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Research Article

APPROACHES TO THE DESIGN OF A PLANAR PARALLEL MANIPULATOR

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ABSTRACT
Manipulator is an important part of a whole robot assembly, forming the mechanical infrastructure of a mechatronic system. Selection of the manipulator affects a broad area extending from modelling to design, from control to operation and furthermore from accuracy to its economy. This study aimed methods of design and operation of a parallel planar robotic assembly have been demonstrated. Modules of the assembly with two degrees of freedom have been designed on a two-point, two-velocity and three point-position bases. Ways of actuating and controlling the motion of the assembly have been shown. Efficiency and effectiveness of the approaches have been illustrated numerically.

Keywords: *Mechanism, Manipulator, Robotic*

1. INTRODUCTION

Manipulator is an important part of a whole robot assembly, forming the mechanical infrastructure of a mechatronic system. Selection of the manipulator affects a broad area extending from modelling to design, from control to operation and furthermore from accuracy to its economy. Manipulator can be constructed by bringing together rotary and/or sliding elements or a combination of these in a suitable manner. Within this context, open or closed kinematic chains formed as such will result in the so-called serial or parallel robotic structures, (Duffy, 1996).

The most fundamental manipulator types like cartesian, cylindrical, spherical, articulated arm and scara are the most widespread examples of serial manipulator. Most classical works rely upon the serial manipulator, which is based on the open chain, (Koren, 1987; Stadler, 1995; Fu *et al.*, 1987, Groover *et al.*, 1986). Although serial manipulators have a high maneuverability within a large workspace, they are subject to significant limitations. First of all, they have limited load carrying capacities due to their highly deformable structures, which are prone to vibrations under large velocities. Additionally, it is probable that the fact that an actuator is needed at each joint, for a serial manipulator having many joints in an open chain, might lead to a rise in the initial and operating costs of the robotic assembly. On the other hand, most of the issues mentioned above are solved by using parallel manipulators facing only the limitation of having small workspaces. Thus, such advantages have invited the research attention on parallel manipulators in recent years, (Innocenti and Castelli, 1990; Bernier *et al.*, 1995, Harris, 1995; Liu, 1995).

Since the workspace of a parallel manipulator is limited, the problem of overlapping the actual space in which the physical tasks like welding, cutting conveying etc. Are to be fulfilled with that of the manipulator becomes significant. The solution of the problem passes through the accurate positioning of the manipulator. Thus, determining optimum values of all the adjustable parameters in a robot assembly constitutes a design task.

Here in this work, approaches to the design of a parallel planar manipulator with two degrees of freedom have been shown. The design of the manipulator has been reduced to the design of modules which considerably dissolve the complexity of the original assembly, mathematically and physically, thus always assuring closed-form solution. Then how actuators may be utilized to operate the assembly and to form an analog robot have been demonstrated.

2. THEORY

The kinematic scheme of the parallel manipulator in consideration is drawn in Fig. 1.

It can be seen that the parallel manipulator in question can be constituted by bringing together two of the basic module shown in Fig. 2.

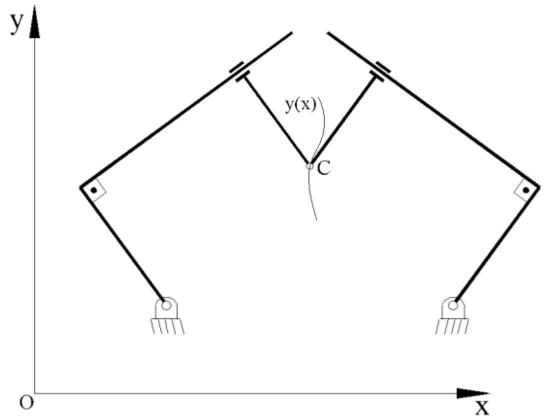


Fig. 1. Kinematic scheme

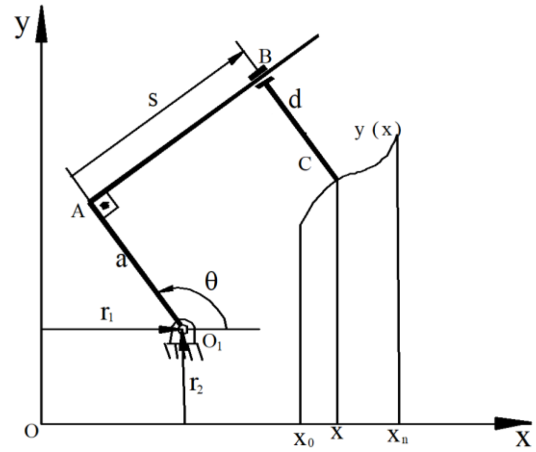


Fig. 2. Module parameters

The fundamental problem here is to determine the most appropriate values of the module parameters involved such that the end point C of the module follow a desired trajectory y (x) within [x_0, x_n] domain. To this end, the following can be written from Fig. 2:

$$x = r_1 + (a-d)\cos\theta + s\sin\theta \qquad (1)$$

$$y = r_2 + (a-d)\sin\theta - s\cos\theta \qquad (2)$$

If (a-d) is designated by b and s is eliminated from the above equations, then a displacement function G(x, y, θ) characterizing the motion of the module on the trajectory is obtained:

$$G(x, y, \theta) = y\sin\theta + x\cos\theta - r_1\cos\theta - r_2\sin\theta - b = 0 \qquad (3)$$

