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THE NEUTROSOPHIZE OF NEW CONTINUITY SPECIES

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Abstract. In this study, after explaining the process that makes the study necessary in the introduction and giving the necessary definitons in the preliminaries, in the third section, some types of open set that were previously defined in general topology and various non-standard topological spaces are presented and the relationships between them are explained with the help of a diagram. Then, the concept of neutrosophic af-open set is defined and its relations with other open set types are examined and their properties are investigated in neutrosophic topology. In the following sections, the concept of af-open set is generalized and different types of continuities are introduced using these new concepts of open set, and the connections between them are illustrated with examples and diagrams.

1. Introduction

The concept of open set has always been one of the indispensable characters of the world of topology. This concept has been divided into many different types in itself as a result of the constant change of social life and the constant change of people's needs. For example, M. H. Alqahtani [4, 5] presented the concepts of F-open set and C-open set in 2023. Later, new ones continued to be added to these open set varieties. These open set types provided scientists with the opportunity to re-approach many concepts from different perspectives that had been previously introduced in topology and to examine their properties in general topology and some other non-standard topologies as in [6, 7, 8, 9, 17, 18]. Furthermore, these open set variants led to the introduction of many types of functions, mappings and continuities as in [13, 21, 22, 25].
Smarandache's introduction of the concept of neutrosophic set in [27] created facilities to make contributions to some other disiplicines as in [10, 11, 12, 14, 16, 19, 20, 24, 28] and allowed the introduction of different non-standard topological spaces. Like uncultivated fields, these new non-standard topological spaces enabled scientists to give products to the world of science unlike anything done until then as in [1, 2, 3, 23]. In this study, we aimed to join these scientists by introducing the concept of neutrosophic af-open set. Also, topological properties of af-interior, af-closure operators are presented by using the

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concept of neutrosophic af-open sets. Moreover, the notions of neutrosophic af-continuous functions and some other continuity types are introduced and the connections between them are illustrated with diagrams.

2. Preliminaries

In this section, we present the basic definitions related to neutrosophic set theory.

Definition 2.1. [27] *A neutrosophic set A on the universe set X is defined as:*

$$A = \{\langle x, T_A(x), I_A(x), F_A(x)\rangle : x \in X\},$$

where $T, I, F : X \to]^-0, 1^+[$ *and* $^-0 \leq T_A(x) + I_A(x) + F_A(x) \leq 3^+$.

Scientifically, membership functions, indeterminacy functions and non-membership functions of a neutrosophic set take value from real standart or nonstandart subsets of $]^-0, 1^+[$. However, these subsets are sometimes inconvenient to be used in real life applications such as economical and engineering problems. On account of this fact, we consider the neutrosophic sets, whose membership function, indeterminacy functions and non-membership functions take values from subsets of $[0, 1]$.

Definition 2.2. [15] *Let X be a nonempty set. If r, t, s are real standard or non standard subsets of* $]^-0, 1^+[$ *then the neutrosophic set* $x_{r,t,s}$ *is called a neutrosophic point in X given by*

$$x_{r,t,s}(x_p) = \begin{cases} (r, t, s), & \text{if } x = x_p \\ (0, 0, 1), & \text{if } x \neq x_p \end{cases}$$

For $x_p \in X$*, it is called the support of* $x_{r,t,s}$*, where r denotes the degree of membership value, t denotes the degree of indeterminacy and s is the degree of non-membership value of* $x_{r,t,s}$.

Definition 2.3. [26] *Let A be a neutrosophic set over the universe set X. The complement of A is denoted by* A^c *and is defined by:*

$A^c = \left\{\left\langle x, F_{\tilde{F}(e)}(x), 1 - I_{\tilde{F}(e)}(x), T_{\tilde{F}(e)}(x)\right\rangle : x \in X\right\}$.

It is obvious that $[A^c]^c = A$.

Definition 2.4. [26] *Let A and B be two neutrosophic sets over the universe set X. A is said to be a neutrosophic subset of B if* $T_A(x) \leq T_B(x), I_A(x) \leq I_B(x), F_A(x) \geq F_B(x)$*, every xinX. It is denoted by* $A \subseteq B$*. A is said to be neutrosophic equal to B if* $A \subseteq B$ *and* $B \subseteq A$*. It is denoted by* $A = B$.

Definition 2.5. [26] *Let* F_1 *and* F_2 *be two neutrosophic sets over the universe set X. Then their union is denoted by* $F_1 \cup F_2 = F_3$ *is defined by:*

$$F_3 = \{\langle x, T_{F_3}(x), I_{F_3}(x), F_{F_3}(x) : x \in X\rangle\},$$

where

$$T_{F_3(x)} = \max\{T_{F_1(x)}, T_{F_2}(x)\},$$

$$I_{F_3(x)} = \max\{I_{F_1(x)}, I_{F_2}(x)\},$$

$$F_{F_3(x)} = \min\{F_{F_1(x)}, F_{F_2}(x)\}.$$

Definition 2.6. [26] *Let* F_1 *and* F_2 *be two neutrosophic sets over the universe set X. Then their intersection is denoted by* $F_1 \cap F_2 = F_4$ *is defined by:*

$$F_4 = \{\langle x, T_{F_4}(x), I_{F_4}(x), F_{F_4}(x) : x \in X\rangle\},$$

where

$$T_{F_4(x)} = \min\{T_{F_1(x)}, T_{F_2}(x)\},$$

$$I_{F_4(x)} = \min\{I_{F_1(x)}, I_{F_2}(x)\},$$

$$F_{F_4(x)} = \max\{F_{F_1(x)}, F_{F_2}(x)\}.$$

Definition 2.7. [26] *A neutrosophic set F over the universe set X is said to be a null neutrosophic set if* $T_F(x) = 0$, $I_F(x) = 0$, $F_F(x) = 1$, *every* $x \in X$. *It is denoted by* 0_X.

Definition 2.8. [26] *A neutrosophic set F over the universe set X is said to be an absolute neutrosophic set if* $T_F(x) = 1$, $I_F(x) = 1$, $F_F(x) = 0$, *every* $x \in X$. *It is denoted by* 1_X.

Clearly $0_X^c = 1_X$ and $1_X^c = 0_X$.

Definition 2.9. [26] *Let* $NS(X)$ *be the family of all neutrosophic sets over the universe the set X and* $\tau \subset NS(X)$. *Then* τ *is said to be a neutrosophic topology on X if:*
1) 0_X *and* 1_X *belong to* τ*;*
2) The union of any number of neutrosophic sets in τ *belongs to* τ*;*
3) The intersection of a finite number of neutrosophic sets in τ *belongs to* τ.
Then (X, τ) *is said to be a neutrosophic topological space over X. Each member of* τ *is said to be a neutrosophic open set* [26].

Definition 2.10. [21] *Let* (X, τ) *be a neutrosophic topological space over X and F be a neutrosophic set over X. Then F is said to be a neutrosophic closed set iff its complement is a neutrosophic open set.*

Definition 2.11. [1] *A neutrosophic point* $x_{r,t,s}$ *is said to be neutrosophic quasi-coincident (neutrosophic q-coincident, for short) with F, denoted by* $x_{r,t,s}\ q\ F$ *if and only if* $x_{r,t,s} \nsubseteq F^c$. *If* $x_{r,t,s}$ *is not neutrosophic quasi-coincident with F, we denote by* $x_{r,t,s}\ \tilde{q}\ F$.

