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A Unified Approach for Out-of-Plane Forced Vibration of Axially Functionally Graded Circular Rods

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Abstract

Out-of-plane forced vibration of axially inhomogeneous curved rods is examined by the Complementary Functions Method (CFM) in the Laplace domain. The material properties, Young's modulus and density, of the circular rods are graded in the axial direction according to a power law distribution while the Poisson's ratio is supposed to be constant. The effectiveness and accuracy of the proposed method are confirmed by comparing its numerical results with those obtained by ANSYS. Application of this unified approach provides accurate results of transient response for axially functionally graded (AFG) circular rods with different variations of material properties along the rod axis.

Keywords: Axially functionally graded circular rods; Laplace Transform; Complementary Functions Method (CFM); Forced Vibration.

1. INTRODUCTION

Circular rods are widely used in practical applications of civil, mechanical and aerospace engineering. In addition, curved rods constructed by using axially functionally graded materials (AFG) are commonly used in modern engineering industries such as nuclear engineering and reactors. Because of their extensive usage, there are many research studies on the predicting and determining of the static and dynamic response of the AFG circular rods.

Elishakoff and Guede [1] studied the vibration and buckling of simply supported AFG beams. Aydogdu and Taskin [2] investigated the free vibration of simply supported FG beam. The semi-inverse method is applied to the vibration and the buckling of the axially graded simply supported beam by Aydogdu [3]. The static and dynamic behaviors of FG beams is investigated by Li [4] with the aid of 4th order governing partial differential equation. Sina et al. [5] applied a new beam theory for free vibration response of FG beams. Şimşek and Kocatürk [6] analyzed the free vibration and the dynamic response of a FG simply-supported beam subjected to a concentrated moving harmonic load. Huang and Li [7] presented a new approach for free vibration of FG beam with non-uniform cross section. Filipich and Piovan [8] deduced a technical theory for the dynamic behavior of thick FGM curved beams. Malekzadeh et. al. [9] presented the out-of-plane free vibration response of FGM circular curved beams in thermal environment. Free vibration of FG spatial curved beams is examined by Yousefi and Rastgoo [10].

The free vibration of FGM beam of non-uniform cross-section is considered by Atmane et. al. [11]. Vibrations of AFG beams of non-uniform cross-section are investigated by Hein and Feklistova [12] with using the Euler–Bernoulli theory and Haar matrices. Piovan et. al. [13] examined the in-plane and out-of-plane dynamics and buckling of FG circular curved beams. The free vibration and stability of AFG tapered Euler–Bernoulli beams are examined by Shahba and Rajasekaran [14]. Bambill et. al. [15] examined the vibrations of stepped AFG beams on the basis of the Timoshenko beam theory. Free vibration of AFG beams is examined by Li et. al. [16] for exponentially graded beams, characteristic equations are derived in closed form. Huang et. al. [17] presented a new approach to investigate the vibration behaviors of AFG Timoshenko beams of non-uniform cross-section.

Sarkar and Ganguli [18] studied the free vibration of AFG Timoshenko beams of uniform cross-section and having fixed–fixed boundary condition. Dynamics of curved AFG tapered beams are studied by Li and Zhang [19] based on a new dynamic model by using the B-spline method (BSM). Pradhan and Chakraverty [20] examined the free vibration of FG beams subjected to different of boundary conditions. Wang et. al. [21] determined that the mid-plane formulation is perfectly fine for the analysis of FGM beams.

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Çalım [22] examined the free vibration and forced vibration of AFG Timoshenko beams on two-parameter viscoelastic foundation by using the CFM, in the Laplace domain. The forced vibration behavior of AFG Timoshenko beams of varying cross-section is studied by Çalım [23] through the unified approach of the CFM and Laplace transform. Aslan et. al. [24] investigated the transient response of homogeneous, isotropic and elastic stepped circular rods under various out-of-plane dynamic loads in the Laplace domain by the CFM. Temel et. al. [25] examined the forced vibration of planar arches with the CFM in the Laplace domain.s

When the literature is reviewed, it can be clearly seen that most of the research papers are addressed to the free vibration of AFG beam. However, to the best of the author's knowledge the available literature for the forced vibration of the AFG circular rods is still very limited.

