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Dynamical Analysis of the Multispan Beams with Method of Multiple Scales

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Abstract

Multi-span beams are statically indeterminate structures in general. They have many applications in civil engineering, mechanism, navigation engineering and so on. For example, multi-span bridges have been widely used in highway and railway. It is of great importance to study the dynamic characteristic of the multi-span beams for engineering design and scientific research. Many engineers and scientists have contributed to the solution of the problem with their innovations, and still the subject draws considerable attention from researchers by now. In this study, we investigate primary resonance case of multi-span beam subject to axial load. Firstly, the mathematical model of the problem is derived by using extended Hamilton principle. This model has geometric nonlinearity. Here, two system of partial differential equations are obtained for axial direction and transverse direction. The numbers of equations and boundary conditions depends on span number. After coupling equations in transverse and axial directions, the system of nonlinear integro-differential equations are obtained and solved using the method of multiple time scales.

Keywords: Method of multiple scales, nonlinear vibration, multi-span beam, primary resonance.

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1. Introduction

There are many studies related to multi-span beam structures in the literature. Multi-supported (multi-span) beams can be applied to some different engineering areas. For example, Henchi et. al. [1] consider dynamic response of multi-span beams as bridges. The beams are under a convoy of moving loads. In another interesting study, the transverse vibrations of an axially accelerating Euler–Bernoulli beam with multiple simple supports are examined [2]. Kesimli et. al. [3] analyse nonlinear vibrations of an axially accelerating multi-supported spring and determine both stable and unstable areas. In some studies, multi span beams is considered as pipes conveying fluid [4]. They present the Timoshenko beam model instead of the Euler Bernoulli beam.

In this study, the solution of the nonlinear mathematical model of the multi-span Euler-Bernoulli beam is presented. As a solution technique, the method of multiple scales is preferred. Applying the method of multiple scales and separating the equations at each power of small parameter, linear differential equations are obtained at each order of small parameter instead of the system of integro-nonlinear differential equation. The principal primary resonances case is analyzed and the stability boundaries and regions are investigated. The obtained results are shown by graphs.

2. Solution Using Method of Multiple Scales

Adding a visco-elastic damping term into the equation of motion in Ref. [5], one obtains

$$\ddot{w}_m+\bar{\mu}\dot{w}_m+\frac{\partial^4 w_m}{\partial x_1^4}-\frac{\partial^2 w_m}{\partial x_1^2}\left[\sum_{m=1}^{M}\int_{x_{m-1}}^{x_m}\left(\frac{\partial w_m}{\partial x_1}\right)^2 d\hat{x}_1\right]-\lambda^2\frac{\partial \ddot{w}_m}{\partial x_1^2}+P\frac{\partial^2 w_m}{\partial x_1^2}=\bar{F}_m\cos\Omega T_0 \tag{1}$$

where m, λ, P and $\bar{F}$ denote different support locations, the fineness coefficient, the axial compressive force and external excitation with amplitude, respectively. In this study, we consider the equation of motion for two span. The method of multiple scales is directly applied to the governing equation to find the general solution of Eq. (1). The perturbation expansion for $w_m(x,t)$ is assumed

$$w_m\left(x,\mathrm{T}_0,\mathrm{T}_2;\varepsilon\right)=\varepsilon w_{m1}\left(x,\mathrm{T}_0,\mathrm{T}_2;\varepsilon\right)+\varepsilon^3 w_{m3}\left(x,\mathrm{T}_0,\mathrm{T}_2;\varepsilon\right)+\ldots\ ;\ m=1,2 \tag{2}$$

where ε is a small parameter artificially inserted into the equations; $\mathrm{T}_0=t$ is usual fast time scale, $\mathrm{T}_1=\varepsilon t$ is slow time scale. We consider only the primary resonance case and hence, the forcing and damping terms are ordered so that they counter the effect of nonlinear terms: that is