Definition 2.12. [1] *A neutrosophic set F in a neutrosophic topological space* (X, τ) *is said to be a neutrosophic q-neighborhood of a neutrosophic point* $x_{r,t,s}$ *if and only if there exists a neutrosophic open set G such that*
$x_{r,t,s}\ q\ G \subset F$.

Definition 2.13. [1]*) A neutrosophic set G is said to be neutrosophic quasi-coincident (neutrosophic q-coincident, for short) with F, denoted by* $G\ q\ F$ *if and only if* $G \nsubseteq F^c$. *If G is not neutrosophic quasi-coincident with F, we denote by* $G\ \tilde{q}\ F$.

Definition 2.14. [3] *A neutrosophic point* $x_{r,t,s}$ *is said to be a neutrosophic interior point of a neutrosophic set F if and only if there exists a neutrosophic open q-neighborhood G of* $x_{r,t,s}$ *such that* $G \subset F$. *The union of all neutrosophic interior points of F is called the neutrosophic interior of F and denoted by* F°.

Definition 2.15. [1] *A neutrosophic point* $x_{r,t,s}$ *is said to be a neurosophic cluster point of a neutrosophic set F if and only if every neutrosophic open q-neighborhood G of* $x_{r,t,s}$ *is q-coincident with F. The union of all neutrosophic cluster points of F is called the neutrosophic closure of F and denoted by* $\overline{F}$.

Definition 2.16. [1] *Let f be a function from X to Y. Let B be a neutrosophic set in Y with members hip function* $T_B(y)$*, indeterminacy function* $I_B(y)$ *and non-membership function* $F_B(y)$. *Then, the inverse image of B under f, written as* $f^{-1}(B)$*, is a neutrosophic subset of X whose membership function, indeterminacy function and non-membership function are*

defined as $T_{f^{-1}(B)}(x) = T_B(f(x))$, $I_{f^{-1}(B)}(x) = I_B(f(x))$ *and* $F_{f^{-1}(B)}(x) = F_B(f(x))$ *for all x in X, respectively.*
Conversely, let A be a neutrosophic set in X with membership function $T_A(x)$, *indeterminacy function* $I_A(x)$ *and non-membership function* $F_A(x)$. *The image of A under f, written as* $f(A)$, *is a neutrosophic subset of Y whose membership function, indeterminacy function and non-membership function are defined as*

$$T_{f(A)}(y) = \begin{cases} sup_{z\in f^{-1}(y)}\{T_A(z)\}, & if\ f^{-1}(y)\ is\ not\ empty,\\ 0, & if\ f^{-1}(y)\ is\ empty,\end{cases}$$

$$I_{f(A)}(y) = \begin{cases} sup_{z\in f^{-1}(y)}\{I_A(z)\}, & if\ f^{-1}(y)\ is\ not\ empty,\\ 0, & if\ f^{-1}(y)\ is\ empty,\end{cases}$$

$$F_{f(A)}(y) = \begin{cases} sup_{z\in f^{-1}(y)}\{F_A(z)\}, & if\ f^{-1}(y)\ is\ not\ empty,\\ 0, & if\ f^{-1}(y)\ is\ empty,\end{cases}$$

for all y in Y, where $f^{-1}(y) = \{x : f(x) = y\}$, *respectively.*

3. Neutrosophic af-open sets

This section provides some new definitions that form the cornerstones of the sections that follow.

Definition 3.1. *A neutrosophic set F in a neutrosophic topological space* (X, τ) *is said to be*

a) Neutrosophic semiopen, if $F \subseteq \overline{F^\circ}$,
b) Neutrosophic preopen, $F \subseteq (\overline{F})^\circ$,
c) Neutrosophic β-open, $F \subseteq \overline{(\overline{F})^\circ}$,
d) Neutrosophic α-open, if $F \subseteq (\overline{(F^\circ)})^\circ$.

By Definition 17, the following diagram is obtained:

$$\begin{array}{ccccc} neutrosophic\ open & \rightarrow & neutrosophic\ \alpha-open & \rightarrow & neutrosophic\ pre-open \\ & & \downarrow & & \downarrow \\ & & neutrosophic\ semi-open & \rightarrow & neutrosophic\ \beta-open \end{array}$$

Diagram I

Definition 3.2. *If, F be a neutrosophic set in neutrosophic topological space* (X, τ) *then,* $\overline{F}_s = \bigcap\{F : F \subseteq A, A is neutrosophic semiclosed\}$ *(resp.* $F_s^\circ = \bigcup\{F : F \subseteq A, A is neutrosophic semiopen\}$*) is called a neutrosophic semiclosure of F (resp. neutrosophic semi-interior of F).*

Definition 3.3. *If, F be a neutrosophic set in neutrosophic topological space* (X, τ) *then,* $\overline{F}_p = \bigcap\{F : F \subseteq A, A is neutrosophic preclosed\}$ *(resp.* $F_p^\circ = \bigcup\{F : F \subseteq A, A is neutrosophic preopen\}$*) is called a neutrosophic preclosure of F (resp. neutrosophic pre interior of F).*

Definition 3.4. *If, F be a neutrosophic set in neutrosophic topological space* (X, τ) *then,* $\overline{F}_\beta = \bigcap\{F : F \subseteq A, A is neutrosophic \beta closed\}$ *(resp.* $F_s^\circ = \bigcup\{F : F \subseteq A, A is neutrosophic \beta open\}$*) is called a neutrosophic β closure of F (resp. neutrosophic β interior of F).*

Definition 3.5. *If, F be a neutrosophic set in neutrosophic topological space* (X, τ) *then,* $\overline{F}_\alpha = \bigcap\{F : F \subseteq A, A is neutrosophic \alpha closed\}$ *(resp.* $F_\alpha^\circ = \bigcup\{F : F \subseteq A, A is neutrosophic \alpha open\}$*) is called a neutrosophic α closure of F (resp. neutrosophic α interior of F).*

Definition 3.6. *Let (X, τ) be a neutrosophic topological space. A neutrosophic set λ is af-open set if $\lambda \subseteq (\lambda \cup \mu)^\circ$ for every μ is neutrosophic open set such that $0_X \neq \mu \neq 1_X$. The complement of the neutrosophic af -open set is called neutrosophic af-closed. We denote the family of all neutrosophic af-open (resp. neutrosophic af-closed) sets of a neutrosophic topological spece (X, τ) by $NafO(X, \tau)$(resp. $NafC(X, \tau)$).*

Problem 3.1. *Let (X, τ) be a neutrosophic topological space. In Definition 22, for every $\mu \in \tau$ such that $0_X \neq \mu \neq 1_X$, can we obtain a new type of neutrosophic af-open sets by taking the neutrosophic closure of μ instead of μ?*

Theorem 3.2. *Every neutrosophic open set in a neutrosophic topological space (X, τ) is neutrosophic af-open set.*