The aim of this study is to give a unified approach for the dynamic response of AFG curved rods. The material properties of the rods are considered to change continuously in the axial direction based on a simple power law. We examined the forced vibration of AFG circular rods made of Al and ZrO_2 subjected to time-dependent loads. The influences of the material gradient parameter on the transient response are remarked. The theoretical formulations and governing equation of motion are obtained based on the Timoshenko's beam theory in the time domain. Laplace transform is then applied, and the derived equations are solved by the CFM. Thereafter the differential equations are solved numerically by the fifth-order Runge –Kutta (RK5) method. An effective inverse Laplace transform method is implemented to retransfer the results to the time domain. To determine the transient response of AFG circular rods by the presented procedure a computer program is coded in FORTRAN. In order to demonstrate the validity and accuracy of the current procedure, some of present results are compared with the results of ANSYS. Accuracy of the presented method is observed.

2. MATERIAL AND METHOD

2.1 Functionally Graded Material

Assume an elastic AFG circular rod with a uniform circular cross-section. The Young's modulus, and mass density of the rod is considered to vary continuously only in the axial direction along the curvature of the rod *(ϕ)* as follows, while the Poisson's ratio is accepted to be constant.

$$E(\varphi)=(E_{al}-E_{zr})(1-\frac{\varphi}{\varphi_0})^n+E_{zr} \quad ; \quad \rho(\varphi)=(\rho_{al}-\rho_{zr})(1-\frac{\varphi}{\varphi_0})^n+\rho_{zr} \tag{1}$$

In the above equations E_{al} and E_{zr} are the Young's modulus of Aluminum and Zirconia and ρ_{al} and ρ_{zr} , are the mass density values of Aluminum and Zirconia. The material properties are assumed to be Aluminum at the $\phi = 0$ and Zirconia at $\phi=\phi_0$. n is a constant describing the variation of material properties along the axial direction of the rod, and (n = 0) corresponds to an isotropic homogeneous material.

2.2 Governing Equations

The time and location depended partial differential equations of AFG circular rods subjected to dynamic loads are given as follows:

$$\frac{\partial U_b}{\partial \varphi}=-r\Omega_n+r\frac{T_b\alpha_b}{G(\varphi)A} \tag{2}$$

$$\frac{\partial \Omega_t}{\partial \varphi}=\Omega_n+r\frac{M_t}{G(\varphi)I_t} \tag{3}$$

$$\frac{\partial \Omega_n}{\partial \varphi}=-\Omega_t+r\frac{M_n}{E(\varphi)I_n} \tag{4}$$

$$\frac{\partial T_b}{\partial \varphi}=r\rho(\varphi)A\frac{\partial^2 U_b}{\partial t^2}-rp_b \tag{5}$$

$$\frac{\partial M_t}{\partial \varphi}=r\rho(\varphi)I_t\frac{\partial^2 \Omega_t}{\partial t^2}+M_n-rm_t \tag{6}$$

$$\frac{\partial M_n}{\partial \varphi}=r\rho(\varphi)I_n\frac{\partial^2 \Omega_n}{\partial t^2}-M_t+rT_b-rm_n \tag{7}$$

$$G(\varphi)=\frac{E(\varphi)}{2(1+v)} \tag{8}$$

Here, $E(\phi)$, $\rho(\phi)$, h, A, I_t, I_n, α_b, r, p_b, m_t and m_n indicate the modulus of elasticity, mass density, radius of the cross-section, cross-sectional area, torsional moment of inertia, bending moment of inertia, shear correction factor, radius of curvature and distributed vertical load, distributed moment of torsion and bending of the rod respectively.

