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Research Article

Unveiling Strategies and Difficulties: Investigating Secondary School Students' Approaches to Area Measurement Problems*

Strateji ve Zorlukları Ortaya Çıkarma: Ortaokul Öğrencilerinin Alan Ölçme Problemlerine Yaklaşımlarının İncelenmesi

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ABSTRACT

This study aims to explore the strategies that secondary school students employ and the difficulties they encounter when solving area measurement problems. The participants consist of 75 seventh and eighth-grade students from southeast Turkey. Data were obtained through a form comprising six open-ended problems, designed to uncover the "nature of justifications". Analysis of the students' responses revealed 11 distinct strategies and 11 difficulties. The most frequently employed strategies for solving area problems were reasoning through drawing shapes and applying the area formula (axb). Students struggled the most with distinguishing changes in the area from changes in the perimeter. It was observed that the root of the difficulties experienced by the students was challenges in measuring length. Notably, when presented with contextual problems, students focused on the context and justified their solutions based on cultural factors. As such, it is recommended that the process should be designed while considering cultural factors (both facilitators and inhibitors) in teaching subjects such as area measurement, which are closely related to real life.

Keywords: Area measurement, problem-solving strategies, students' difficulties, justification

ÖZ

Bu araştırma ortaokul öğrencilerinin alan problemleri çözümlerine yansıyan stratejileri ve karşılaştıkları zorlukları ortaya koymak amacı ile gerçekleştirilmiştir. Çalışmaya Türkiye'nin güney doğusundaki bir büyük şehirden 7 ve 8. sınıflar öğrencilerinden oluşan 75 kişi katılmıştır. Farklı gerekçelendirme türleri dikkate alınarak oluşturulan 6 açık uçlu sorudan 3 bağlam içerisinde sorulmuştur. Öğrencilerin çözümleri incelendiğinde 11 farklı strateji ve 11 farklı zorlukla karşılaşıldığı tespit edilmiştir. Öğrencilerin alan problemi çözümünde en sık kullandıkları stratejinin şekil çizerek muhakeme etmek ve alan formülü (axb) kullanmak olduğu görülmüştür. Yapılan incelemeler ışığında en sık yaşanan zorluk ise öğrencilerin alandaki değişim ile çevrede gerçekleşen değişimi ayırt etmek olduğu tespit edilmiştir. Ayrıca öğrencilerin yaşadıkları zorlukların temelinde uzunluğu hesaplama ve tespit etmeye dair zorluklar olduğu görülmüştür. Son olarak bağlam içinde sorulan problemlerde öğrencilerin bağlama odaklandıkları ve kültürel faktörler ışığında çözüm gerekçeleri sundukları tespit edilmiştir. Bu bağlamda günlük hayata teması çok olan alan ölçme /hesaplama gibi konuların öğretiminde kültürel faktörler (kolaylaştırıcı ve engelleyici) dikkate alınarak sürecin tasarlanması önerilmektedir.

Anahtar Kelimeler: Alan ölçme, problem çözme stratejileri, öğrenci zorlukları, gerekçelendirme

*This study originated as an artefact of the first author's doctoral thesis, supervised by the second author.

INTRODUCTION

Measuring the physical world is one of humankind's oldest pursuits (Drake, 2014). The quantification of concepts such as length, area, and volume, which are involved in science, commerce, and daily life, has emerged as an inherent necessity to prevent disputes and injustices. Smith and Barret (2017) define *measurement* as quantifying the measurable characteristics of objects by comparing them with a unit. Measurement is essential for developing and applying science and technology as it provides tools for measuring and controlling physical attributes (Crosby, 1997). Measurement is also a fundamental component of primary and secondary school mathematics worldwide (Ministry of National Education [MEB], 2018; Van de Walle, et al., 2014). In addition, measurement plays a vital role in teaching mathematics as it acts as a bridge between the learning fields of "geometry" and "numbers and operations" (Sarama & Clement, 2009). Education in measurement begins in the early years of school and continues to develop over time. Although measurement is a frequently used facet of mathematics in daily life and may be intuitive for students, research shows that students' understanding of measurement is less developed compared to other learning domains in mathematics (Smith et al.,2013; Thompson & Preston, 2004).

Researchers have studied how best to develop a conceptual understanding of measurement through instruction (Barrett et al., 2012; Huang & Witz, 2011). Students need both knowledge of two-dimensional (2D) geometry and numerical calculation skills to understand the area formulas of basic shapes—such as squares, rectangles, and triangles—and to then use this understanding to measure the area of a polygon (Burns & Brade, 2003; Fuys, et al., 1988). As such, a certain level of readiness is necessary for students to become adept problem solvers in area measurement. In the context of mathematics education, gaps in subject knowledge and misunderstandings of concepts can lead to a range of different learning difficulties. Research indicates that students encounter many difficulties and have misconceptions when it comes to calculating areas (Battista, 1982; Clements & Stephan, 2004; Jirotková et al., 2019; Nitabach & Lehrer, 1996; Outhred & Mitchelmore, 1996).

In area measurement, as in all mathematics teaching, open-ended problems are helpful and serve as valuable tools to reveal the difficulties that students experience, the errors and correct solutions they produce, and their understanding of the subject (NCTM, 2000). The emphasis given to various problem types and solution strategies, both in textbooks and in the wider curriculum, indicates an increased recognition of their importance. In addition, the Turkish Ministry of National Education's exam directive for 2023 includes guidance on using open-ended questions in standard school exams. In exams that include open-ended problems, the Turkish MEB (2023, p.2) states that written exam analyses are used to grade student answers and report students' learning deficiencies. This allows teachers to help address common deficiencies by providing feedback to students.

From the Ministry's statement, it is clear that open-ended questions are preferred as appropriate tools to reveal learning deficiencies and students' misconceptions. Analyzing the strategies used by students while solving problems can provide valuable insights into student understanding and highlight any errors and difficulties. In this context, Cai (2003) states that although students' knowledge of problem-solving strategies does not guarantee that students will arrive at the correct solution, it is nonetheless associated with successful problem solving.

Problem-solving strategies (Schoenfeld, 1999), which play a crucial role in solving problems, provide important opportunities to get to know students. For example, the arguments students employ in problem-solving strategies can provide the teacher with useful information about the student. This information may include the student's understanding of a particular concept, their mathematical communication skills, and their ability to make connections between different learning and sub-learning areas. One of the methods to obtain answers to these problems is to examine the justifications students provide for their solutions to the problems. Mathematical justification is vital to determining and explaining the veracity of a mathematical assumption or claim (Balacheff, 1988; Harel & Sowder, 2007; Simon & Blume, 1996). Staples and Bartlo (2010) state that asking students for justifications encourages them to think more deeply about mathematical concepts, as they grapple with various ideas and look for mathematical connections to generate new insights. When asked to justify, students are expected to provide reasoned arguments that validate their assumptions and confirm the accuracy of their solutions (Pamungkas et al., 2018). Understanding how students think through justifications can help teachers analyze the strategies students use and any problems or challenges they may encounter. This study seeks to uncover the solution strategies used by secondary school students to solve open-ended area problems and the mathematical difficulties that arise as a result. Within the scope of this study, we aim to answer the following questions:

RQ1: While students solve open-ended area problems, what strategies do they use?

RQ2: While students solve open-ended area problems, what mathematical difficulties do students face?

RQ3: While students solve open-ended area problems, how are students' difficulties and strategies related?