Considering that the robot arm will rotate about O according to the θ' motion variable starting from an initial position θ_0, the following relationships can be written to meet the motion co-ordination requirements:

$$\theta = \theta_0 + \theta' \qquad (4)$$

$$\theta' = r_x(x - x_0) \qquad (5)$$

$$r_x = \Delta\theta / \Delta x,\ \Delta\theta = \theta_n - \theta_0,\ \Delta x = x_n - x_0 \quad (6)$$

where θ_n is the final position of the robot arm corresponding to the point (x_n) of the given trajectory. Evaluating the above relationships together with the trigonometric identities and rearranging will yield the following expression:

$$G(x,y,r_1,r_2,b,\theta_0,\theta') = \sin\theta_0(y\cos\theta' - x\sin\theta')$$

$$+\cos\theta_o(y\sin\theta' + x\cos\theta') + r_1(\sin\theta_o \sin\sin\theta' - \cos\theta_o\cos\theta')$$

$$-r_2(\sin\theta_o\cos\theta' + \cos\theta_o\sin\theta') - b = 0 \quad (7)$$

Now, it is possible to obtain velocity relationships by taking the first derivative of the displacement function G with respect to time:

$$\frac{dG}{dt} = \frac{dG}{d\theta}.\frac{d\theta}{dt} \quad (8)$$

If the velocities of the end point of the manipulator in the x and y directions are represented by V_x and V_y, respectively, and the angular speed of the rotating arm is designated by ω, then the following will come out of (8):

$$\frac{1}{\omega}.\frac{dG}{dt} = \sin\theta_0(\frac{V_y}{\omega}\cos\theta' - \frac{V_x}{\omega}\sin\theta' - y\sin\theta' - x\cos\theta')$$
$$+\cos\theta_0(\frac{V_y}{\omega}\sin\theta' + \frac{V_x}{\omega}\cos\theta' + y\cos\theta' - x\sin\theta')$$
$$+r_1(\cos\theta_0\sin\theta' + \sin\theta_0\cos\theta')$$
$$+r_2(\sin\theta_0\sin\theta' - \cos\theta_0\cos\theta') = 0 \quad (9)$$

where

$$\omega = \frac{d\theta}{dt} = \frac{d\theta'}{dt};\ V_y = \frac{dy}{dt};\ V_x = \frac{dx}{dt} \quad (10)$$

Examination of the basic displacement and velocity functions (7) and (9) will reveal that there are four available parameters (r_1, r_2, θ_0, b) for formulating a design. One approach for a formulation of manipulator design is to require that the end point of the manipulator fit to specified two-position and two-velocity values. In such a context, if Precision-Point or Accuracy-Point (Hartenberg and Denavit, 1964) Subdomain (Akçalı and Dittrich, 1989a) and Galerkin (Akçalı and Dittrich, 1989b) methods are applied to displacement and velocity functions, then the following will result as the basic design equations:

$$P_i(\theta_0) + r_1 R_i(\theta_0) - r_2 Q_i(\theta_0) - bE_i. = 0 \quad i = 1,2 \quad (11)$$

$$H_i(\theta_0) + r_1 Q_i(\theta_0) + r_2 R_i(\theta_0) = 0 \ \ i = 1,2 \quad (12)$$

where:

$$Q_i(\theta_0) = D_i\sin\theta_0 + C_i\cos\theta_0 \quad ;$$
$$R_i(\theta_0) = C_i\sin\theta_0 - D_i\cos\theta_0$$

$$H_i(\theta_0) = K_i\sin\theta_0 + L_i\cos\theta_0 \quad ;$$
$$P_i(\theta_0) = A_i\sin\theta_0 + B_i\cos\theta_0 \quad i = 1,2 \quad (13)$$

The coefficients contained in (13) are defined is accordance with each method as follows: Accuracy-Point method takes into account points x_i i =1,2:

$$A_i = y_i\cos_i' - x_i\sin\theta_i';\ B_i = y_i\sin\theta_i' + x_i\cos\theta_i'$$

$$C_i = \sin\theta_i' \ ;\ D_i = \cos\theta_i';$$
$$E_i = 1.0 \quad \} \quad i = 1,2 \quad (14)$$
$$K_i = (\frac{V_y}{\omega})_i D_i - (\frac{V_x}{\omega})_i C_i - B_i \quad ;$$
$$L_i = (\frac{V_y}{\omega})_i C_i + (\frac{V_x}{\omega}) D_i + A_i$$