The unknown column matrix, $\{Y(\phi,t)\}$, for the transient response of AFG circular rods is given as:

$$\{Y(\varphi,t)\}=\{U_b,\Omega_t,\Omega_n,T_b,M_t,M_n\}^T \tag{9}$$

The Laplace transform of a time-dependent function $f(t)$ is the function "$\overline{F}(s)$".

$$L[f(t)]=\overline{F}(s)=\int_0^\infty f(t)e^{-st}dt \tag{10}$$

where "s" is the Laplace variable also known as operator variable in the Laplace domain. The Laplace transform of first and second derivatives with respect to time are as follows:

$$L\left[\dot{f}(t)\right]=s\overline{F}(s)-f(0) \tag{11}$$

$$L\left[\ddot{f}(t)\right]=s^2\overline{F}(s)-sf(0)-\dot{f}(0) \tag{12}$$

Applying the Laplace transform to equations (2-7), converts these partial differential equations to variable-coefficient ordinary differential equations. Thereby, the governing ordinary differential equations of the dynamic behavior of out-of-plane loaded AFG circular rods can be obtained in the Laplace domain as follows:

$$\frac{d\overline{U}_b}{d\varphi}=-r\overline{\Omega}_n+r\frac{\overline{T}_b\alpha_b}{G(\varphi)A} \tag{13}$$

$$\frac{d\overline{\Omega}_t}{d\varphi}=\overline{\Omega}_n+r\frac{\overline{M}_t}{G(\varphi)I_t} \tag{14}$$

$$\frac{d\overline{\Omega}_n}{d\varphi}=-\overline{\Omega}_t+r\frac{\overline{M}_n}{E(\varphi)I_n} \tag{15}$$

$$\frac{d\overline{T}_b}{d\varphi}=rs^2\rho(\varphi)A\overline{U}_b-r\overline{p}_b \tag{16}$$

$$\frac{d\overline{M}_t}{d\varphi}=rs^2\rho(\varphi)I_t\overline{\Omega}_t+\overline{M}_n-r\overline{m}_t \tag{17}$$

$$\frac{d\overline{M}_n}{d\varphi}=rs^2\rho(\varphi)I_n\overline{\Omega}_n-\overline{M}_t+r\overline{T}_b-r\overline{m}_n \tag{18}$$

Where the terms shown by $(\overline{\bullet})$ indicates the Laplace transform of the quantities.

$$\begin{aligned}
L\left[\rho(\varphi)A\frac{\partial^2 U_b}{\partial t^2}\right]&=\rho(\varphi)A\left[s^2\overline{U}_b-sU_b(\varphi,0)-\frac{\partial U_b(\varphi,0)}{\partial t}\right]\\
L\left[\rho(\varphi)I_t\frac{\partial^2 \Omega_t}{\partial t^2}\right]&=\rho(\varphi)I_t\left[s^2\overline{\Omega}_t-s\Omega_t(\varphi,0)-\frac{\partial \Omega_t(\varphi,0)}{\partial t}\right]\\
L\left[\rho(\varphi)I_n\frac{\partial^2 \Omega_n}{\partial t^2}\right]&=\rho(\varphi)I_n\left[s^2\overline{\Omega}_n-s\Omega_n(\varphi,0)-\frac{\partial \Omega_n(\varphi,0)}{\partial t}\right]
\end{aligned} \tag{19}$$

In this study the initial conditions for $t = 0$ are considered to be zero, which are the second and third terms on the right-hand side of the Eq. (19). Here ϕ is independent variable and s indicates the parameter of the Laplace transform. In the numerical solution of the initial-value problem based on the CFM, the fifth-order Runge–Kutta (RK5) algorithm is used which is one of the most efficient numerical methods (see Aslan et. al. [24]).

The results, obtained in the Laplace domain, are transformed to the time domain with the help of modified Durbin's numerical inverse Laplace transform method (Durbin [26], Temel et. al. [27]).

3. NUMERICAL EXAMPLES AND DISCUSSION

To determine the forced vibration behavior of the AFG circular rods, a computer program is coded in FORTRAN. Results of the presented method are compared with results of ANSYS to verify the validity of suggested procedure. Comparisons are shown in graphics. In this problem the effect of shear deformation is taken into account.