2. Conceptual framework

2.1. Mathematical difficulties and the difficulties experienced by students in measuring, calculating and comparing areas

The Oxford English Dictionary defines ‘difficulty' as "The quality, fact, or condition of being hard to accomplish or perform" (OED, 2024). When considered in this context, the difficulties students face while doing mathematics can be termed as mathematical difficulties. As per the literature in mathematics education, the term "difficulty" is a comprehensive concept that encompasses the difficulties students experience in learning mathematics, including "errors" and "misconceptions" (Bingölbali & Özmantar, 2015). A review of studies focused on area measurement reveals that there are a range of difficulties that students typically experience. One of the primary difficulties highlighted in research is that rather than using the idea of covering the surface to be measured with a unit, the area = base × height algorithm is used for each geometric shape without taking into account the characteristics of the shape (e.g., Battista, 1982; Clements & Stephan, 2004; Kidman & Cooper, 1997; Nitabach & Lehrer, 1996; Outhred & Mitchelmore, 1996). Another difficulty that students face when measuring area is the difficulty arising from the confusion between the concepts of perimeter and area. This confusion may stem from an inability to distinguish between these two concepts (Asil-Güzel, 2018; Jirotková et al., 2019; Machaba, 2016), or from the misconception that rectangles with identical perimeters should have the same area (Baturo & Nason, 1996). Another common misunderstanding is the belief that an increase in area always corresponds to an increase in perimeter. Tsamir (2003) describes this confusion as a "more A, more B" situation. Another issue referred to in the literature arises from the definition of basic geometric shapes. Shape identification forms the basis for developing the area formula of geometric shapes (Owens & Outhred, 2006). For this reason, students’ understanding of the properties of geometric shapes can influence their performance in area measurement tasks.

The abovementioned difficulties can also lead to various errors and misconceptions. Based on the answers given to the open-ended area measurement and comparison questions presented to the students posed by this study, it is difficult to determine whether the difficulties experienced by the students were due to careless mistakes they made or mathematical misconceptions. *Misconceptions* are defined as unscientific understandings that lead to systematic errors and contradict the fundamental nature of mathematics (Zembat, 2014). Therefore, within the scope of the study, these struggles identified in the students' responses were generally interpreted as the difficulties that students experience in field education.

2.2. Problem, problem-solving strategies and area problem-solving strategies.

Van De Walle et al. (2014) define doing mathematics as not merely solving numerous examples and imitating the methods provided by the teacher, but rather as the development and application of strategies to solve problems, and verifying whether these applications lead to the correct result. According to this definition, there is a close connection between the process of doing mathematics and the process of problem-solving (Polya, 1973). Especially when tackling open-ended and non-routine problems, students learn to use the operations in the right place following the requirements of the problem, rather than merely memorizing them (Olkun et al., 2009). When solving a problem, it is necessary to establish a connection between the information given and the information requested. This link can be made by effectively employing the chosen strategy. Pressley and Hilden (2006) characterizes strategy as conscious and controllable activities carried out to achieve a result. Cai (2003) states that student success in solving open-ended, non-routine problems parallels the strategies students use while solving problems. Ramnarain (2014) posits that revealing the problem-solving strategies used by students can aid in elucidating their cognitive processes, while Woodward et al. (2012) argue that it can contribute to improved awareness of students' ideas and approaches. In recent years, studies on problem-solving have focused on open-ended problems (Elçi, 2022; Gür & Hangül, 2015; Probosiwi et al., 2021; Yazgan & Bintaş, 2005). Such problems, which appear to have no solution, involve a process that requires students to think and work, necessitating the use of different strategies (Aydurmuş, 2013). An examination of the literature reveals that although there are many studies on students' problem-solving strategies, there is no shared definition of what a problem-solving strategy is in these studies. In this study, problem-solving strategies are understood as all the relationships students establish, the paths they follow, and the actions they undertake to solve a given problem. Studies that examine students' strategies frequently observe the inclusion of systematic listing, using diagrams, predicting and checking, working backward, reasoning, using known information (formulas), and using variables (Buckley et al., 2019; Gür & Hangül, 2015; Bülbül et al., 2021.).

Studies that explore the strategies students use when solving area problems often find that the most common strategy is the use of the area formula (Huang & Witz, 2013; Zacharos, 2006; Kospentaris et al., 2011). During their educational journey, students learn formulas to calculate the area of squares, rectangles, triangles, and parallelograms, and they use these formulas to solve different problems. Another strategy that students use when calculating area is the unit counting strategy (Outhred & Mitchelmore, 2000; Zacharos, 2006). This strategy, which uses the idea of surface covering, also allows students to interpret the area visually. Other strategies, built on students' visual interpretation of shapes, involve completing the given shape or dividing it into pieces. In these strategies, students transform shapes that lack a formula to calculate the area into familiar shapes by completing or dividing them. Gürefe (2018) categorized these two strategies as multi-step strategies. Most research focuses on the abovementioned strategies (Gürefe, 2018; Zacharos, 2006). Upon reviewing the studies conducted in this context, it becomes evident that most questions asked to reveal students' strategies are direct and not contextualized to everyday life. Although some studies emphasize approaches based on sociocultural perspectives, these studies are limited to underlining the role of these cultural "tools" in the teaching process (Zacharos, 2006). However, area measurement and calculation have broad applications in daily life. To encourage students to use different strategies (e.g., visual, dynamic), questions of this nature should be posed to students (Presmeg, 2014; Vale & Barbosa, 2018). The problems prepared within the scope of the study were selected and arranged to be both direct and relevant to everyday life contexts, thereby facilitating the use of different strategies by students.

METHODOLOGY

This study is qualitative in nature, and a survey model was preferred. In research, the survey model is often preferred when the goal is to describe an existing phenomenon as it exists (Patton, 2014). In this study, an examination of students' justifications for solving open-ended field problems was conducted to identify the strategies they use and the difficulties they encounter.

3.1. The background of the study

The data for the study were collected within the scope of a doctoral thesis written by the first author under the supervision of the study's second author. The purpose of collecting these data within the scope of the thesis was to create "observer research tools" derived from the authentic environment in which different student understandings occur. In this thesis, only two out of the six problems evaluated in this study were utilized, along with three student solutions for each problem.

3.2. Participants

The study participants consist of 75 secondary school students from a metropolitan center in the Southeastern region of Turkey. The selection of the school was informed by a number of factors, including the diversity of socio-economic levels, academic achievement and gender of the students, the willingness of the school administration to cooperate, and the researcher's employment at that school. Open-ended questions were presented to the student groups who volunteered to participate in the study. This group comprises 27 students from the 7th grade and 48 from the 8th grade. The main reason for this selection is that the target achievements, which include the basic concepts and skills of area measurement in the mathematics curriculum, are included in the 3rd through 7th grades (MEB, 2018). Consequently, students in the 7th and 8th grades are expected to know the target outcomes related to the concept of area measurement. Thirty of the participants are female students, and 45 are male students. The participants mainly consist of eighth-grade students.

3.3. Data Collection

The data collection tool used in this study consists of six open-ended area measurement and comparison problems. Three key elements were taken into consideration during the design of these problems. The first of these is the DIVINE framework, which offers different types of justifications depending on their nature. DIVINE is an acronym for four types of justification tasks: **D**ecision-making, **I**nference, **V**al**I**datio**N,** and **E**laboration (Chua, 2017). The second element is the common difficulties students encounter in the area measurement context, and the third is the various problem-solving strategies. While selecting the problems, studies conducted within the context of area measurement were reviewed, and problems from various sources (textbooks, theses, articles, and suggestions from experienced teachers) were selected and organized. Two academicians specializing in mathematics education examined the prepared problems, and adjustments were made based on the feedback from two teachers with at least ten years of experience. A pilot application of the revised problems was conducted with 28 seventh-grade students. Following the pilot application, necessary adjustments were made, resulting in the final version of the data collection tool. The nature and purpose of the problems included in the data collection tool are detailed in Table 1.

Table1

The Nature and Purpose of The Problems Included In The Data Collection Tool

Problems	Nature of justification tasks	Mathematical purpose of the problem
1	• Elaboration • Validation	• Area measurement • Area and edge length relationship • The effect of the change in the sides of polygons on the area
2	• Inference • Validation	• Meaning of the concept of area • Elements needed to measure the area
3	• Making Decision • Validation	• The relationship between areas and perimeters of polygons
4	• Inference • Validation	• Basic properties of polygons • Part-whole relationship • Area measurement
5	• Inference • Validation	• Area measurement • Conservation of area
6	• Inference • Validation	• Area measurement • Conservation of area • Relationship between edge length and area

The purpose of each problem used in the study is shown in Table 1. In line with these purposes, the researcher formulated two problems (problems 4 and 6). The first and second problems employed in the study were adapted from the interview problems found in Çavuş Erdem (2018). The third problem was adapted from the 7th Grade Mathematics Applications Textbook (2015, p.41). The fifth problem used in the study is an adaptation in Turkish of a problem from Driscoll et al. (2007). The form containing open-ended questions was distributed to the students and they were given 40 minutes. The student answers were examined on the same day and for the answers that were not understood or were found interesting, individual interviews were conducted with the students and they were asked to explain their solutions.

