx 0.0678$. It is very interesting that the temperature-dependent thermal conductivity is to lower the maximum temperature in the fluid. It is due to increasing the rate of heat transfer from fluid to both upper and plate.

After these verifications, we analyze heat flux and the effects of some physical applicable parameters in this problem such as suction parameter, Brinkman number, magnetic parameter and Prandtl number. The effects of Brinkman number, $Br$, on heat transfer for fixed values of $S$, Pr and $M$ are shown in Fig. 3. As expected, heat transfer is maximum at the surface for small values of $Br$ and decreases with the increasing vertical distance $\zeta$. It is because the fact that as one goes further from the plate, the overall thermal conductivity of the fluid increases and it reduce heat transfer to the specific point. Therefore, the temperature of this point decreases. However, for large values of $Br$, due to large value of viscosity or annular velocity from Eq. (4), a significant amount of heat is generated near the plate. Because of this interesting effect, heat transfer at the surface increases with the increase in vertical distance at first, and follows with the decrease by increasing in vertical distance.

**Table 1.** The results of VIM and Numerical Solution for $Br = 1$, $M = 1$, $S = 1$, $\mathrm{Pr} = 1$ and $\varepsilon = 0.1$.

| $\zeta$ | $\theta(\zeta)_{VIM}$ | $\theta(\zeta)_{NS}$ | $Error = \lvert\theta(\zeta)_{VIM} - \theta(\zeta)_{NS}\rvert$ |
|---|---|---|---|
| 0 | 1 | 1 | 0.0000000000 |
| 0.3 | 0.7335135560 | 0.7328648312 | 0.0006487248 |
| 0.6 | 0.4504575545 | 0.4502151854 | 0.0002423691 |
| 0.9 | 0.2595377529 | 0.2596492175 | 0.0001114646 |
| 1.2 | 0.1456206364 | 0.1457845375 | 0.0001639011 |
| 1.5 | 0.0807542106 | 0.0808593755 | 0.0001051649 |
| 1.8 | 0.0445432818 | 0.0445825905 | 0.0000393087 |
| 2.1 | 0.0245075613 | 0.0244998753 | 0.0000076860 |
| 2.4 | 0.0134673684 | 0.0134306850 | 0.0000366834 |
| 2.7 | 0.0073960122 | 0.0073425221 | 0.0000534901 |
| 3 | 0.0040604612 | 0.0039975112 | 0.0000629500 |