A fixed-ended AFG circular rod, demonstrated in figure 1.a, is now assumed under an out of plane point load applied to its crown point. Material properties mass density values of Aluminum and Zirconia, ρ_{al}=2700x10^{-6} *kg/cm^3*, ρ_{zr}=5700x10^{-6} *kg/cm^3* Poisson's ratio, ν = 0.3, and Young's modulus of Aluminum and Zirconia, E_{al}=7.0x10^5 *kgf/cm^2* and E_{zr}=2.0x10^6 *kgf/cm^2* . One type of time depended point load, shown in figure 1.b, with the amplitude *p0* = 1 *kgf* is implemented to the crown point of the rod. The equations (13-18) given in canonical form are solved numerically in the Laplace domain by the CFM. Here the torsional, flexural rigidity and cross-sectional area of the rod are;

$$G(\varphi)I_t = G(\varphi)\frac{\pi h^4}{2}; \qquad E(\varphi)I_n = E(\varphi)\frac{\pi h^4}{4}; \qquad A = \pi h^2$$

The boundary conditions of the symmetric point and the fixed-end are given as follows:

$$\varphi = 0 \rightarrow \begin{cases} \Omega_n = 0 \\ T_b = p/2 \\ M_t = 0 \end{cases} \quad ; \quad \varphi = \varphi_0 \rightarrow \begin{cases} U_b = 0 \\ \Omega_t = 0 \\ \Omega_n = 0 \end{cases}$$

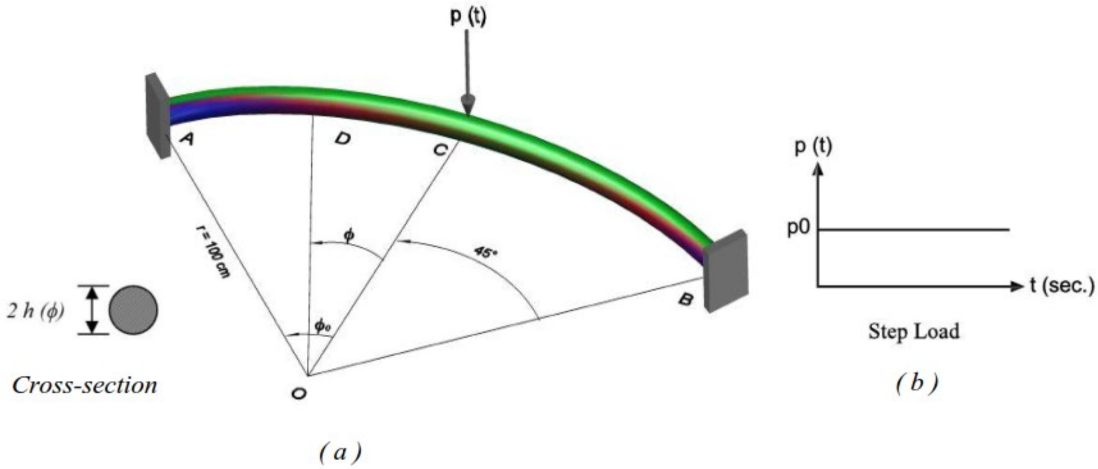

Figure 1: (a) Fix-ended AFG circular rod; (b) Time-dependent step load;

Shear correction factor α_b = 1.11, radius of the circular rod *r* = 100 cm and $\phi_0 = \pi/4$.

To examine the dynamic response of the AFG circular with the traditional axially layered model in ANSYS a parametric study has been done to determine the sufficient number of finite elements and axial layers for having a approximate continuous variation of the material properties. To obtain the accurate forced vibration behavior, the AFG rod is divided into 30, 90 and 270 elements and comparisons are given in figures 2 – 3. The material gradient index is assumed to be n =2.