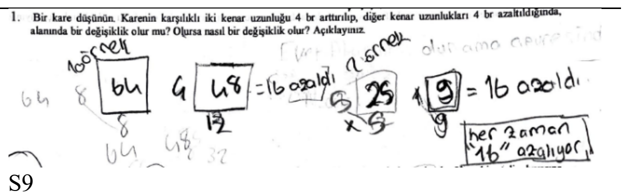
The ethical approval for the present study was granted by Dokuz Eylul University Scientific Research and Publication Ethics Committee on 03.01.2023 with the document number E-87347630-659-478309.

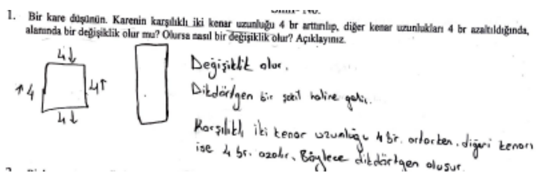
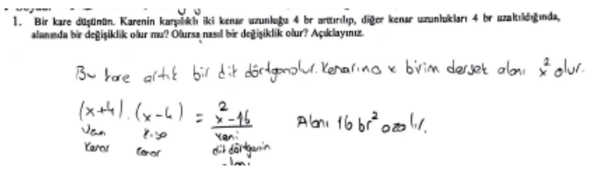
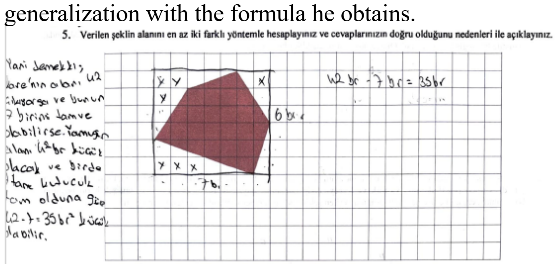
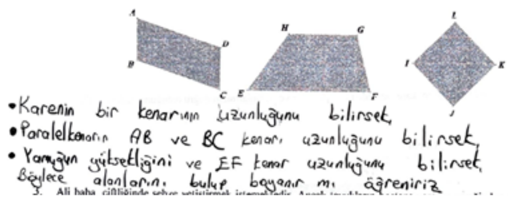
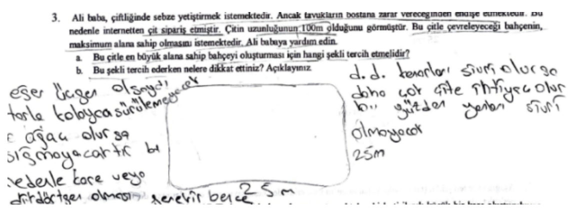
3.4. Data Analysis

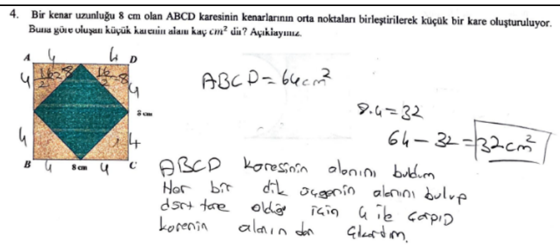
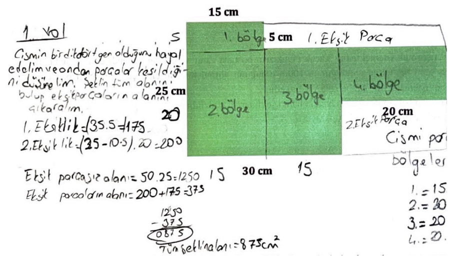
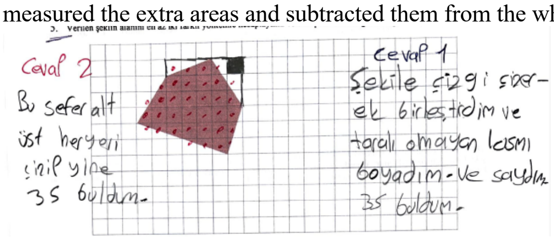
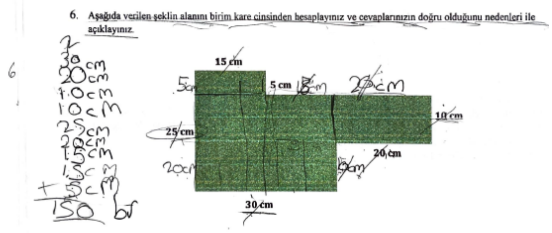
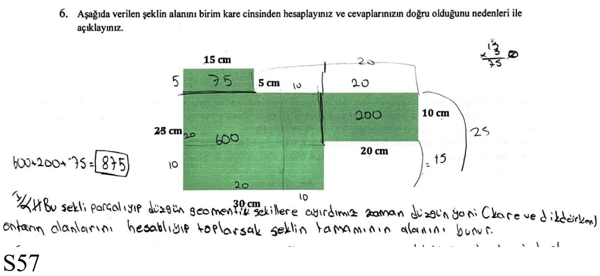
Within the scope of the study, the data obtained from the authentic expressions students used while solving area problems were evaluated through content analysis. This analysis aimed to uncover student strategies and difficulties. In the context of these analyses, the codes that emerged for student strategies, along with their definitions and sample answers, are presented in Table 2. The codes that emerged for the difficulties, their definitions, and sample answers are presented in Table 3.

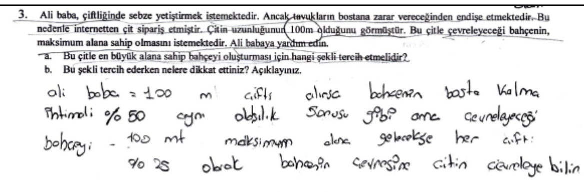
Table 2

Students' Area Measurement Strategies and Sample Answers

Number	Strategy	Description of strategy	Sample Students' Answer
1	Trial and error / systematic value-giving	This strategy involves solving the problem through trial and error.	 S9

			The student gives different number values (8 and 5) for the sides of the square. He sees that the change is 16 square centimeters and generalizes it.
2	Draw/redraw a shape	This strategy involves problem-solving through the identification of variants, invariants, or structures by drawing figures and subsequently presenting these observations as supporting arguments for the solution	 S44 The student interprets the change by drawing a shape but mentions the change in the shape instead of the change in the area.
3	Using a variable	This strategy involves solving the problem through the use of variables.	 S58 To interpret the change in area, the student expresses the length of one edge of the square as x and reaches a generalization with the formula he obtains.
4	Estimating	This strategy involves estimating the answer	 S57 The student finds the area of the rectangle surrounding the shape and subtracts the exact unit squares. S/he also estimates the remaining area.
5	Reasoning	This strategy involves solving problems primarily using logical and quantitative reasoning. The responses include contextual reasoning (reasoning based on the difference between input and output values) and imperfect reasoning.	 S32 The student reasons to find the area by considering the properties of geometric shapes.
8	Cultural/ Contextual approach	This strategy involves non-mathematical reasoning and focusing the contextual features.	 S3 The student states that the arable fields should be rectangular and explains the reason culturally, not mathematically.