$$\bar{\mu}=\varepsilon^2\mu \qquad\qquad \bar{F}_m=\varepsilon^3 F_m \tag{3}$$

$D_n=\partial/\partial T_n$, its derivatives have been expressed in time. Substituting Eqs. (3) into Eq. (1) and separating at each order of ε, one obtains

$$O\left(\varepsilon^1\right): D_0^2 w_{m1}-D_0^2\lambda^2 w''_{m1}+P'' w_{m1}+w_{m1}^{iv}=0 \tag{4}$$

$$O\left(\varepsilon^3\right): D_0^2 w_{m3}-D_0^2\lambda^2 w''_{m3}+P'' w_{m3}+w_{m3}^{iv}=-2D_0D_1w_{m2}-\mu D_0 w_{m1}+2\lambda^2 D_0 D_1 w''_{m2}-\left(D_1^2+2D_0D_2\right)w_{m1}$$
$$+\frac{1}{2\lambda^2}w''_{m1}\left[\sum_{m=1}^{2}\int_{x_{m-1}}^{x_m} {w'_{m1}}^2 dx_1\right]+F_m\cos\Omega T_0 \tag{5}$$

The problem at orders ε is linear. The generating solution at order ε are assumed as

$$w_{m1}=\left(A_m(T_2)e^{i\omega_m T_0}+\bar{A}_m(T_2)e^{-i\omega_m T_0}\right)X_m\left(x_1\right);\quad m=1,2 \tag{6}$$

where A_m and $\bar{A}_m$ are complex amplitudes and their conjugates, respectively. Substituting w_{m1} solution into this relation, we obtain

$$O\left(\varepsilon^3\right): D_0^2 w_{m3}-D_0^2\lambda^2 w''_{m3}+P'' w_{m3}+w_{m3}^{iv}=-\mu i\omega_m X_m\left(x_1\right)\left(A_m(T_2)e^{i\omega_m T_0}-\bar{A}_m(T_2)e^{-i\omega_m T_0}\right)$$
$$-2i\omega_m X_m\left(x_1\right)\left(D_2A_m(T_2)e^{i\omega_m T_0}-D_2\bar{A}_m(T_2)e^{-i\omega_m T_0}\right)+F_m\cos\Omega T_0$$
$$+\frac{b}{2\lambda^2}\left(A_m(T_2)e^{i\omega_m T_0}+\bar{A}_m(T_2)e^{-i\omega_m T_0}\right)X''_m\left(x_1\right) \tag{7}$$

where

$$b=\sum_{m=1}^{2}\int_{x_{m-1}}^{x_m}\left[\left(A_m(T_2)e^{i\omega_m T_0}+\bar{A}_m(T_2)e^{-i\omega_m T_0}\right)X'_m\left(x_1\right)\right]^2 dx_1 \tag{8}$$

We consider the primary resonance case. Besides, we assume that the external excitation frequency approximately the natural frequencies of the system, that is $\Omega\approx\omega_m+\varepsilon^2\sigma$ where σ and ω_m denote the detuning parameter and the natural frequency. The detuning parameter represents the nearness of the external excitation frequency to the natural frequency of the system. Applying the normalization as

$$\sum_{m=1}^{2}\int_{x_{m-1}}^{x_m} X_m^2 dx_1=1, \qquad F=\sum_{m=1}^{2}\int_{x_{m-1}}^{x_m} F_m X_m dx_1 \tag{9}$$

eliminating secular terms, then, the resulting equation is obtained as

$$D_0^2 w_{m3}-D_0^2\lambda^2 w''_{m3}+P'' w_{m3}+w_{m3}^{iv}=2i\omega_m\left(\lambda^2 X''_m\left(x_1\right)-X_m\left(x_1\right)\right)D_2A_m(T_2)+F_m e^{i\sigma T_2}$$
$$-\mu i\omega_m X_m\left(x_1\right)A_m(T_2)+\frac{b}{2\lambda^2}X''_m\left(x_1\right) \tag{10}$$