Proof. Let (X, τ) be any neutrosophic topological space and let $\lambda \subseteq X$ be any neutrosophic open set. Therefore, $\lambda = \lambda^\circ \subseteq (\lambda \cup \mu)^\circ$ is neutrosophic open set such that $0_X \neq \mu \neq 1_X$. Thus, $\lambda$is neutrosophic af-open set. Then for the collection of $NafO(X, \tau)$, $\tau \subseteq NafO(X, \tau)$. □

Remark. *The converse of Theorem 3.2. is not always true as shown by the following example.*

Example 3.1. *Let (X, τ) be a neutrosophic topological space, with $X = \{a, b, c\}$, $\tau = \{0_X, \lambda, 1_X\}$, where λ, μ are two neutrosophic sets defined as $\lambda = \{\langle a, 0.5, 0.5, 0.5\rangle,$ $\langle b, 0.7, 0.7, 0.3\rangle, \langle c, 0.9, 0.9, 0.1\rangle\}$ and $\mu = \{\langle a, 0.4, 0.4, 0.6\rangle, \langle b, 0.3, 0.3, 0.7\rangle, \langle c, 0.9, 0.9, 0.1\rangle\}$. Then, $\mu \in NafO(X, \tau)$, and but the set μ is not neutrosophic open.*

Theorem 3.3. *Let (X, τ) be any neutrosophic topological space and λ, μ be two neutrosophic af-open sets. Then, the following properties are hold:*

(1) $\lambda \cap \mu$ is neutrosophic af-open set.
(2) $\lambda \cup \mu$ is neutrosophic af-open set.

Proof. (1) Let λ and μ be two neutrosophic af-open sets. Then from Definition 3.6, $\lambda \subseteq (\lambda \cup \beta)^\circ$ and $\mu \subseteq (\mu \cup \beta)^\circ$ for every β is neutrosophic open set and $0_X \neq \beta \neq 1_X$. Then, $\lambda \cap \mu \subseteq (\lambda \cup \beta)^\circ \cap (\mu \cup \beta)^\circ = ((\lambda \cup \beta) \cap (\mu \cup \beta))^\circ \subseteq ((\lambda \cap \mu) \cup \beta)^\circ$.
(2) Let λ and μ be two neutrosophic af-open sets. Then from Definition 3.6, $\lambda \subseteq (\lambda \cup \beta)^\circ$ and $\mu \subseteq (\mu \cup \beta)^\circ$ for every β is neutrosophic open set and $0_X \neq \beta \neq 1_X$. Then, $\lambda \cup \mu \subseteq (\lambda \cup \beta)^\circ \cup (\mu \cup \beta)^\circ = ((\lambda \cup \beta) \cup (\mu \cup \beta))^\circ \subseteq ((\lambda \cup \mu) \cup \beta)^\circ$. □

Proposition 3.4. *Let (X, τ) be any neutrosophic topological space. If, for every $\alpha \in \Delta$, $\lambda_\alpha \in NafO(X, \tau)$, then $\bigcup_{\alpha\in\Delta} \lambda_\alpha \in NafO(X, \tau)$.*

Proof. Let $\lambda_\alpha \in NafO(X, \tau)$ for every $\alpha \in \Delta$. Then, $\lambda_\alpha \subseteq \bigcup_{\alpha\in\Delta} \lambda_\alpha$, for every $\alpha \in \Delta$. For any β is neutrosophic open $(0_X \neq \beta \neq 1_X)$ and each $\alpha \in \Delta$, we get $\lambda_\alpha \subseteq (\lambda_\alpha \cup \beta)^\circ \subseteq ((\bigcup_{\alpha\in\Delta} \lambda_\alpha) \cup \beta)^\circ$. Hence we have $\bigcup_{\alpha\in\Delta} \lambda_\alpha \subseteq ((\bigcup_{\alpha\in\Delta} \lambda_\alpha) \cup \beta)^\circ$. Therefore $\bigcup_{\alpha\in\Delta} \lambda_\alpha \in NafO(X, \tau)$. □

Theorem 3.5. *Let (X, τ) be any neutrosophic topological space and $\tau_{NafO} = \{\lambda : \lambda \in NafO(X, \tau)\}$. Then is τ_{NafO} a neutrosophic topology such that $\tau \subseteq \tau_{NafO}$.*

Proof. According to Theorem 3.2, we have $\tau \subseteq \tau_{NafO}$. We show that τ_{NafO} is a neutrosophic topology.

(1) It is clear that $0_X, 1_X \in \tau_{NafO}$.
(2) and (3) are seen that from Theorem 3.3 and Proposition 3.4.

□

4. Generalizations of Neutrosophic af-open sets

Definition 4.1. *A subset λ of a neutrosophic topological space (X, τ) is said to be*

(i) *neutrosophic $af\alpha-open$ if $\lambda \subseteq (\lambda \cup \mu)^{\circ}_{\alpha}$, for every μ is neutrosophic open and $0_X \neq \mu \neq 1_X$,*
(ii) *neutrosophic $afp-open$ if $\lambda \subseteq (\lambda \cup \mu)^{\circ}_{p}$, for every μ is neutrosophic open and $0_X \neq \mu \neq 1_X$,*
(iii) *neutrosophic $afs-open$ if $\lambda \subseteq (\lambda \cup \mu)^{\circ}_{s}$, for every μ is neutrosophic open and $0_X \neq \mu \neq 1_X$,*
(iv) *neutrosophic $af\beta-open$ if $\lambda \subseteq (\lambda \cup \mu)^{\circ}_{\beta}$, for every μ is neutrosophic open and $0_X \neq \mu \neq 1_X$.*

The complement of a neutrosophic afα-open (resp. neutrosophic afp-open, neutrosophic afs-open, neutrosophic afβ-open) set is said to be neutrosophic afα-closed (resp. neutrosophic afp-closed, neutrosophic afs-closed, neutrosophic afβ-closed). The family of all f neutrosophic afα-open (neutrosophic afα-closed) (resp. neutrosophic afp-open (neutrosophic afp-closed), neutrosophic afs-open (neutrosophic afs-closed), neutrosophic afβ-open (neutrosophic afβ-closed)) sets in a neutrosophic topological space (X, τ) is denoted by $Naf\alpha O(X, \tau)(Naf\alpha C(X, \tau))(resp. NafPO(X, \tau)$ $(NafPC(X, \tau)), NafSO(X, \tau)(NafSC(X, \tau)), Naf\beta O(X, \tau)(Naf\beta C(X, \tau)))$.