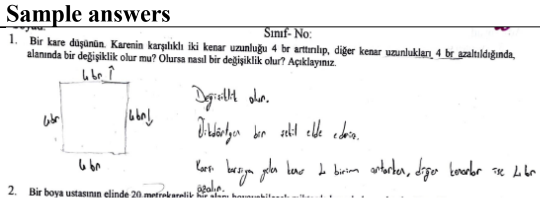
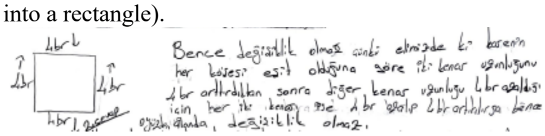
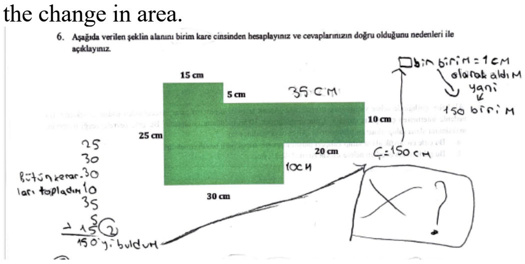
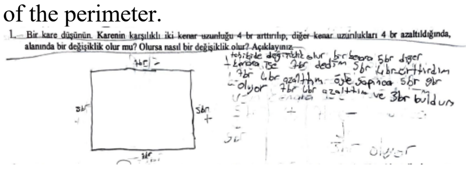
9	Using (axb) formula	This strategy involves solving problem with axb formula.	 S31 The student gives different number values (8 and 5) for the sides of the square. He sees that the change is 16 square centimeters and generalizes it.
11	Completing shape	This strategy involves solving the problem by completing the shape or transforming it into a more familiar shape.	 S58 In this solution, the student completed the shape into a rectangle and measured the whole area. Then he/she measured the extra areas and subtracted them from the whole.
12	Counting unit squares	This strategy involves dividing the shape into units and count, or if the units are given, counting directly.	 S34 The student calculates the area by counting the painted unit squares.
13	Measuring area by counting edge length unit / Measuring perimeter instead of area (a+b)	This strategy involves measuring the perimeter instead of the area.	 S52 The student measured the perimeter instead of the area.
14	Dividing the shape into parts	This strategy involves measuring area by partitioning the figure into familiar geometric shapes.	 S57 The student divided the shape into rectangles, then measured the areas of the rectangles separately, and finally summed the areas of these three rectangles.

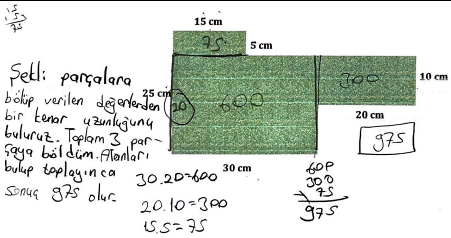
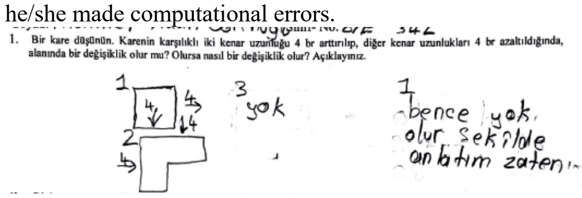
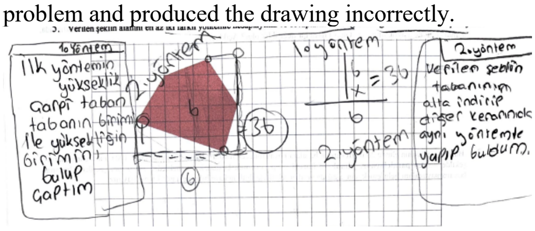
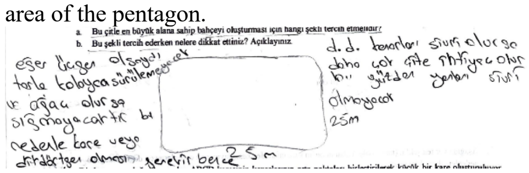
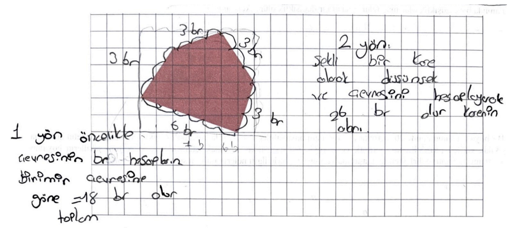
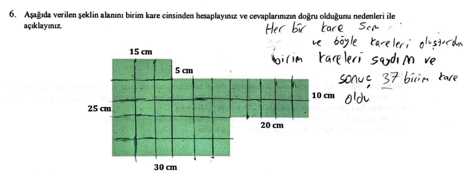
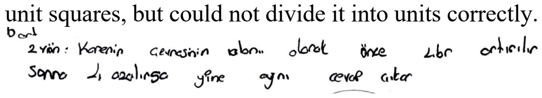
İ	Irrelevant Answer	These answers lack detectable connections to any problem-solving methodology or strategy.	 S64 The student gave an answer that was not related to field measurement. The student mentioned probability.

As indicated in Table 3, students used 11 distinct strategies while solving problems. These strategies, each accompanied by an explanation and an example representing that strategy, are detailed in the table. The strategies that emerged within the scope of the study are trial and error, drawing a shape, using variables, estimation, reasoning, cultural/contextual approach, using a known information/formula (^{axb}), completing a shape, counting unit squares, measuring area by counting unit of edge lengths /measuring perimeter instead of area (a+b) and dividing the shape into parts. In addition, explanations that included a justification but could not be classified under any specific strategy were deemed as irrelevant or incomprehensible answers. The difficulties students faced while solving these problems are included in Table 3.

Table 3

Difficulties While Solving Area Problems and Sample Answers

Number	Difficulties	Sample answers
D1	Confusion between shape change and area change	 S48 The student interpreted the change in the properties of the geometric shape as the change in area (The square transformed into a rectangle).
D2	Confusion between edge length/length and area concepts	 S33 The student interpreted the change in edge lengths instead of the change in area.
D3	Confusion between the concepts of perimeter and area	 S52 Instead of measuring the area, the student measured the length of the perimeter.
D5	Confusion about the basic properties of the shape (difficulty identifying the shape)	 S26 The student did not correctly state the properties of the square shape (one edge 4br and the other edge 7br) and misinterpreted the change.

D6	Difficulty in computing	 S32 Although the student chose an appropriate way to find the area, he/she made computational errors.
D7	Difficulty with reading, understanding what is read, and using information in a problem	 S55 The student misinterpreted the information given in the problem and produced the drawing incorrectly.
D8	Difficulty using area formulas	 S9 The student used the base x height algorithm to measure the area of the pentagon.
D10	Difficulty relating mathematics to context/Too much focus on cultural elements	 S3 Instead of calculating the shape with the largest area, the student chose the most suitable shape for plowing the farm with a tractor and placing a fence. The student focused too much on contextual elements.
D11	Difficulty determining/measuring/conservation of length	 S12 The student wanted to measure lengths to measure area, but he/she chose the wrong units to measure edge lengths.
D12	Difficulty measuring non-square units /Difficulty separating into units	 S1 The student tried to measure the area by dividing the shape into unit squares, but could not divide it into units correctly.
D13	Difficulty understanding that the area can change even when the perimeter length does not change.	 S64 The student stated that the shape's perimeter remaining constant implies that there will be no change to its area.

As detailed in Table 3, students encounter 11 distinct difficulties while solving domain problems. Student difficulties, codes of difficulties, and sample student solutions are presented in Table 3.

3.5. Trustworthiness and credibility

To determine the codes, we first examined the data. We noted each code and constructed a codebook once the existing codes had been reiterated. Subsequently, we provided an independent researcher with 120 solutions from 20 randomly selected students, along with the codebook. The independent researcher and an author then analyzed the selected data separately. Following this, the independent researcher and the author convened to compare their coding. In the comparison, the agreement percentages given by Miles and Huberman (1994) were calculated, resulting in a 91% agreement for strategies and a 96% agreement for difficulties. Although these rates were considered sufficient, the researcher and the expert discussed the answers where there was disagreement until a consensus was reached. The researcher then proceeded to complete the analysis of the remaining answers accordingly.

3.6. The Roles of the Researchers

During the course of the study, a number of roles for the researchers were identified. The data collection tool was prepared by the first and second authors, while the data was collected by the first author. The initial data was collected from the students using written forms, and preliminary analyses were conducted on the same day. Interviews with the students were conducted within two days, based on this data. The analyses were performed by the researchers, and the results were reported.

FINDINGS

4.1. Strategies used by students while solving area problems.

While coding the students' solution strategies, the dominant approach that led the students to the problem's solution was considered. Some students used multiple strategies to solve the problem. A portion of these students began to tackle the problem using one strategy and, upon failing to reach a solution, proceeded with an alternate strategy. Other students who utilized dual strategies started with one strategy and attempted to solve the problem by applying the data derived from the first strategy to the second strategy. As 135 of the analyzed solutions were either blank and irrelevant/incomprehensible, they were not classified under any solution strategy. Table 4 presents the frequencies of the strategies used in the solutions, along with the frequencies of blank or incomprehensible/irrelevant answers.