Subdomain Method considers subintervals [x_{i-1}, x_i] i = 1,2:

$$A_i = \int_{\theta_{i-1}'}^{\theta_i'} (y\cos\theta' - x\sin\theta')d\theta' \quad ;$$
$$B_i = \int_{\theta_{i-1}'}^{\theta_i'} (y\sin\theta' + c\cos\theta')d\theta'$$
$$\} \quad i = 1,2$$
$$C_i = \int_{\theta_{i-1}'}^{\theta_i'} \sin\theta' d\theta' = \cos\theta_{i-1}' - \cos\theta_i' \quad ;$$
$$D_i = \int_{\theta_{i-1}'}^{\theta_i'} \cos\theta' d\theta' = \sin\theta_i' - \sin\theta_{i-1}'$$

$$E_i = \int_{\theta_{i-1}'}^{\theta_i'} d\theta' = \theta_i' - \theta_{i-1}' \quad ;$$
$$K_i = \int_{\theta_{i-1}'}^{\theta_i'} \left[\frac{1}{\omega}\frac{d}{dt}(y\cos\theta' - x\sin\theta')\ \right]d\theta' \quad i = 1,2 \quad (15)$$

$$L_i = \int_{\theta_{i-1}'}^{\theta_i'} \left[\frac{1}{\omega}\frac{d}{dt}(y\sin\theta' + x\cos\theta')\ \right]d\theta'$$

The coefficients appearing in (13) are evaluated by Galerkin method, with reference to selected weighting functions w_i i = 1,2 as shown below:

$$A_i = \int_{\theta_0'}^{\theta_n'} (y\cos\theta' - x\sin\theta')w_i d\theta' \quad ;$$
$$B_i = \int_{\theta_0'}^{\theta_n'} (y\sin\theta' + x\cos')w_i d\theta'$$
$$C_i = \int_{\theta_0'}^{\theta_n'} \sin\theta' w_i d\theta' \ ;\ D_i = \int_{\theta_0'}^{\theta_n'} \cos\theta' w_i d\theta'$$
$$E_i = \int_{\theta_0'}^{\theta_n'} w_i d\theta' \quad \} \ i = 1,2 \quad (16)$$

$$K_i = \int_{\theta_n^{'}}^{\theta_0^{'}} \left[(\frac{V_y}{w})\cos\theta' - (\frac{V_x}{w})\sin\theta' - y\sin\theta' - x\cos\theta' \right] w_i d\theta'$$

$$L_i = \int_{\theta_0^{'}}^{\theta'n} \left[(\frac{V_y}{\omega})\sin\theta' + (\frac{V_x}{\omega})\cos\theta' + y\cos\theta' - x\sin\theta' \right] w_i d\theta'$$

In the solution phase of design formulation, which consists of four non-linear equations, b, r_2 and r_1 are eliminated, reducing the set (11)-(12) to the following:

$$P_{ss}\sin^2\theta_0 + P_{sc}\sin\theta_0\cos\theta_0 + P_{cc}\cos^2\theta_0 = 0 \quad (17)$$

In the general case, there are two solutions given by:

$$\theta_{0_+^-} = \tan^{-1}\left(\frac{-P_{sc}\,{}^{-}_{+}(P_{sc}^2 - 4P_{ss}P_{cc})^{1/2}}{2P_{ss}}\right) \quad (18)$$

where:

$$P_{ss} = P_{1ss}R_2^{''} - P_{2ss}R_1^{''} \quad ; \quad P_{sc} = P_{1sc}R_2^{''} - P_{2sc}R_1^{''} \quad ;$$
$$P_{cc} = P_{1cc}R_2^{''} - P_{2sc}R_1^{''} \quad (19)$$

$$\left.\begin{aligned} A_1' &= A_1E_2 - A_2E_1 \;;\; B_1^{'} = B_1E_2 - B_2E_1 \\ C_1' &= C_1E_2 - C_2E_1 \;;\; D_1^{'} = D_1E_2 - D_2E_1 \end{aligned}\right\} \quad (20)$$

$$\left.\begin{aligned} R_j^{''} &= C_1^{'}C_j + D_1^{'}D_j \;;\; P_{jss} = A_1^{'}C_j + D_1^{'}K_j \\ P_{jsc} &= B_1^{'} - C_j - A_1^{'}D_j + C_1^{'}K_j + D_1^{'}L_j \;;\; P_{jcc} = C_1^{'}L_j - B_1^{'}D_j' \end{aligned}\right\} j = 1,2 \quad (21)$$

Since for a given tangent value, there exist two angles separated by 180°, four possible angles might satisfy equation (17). Thus, in order to decide on the technically meaningful ones as well as on the quality of outcome, a motion analysis should be carried out.