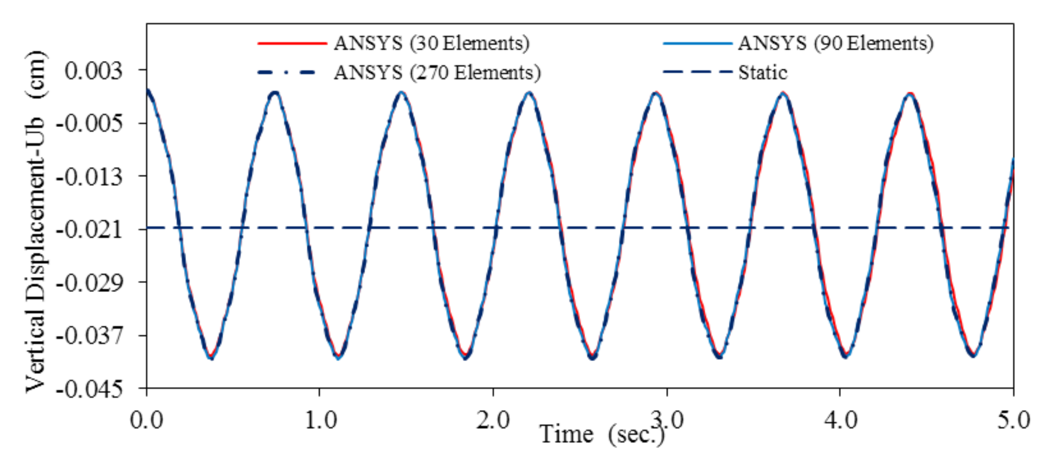

Figure 2: Vertical displacement (U_b) of the midpoint of the AFG circular rod.

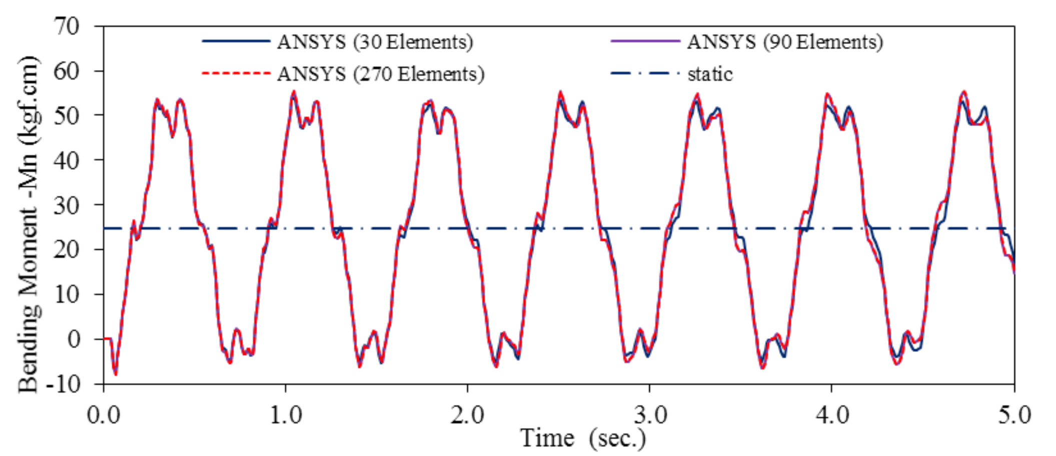

Figure 3: Moment of bending (M_n) at the rod fixed-end.

It is manifest in the figures 2 and 3 that the AFG rod must be meshed into 270 or more finite elements to acquire accurate result in ANSYS. For this reason, 270 finite elements will be used to mesh the AFG arch in ANSYS.

Temel et. al. [25] indicated the efficiency and exactness of the unified approach of the Laplace transform and the CFM by showing that results of the presented method for a coarse time increment along with fewer Laplace transform parameters overlap the results of ANSYS which are obtained with finer time increments. Because in the dynamic analysis the accuracy of conventional numerical step-by-step time integration methods depends on the appropriate selection of the optimum time increment. Thus a small time increment is needed to obtain efficient results. In this research, 64 and 512 of time sub-steps are used by the presented approach and ANSYS respectively. By using the presented method, highly accurate results can be obtained, even with a coarse time increment size.