From Definition 4.1, we have the following diagram:

$$\begin{array}{ccccc} neutrosophic\ open & & & & \\ \downarrow & & & & \\ neutrosophic\ af-open & \rightarrow & neutrosophic\ af\alpha-open & \rightarrow & neutrosophic\ afp-open \\ & & \downarrow & & \downarrow \\ & & neutrosophic\ afs-open & \rightarrow & neutrosophic\ af\beta-open \end{array}$$

Diagram II

Problem 4.1. *In the above definition, for every $\mu \in \tau$ such that $0_X \neq \mu \neq 1_X$, can a new types of neutrosophic af-open set be given by taking the neutrosophic closure of μ instead of μ?*

Remark. *The inverses of the requirements in the diagram above may not always be true.*

Example 4.1. *It can be seen from Example 3.1 that not every neutrosophic af-open set is a neutrosophic open set.*

Example 4.2. *Let (X, τ) be a neutrosophic topological space, with $X = \{a, b, c\}$, $\tau = \{0_X, \lambda, 1_X\}$, where λ, μ are two neutrosophic sets defined as $\lambda = \{\langle a, 0.2, 0.2, 0.8\rangle, \langle b, 0.7, 0.7, 0.3\rangle, \langle c, 0.4, 0.4, 0.6\rangle\}$ and $\mu = \{\langle a, 0.7, 0.7, 0.3\rangle, \langle b, 0.9, 0.9, 0.1\rangle, \langle c, 0.1, 0.1, 0.9\rangle\}$. Then, $\mu \in Naf\alpha O(X, \tau)$, and but the set μ is not neutrosophic af-open.*

Example 4.3. *Let (X, τ) be a neutrosophic topological space, with $X = \{a, b, c\}$, $\tau = \{0_X, \mu, 1_X\}$, where λ, μ are two neutrosophic sets defined as $\lambda = \{\langle a, 0.2, 0.2, 0.8\rangle, \langle b, 0.3, 0.3, 0.7\rangle, \langle c, 0.7, 0.7, 0.3\rangle\}$ and $\mu = \{\langle a, 0.1, 0.1, 0.9\rangle, \langle b, 0.2, 0.2, 0.8\rangle, \langle c, 0.2, 0.2, 0.8\rangle\}$. Then, $\lambda \in NafSO(X, \tau)$, and but the set λ is neither neutrosophic afα-open nor neutrosophic afp-open.*

Example 4.4. *Let (X, τ) be a neutrosophic topological space, with $X = \{a, b, c\}$, $\tau = \{0_X, \mu, 1_X\}$, where λ, μ are two neutrosophic sets defined as $\lambda = \{\langle a, 0.3, 0.3, 0.7\rangle, \langle b, 0.8, 0.8, 0.2\rangle, \langle c, 0.7, 0.7, 0.3\rangle\}$ and $\mu = \{\langle a, 0.1, 0.1, 0.9\rangle, \langle b, 0.3, 0.3, 0.7\rangle, \langle c, 0.4, 0.4, 0.6\rangle\}$.*

Then, $\lambda \in NafPO(X,\tau)$, and but the set λ is neither neutrosophic afα-open nor neutrosophic afs-open.

Remark. *From Example 4.3 and Example 4.4, neutrosophic afp-open sets and neutrosophic afs-open sets are independent of each other.*

Example 4.5. *Let (X,τ) be a neutrosophic topological space, with $X = \{a,b,c\}$, $\tau = \{0_X, \lambda, 1_X\}$, where λ, μ are two neutrosophic sets defined as $\lambda = \{\langle a, 0.1, 0.1, 0.9\rangle, \langle b, 0.3, 0.3, 0.7\rangle, \langle c, 0.1, 0.1, 0.9\rangle\}$ and $\mu = \{\langle a, 0.3, 0.3, 0.7\rangle, \langle b, 0.5, 0.5, 0.5\rangle, \langle c, 0.7, 0.7, 0.3\rangle\}$. Then, $\mu \in Naf\beta O(X,\tau)$, and but the set μ is not neutrosophic afp-open.*

Example 4.6. *Let (X,τ) be a neutrosophic topological space, with $X = \{a,b,c\}$, $\tau = \{0_X, \lambda, 1_X\}$, where λ, μ are two neutrosophic sets defined as $\lambda = \{\langle a, 0.2, 0.2, 0.8\rangle, \langle b, 0.8, 0.8, 0.2\rangle, \langle c, 0.5, 0.5, 0.5\rangle\}$ and $\mu = \{\langle a, 0.6, 0.6, 0.4\rangle, \langle b, 0.5, 0.5, 0.5\rangle, \langle c, 0.4, 0.4, 0.6\rangle\}$. Then, $\mu \in Naf\beta O(X,\tau)$, and but the set μ is not neutrosophic afs-open.*

5. Neutrosophic af-interior and neutrosophic af-closure operators

Definition 5.1. *Let (X,τ) be a neutrosophic topological space and λ a neutrosophic subset of X. The neutrosophic af-interior, λ°_{af}, is defined as follows : $\lambda^{\circ}_{af} = \bigcup\{\mu : \mu \in NafO(X,\tau), \mu \subseteq \lambda\}$.*

Theorem 5.1. *Let (X,τ) be a neutrosophic topological space and λ, μ neutrosophic subsets of X. Then the following statements are hold:*

(1) λ°_{af} is neutrosophic af-open set,
(2) $\lambda^{\circ}_{af} \subseteq \lambda$,
(3) λ°_{af} is the largest neutrosophic af-open subset contained in the set λ,
(4) $(\lambda^{\circ}_{af})^{\circ}_{af} = \lambda^{\circ}_{af}$,
(5) If $\lambda \subseteq \mu$, $\lambda^{\circ}_{af} \subseteq \mu^{\circ}_{af}$,
(6) $\lambda^{\circ}_{af} \cup \mu^{\circ}_{af} \subseteq (\lambda \cup \mu)^{\circ}_{af}$,
(7) $\lambda^{\circ}_{af} \cap \mu^{\circ}_{af} = (\lambda \cap \mu)^{\circ}_{af}$.

Proof. 1) λ°_{af} is neutrosophic af-open set. Indeed, the union of neutrosophic af-open sets belonging to the neutrosophic topological space τ is neutrosophic af-open from the Proposition 3.4.
2) It is clear from Definition 5.1.
3) Let's assume the opposite, that is, a neutrosophic af-open set β that is larger than the set λ°_{af} that the set λ contains. That is, $\lambda^{\circ}_{af} \subseteq \beta\lambda$. On the other hand, for every $\mu \subseteq \lambda$ neutrosophic af-open set from Definition 5.1, $\mu \subseteq \lambda^{\circ}_{af}$. If we take $\mu = \beta$ specifically, we find $\beta \subseteq \lambda^{\circ}_{af}$. Then $\beta = \lambda^{\circ}_{af}$ is obtained. Thus, the neutrosophic set λ°_{af} is the largest neutrosophic af-open subset contained in the set *lambda*.
4) Let $\beta = \lambda^{\circ}_{af}$. By (2) and Definition 5.1, $\beta = \beta^{\circ}_{af}$. Then, $\lambda^{\circ}_{af} = (\lambda^{\circ}_{af})^{\circ}_{af}$.
5) Since $\lambda \subseteq \mu$ and $\lambda^{\circ}_{af} \subseteq \lambda$, $\lambda^{\circ}_{af} \subseteq \mu$. By (2), $\mu^{\circ}_{af} \subseteq \mu$. From (3), since μ°_{af} is the largest neutrosophic open set contained in μ neutrosophic sets, $\lambda^{\circ}_{af} \subseteq \mu^{\circ}_{af} \subseteq \mu$. In that case $\lambda^{\circ}_{af} \subseteq \mu^{\circ}_{af}$.
6) $\lambda \subseteq \lambda \cup \mu$ and $\mu \subseteq \lambda \cup \mu$ always hold. From (5), $\lambda^{\circ}_{af} \subseteq (\lambda \cup \mu)^{\circ}_{af}$ and $\mu^{\circ}_{af} \subseteq (\lambda \cup \mu)^{\circ}_{af}$, respectively. Therefore, $\lambda^{\circ}_{af} \cup \mu^{\circ}_{af} \subseteq (\lambda \cup \mu)^{\circ}_{af}$.
7) It is always hold that $\lambda \cap \mu \subseteq \lambda$ and $\lambda \cap \mu \subseteq \mu$. From (5), we obtain $(\lambda \cap \mu)^{\circ}_{af} \subseteq \lambda^{\circ}_{af}$ and $(\lambda \cap \mu)^{\circ}_{af} \subseteq \mu^{\circ}_{af}$, respectively. Hence, $(\lambda \cap \mu)^{\circ}_{af} \subseteq \lambda^{\circ}_{af} \cap \mu^{\circ}_{af}$. On the other hand, $\lambda^{\circ}_{af} \subseteq \lambda$ and $\mu^{\circ}_{af} \subseteq \mu$. From here $\lambda^{\circ}_{af} \cap \mu^{\circ}_{af} \subseteq \lambda \cap \mu$. Since $\lambda^{\circ}_{af} \cap \mu^{\circ}_{af}$ are neutrosophic af-open sets and