If the actuation of the manipulator is based upon (θ, s) which are computed by (4)-(6) and (22) given below, then position error e is evaluated by means of (1), (2), (22) and (23), where $x_{th}, y_{th}, x_{ac}, y_{ac}$ are theoretical and actual co-ordinates, respectively,

$$s = \left[(x_{th} - r_1)^2 + (y_{th} - r_2)^2 - b^2\right]^{1/2} \quad (22)$$

$$e = \left[(x_{th} - x_{ac})^2 + (y_{th} - y_{ac})^2\right]^{1/2} \quad (23)$$

In order to determine error in velocity, first theoretical velocities V_{xth}, V_{yth} and V_{th}, then actual velocities V_{xac}, V_{yac} and V_{ac} are calculated with reference to (24)-(26), finally ending in (27).

$$\frac{V_{xth}}{\omega} = \frac{1}{r_x};\; \frac{V_{yth}}{\omega} = (\frac{dy}{dx}).\frac{1}{r_x};\; \frac{V_{th}}{\omega} = \left[(\frac{V_{xth}}{\omega})^2 + (\frac{V_{yth}}{\omega})^2\right]^{1/2} \quad (24)$$

$$\frac{1}{\omega}\frac{ds}{dt} = \frac{1}{s}\left[(x_{th} - r_1)\frac{V_{xth}}{\omega} + (y_{th} - r_2)\frac{V_{yth}}{\omega}\right] \quad (25)$$

$$\frac{V_{xac}}{\omega} = -b\sin\theta + \frac{1}{\omega}(\frac{ds}{dt})\sin\theta + s\cos\theta$$

$$\frac{V_{yac}}{\omega} = b\cos\theta - (\frac{ds}{dt})\frac{1}{\omega}\cos\theta + s\sin\theta \quad ;$$

$$\frac{V_{ac}}{\omega} = (V_{xac}^2 + V_{yac})^{1/2} \quad (26)$$

$$\frac{e_v}{\omega} = \left[(V_{xth} - V_{xac})^2 + (V_{yth} - V_{yac})^2\right]^{1/2} \quad (27)$$

3. SIMPLIFIED APPROACH

By letting a = d or b = 0 in Fig. 2, and by requiring that three point-positions on the specified trajectory be satisfied in the sense of Accuracy-Point, Subdomain and Galerkin methods, by the end point of the robot arm, a simplified approach can be made to the problem. In that case, the displacement function G becomes:

$$G(x, y, \theta) = y - r_2 - (x - r_1)\tan\theta = 0 \quad (28)$$

Then the design equations take the form of a set of three linear equations as shown below:

$$A_i - B_i t_0 + C_i t_1 + D_i t_2 = 0 \text{ i} = 1,2 \quad (29)$$

where:

$$t_0 = \tan\theta_0;\; t_1 = a_1 + a_2 t_0;\; t_2 = a_1 t_0 - a_2 \quad (30)$$

Coefficients in Accuracy-Point Method are as follows:

$$A_i = y_i - x_i\tan\theta_i^{'};\quad B_i = x_i + y_i\tan\theta_i^{'};\quad C_i = \tan\theta_i^{'};$$
$$D_i = 1 \quad (31)$$

These in Subdomain Method are:

$$\left.\begin{aligned} A_i &= \int_{\theta i-1}^{\theta_{i_i}^{'}}(y - x\tan\theta')d\theta';\; B_i = \int_{\theta i-1}^{\theta_{i_i}^{'}}(x + y\tan\theta')d\theta'; \\ C_i &= \int_{\theta i-1}^{\theta_{i_i}^{'}}\tan\theta' d\theta';\; D_i = \int_{\theta i-1}^{\theta_i^{'}}d\theta' \end{aligned}\right\} \quad (32)$$

In Galerkin Method, the coefficients are defined as such:

$$A_i = \int_{\theta_{i-1}}^{\theta_i'} (y - x\tan\theta')w_i d\theta';$$
$$B_i = \int_{\theta_{i-1}}^{\theta_i'} (x + y\tan\theta')w_i d\theta';$$
$$C_i = \int_{\theta_{i-1}}^{\theta_i'} \tan\theta' w_i d\theta'; \; D_i = \int_{\theta_{i-1}}^{\theta_i'} w_i d\theta' \qquad (33)$$

Solution of the design equations yields the sought set (r_1, r_2, θ_0) as given below:

$$\theta_0 = \tan^{-1}(t_0) \;\; ; \;\; r_2 = (t_1 t_0 - t_2)/(1+t_0^2) \;\; ; \;\; r_1 = t_1 - r_2 t_0 \qquad (34)$$

where:

$$t_0 = -A''/B'' \;\; ; \;\; t_1 = (B_2' t_0 - A_2')/C_i \;\; ; \;\; t_2 = \left[-A_1 + B_1 t_0 - C_1(B_2' - A_2')/C_2' \;\right]/D_1 \;\; \} \qquad (35)$$