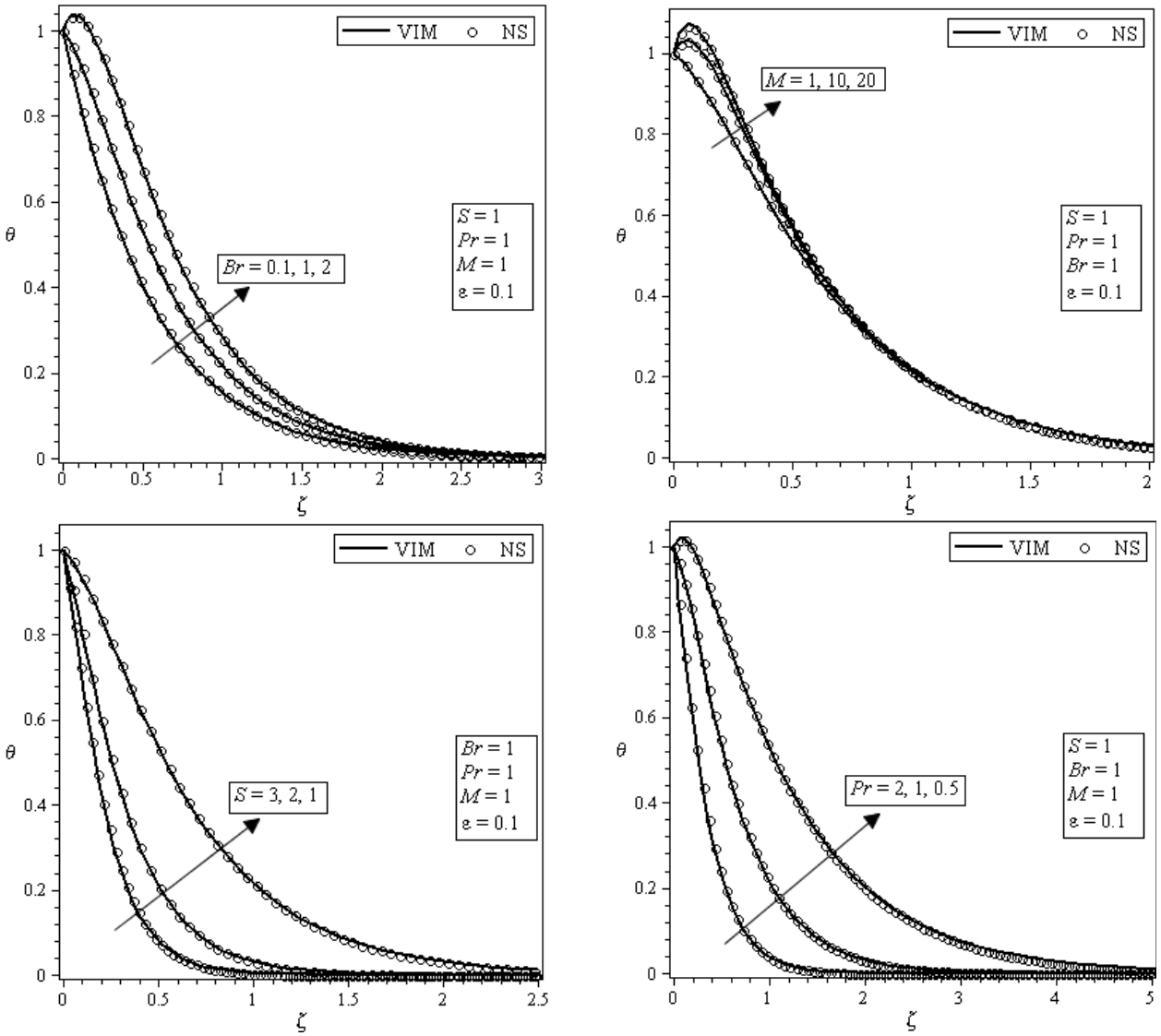


**Figure 2.** Comparison of dimensionless temperature variation obtained by the VIM and numerical solution

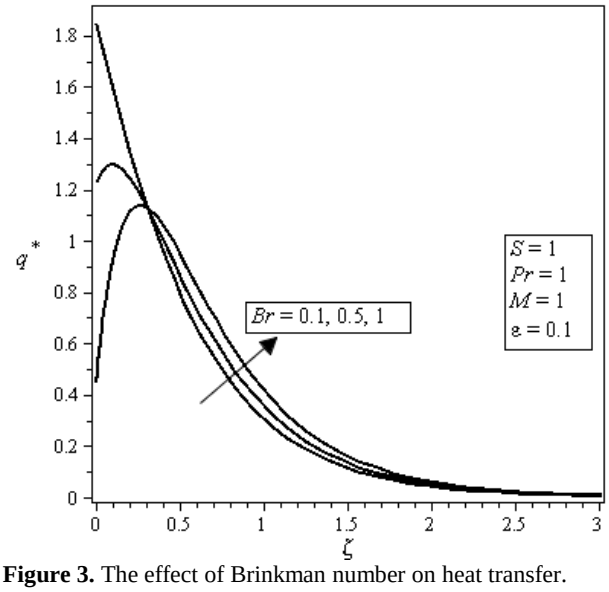


**Figure 3.** The effect of Brinkman number on heat transfer.

Figure 4 shows the effects of magnetic parameter on heat transfer for fixed values of $S$, Pr and $Br$. It is obvious from Fig. 4 that the heat transfer decreases with the vertical distance from the plate, and with the thermal conductivity as well. It is also seen from this figure that the heat flux is almost equal for $\zeta \geq 0.8$. If one wants to find the maximum heat transfer rate and its location, it is easily achievable by differentiating the Eq. (17) or (18) and solving the resultant equation for $\zeta$. For example, using $S = \Pr = 1$, $Br = 0.5$, $M = 5$ and $\varepsilon = 0.1$ the maximum value of heat flux is $q^* \approx 1.2500$, which occurs at $\zeta \approx 0.2109$.

For fixed values of $M$, $Br$ and Pr, the effect of suction parameter on heat transfer is shown in Fig. 5. In this case, the suction parameter is varied, and the Brinkman number is kept low. For low values of suction parameter, i.e. $S = 0.5$ the heat transfer along the $z$ axis firstly increase, reaches to a maximum number and then decreases by increasing in the value of $\zeta$.