$(\lambda \cap \mu)^\circ_{af}$ is the largest neutrosophic af-open set contained in the $\lambda \cap \mu$ neutrosophic set, we have $\lambda^\circ_{af} \cap \mu^\circ_{af} \subseteq (\lambda \cap \mu)^\circ_{af} \subseteq \lambda \cap \mu$. Thus, $\lambda^\circ_{af} \cap \mu^\circ_{af} = (\lambda \cap \mu)^\circ_{af}$. □

Theorem 5.2. *Let (X, τ) be a neutrosophic topological space and and a neutrosophic subset λ of X. Then, λ neutrosophic set to be af-open set if and only if, $\lambda^\circ_{af} = \lambda$.*

Proof. $\Rightarrow$ Let λ be a neutrosophic af-open set. From Theorem 5.1 (2), $\lambda^\circ_{af} \subseteq \lambda$. On the other hand, since λ is a neutrosophic af-open set, $\lambda \subseteq \lambda$ and by Definition 5.1, $\lambda \subseteq \lambda^\circ_{af}$. In that case $\lambda = \lambda^\circ_{af}$.
$\Leftarrow$ According to the hypothesis, let's take $\lambda = \lambda^\circ_{af}$. Since λ°_{af} is a neutrosophic af-open set and $\lambda = \lambda^\circ_{af}$, so λ is a neutrosophic af-open set. □

Lemma 5.3. *For 1_X and 0_X neutrosophic af-open sets, then $(1_X)^\circ_{af} = 1_X$ and $(0_X)^\circ_{af} = 0_X$.*

Definition 5.2. *Let (X, τ) be a neutrosophic topological space and a neutrosophic subset λ of X. The neutrosophic af-closure of λ, $\overline{\lambda}_{af}$, is defined as follows : $\overline{\lambda}_{af} = \bigcap\{\beta : \lambda \subseteq \beta, \beta \in NafC(X, \tau)\}$.*

Theorem 5.4. *Let (X, τ) be a neutrosophic topological space and λ, μ neutrosophic subsets of X. Then the following statements are hold:*

(1) $\overline{\lambda}_{af}$ is neutrosophic af-closed set,
(2) $\lambda \subseteq \overline{\lambda}_{af}$,
(3) $\overline{\lambda}_{af}$ is the smallest neutrosophic af-closed set containing λ,
(4) $\overline{(\overline{\lambda}_{af})}_{af} = \overline{\lambda}_{af}$,
(5) If $\lambda \subseteq \mu$, $\overline{\lambda}_{af} \subseteq \overline{\mu}_{af}$,
(6) $\overline{(\lambda \cap \mu)}_{af} \subseteq \overline{\lambda}_{af} \cap \overline{\mu}_{af}$,
(7) $\overline{(\lambda \cup \mu)}_{af} = \overline{\lambda}_{af} \cup \overline{\mu}_{af}$,
(8) $\overline{(1_X)}_{af} = 1_X$ and $\overline{(0_X)}_{af} = 0_X$.

Theorem 5.5. *Let λ be any neutrosophic set in a neutrosophic topological space (X, τ). Then, $\overline{(\lambda^c)}_{af} = (\lambda^\circ_{af})^c$ and $(\lambda^c)^\circ_{af} = (\overline{\lambda}_{af})^c$.*

Proof. We see that a neutrosophic af-open set $\beta \subseteq \lambda$ is precisely the complement of a neutrosophic af-closed set $\nu = \beta^c \supseteq \lambda^c$. Thus
$(\lambda)^\circ_{af} = \bigcup\{\nu^c : \nu \text{ is neutrosophic } af-closed \text{ and } \nu \supseteq \lambda^c\}$
$=\bigcap(\{\nu : \nu \text{ is neutrosophic } af-closed \text{ and } \nu \supseteq \lambda^c\})^c$
$= (\overline{(\lambda^c)}_{af})^c$
whence
$\overline{(\lambda^c)}_{af} = (\lambda^\circ_{af})^c$.

Next let β be any neutrosophic af-open set. Then for a neutrosophic af-closed set $\mu \supseteq \lambda$, $\beta = \mu^c \subseteq \lambda^c$. $\overline{\lambda}_{af} = \bigcap\{\beta^c : \beta \text{ is neutrosophic } af-open \text{ and } \beta \subseteq \lambda^c\}$
$= \bigcup(\{\beta : \beta \text{ is neutrosophic } af-open \text{ and } \beta \subseteq \lambda^c\})^c$
$= ((\lambda^c)^\circ_{af})^c$.
As a result
$(\lambda^c)^\circ_{af} = (\overline{\lambda}_{af})^c$. □

Definition 5.3. *Let β be a neutrosophic set in a neutrosophic topological space (X, τ) and $x_{r,t,s}$ is a neutrosophic point of X. β is called:*

(i) af-neighbourhood of $x_{r,t,s}$, if there exista a neutrosophic af-open set μ such that $x_{r,t,s} \in \mu \subseteq \beta$.
(ii) af-q-neighbourhood of $x_{r,t,s}$ if there exista a neutrosophic af-open set μ such that $x_{r,t,s} \in q\mu \subseteq \beta$.