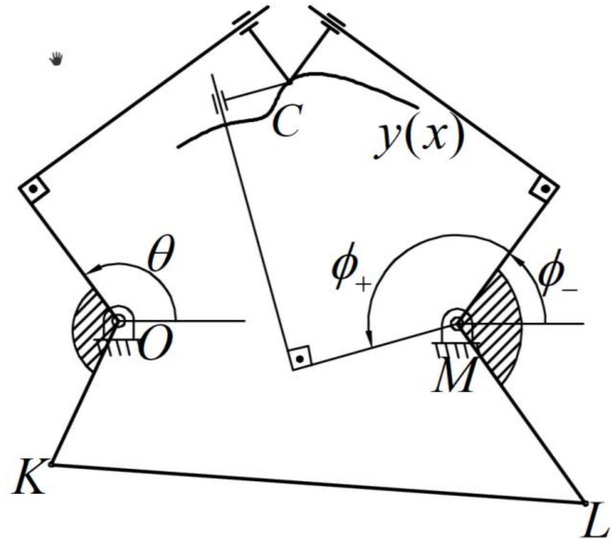
$$A_j' = D_k A_1 - D_1 A_k \;\; ; \;\; B_j' = D_k B_1 - D_1 B_k \;\; ; \;\; C_j' = D_k C_1 - D_1 C_k \quad j = 1,2 \quad k = j + 1 \qquad (36)$$

$$A'' = C_2' A_1' - C_1' A_2' \; ; \; B'' = B_2' C_1' - B_1' C_2' \qquad (37)$$

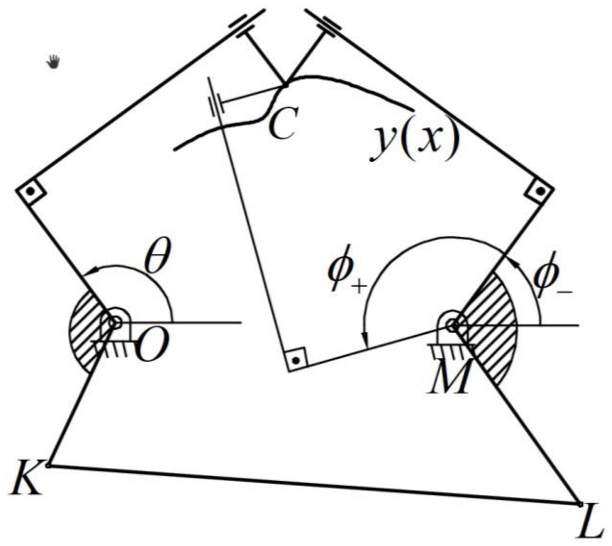
To secure a pair of solutions needed in the manipulator, the solution process is implemented twice by a change in method or some input parameters like the amount of arm rotation or the sense of rotation, if necessary.

4. ACTUATION POSSIBILITIES

The basic module is, in fact, a serial manipulator, Fig. 2, while the manipulator constructed out of two or possibly more basic modules will be of parallel type, securing more stiffness by its mechanical structure. One disadvantage of the serial manipulator is that one actuator should be available at each joint to the loss of payload capacities of the robot assembly. Thus, the advantageous feature of the parallel robot, namely having less actuators than the number of joints, offers possibilities of using analogously programmable actuators attached to the ground. Since actuation of the manipulator under consideration depends on slider displacement (s) and arm rotation (θ), possible analogously programmable actuators are either a function generating four-bar or an inverted slider-crank mechanism, the dimensions of which are continuously adjustable according to the design of the generator, which changes by trajectory (y), Fig. 3 (a), (b).



(a)



(b)

Fig. 3. Actuators of a parallel manipulator

The functions of the 4-bar OKML in Fig. 3(a) and the inverted slider-crank QOD in Fig. 3(b) are to provide the necessary rotation ϕ required about M and the needed slider displacement (s) along AB at the right time for the fulfilment and control of the trajectory task, just like the supply of right voltages at the right time in the case of electrical drive. In other words, two-degree –of-freedom manipulator receives one rotary actuation (θ) from the motor at ground pivot O, and the other actuation either at ground pivot M through OKML 4-bar (ϕ) angle generator, instead of a motor, Fig. 3(a), or along motion direction AB through QOD slider-crank (s) displacement generator instead of a hydraulic or pneumatic drive on the moving arm, Fig. 3(b). In this manner, the dimensions of the 4-bar or the slider-crank function generators can be viewed as an information storage medium transforming input data like trajectory, manipulator design values and motor rotation into new actuation variables like ϕ or s in accordance with the following mathematical relationships

$$s = \frac{r_2 + b\sin\theta - y}{\cos\theta} \tag{38}$$

$$\tan\frac{\phi}{2} = \frac{A_{+}^{-}\sqrt{A^2 + B^2 - C^2}}{B + C} \tag{39}$$

with $A = (r_4 - y)\cos\theta$; $C = -h\cos\theta$;

$$B = (y - r_2)\sin\theta + (r_3 - r_1)\cos\theta - b \tag{40}$$

where (r_3, r_4, h) are the parameters of the second serial manipulator like (r_1, r_2, b) in the first serial manipulator.