Table 4

Strategies Used in Solutions and Their Distribution According to Problems

Problems	St1	St2	St3	St4	St5	St8	St9	St11	St12	St13	St14	Irrelevant
1	26	70	8	1	6	-	-	-	-	5	-	2
2	6	-	-	1	34	1	1	-	-	-	-	33
3	16	15	-	-	24	11	-	-	-	-	-	22
4	-	1	1	-	6	-	45	-	-	-	1	21
5	2	10	-	2	-	-	4	12	20	6	5	30
6	1	-	-	1	-	-	-	8	9	24	10	27
Total	51	96	9	5	70	12	50	20	29	35	16	135

The shape drawing strategy is the most frequently used in solving area problems (st2). Students tend to resort to drawing to interpret the change when no shape is provided in the problem. Notably, in the first problem, 70 out of 75 students opted to draw figures to facilitate problem solving. The second most frequently used strategy is logical reasoning (st5). Although logical reasoning is a component of every solution, it serves as the primary strategy for solving certain problems. The strategy was particularly observed in problems that required an understanding of the concept of area and the information necessary for its measurement.

Another strategy observed with high frequency (51 instances) was trial and error (st1). This strategy is frequently used in problems that provide information about the perimeter and require an interpretation of the area. At times, students chose values that were appropriate to the problem, while at other times, they were unable to do so. Even when students selected appropriate values, they often struggled to make correct interpretations due to various mathematical difficulties they encountered. Figure 1 provides an example of a response where the change in area is detected by selecting appropriate values.

Figure1

Trial and Error Strategy/Systematic Value-Giving (Left Figure (S27): Correct Interpretation; Right Figure (S2): Wrong Interpretation)

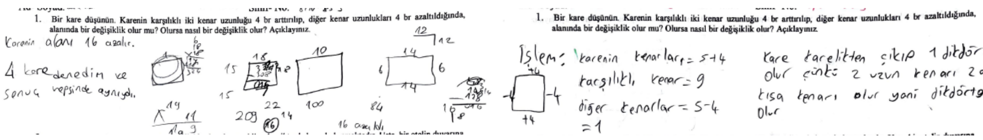
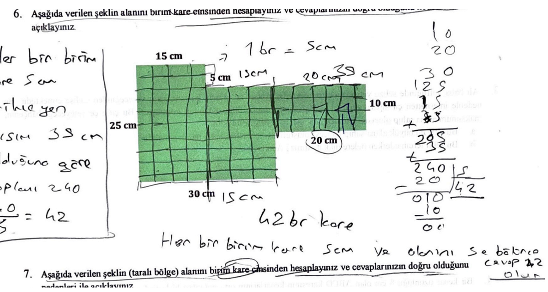


Figure 1 presents two sample solutions that employed the systematic valuing method. In the solution on the left, appropriate reasoning was used to measure the area, whereas, in the solution on the right, the focus was on interpreting the change in the geometric shape rather than the area. The strategy of using known information/formula (st9) was primarily coded in problems where students either preferred the formula or directly used the formula without manipulating the shape. In other problems where students were tasked with measuring area, they utilized area or perimeter formulas. However, they used these formulas as tools for the implementation of the basic strategy. For this reason, answers that focused on the formula in the solution were evaluated under a different code. This strategy, identified in 50 solutions, was noted as the most frequently used solution, especially in the 4th problem.

Students also used the unit square counting/dividing into unit strategy (st12) to measure area. Notably, in problem 5, which was drawn on grid paper, 20 students preferred st12 to solve the problem. In problem 6, although no grid paper was used, nine students tried to measure the area by dividing the given shape into unit squares (Figure 2). However, none of the students correctly solved problem 6 using st12.

Figure2

Dividing into unit squares and counting unit strategy (S8)



Another strategy that students employed to solve area problems is the shape-completing strategy (st11). In this strategy, students transform unfamiliar shapes into more recognizable shapes, such as squares and rectangles, by completing them. The process of completing the figure varies among students. For example, some students calculated the area of the completed shape and subtracted the area outside the requested area from this total, while another group determined the area of the smallest rectangle that encloses the asked shape. Yet another group of students aimed to solve the problem by completing the shape, but they measured the perimeter instead of the area. As these examples demonstrate, students do not necessarily arrive at the same result even when they choose the same strategy.

Another strategy that students employ to solve problems by manipulating the shape is the "dividing the shape into parts" strategy (st14). In this strategy, students attempt to solve the problem of an unfamiliar shape by breaking it down into familiar shapes. Students adopted this solution strategy in 16 of the problems examined. The students successfully applied this strategy, especially in the sixth problem. However, some students who incorrectly solved the problem divided the shape into parts but were unable to determine the dimensions, such as sides and heights, required to measure the areas of these parts. These students could not divide the area into easy-to-measure pieces due to their inability to reason correctly while breaking down the shape.

One of the strategies often used by students is measuring the area by counting the edge length unit or measuring the perimeter instead of the area (st13). Initially, solutions created in this manner were evaluated using another strategy. However, due to their frequent usage (35 instances), it was decided to categorize them as a separate strategy. Students using this strategy attempted to calculate the area by adding the side lengths given in the figure or by counting units if the length was not specified. Although this unit counting process was occasionally correct, students were generally unable to measure the perimeter length correctly.

When we examined the solutions to contextual domain problems, we found an approach that was not encountered in other problems. When tackling real-life problems, students tended to focus on their contextual experiences rather than the specific result requested in the problem. In some cases, students focused on non-mathematical aspects, such as the practicality or usability of the shapes. For instance, in the 3rd problem, where they were asked to find the maximum area that could be constructed with a fixed length, students considered the shape of the field rather than maximizing the area. Some attempted to design a field shape that would be easy to plow. Others considered factors of comfort and security. The shape was thus interpreted in various ways. In this context, in 12 solutions, students tried to solve the problem with a cultural/contextual approach (st8) rather than a mathematical solution.

Nine students used the variable use strategy (st3) while solving the area problem, and two were able to derive a generalization inherent to the structure of the problem. Five students formulated solutions using the estimating strategy (st4).

4.2. Difficulties experienced by students while solving area problems

In the analysis conducted to reveal students' difficulties while solving area problems, the students' solutions and the justifications they provided were carefully examined. As a result of this review, no difficulties (Nd) were detected for 94 answers. In these instances, the student either solved the problem correctly without encountering any difficulty, or the answer could not be categorized under any difficulty because it could not be understood. Some students had multiple difficulties with a single problem. The difficulties identified and their distribution across the problems are presented in Table 5.

Table5

Identified difficulties and distribution according to problems

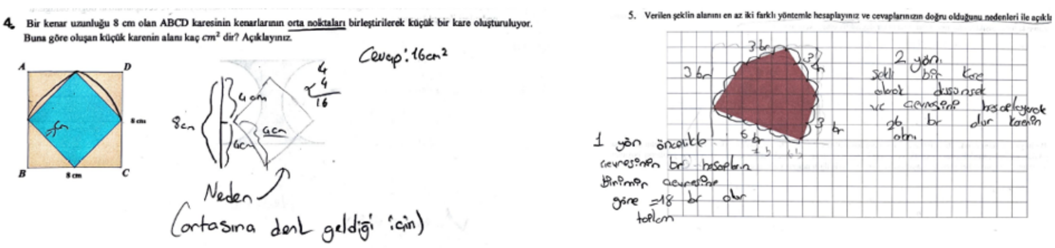
Problems	D1	D2	D3	D5	D6	D7	D9	D10	D11	D12	D13	Nd
1	19	6	12	5	8	10	-	-	-	-	5	24
2	1	2	7	2	2	2	12	2	-	-	-	22
3	-	-	7	1	3	-	-	17	1	-	-	28
4	1	6	17	-	1	-	-	-	30	-	-	12
5	-	-	11	-	3	-	6	-	22	15	-	3
6	-	1	23	-	9	-	2	-	2	7	2	7
Total	21	15	77	8	26	12	20	19	55	22	7	96

Table 5 shows the difficulties encountered across all six problems. The most frequently observed difficulty, identified 77 times, was the confusion between perimeter and area (D3). Another difficulty, detected at least once in all six problems and 26 times in total, was the difficulties students encountered or the mistakes they made during computation (D6). These errors manifested in two forms. In the first scenario, both the strategy and the computation could be incorrect. In the second scenario, students may choose the correct strategy but still arrive at an incorrect answer due to computational errors.