Structural displacements, forces and moments of the AFG rod subjected to a point step loads are shown in figures 4 – 6. The material gradient index is considered to be n =2.

It can be clearly seen in figures 4 – 6 that the time-dependent vertical deformation, structural forces and moment results of AFG circular rods, obtained for a few time sub steps by the combination of Laplace transform and the CFM are in a good agreement with the results of conventional step-by-step time integration method (ANSYS) for a great number of time sub steps.

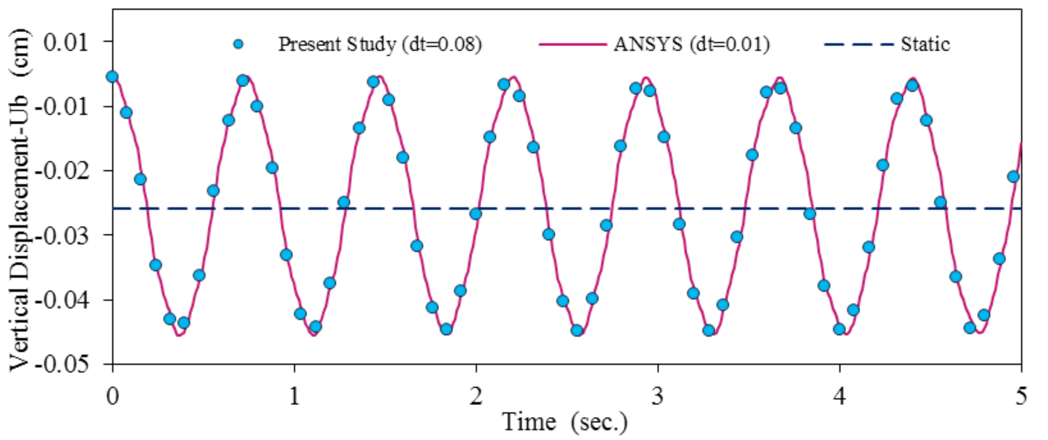

Figure 4: Comparison of the vertical displacement of the midpoint of the AFG rod.

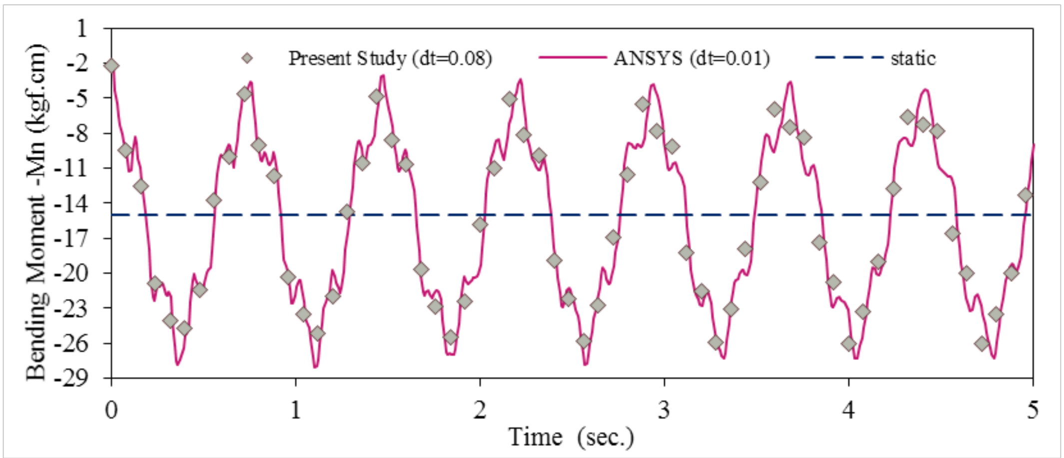

Figure 5: Comparison of the bending moment (M_n) at the rod midpoint.