From the discussion above, it is to be understood that next to the rotary actuator at O, if the two-degree-of-freedom robot assembly is to be brought into motion to follow a trajectory y by means of a programmed motion of the slider, then the slider-crank is designed with a suitable method like (Akçalı, 1987) such that functional relationship (38) is generated between the reverse rotations (-θ) of the motor at O and slider translation (s). If two-rotary actuators are to be used for the same purpose, in that case a 4-bar is designed by means of (Akçalı and Dittrich, 1989b) to generate (39) function between θ and ϕ in place of a motor at M in addition to the one located at O. Of course, in construction of the 4-bar or the inverted slider-crank, adjustability of dimensions is foreseen.

One advantage of these analogous designs is that the sense of actuation (θ, ϕ) will be kept same within long intervals as opposed to possibly frequent changes in the sign of actuation in digital applications. In the direct kinematic analysis, corresponding to (θ, ϕ) actuation, the following (x,y) trajectory co-ordinates will be generated:

$$x = r_1 + b\cos\theta + \frac{\sin\theta}{\sin(\theta - \phi)}[(r_3 - r_1)\cos\phi + (r_4 - r_2)\sin\phi + h - b\cos(\theta - \phi)] \tag{41}$$

$$y = r_2 + b\sin\theta - \frac{\cos\theta}{\sin(\theta - \phi)}[(r_3 - r_1)\cos\phi + (r_4 - r_2)\sin\phi + h - b\cos(\theta - \phi)] \tag{42}$$

5. ON APPLICATIONS

The design and operation of the manipulators proposed here are primarily based on analogous variables. As is well known, analogous variables are continuous in contrast with the discrete nature of digital variables, (raven 1987). Hence there are no zigzags in the functioning of actuators in driving the manipulators considered here. This aspect is consistently taken into considerations when applying the techniques presented here. Take, for instance, the problem of transporting an article from the point with (x_0, y_0) co-ordinates on a conveyor moving with velocity V_0 to the point with (x_n, y_n) co-ordinates on another conveyor with a linear velocity of V_n. In order to associate this problem with both general theory and simplified approach, it is sufficient to find a trajectory function $y(x)$ satisfying the velocity and the given end points. For instance, the cubic polynomial

$$y = ax^3 + bx^2 + cx + d \tag{43}$$

with the following computed coefficients (a, b, c, d) will be an answer to the requirements.

$$m_0 = \sqrt{(\frac{V_0 r_x}{\omega})^2 - 1}\,; m_n = \sqrt{(\frac{V_n r_x}{\omega})^2 - 1} \tag{44}$$

$$c_0 = \frac{y_n - y_0}{x_n - x_0}\,; c_1 = x_n^2 + x_n x_0 + x_0^2;\; c_2 = x_n + x_0 \tag{45}$$

$$k_1 = 3x_0^2 - c_1 \;;\; k_2 = 2x_0 - c_2 \;;\; l_1 = 3x_n^2 - c_1 \;;\; l_2 = 2x_n - c_2 \tag{46}$$

$$a = \frac{l_2(m_0 - c_0) - k_2(m_n - c_0)}{k_1 l_2 - k_2 l_1} \;; \quad b = \frac{l_1(m_0 - c_0) - k_1(m_n - c_0)}{k_2 l_1 - k_1 l_2} \tag{47}$$

$$c = c_0 - ac_1 - bc_2 \;;\; d = y_n - ax_n^3 - bx_n^2 - cx_n \tag{48}$$

where r_x, ω are selected as required in the techniques.

6. NUMERICAL RESULTS AND DISCUSSION

Analytical thoughts developed have been transformed into computer programs under Fortran 77 coding. Utilizing these programs, comprehensive illustrations are presented here to review the numerical results and to discuss their significance.

Example 1 A trajectory described by y = 0.5 x+ 0.5 $0 \leq x \leq 1$ is to be followed under a uniform velocity requirement matching a 10s. travel on the part of a two-arm manipulator with the sliding directions passing through the fixed revolute joints.

The solution lies in the implementation of the simplified approach twice. Data for the first implementation are in case of Precision-Point Method x_i, i=1,2,3 being [0.2;0.6;1.0] in case of Subdomain Method subintervals being [x_{i-1},x_i] i = 1,2,3 [0.0;0.40],[0.40;0.75],[0.75;1.00] and finally for Galerkin Method weighting functions w_i i =1,2,3 being [x,x^2,x^3]. The amount of arm rotation Δθ is taken to be 45°, for every method. The numerical results are displayed in Table 1, indicating also max absolute error, e_{max}.