Students also struggled with measuring and conserving length while solving area problems (D11). They found it challenging to determine the side lengths of various shapes, which are crucial for measuring the area.

Figure3

Detecting the edge length(left) S12 andcounting the edge length unit (right) S64

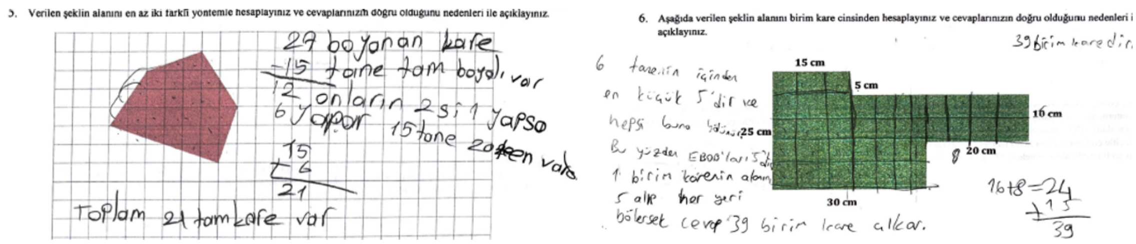


As depicted in Figure 3 (left), the student was unable to correctly interpret the edge lengths of an isosceles right triangle. Of the 75 students who participated in the study, 30 solved the problem in a manner similar to S12, mistakenly treating the isosceles right triangle as an equilateral triangle. Some students had difficulty counting units of length that were not aligned with the horizontal and vertical lines on the gridded paper (Figure 3, right). They counted these lengths as if they were on the horizontal and vertical lines, indicating a flawed understanding of length conservation.

Additionally, students faced difficulties in measuring non-square units and separating them into units (D12) while solving two problems. This difficulty was identified 22 times in total. As shown in Figure 4 (left), the student was able to count unit squares but struggled with non-square units. In Figure 4 (right), the student had difficulty dividing the given shape into units.

Figure4.

S29 (left) difficulty measuring non-square units and S27 (right) difficulty separating into units

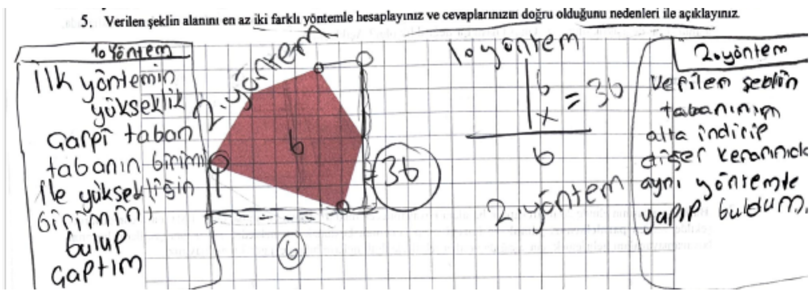


Upon examining the first problem, which required students to interpret the change made on the opposite edges of a square, it was found that the students were confused (D1) by the concepts of shape change and area change. In these and similar answers, 19 students developed an incorrect understanding that a change in shape equates to a change in area.

Another challenge faced by students while solving area problems was the difficulty in using the area formula (D9). In the process of solving 20 problems, students struggled with the application of the area formula. Some students had trouble in choosing the appropriate elements used in the formula, while others had difficulty in determining in which shapes and situations the formula was applicable. As seen in Figure 5, to find the pentagon's area on a grid paper, the student expressed one edge as the base of the shape, chose a height, and believed they could find the area by multiplying these. In addition, the only difficulty experienced by S9 in this figure was not related to using the area algorithm; S9 also struggled to determine the edge length.

Figure5

Difficulty using area formulas (s9)



Students had difficulty relating mathematics to contextual problems, and for some, the cultural perspective overshadowed the mathematical expectations of the problem. This difficulty, encountered in problems 2 and 3, was coded as ‘Difficulty relating mathematics to context/Too much focus on cultural elements’ (D10). It was also observed that students confused area with side length (D2). These students perceived the change in edge length as a change in area. Another challenge students encountered related to using the information provided in the problem to solve it (D7). Although the exact reason (difficulty in reading, reading comprehension, or mathematizing the problem, etc.) could not be determined within the scope of this study, 12 answers were found in which students either could not use the given information in solving the problem or used it incorrectly.

Furthermore, students had difficulties in identifying the given shape and understanding its basic properties (D5) while solving the area problem. This difficulty, encountered in the solutions of 8 students, manifested as struggles in determining the shape and its essential features. In the given open-ended area problems, when students were asked to interpret the change by manipulating the shape, seven answers indicated that the students believed the area would only change if there were changes in the perimeter (D13).

The challenges were divided into three basic structures. The first pertains to student difficulties regarding the confusion between area and other geometric concepts and structures. Students often confuse area with other geometric concepts such as perimeter, change in shape, and edge length. The second category of difficulties relates to the definition and formula of the concept of area. In this context, students struggle with defining the area as a covered surface, choosing/counting units appropriately, determining when the area calculation algorithm may be suitable for which shapes, determining the length required to measure the area, and discerning in which cases the area is conserved and in which cases it changes. The final difficulty encountered is processing errors, which can be described as general difficulties, an excessive focus on the context rather than the desired mathematics, and difficulty in completing the problems.

DISCUSSION, CONCLUSION AND RECOMMENDATIONS,

This study reveals the solution strategies used by secondary school students when tackling open-ended area problems and the mathematical difficulties that arise; 11 distinct strategies and 11 different mathematical difficulties were identified in the students' solutions. The most frequently used strategy by students is the shape drawing/shape redrawing strategy. A significant number of students when solving verbal problems, preferred to solve them by drawing figures. Drawing shapes is the initial step for students to mathematically express and model the problem (Duval, 1998). However, drawing shapes has brought about some difficulties. The first of these is the improper use of the information given in the problem. Sulistiowati et al. (2019) suggest that the most common difficulty students with low academic success encounter in solving geometry problems is the inability to interpret the problem within a mathematical model. Therefore, the difficulty experienced by the students in drawing the shape led them to misinterpret the problem at the first step of geometric reasoning and to produce a shape utterly different from the desired structure in the context of the problem, thus failing to arrive at the answer to the problem (Table 3, D7). Some students produced the drawing correctly but interpreted every change they observed in the drawing (change in geometric shape, perimeter, edge length) as a change in area. Upon reviewing the literature, the confusion experienced by students between the concepts of area and perimeter is a common misconception (Asil-Güzel, 2018; Machaba, 2016; Tan et al., 2016; Jirotková et al., 2019). The difficulties experienced by the students regarding the concepts of perimeter and area emerged in all the problems used in this study. Lin and Tsai (2003) revealed that approximately 50% of the 1,601 study participants confused perimeter and area. This indicates that students' understanding of the concepts of perimeter and area is limited. Moyer (2001) states that conceptual understanding of the relationship between space and perimeter depends on the ability to differentiate these qualities and explain the processes of two measurements.

In problems where students were asked to go beyond the context and arrive at generalizations (problems 1 and 3), the trial strategy by assigning value (st1) was the most prevalent. Many students, particularly those in the seventh grade who are just being introduced to the subject of equations, lack the foundation to solve the problem using variables (MEB, 2018), hence their preference for this method. Students who use this strategy typically reach a generalization and interpret the area after one or a few attempts. Some students made different and correct value assignments but confused the area with different geometric concepts, interpreting the shape, edge lengths, or perimeter instead of the area (D1. D2, D3). In addition, students experimenting with value assignment often drew the shape and evaluated the change simultaneously. Özdemir et. al (2018) noted in their study with secondary school mathematics teachers that their participants frequently used visual elements and predominantly employed the trial-and-error strategy when solving problems without variables. Some students attempted to solve the problem using the variable use strategy(st6), but only two achieved the correct result. When the results of the students who failed to arrive at the correct answer were examined, it was found that they had difficulty computing variables (D6) or calculating the perimeter instead of the area using variables (D3).