Theorem 5.6. *A neutrosophic set β is neutrosophic af-open set if and only if, for each neutrosophic point $x_{r,t,s} \in \beta$, β is a af-neighbourhood of $x_{r,t,s}$.*

Proof. Straightforward. □

Definition 5.4. *Let (X, τ) be the neutrosophic topological space, λ be a neutrosophic set in (X, τ) and $x_{r,t,s}$ be a neutrosophic point. If, every af-q-neighborhood of $x_{r,t,s}$ is quasi-coincident with λ, then $x_{r,t,s}$ is said to be a af-cluster point of λ.*

Theorem 5.7. *Let β be a neutrosophic set and $x_{r,t,s}$ a neutrosophic point in a neutrosophic topological space (X, τ). Then, $x_{r,t,s} \in \overline{\beta}_{af}$ if and only if, every af-q-neighbourhood of $x_{r,t,s}$ is quasi-coincident with β.*

6. Neutrosophic af-continous functions

Definition 6.1. *A function $f : (X, \tau) \to (Y, \sigma)$ is said to be neutrosophic af-continuous, if, for each $\lambda \in \sigma$, $f^{-1}(\lambda)$ is neutrosophic af-open in (X, τ).*

Theorem 6.1. *Every neutrosophic continuous function is neutrosophic af-continuous.*

Proof. By Theorem 3.2, every neutrosophic open set is neutrosophic af-open and the proof is obvious. □

Example 6.1. *Let (X, τ), (Y, σ) be a neutrosophic topological spaces, with $X = \{a, b\}$, $Y = \{0.1, 0.4\}$, $\tau = \{0_X, \mu, 1_X\}$, $\sigma = \{0_Y, \beta, 1_Y\}$, where β, μ are two neutrosophic sets defined as $\mu = \{\langle a, 0.3, 0.3, 0.7\rangle, \langle b, 0.7, 0.7, 0.3\rangle\}$ and $\beta = \{\langle 0.1, 0.2, 0.2, 0.8\rangle,$ $\langle 0.4, 0.2, 0.2, 0.8\rangle\}$ in neutrosophic topological spaces (X, τ), (Y, σ), respectively. Then, a function $f : (X, \tau) \to (Y, \sigma)$ defined as f(a)=0.1, f(b)=0.4 is neutrosophic af-continuous but not neutrosophic continuous.*

Definition 6.2. *A function $f : (X, \tau) \to (Y, \sigma)$ is said to be $neutrosophic af\alpha - continuous$ (resp. neutrosophic afp-continuous, neutrosophic afs-continuous, neutrosophic afβ-continuous) if for each $\lambda \in \sigma$, $f^{-1}(\lambda)$ is neutrosophic afα-open (resp. neutrosophic afp-open, neutrosophic afs-open, neutrosophic afβ-open) in (X, τ).*

By Definitions 6.1 and 6.2, the following implications hold:

$neutrosophic\ - cont$
$\downarrow$
$neutrosophic\ af - cont \to neutrosophic\ af\alpha - cont \to neutrosophic\ afp - cont$
$\downarrow \quad \downarrow$
$neutrosophic\ afs - cont \to$
$neutrosophic\ af\beta - cont$

Diagram III

Remark. *None of the implications in Diagram III is reversible as shown by examples stated below.*

Example 6.2. *It can be seen from Example 6.1 that not every neutrosophic af-continuous function is a neutrosophic continuous.*

Example 6.3. *Let* (X,τ), (Y,σ) *be a neutrosophic topological spaces, with* $X = \{a,b\}$, $Y = \{0.2, 0.5\}$, $\tau = \{0_X, \lambda, 1_X\}$, $\sigma = \{0_Y, \beta, 1_Y\}$, *where* λ, β *are two neutrosophic sets defined as* $\lambda = \{\langle a, 0.7, 0.7, 0.3\rangle, \langle b, 0.4, 0.4, 0.6\rangle\}$ *and* $\beta = \{\langle 0.2, 0.9, 0.9, 0.1\rangle,$ $\langle 0.5, 0.1, 0.1, 0.9\rangle\}$ *in neutrosophic topological spaces* (X,τ), (Y,σ), *respectively. Then, a function* $f : (X,\tau) \to (Y,\sigma)$ *defined by* $f(a)$*=0.2,* $f(b)$*=0.5 neutrosophic af*α*-continuous but not neutrosophic af-continuous.*

Example 6.4. *Let* (X,τ), (Y,σ) *be a neutrosophic topological spaces, with* $X = \{a,b\}$, $Y = \{0.1, 0.4\}$, $\tau = \{0_X, \mu, 1_X\}$, $\sigma = \{0_Y, \beta, 1_Y\}$, *where* μ, β *are two neutrosophic sets defined as* $\mu = \{\langle a, 0.2, 0.2, 0.8\rangle, \langle b, 0.2, 0.2, 0.8\rangle\}$ *and* $\beta = \{\langle 0.1, 0.3, 0.3, 0.7\rangle,$ $\langle 0.4, 0.7, 0.7, 0.3\rangle\}$ *in neutrosophic topological spaces* (X,τ), (Y,σ), *respectively. Then, a function* $f : (X,\tau) \to (Y,\sigma)$ *defined as* $f(a)$*=0.1 and* $f(b)$*=0.4 is neutrosophic afs-continuous but neither neutrosophic af*α*-continuous nor neutrosophic afp-continuous.*

Example 6.5. *Let* (X,τ), (Y,σ) *be a neutrosophic topological spaces, with* $X = \{a,b,c\}$, $Y = \{0.1, 0.3, 0.5\}$, $\tau = \{0_X, \mu, 1_X\}$, $\sigma = \{0_Y, \beta, 1_Y\}$, *where* μ, β *are two neutrosophic sets defined as* $\mu = \{\langle a, 0.2, 0.2, 0.8\rangle, \langle b, 0.4, 0.4, 0.6\rangle, \langle c, 0.5, 0.5, 0.5\rangle\}$ *and* $\beta = \{\langle 0.1, 0.4, 0.4, 0.6\rangle, \langle 0.3, 0.9, 0.9, 0.1\rangle, \langle 0.5, 0.8, 0.8, 0.2\rangle\}$ *in neutrosophic topological spaces* (X,τ), (Y,σ), *respectively. Then, a function* $f : (X,\tau) \to (Y,\sigma)$ *defined as* $f(a)$*=0.1,* $f(b)$*=0.3 and* $f(c)$*=0.5 is neutrosophic afp-continuous but neither neutrosophic af*α*-continuous nor neutrosophic afs-continuous.*

Example 6.6. *Let* (X,τ), (Y,σ) *be a neutrosophic topological spaces, with* $X = \{a,b,c\}$, $Y = \{0.2, 0.5, 0.6\}$, $\tau = \{0_X, \lambda, 1_X\}$, $\sigma = \{0_Y, \beta, 1_Y\}$, *where* λ, β *are two neutrosophic sets defined as* $\lambda = \{\langle a, 0.1, 0.1, 0.9\rangle, \langle b, 0.4, 0.4, 0.6\rangle, \langle c, 0.1, 0.1, 0.9\rangle\}$ *and* $\beta = \{\langle 0.2, 0.3, 0.3, 0.7\rangle, \langle 0.5, 0.5, 0.5, 0.5\rangle, \langle 0.6, 0.8, 0.8, 0.2\rangle\}$ *in neutrosophic topological spaces* (X,τ), (Y,σ), *respectively. Then, a function* $f : (X,\tau) \to (Y,\sigma)$ *defined as* $f(a)$*=0.2,* $f(b)$*=0.5 and* $f(c)$*=0.6 is neutrosophic af*β*-continuous but not neutrosophic afp-continuous.*