Table 1. First Robot Arm for Trajectory y = 0.5 x+ 0.5 $0 \le x \le 1$

Method	r_1	r_2	$\theta_0(^o)$	Emax
Precision-Point	-0.0155	2.0311	-90.44	0.0035
Subdomain	-0.0768	2.0064	-87.74	0.0060
Galerkin	-0.0414	2.0306	-89.47	0.0042

Input for the second implementation are in Precision-Point Method x_i = 1, 2, 3 being [0.03;0.40;0.62] in Subdomain Method subintervals being [0.00;0.35], [0.35; 0.40], [0.40; 1.00]; in Galerkin Method weighting functions w_i = 1, 2, 3 being [x, x^2, x^3]. Arm rotation angle ($\Delta\theta$) has been taken as 45° in Precision-Point and Subdomain Methods but in Galerkin Method as 42°. The outcome has been presented in Table 2. When the two arms are put together to form the parallel manipulator in question, then the maximum absolute (e_{max}) error turns out to be 0.0034 in Precision-Point, 0.0056 in Subdomain and 0.0489 in Galerkin methods. Theoretical velocity on the trajectory is supposed to be 0.1118, constant throughout the motion. Maximum velocity errors in Precision-Point, Subdomain and Galerkin designs become 0.0152; 0.0183 and 0.0165, respectively.

Table 2. Second Robot Arm for Trajectory y = 0.5 x+ 0.5 0≤x≤1

Method	r_3	r_4	$\phi_0(^o)$	e_{max}
Precision-Point	-0.3934	1.8573	-74.90	0.0681
Subdomain	-0.1794	1.9583	-83.66	0.0163
Galerkin	-0.0891	2.1232	-87.82	0.0036

Example 2 Requirements of Example 1 are to be met by the general parallel manipulator of Fig. 1.

General theory is applied once, here. When data for precision points x_i, i=1,2, [0.4,0.7], for subdomains, [x_{i-1},x_i], [0.2;0.6],[0.6;0.9] for weighting functions w_i,i=1,2;[sin x,cos x] and $\Delta\theta$=45° are taken into account, the results shown in Table 3 are obtained.

Table 3. Modules For Trajectory y = 0.5 x+ 0.5 0≤x≤1

Method	r_1	r_2	$\theta_0(^o)$	b	e_{max}
Precision-Point	-0.7203	3.3156	-88.18	2.8405	0.0035
	-0.8205	2.0423	-24.75	1.3698	0.0653
Subdomain	-0.7082	3.3087	-88.49	2.8290	0.0026
	-0.8216	2.0405	-25.50	1.3870	0.0514
Galerkin	-0.7529	3.2559	-85.94	2.8022	0.0006
	-0.5659	1.9826	-22.50	1.0708	0.2228

Two significantly different module designs are brought together for the construction of the parallel manipulator. After this process, b values are slightly affected, leading to 1.4439, 1.3496 and 1.0902, in Precision-Point, Subdomain and Galerkin methods, respectively, leaving all others same. Maximum errors in the resulting parallel manipulators become 0.0026, 0.0026, 0.0007 in the aforementioned methods, respectively. Maximum deviations from the constant 0.1118 velocity value in Precision-Point, Subdomain and Galerkin methods turn out to be 0.0023, 0.0024, 0.0013, respectively.

If the results of two examples above are compared, it will be seen that the chances of getting more refined designs are always much more in the general theory against simplified approach. While more than one application is needed in the simplified approach for the formation of a parallel manipulator, only one implementation of general theory is sufficient. Another advantage of the general theory is that the mirror-image manipulator (with ϕ_+) is a good alternative when space for the manipulator (with ϕ_-) is not appropriate, since they both produce the same trajectories with the same end velocities.

The robotic assembly designed in this work differs from the classical constrained-motion mechanisms in that it has a flexible structure. While a classical one-degree of frecdom mechanism generates only one and constant curve, the robotic assembly under consideration can produce as many curves as the intervals of adjustability of parameters permit. If a change in the positions of fixed pivots of the two manipulators is not seen practical, then following the design of manipulators, the dimensions of the function-generating four-bar relating θ'_c rotations of the first manipulator to the ϕ'_c rotations of the second one can easily be made adjustable. In that case, the design of the function-generating 4-bar is realised with respect to a transformed function (θ'_c, ϕ'_c) corresponding to trajectory $y_c\ (x_c)$ in a co-ordinate system $x_c - y_c$ located at the ground pivot (O) of the first manipulator in the following way : First, $\theta_M = \tan^{-1}[r_4 - r_2)/(r_3 - r_1)]$ is computed, then co-ordinate transformations x_c=(x-r_1)cosθ_M+(y-r_2)sinθ_M, y_c= -(x-r_1)sinθ_M+(y-r_2)cosθ_M $\theta'_c = \theta_c - \theta^0_c$, $\phi'_c = \phi_c - \phi^0_c$,where $\theta^0_c = \theta_0 - \theta_M$, $\phi^0_c = \phi_0 - \theta_M$ and $\theta_c = \tan^{-1}\frac{y_c}{x_c} + \cos^{-1}\frac{b}{\sqrt{x_c^2 + y_c^2}}$, $\phi_c = \pi - \tan^{-1}\frac{y_c}{OM - x_c} \mp \cos^{-1}\frac{h}{\sqrt{y_c^2 + (OM - y_c)^2}}$