One of the strategies most frequently used by students is the reasoning strategy (st5). Students attempted to solve the problems by establishing relationships between the information presented to them in the problems and their pre-existing knowledge. Reasoning is a broad structure that encompasses mental actions such as an individual's perception of the situation, the creation of quantities related to this situation, and the re-establishment of relationships between the quantities created (Moore et al., 2009; Thompson, 1989). This strategy was particularly favored by students in problems presented in a social context. Jack & Thompson (2017) suggest that students can leverage their everyday life experiences with quantitative reasoning skills, enabling them to solve the problem in a meaningful way. In the process of reasoning through the given problems, some students completely detached from the context, mathematized the relationships given, and established logical relationships. However, it was observed that some students overly focused on the context and based their reasoning entirely on their cultural background (D10). In a problem where the student was asked to create the largest area for a fixed perimeter, rather than focusing on the shapes they observed around them and creating the largest area for the garden, they believed that the most suitable shape for plowing the garden would be more appropriate. Although culture and mathematics may initially appear as distinct structures at first glance, it has been posited that mathematics is born from culture and that culture holds an important place in mathematics teaching and applications, leading researchers such as D'Ambrosio (1995) and Bishop (1988) to propose the concept of ethnomathematics. The relationship between mathematics and culture has not consistently served as a facilitator. As evidenced by the findings, the intricate interplay between culture and mathematics can, in fact, present difficulties. Brousseau (2002) delineated these difficulties as cultural obstacles, representing one of the mathematical obstacles.

Students adopted a formula-based approach when solving area problems (st9). These students immediately resorted to the area formula and aimed to solve problems by substituting the given values. Using this strategy requires specific prior knowledge, much like with other strategies. For example, recognizing two-dimensional geometric shapes, determining that the shape is unsuitable for using this algorithm, selecting the correct height and edge length, and performing the operations correctly are required (Jupri et al., 2020; Huang et al. 2013). However, the findings of the study indicate that students encountered various difficulties in using the area algorithm. Some students chose one side as the base without considering the properties of the shape and then attempted to calculate the area by considering the vertical length between the points furthest from the base as the height. In contrast, others thought that the shape's area could be obtained by multiplying the lengths of two consecutive edges. These answers from the students reveal that they cannot fully establish the relationship between the concept of area and its formula, corroborating this finding obtained in other studies (Tan et al., 2016; Clements & Stephan, 2004; Kidman & Cooper, 1997; Battista, 1982). Even if the students correctly understand the area calculation algorithm and determine which lengths should be chosen to substitute into the calculation algorithm, if the length is not directly provided to the students, it was observed that the students could not calculate the area because they did not measure the length. This difficulty in determining the edge length (D11), the second most common difficulty experienced by students, has been identified as a significant problem students face in calculating the area. Students' lack of length measurement skills poses a substantial difficulty in calculating area. Asil-Güzel (2018), in her study to reveal students' understanding of length measurement and comparison and the difficulties they experienced, found that 7th and 8th-grade students lacked skills such as determining length and counting units. Considering the findings obtained and the information in the literature, students' deficiencies in the preliminary knowledge required for area calculation directly affect area measurement.

When tasked with calculating the area of shapes that lack a direct calculation algorithm, students resort to strategies such as completing the shape they are familiar with (st11), breaking it down into familiar shapes (st14), and dividing it into unit squares/counting units (st12). Although these strategies do not guarantee the resolution of such problems, they are deemed reasonable approaches to solving them (Driscoll et al., 2007). Fuys et al. (1988) assert that if students possess basic geometric knowledge about shapes, they will develop a better understanding of the area formula. Consequently, they can calculate non-regular geometric shapes by applying their known shapes and establishing connections between formulas. However, it is observed that students struggle with the properties of basic geometric shapes. Additionally, other student difficulties encountered when using the St11, St12, and St14 strategies generally include calculation errors, difficulties in using the area formula, difficulties in counting unit squares, and particularly difficulties in determining the side length. Regardless of how correctly a student chooses the strategy that can potentially lead to the solution, the correct strategy only guarantees the attainment of the correct answer if the student has prior knowledge. This finding from the study aligns with Adıgüzel-Doğan's (2021) discovery that students' algebraic reasoning and geometric knowledge structures interact, and that students cannot solve the problem if one is lacking.

Additionally, students required assistance in understanding how a given change affected the area, in which cases the area was conserved, and in which cases it changed (D13). Piaget, et al (1960) define the concept of conservation as the understanding that certain properties of objects remain constant despite changes in their appearance. Although it is acknowledged that this may vary according to societal and individual characteristics, the conservation of the concept of area is expected to be acquired between the ages of 8–10 (Piaget et al., 1960). Although the 7th and 8th-grade students participating in the study have reached this age threshold and possess notions about area measurement, they have difficulty detecting the mutable and immutable features of the shape. Ersoy (2006) emphasizes that the conservation of a feature to be measured should be taught before teaching the measurement of the feature. However, the findings indicated that there needs to be an enhancement in the students' understanding regarding the conservation of area and length.

One notable confusion that students experience between area and perimeter is the student's erroneous understanding that the area of a rectangle, whose perimeter remains unchanged, will not change. Baturo and Nason (1996) suggested that this difficulty experienced by students stems from their limited understanding of the area and that it is necessary to evaluate the area from two perspectives: static and dynamic. However, Batura and Nason (1996) also add that dynamic perspective is generally not incorporated into curricula.

While solving the problems presented to them, students can generally select the appropriate solution strategy to solve the problem. This indicates that students follow a conscious path when choosing an area problem. However, the fact that students encountered various difficulties in applying the strategy they chose revealed that the students possessed procedural knowledge rather than a conceptual understanding of the area, that they had deficiencies in their prior knowledge, and that they were unable to select the information provided and requested in the problems. For this reason, it may be beneficial to base education on the development of conceptual understanding, transcending the memorization and application of procedures in the process of students’ learning area measurement. This may enable students to evaluate the reasonableness of their answers and verify the outcomes of their actions.

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GENİŞ ÖZET

Giriş

Fiziksel dünyayı ölçme, insanoğlunun en eski uğraşlarından biridir (Drake, 2014). Uzunluk, alan, hacim gibi bilimin, ticaretin ve gündelik yaşamın içinde olan kavramların sayısallaştırılması, hem iletişim için hem de anlaşmazlıkların ve haksızlıkların önlenmesi için doğal bir ihtiyaç olarak ortaya çıkmıştır. Smith ve Barrett (2017) ölçmeyi, varlıkların ölçülebilir özelliklerini bir birim ile mukayese ederek sayısallaştırma süreci olarak tanımlamaktadır. Ölçme, fiziksel niteliklerin ölçülmesi ve kontrolü için araçlar sağladığından bilimin ve teknolojinin gelişmesi ve uygulanması için önem arz etmektedir (Crosby, 1997). Ölçme aynı zamanda dünyanın birçok yerinde ilkokul ve ortaokul matematiğin temel ve önemli bir öğrenme alanıdır (MEB, 2018; Van de Walle, vd., 2012). Matematik öğretiminin tamamında olduğu gibi alan ölçme konusunda da öğrencilerin yaşadıkları zorlukları, yaptıkları hataları-doğruları, sahip oldukları bilgileri ortaya koymak için açık uçlu problemler kullanışlı ve kıymetli araçlardır (NCTM, 2000). Farklı problem türleri ve bu problem türleri için farklı çözüm stratejilerine verilen önemin arttığı gerek ders kitaplarında gerekse öğretim programında yapılan vurgulardan fark edilmektedir. Ayrıca MEB (2023) tarafından yayınlanan sınav yönergesinde, okullarda yapılacak ortak sınavlarda kullanılmak üzere açık uçlu sorular kullanılması hususunda bilgilendirmeler yapıldığı görülmüştür. Açık uçlu sorular öğrencilerin yalnızca ne bildiklerinin değil, yaptıkları gerekçelendirmeler yolu ile eksiklerinin neler olduğu, sahip oldukları kavram yanılgıları ve yaşadıkları matematiksel zorlukların ortaya çıkarılmasına da olanak tanımaktadır. Matematiksel gerekçelendirme, matematiksel bir varsayımın veya iddianın doğruluğunu belirlemek ve açıklamak için önemli bir enstrümandır (Balacheff 1988; Harel & Sowder, 2007; Simon ve Blume, 1996). Staples ve Bartlo (2010) öğrencilerden gerekçelendirme talep edildiğinde, çeşitli fikirlerle boğuşarak yeni fikirler elde etmek için matematiksel bağlantılar araması gerektiğinden öğrencilerin matematiksel kavramlar üzerinde daha derin düşünmeye yönlendirildiğini ifade etmektedir. Gerekçelendirme talep edildiğinde öğrencilerden varsayımlarını veya yaptıkları bir çözümün doğruluğunu argümanlarla kanıtlamaları ya da açıklamaları beklenir (Pamungkas vd., 2018). Gerekçelendirmeler yoluyla öğrencilerin düşünme akışını bilmek öğretmenlerin, öğrencilerin kullandıkları stratejileri ve karşılaşabilecekleri sorunları veya zorlukları analiz edebilmelerine yardımcı olabilir. Bu kapsamda bu çalışmanın amacı ortaokul öğrencilerinin açık uçlu alan problemleri çözerken kullandıkları çözüm stratejilerini ve ortaya çıkan matematiksel zorlukları ortaya koymaktır. Bu çalışma bağlamında aşağıdaki sorulara cevap verilmeye çalışılacaktır. Öğrencilerin açık uçlu alan problemleri çözerken:

Kullandıkları stratejiler nelerdir?
Karşılaştıkları matematiksel zorluklar nelerdir?
Bu zorluk ve stratejiler ne şekilde ilişkilidir?

Yöntem

Bu çalışma doğası gereği nitel bir çalışma olup tarama modeli benimsenmiştir. Araştırmalarda mevcut bir olgu olduğu haliyle betimlenecekse tarama modeli tercih edilebilir (Patton, 2014). Bu çalışmada da öğrencilerin açık uçlu alan problemlerinin çözümlerindeki gerekçelendirmeleri incelenerek kullandıkları stratejiler ve yaşadıkları zorluklar tespit edilmeye çalışılmıştır. Çalışmanın katılımcıları Türkiye'nin Güneydoğu bölgesinde bulunan bir büyükşehir merkezinde eğitimlerine devam eden 75 ortaokul öğrencisinden oluşmaktadır. Bu öğrencilerin 27 tanesi 7. sınıf, 48 tanesi 8. sınıf öğrencisidir. Bu çalışmanın veri toplama aracı 6 tane açık uçlu alan ölçme/hesaplama ve karşılaştırma sorusundan oluşmaktadır. Veri toplama aracının problemleri tasarlanırken üç unsur dikkate alınmıştır. Bunlardan ilki DIVINE çerçevesinde yer alan amaçlarına göre gerekçelendirme türleridir. İkincisi ise alan ölçme/hesaplama bağlamında yaygın olarak görülen öğrenci zorlukları ve üçüncüsü farklı problem çözme stratejileridir. Çalışma kapsamında öğrencilerin alan problemi çözerken kullandıkları kendilerine özgü ifadelere dayalı olarak elde edilen veriler, içerik analizi ile değerlendirilmiştir.

Sonuç ve Tartışma

Ortaokul öğrencilerinin açık uçlu alan problemleri çözerken kullandıkları çözüm stratejilerini ve ortaya çıkan matematiksel zorlukları ortaya koymak amacı ile yapılan bu çalışmada öğrencilerin çözümlerinde, 11 farklı strateji ve 11 farklı matematiksel zorluk tespit edilmiştir. Öğrencilerin en sık kullandığı strateji şekil çizme/şekli yeniden çizme stratejisidir. Çok sayıda öğrencinin özellikle sözel olarak ifade edilen problemleri çözerken şekil çizmeleri dikkat çekici bir bulgudur. Şekil çizme, öğrencilerin problemi matematiksel olarak ifade etme ve modellemelerinin ilk adımı olarak görülmektedir (Duval, 1998). Ancak şekil çizme beraberinde bazı zorlukları ortaya çıkarmıştır. Bunlardan ilki soruda verilen bilgileri şekle doğru yerleştirememektir. Sulistiowati, vd., (2019) özellikle akademik başarısı düşük öğrencilerin geometri problemleri çözmede en sık yaşadıkları zorluğun, problemi matematiksel bir model içinde yorumlayamama olduğunu ifade etmektedir.

Öğrencilerin en sık kullandıkları stratejilerden biri muhakeme yapma stratejisidir. Öğrenciler, problemde kendilerine sunulan bilgiler ve var olan bilgileri arasında ilişkiler kurarak problemleri çözmeye çalışmışlardır. Muhakeme geniş bir yapı olup bireyin durumu algılaması, bu durumla ilişkili nicelikler oluşturması, oluşturduğu nicelikler arasında tekrar ilişki kurması gibi zihinsel eylemleri içermektedir (Moore, Carlson ve Oehrtman, 2009; Thompson, 1989). Öğrencilerin özellikle sosyal bir bağlam içerisinde sunulan problemlerde bu stratejiyi yoğun olarak tercih ettikleri gözlemlenmiştir. Smith ve Thompson (2007), niceliksel muhakeme becerisi ile öğrencilerin günlük hayattaki tecrübelerinden yararlanılabileceğini belirtmiştir ve problemi onlar için anlamlı bir şekilde çözülmesine imkân vereceğini ifade etmiştir. Bazı öğrenciler verilen problemlerde muhakeme yaparken bağlamdan tamamen sıyrılıp verilen ilişkileri matematikleştirmiş ve mantıksal ilişkiler kurmuştur. Ancak bazı öğrencilerin bağlama fazla odaklandıkları ve muhakemelerini tamamen içinde bulundukları kültüre bağlı olarak yaptıkları görülmüştür (Z10). Sabit bir çevre için en geniş alanın oluşturulması istenen problemde öğrenci bahçe için çevresinde gördüğü şekillere odaklanarak en geniş alanı oluşturmak yerine bahçenin sürülebilmesi için uygun şeklin daha doğru olabileceğini düşünmüşlerdir. Kültür ve matematik ilk bakışta farklı yapılar gibi gelse de matematiğin kültürle doğduğu, matematik öğretiminde ve uygulamalarında önemli yeri olduğu ifade edilmişmiş ve D'Ambrosso (1995) ve Bistop (1995) gibi araştırmacılar tarafından etnomatematik kavramı ortaya konmuştur. Matematik ve kültür arasında ki bu ilişki bulgulardan da görüldüğü gibi her zaman kolaylaştırıcı bir rol üstlenmemiştir. Bulgularda da görülen bu zorluk Brousseau (2002) tarafından matematiksel engellerden biri olan kültürel engeller olarak literatüre dahil edilmiştir.

Zorlukların genel olarak üç kategoriye ayrıldığı görülmüştür. Bunların ilki alan ile diğer geometrik kavram ve yapılar arasında yaşanan karmaşaya dair öğrenci zorluklarıdır. Öğrencilerin alanı; çevre, şekildeki değişim ve kenar uzunluğu gibi diğer geometrik kavramlarla karıştırmaktadır. İkinci grup zorluk ise alan kavramının tanımına ve formülüne dair zorluktur. Bu kapsamda öğrenci alanı kaplanan bir yüzey olarak tanımlamakta güçlük çekmekte, birim seçme/saymayı uygun şekil de yapamamakta, alan hesaplama algoritmasının ne zaman hangi şekiller için uygun olabileceğini tespit edememekte, alanı hesaplamak için gerekli uzunluğu tayin edememekte ve alanın hangi durumlarda korunduğu hangi durumlarda değiştiğini tespit edememektedir. Son olarak karşılaşılan zorluk ise genel zorluklar olarak ifade edilebilecek işlem hataları, bağlama istenen matematikten daha fazla odaklanma ve soruyu tamamlamakta yaşanan zorluktur.