Example 6.7. *Let* (X,τ), (Y,σ) *be a neutrosophic topological spaces, with* $X = \{a,b,c\}$, $Y = \{0.3, 0.5, 0.7\}$, $\tau = \{0_X, \lambda, 1_X\}$, $\sigma = \{0_Y, \beta, 1_Y\}$, *where* λ, β *are two neutrosophic sets defined as* $\lambda = \{\langle a, 0.2, 0.2, 0.8\rangle, \langle b, 0.8, 0.8, 0.2\rangle, \langle c, 0.5, 0.5, 0.5\rangle\}$ *and* $\beta = \{\langle 0.3, 0.6, 0.6, 0.4\rangle, \langle 0.5, 0.5, 0.5, 0.5\rangle, \langle 0.7, 0.4, 0.4, 0.6\rangle\}$ *in neutrosophic topological spaces* (X,τ), (Y,σ), *respectively. Then, a function* $f : (X,\tau) \to (Y,\sigma)$ *defined as* $f(a)$*=0.3,* $f(b)$*=0.5 and* $f(c)$*=0.7 is neutrosophic af*β*-continuous but not neutrosophic afs-continuous.*

Corollary 6.2. *A function* $f : (X,\tau) \to (Y,\sigma)$ *is neutrosophic af-continuous if and only if,* $f : (X,\tau) \to (Y,\sigma)$ *is neutrosophic continuous.*

Proof. This is an immediate consequence of Theorem 3.5. □

Theorem 6.3. *A function* $f : (X,\tau) \to (Y,\sigma)$ *is neutrosophic af-continuous and* $g : (Y,\sigma) \to (Z,\eta)$ *is neutrosophic continuous, then* $gof : (X,\tau) \to (Z,\eta)$ *is neutrosophic af-continuous.*

Proof. It is clear. □

By using neutrosophic af-neighborhood, neutrosophic af-open sets, neutrosophic af-closed sets, neutrosophic af-interior and neutrosophic af-closure, we obtain characterizations of neutrosophic af-continuous functions.

Lemma 6.4. *Let (X, τ) be a neutrosophic topological space. A neutrosophic subset μ is neutrosophic af-closed if and only if $\overline{(\mu \cap \beta)} \subseteq \mu$ for every neutrosophic closed set β of X such that $0_X \neq \beta \neq 1_X$.*

Proof. μ is neutrosophic af-closed if and only if μ^c is neutrosophic af-open. By Definition 3.6, $\mu^c \subseteq (\mu^c \cup \alpha)^\circ$ for every $\alpha \in \tau$ such that $0_X \neq \alpha \neq 1_X$.
This is equivalent to $((\mu^c \cup \alpha)^\circ)^c \subseteq \mu$. Now, we have $((\mu^c \cup \alpha)^\circ)^c = \overline{((\mu^c \cup \alpha)^c)} = \overline{(\mu \cap \alpha^c)}$. Therefore, we obtain $\overline{(\mu \cap \beta)} \subseteq \mu$ for every neutrosophic closed set β of X such that $0_X \neq \alpha \neq 1_X$. □

Theorem 6.5. *For a function $f : (X, \tau) \to (Y, \sigma)$, the following properties are equivalent:*

(1) f is neutrosophic af-continuous;
(2) For each point $x_{r,t,s} \in X$ and each neutrosophic open set $\mu \in Y$ containing $f(x_{r,t,s})$, there exists $\alpha \in NafO(X)$ such that $x \in \alpha$, $f(\alpha) \subseteq \mu$;
(3) For each point $x_{r,t,s} \in X$ and each neutrosophic open set μ of Y containing $f(x_{r,t,s})$, there exists a neutrosophic af-neighorhood λ of $x_{r,t,s}$ such that $f(\lambda) \subseteq \mu$;
(4) The inverse image of each neutrosophic closed set in Y is neutrosophic af-closed;
(5) For each neutrosophic closed set μ of Y, $\overline{(f^{-1}(\mu) \cap \beta)} \subseteq f^{-1}(\mu)$ for every closed set in X such that $0_X \neq \beta \neq 1_X$;
(6) For each neutrosophic subset μ of Y, $\overline{(f^{-1}(\overline{(\mu)}) \cap \beta)} \subseteq f^{-1}(\overline{(\mu)})$ for every neutrosophic closed set β in X such that $0_X \neq \beta \neq 1_X$;
(7) For each neutrosophic subset λ of X, $f(\overline{(\lambda \cap \beta)}) \subseteq \overline{(f(\lambda))}$ for every neutrosophic closed set β in X such that $0_X \neq \beta \neq 1_X$;
(8) For each neutrosophic subset μ of Y, $\overline{(f^{-1}(\mu))}_{af} \subseteq f^{-1}(\overline{(\mu)})$;
(9) For each neutrosophic subset μ of Y, $f^{-1}((\mu)^\circ) \subseteq (f^{-1}(\mu))^\circ_{af}$.

Proof. (1) ⇒ (2): Let $x_{r,t,s} \in X$ and μ be any neutrosophic open set of Y containing $f(x_{r,t,s})$. Set $\alpha = f^{-1}(\mu)$, then by Definition 5.4, α is a neutrosophic af-open set containing $x_{r,t,s}$ and $f(\alpha) \subseteq \mu$.
(2) ⇒ (3): Every neutrosophic af-open set containing $x_{r,t,s}$ is a neutrosophic af-neighborhood of $x_{r,t,s}$ and the proof is obvious.
(3) ⇒ (1): Let μ be any neutrosophic open set in Y. For each $x_{r,t,s} \in f^{-1}(\mu)$, $f(x_{r,t,s}) \in \mu \in \sigma$. By (3) there exists a neutrosophic af- neighborhood ν of $x_{r,t,s}$ such that $f(\nu) \subseteq \mu$; hence $x_{r,t,s} \in \nu \subseteq f^{-1}(\mu)$. There exists $\alpha_{x_{r,t,s}} \in NafO(X)$ such that $x_{r,t,s} \in \alpha_{x_{r,t,s}} \subseteq \nu \subseteq f^{-1}(\mu)$. Hence $f^{-1}(\mu) = \bigcup\{\alpha_{x_{r,t,s}} : x_{r,t,s} \in f^{-1}(\mu)\} \in NafO(X)$. This shows that f is neutrosophic af-continuous.
(1) ⇒ (4) ⇒ (5) ⇒ (1): By Lemma 6.4, the proof is obvious.
(5) ⇒ (6): For each neutrosophic subset μ of Y, $\overline{(\mu)}$ is neutrosophic closed in Y and the proof is obvious.
(6) ⇒ (7): Let λ be any neutrosophic subset of X. Set $\mu = f(\lambda)$, then by (6) $\overline{(\lambda \cap \beta)} \subseteq \overline{(f^{-1}(\overline{(f(\lambda))}) \cap \beta)} \subseteq f^{-1}(\overline{(f(\lambda))})$ for every neutrosophic closed set β in X such that $0_X \neq \beta \neq 1_X$. Therefore, we obtain for each neutrosophic subset λ of X, $f(\overline{(\lambda \cap \beta)}) \subseteq \overline{(f(\lambda))}$ for every neutrosophic closed set β in X such that $0_X \neq \beta \neq 1_X$.
(7) ⇒ (1): Let μ be any open set of Y. Then μ^c is neutrosophic closed in Y. Set $\alpha = f^{-1}(\mu^c)$, then by (7) $f(\overline{(f^{-1}(\mu^c) \cap \beta)}) \subseteq \overline{(f(f^{-1}(\mu^c)))} = \mu^c$ for every neutrosophic closed set β in X such that $0_X \neq \beta \neq 1_X$. Therefore, we have