$OM = [(r_3 - r_1)^2 + (r_4 - r_2)^2]^{1/2}$ are carried out. To demonstrate the practical nature of the proposition, an experimental model based on the simplified approach has been constructed, Fig. 4. A synchronous motor with 1 rpm located at O and controlled by a timer is used together with light aluminium arms. The pen attached to the generating-point (C) draws desirable curves with insignificant, unnoticeable errors always remaining inside the thickness of the line. By the presence of a mechanical drive, 4-bar, a motor is saved from the second manipulator. Hence, the assembly is suitably termed as analog robot. Two illustrations are shown in Fig. 5, in which straight lines between points having co-ordinates (10, 30) and (24.5, 18) in the first one and between (10, 24) and (24, 28.5) co-ordinates in the second one are traced. Relevant adjustable parameters (x_1, x_2, x_3, x_6) turn out to be (8.5161, 29.9633, 16.0945, 35.0000) in the first design and (33.1220, 11.3563, 14.9201, 35.0000) in the second one.

Conclusively, all numerical results and experimental work indicate that methods of design for a parallel robotic assembly work very well, leading to optimum results.

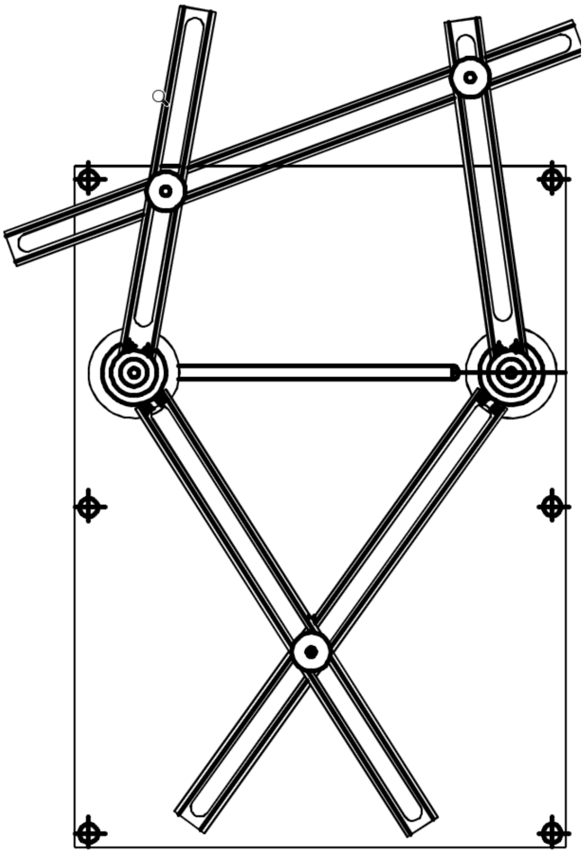
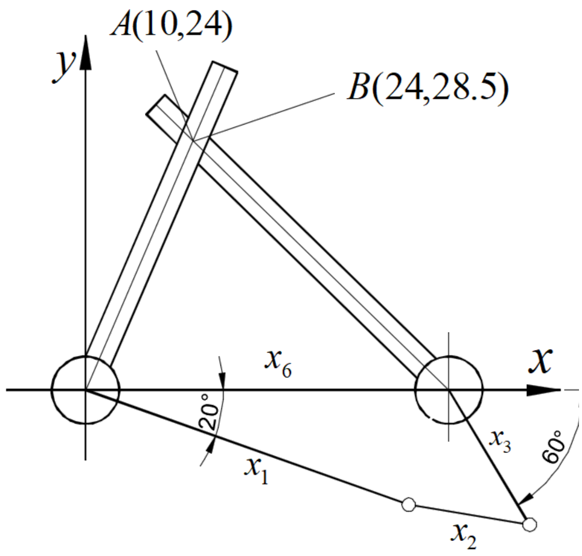


Fig. 4. Experimental model



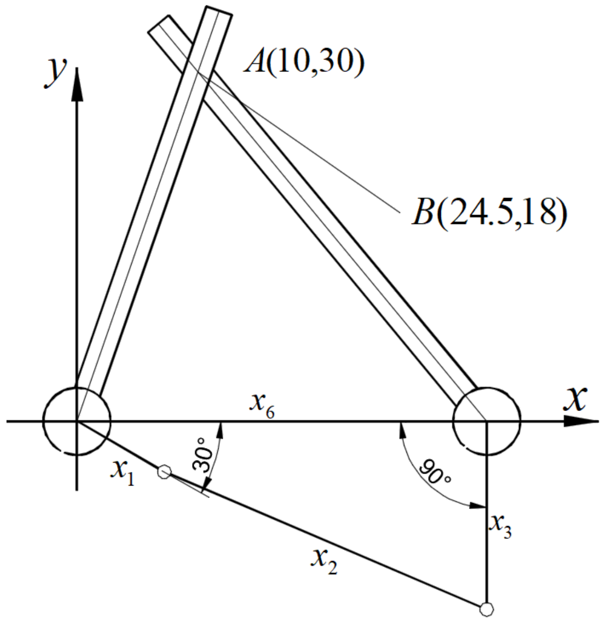


Fig. 5. Two illustrations

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