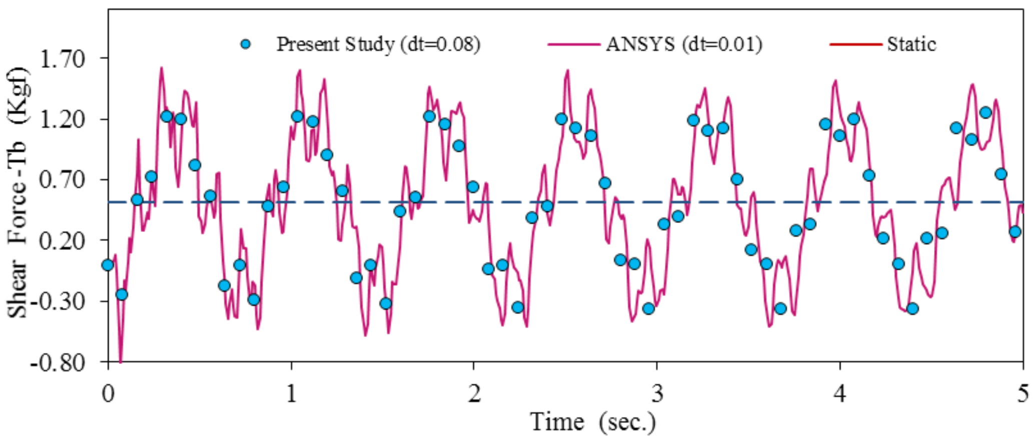

Figure 6: Comparison of the shear force (T_b) at the rod fixed-end.

The forced vibration of the AFG circular rods is now examined for various types of axially varied materials. The material of the rod is supposed as aluminum at the midpoint and zirconia at the fixed end of the rod while the material properties vary continuously in the axial direction with respect to a power-law form. Transient response is evaluated several values of the material gradient index "*n*" to present a parametric study. Comparisons are illustrated in figures 7 – 9.

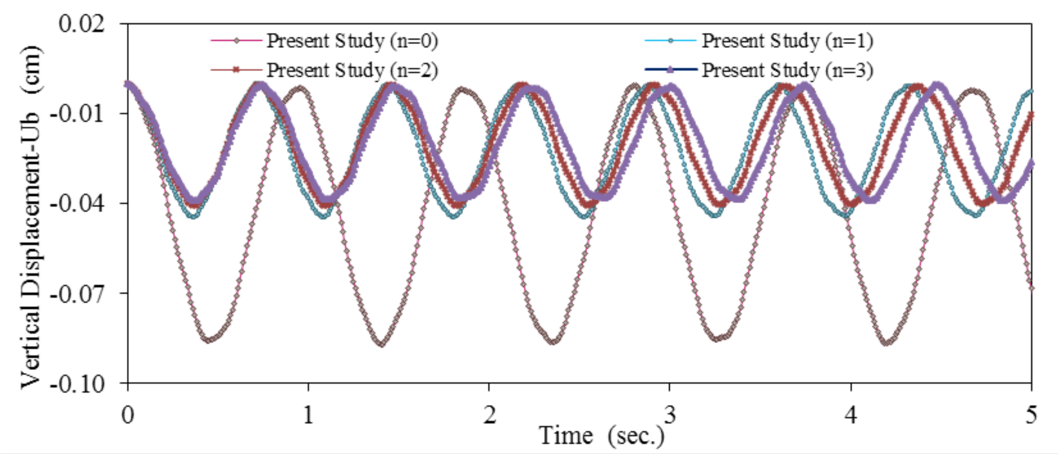

Figure 7: Comparison of the vertical displacement of the midpoint of the AFG rod.