Applying the solvability condition into Eq. (10), one obtains

$$2i\omega_m\left(\lambda^2-1\right)D_2A_m-\mu i\omega_m A_m+\frac{b}{2\lambda^2}A_m^2\bar{A}_m+F_m e^{i\sigma T_2}=0 \tag{11}$$

It is convenient to write the polar form in the following instead of A_m

$$A_m=\frac{1}{2}a_m\left(\mathrm{T}_2\right)e^{i\beta_m\left(\mathrm{T}_2\right)} \tag{12}$$

where a_m and β_m are real functions of T_2. The solution is obtained by substituting the polar form of A_m into the Eq. (11) and separating the resulting equation into imaginary and real parts as

$$\omega_m a_m\left(1-\lambda^2\right)\left(\sigma-\gamma_m'\right)-\frac{b a_m^3}{16\lambda^2}=F_m\cos\gamma_m \tag{13}$$

$$-\omega_m\left(\lambda^2-1\right)a_m'+\frac{1}{2}\mu\omega_m a_m=F_m\sin\gamma_m \tag{14}$$

where $\gamma_m=\sigma T_2-\beta_m$. For steady state case $\gamma_m'=a_m'=0$, with the same mathematical procedures, the stability boundaries are found as

$$\sigma_{1,2}=\frac{1}{a_m\omega_m\left(1-\lambda^2\right)}\left(\frac{b a_m^3}{16\lambda^2}\pm\sqrt{F_m^2-\left(\frac{1}{2}\mu\omega_m a_m\right)^2}\right) \tag{15}$$

3. Numerical Results

In this section, some numerical results are presented for stability boundaries of primary resonances. The nonlinear frequencies for all cases are shown in Fig. 1-3. The region between two lines is unstable region in Fig. 1-3. The dashed lines are unstable boundaries. The solid lines are stability boundaries. When η increases, the frequencies are lower in Fig1. The effect of the values X_m on the stability region is too limited. The nonlinear terms are more effective for smaller slenderness ratio in Fig 2. The frequency–response curves shown for different F values in Fig.3. For larger F values, the unstable region is getting larger (In all figures, the horizontal axis represents σ and the vertical axis denotes a_m).

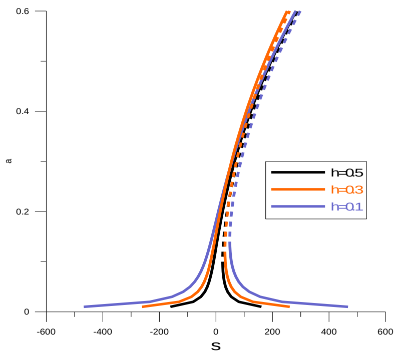

Figure 1. Frequency–response curves

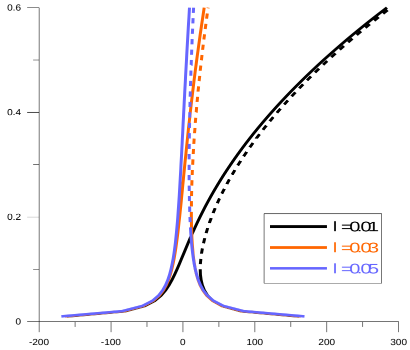

Figure 2. Frequency–response curves for different slenderness ratio curves.

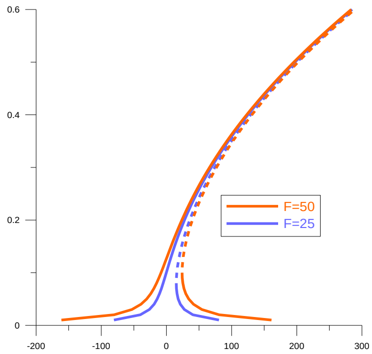

Figure 3. Frequency–response curves for different external excitation with amplitude.

4. Conclusions

The nonlinear mathematical model of the beam having two number of the span is introduced. As a solution technique, the method of multiple scale is used. The primary resonance case of the beam under harmonic external excitation is analysed. The effect of support location on the stability boundaries is presented. The effect of the location of internal support is getting higher for smaller slenderness ratio.

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