$\overline{(f^{-1}(\mu^c) \cap \beta)}$
$\subseteq f^{-1}(f(\overline{(f^{-1}(\mu^c) \cap \beta)})$
$\subseteq f^{-1}(\mu^c) = (f^{-1}(\mu))^c$.
Therefore, $f^{-1}(\mu) \subseteq (\overline{(f^{-1}(\mu^c) \cap \beta)})^c$
$= ((f^{-1}(\mu^c) \cap \beta)^c)^\circ$
$= (f^{-1}(\mu) \cup \beta^c)^\circ$
$= (f^{-1}(\mu) \cup \alpha)^\circ$
for every neutrosophic open set α of X such that $0_X \neq \beta \neq 1_X$.
(4) $\Rightarrow$ (8): Let μ be any neutrosophic subset of Y. By (4) $f^{-1}(\overline{\mu})$ is neutrosophic af-closed in X and
$f^{-1}(\mu) \subseteq f^{-1}(\overline{\mu})$. Therefore, $\overline{(f^{-1}(\mu))}_{af} \subseteq f^{-1}(\overline{\mu})$.
(8) $\Rightarrow$ (9): Let μ be any neutrosophic subset of Y. Then,
$f^{-1}(\mu^\circ) = f^{-1}((\overline{\mu^c})^c)$
$=(f^{-1}(\overline{\mu^c}))^c \subseteq (\overline{((f^{-1}(\mu))^c)}_{af})^c$
$= (f^{-1}(\mu))^\circ_{af}$
(9) $\Rightarrow$ (1): Let μ be any neutrosophic open set of Y. By (9), $f^{-1}(\mu) \subseteq (f^{-1}(\mu))^\circ_{af} \subseteq f^{-1}(\mu)$.
Therefore, we have $(f^{-1}(\mu))^\circ_{af} = f^{-1}(\mu)$ and hence f is neutrosophic af-continuous. □

Definition 6.3. *A function $f : (X, \tau) \to (Y, \sigma)$ is said to be neutrosophic af-irresolute if for each neutrosophic af-open set μ in (Y, σ), $f^{-1}(\mu)$ is neutrosophic af-open in (X, τ).*

Theorem 6.6. *If a function $f : (X, \tau) \to (Y, \sigma)$ is neutrosophic af-irresolute, then f is neutrosophic af-continuous.*

The converse of Theorem 6.6 is not always true as shown by the following example.

Example 6.8. *Let (X, τ), (Y, σ) be a neutrosophic topological spaces, with $X = \{a, b, c\}$, $Y = \{0.1, 0.7, 0.5\}$, $\tau = \{0_X, \lambda, 1_X\}$, $\sigma = \{0_Y, \beta, 1_Y\}$, where λ, β are two neutrosophic sets defined as $\lambda = \{\langle a, 0.3, 0.3, 0.7\rangle, \langle b, 0.2, 0.2, 0.8\rangle, \langle c, 0.5, 0.5, 0.5\rangle\}$ and*
$\beta = \{\langle 0.1, 0.3, 0.3, 0.7\rangle, \langle 0.7, 0.2, 0.2, 0.8\rangle, \langle 0.5, 0.5, 0.5, 0.5\rangle\}$
in neutrosophic topological spaces (X, τ), (Y, σ), respectively. Then, a function $f : (X, \tau) \to (Y, \sigma)$ defined as $f(a)$=0.1, $f(b)$=0.7 and $f(c)$=0.5 is neutrosophic af-continuous but not neutrosophic af-irresolute.

Definition 6.4. *A function $f : (X, \tau) \to (Y, \sigma)$ is said to be neutrosophic $af-open$ (resp. neutrosophic $af\alpha-open$, neutrosophic $afp-open$, neutrosophic $afs-open$, neutrosophic $af\beta-open$), if f(λ) is neutrosophic af-open (resp. neutrosophic afα-open, neutrosophic afp-open, neutrosophic afs-open, neutrosophic afβ-open) in (Y, σ) for every neutrosophic open set λ in (X, τ).*

Proposition 6.7. *Every neutrosophic open function is neutrosophic af-open.*

Proof. It is obvious. □

Remark. *As can be seen from Example 3.1, the converse of Proposition 6.7 may not always be true.*

Theorem 6.8. *A function $f : (X, \tau) \to (Y, \sigma)$ is neutrosophic af-open if and only if for each neutrosophic subset μ in (Y, σ) each neutrosophic closed set β in (X, τ) containing $f^{-1}(\mu)$, there exists a neutrosophic af-closed set ν in (Y, σ) containing μ such that $f^{-1}(\nu) \subseteq \beta$.*

Proof. Necessity. Let $\nu = (f(\beta^c))^c$. Since $f^{-1}(\mu) \subseteq \beta$, we have $f(\beta^c) \subseteq \mu^c$. Since f is neutrosophic af-open, then ν is neutrosophic af-closed and $f^{-1}(\nu) = (f^{-1}(f(\beta^c)))^c \subseteq (\beta^c)^c = \beta$ Sufficieny. Let α be any neutrosophic open set in (X, τ) and $\mu = (f(\alpha))^c$. Then, $f^{-1}(\mu) = (f^{-1}(f(U)))^c \subseteq \alpha^c$ and α^c is neutrosophic closed. By the hypothesis, there exists a neutrosophic af-closed set ν in (Y, σ) containing μ such that $f^{-1}(\nu) \subseteq \alpha^c$. Then, we have $\nu \subseteq (f(\alpha))^c$. Therefore, we obtain $(f(\alpha))^c \subseteq \nu \subseteq (f(\alpha))^c$ and $f(\alpha)$ is neutrosophic af-open in (Y, σ). This shows that f is neutrosophic af-open. □

Proposition 6.9. *A function $f : (X, \tau) \to (Y, \sigma)$ is neutrosophic open and $g : (Y, \sigma) \to (Z, \eta)$ is neutrosophic af-open, then $gof : (X, \tau) \to (Z, \eta)$ is neutrosophic af-open.*

7. Conclusion

Our main aim when starting this study was to offer a new alternative to the open set types that were previously introduced in mathematics and formed the basis of many studies. In the preliminaries section of our study, some definitions that are necessary to introduce this new open set type and that we have used in our previous studies are included. In the third subheading, we redefined some open set types that have been used for a long time in topological spaces from a new perspective, and after illustrating the relationship between these open set types with the help of a diagram, we introduced the new open set type. By examining the properties of this new type of open set that we have introduced, we tried to eliminate the question marks that may arise in the minds of scientists who will conduct future research, with the help of examples, which we hope will inspire our study. In the fourth subheading of our study, we introduced open set types, which we can call sub-types of our new open set type, and after examining their properties and giving examples of these properties, we illustraed the relationship between them with the help of a diagram. In the fifth subheading, we introduced different interior and closure operators and neighborhood types with these operators with the help of our new set types. In the sixth subheading, which is the last subheading of our study, we examined new types of continuity.

Our expectation is that this study will pave the way for new research in topology and other sub-branches of mathematics. In addition, one of our primary goals is to help create new works that will contribute to human life in different branches of science.

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