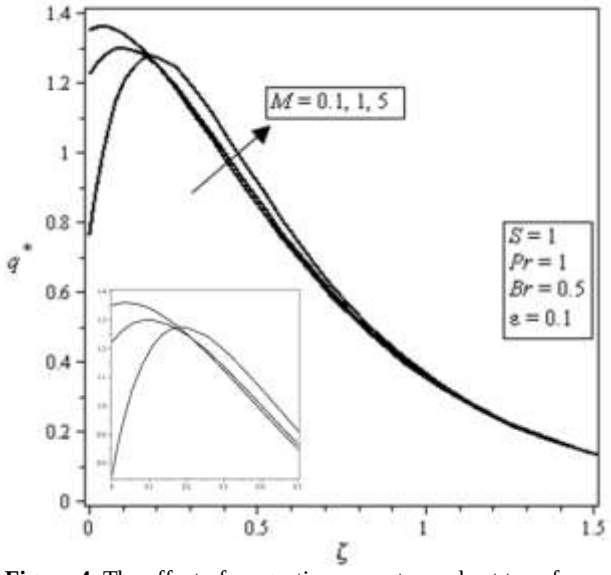


**Figure 4.** The effect of magnetic parameter on heat transfer.

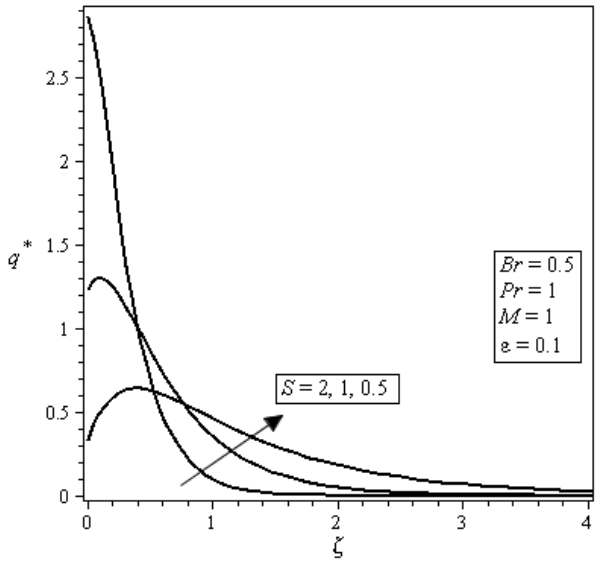


**Figure 5.** The effect of suction parameter on heat transfer.

However, this trend for high values of suction parameter is much simpler. For high values of $S$, i.e. $S = 2$ the heat flux decreases by increase in the value of $\zeta$. This behavior is because the fact that high values of suction parameter can overcome effects of other parameters of the heat flux and becomes dominant compare with other parameters.

The heat transfer from the porous plate also depends on the Prandtl number. This effect is shown in Fig. 6. Again, for large values of Pr, the heat transfer at the surface is maximum and decreases as $\zeta$ increases. However, as the value of Pr decreases, i.e. $\Pr = 0.5$ the heat flux has the maximum. This maximum can be found easily from Eq. (18) same as what we done for Fig. 4.

Herein the value of $\varepsilon$ was taken as 0.1. Moreover, in the wake of the large term of second iteration for the solution, the result of the first iteration is shown; however obtained results are calculated using two iterations.

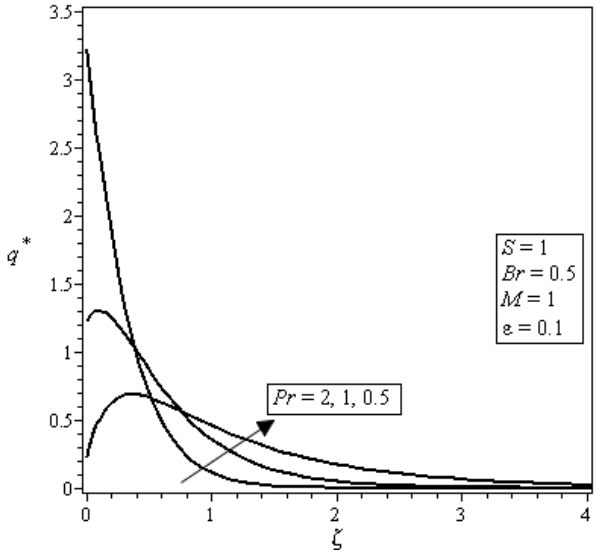


**Figure 6.** The effect of Prandtl number on heat transfer.

## CONCLUSION

An analytical solution for the nonlinear energy equation due to temperature-dependent thermal conductivity is obtained for a hydromagnetic flow of an incompressible viscous electrically conducting fluid past a rotating porous plate. VIM is employed to solve a nonlinear energy equation. It has been shown that there is a very good agreement between the analytical and numerical results. The analysis is performed for different $Br$, $M$, $S$ and Pr numbers. It is demonstrated that for large values of Brinkman number and magnetic parameter a significant amount of heat is generated near the plate. It is interesting to note that, assuming temperature-dependent thermal conductivity for the fluid material, which is more realistic assumption compare with constant thermal conductivity, decreases the maximum temperature in the fluid. This conclusion should be very important for many applications, since in many of them it is very important to accurately estimate the temperature field in the rotating fluid.

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