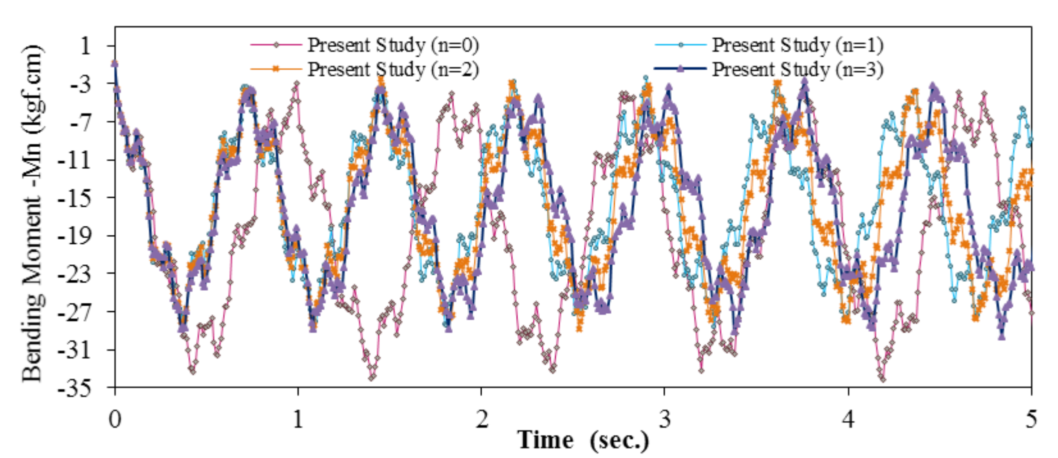

Figure 8: Comparison of the bending moment (M_n) at the rod midpoint.

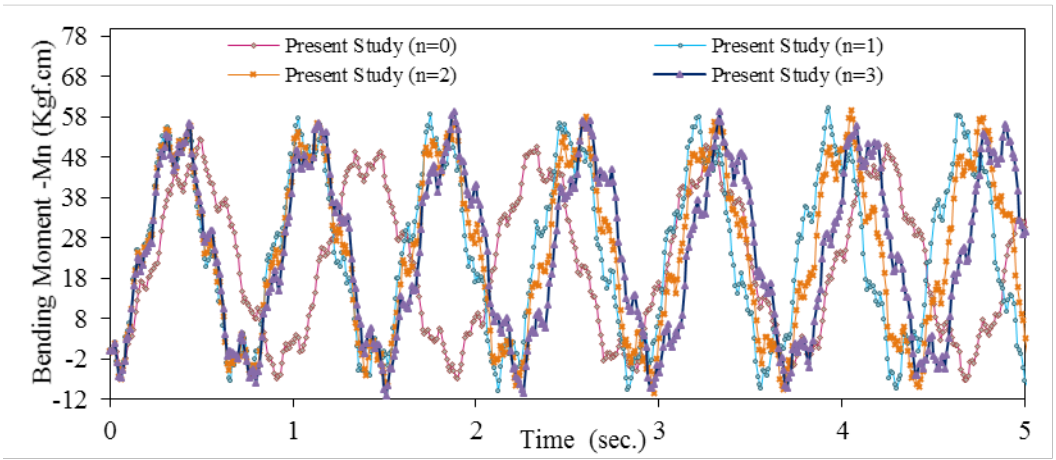

Figure 9: Comparison of the bending moment (M_n) at the rod fixed-end.

It can be seen obviously in figures 7 and 8 that displacement and bending moment of the midpoint of the rod are inversely proportional to the material gradient index, when as the "*n*" becomes larger the amplitudes of vertical displacement and bending moment of the rod midpoint become smaller. While in the fixed support of the rod there is the exact the opposite situation as the material gradient index increases the magnitude of the bending moment increases as well, which can be clearly seen in figure 9.

4. CONCLUSION

This research aims to examine the transient response of AFG circular under dynamic loads. For this purpose the unified method of Laplace transform and the CFM is employed. The effect of shear deformation is considered. RK5 algorithm is used for the numerical solution of the initial value problems.

The governing equations of the related problem are first obtained in the time domain. Laplace transform is then applied and the set of simultaneous linear algebraic equations are solved by the CFM in the Laplace domain for a set of Laplace parameters. Solution results are retransformed to the time domain by an efficient inverse numerical Laplace transform method. To validate the presented procedure, a computer program is coded in FORTRAN. Results of the presented method are compared with those obtained from ANSYS

It is found that present method gives efficient and accurate results even with fewer Laplace parameter and coarse time step size while a finer time increment is required to solve the AFG circular rods by conventional step by step time integration method.

Given comparison evinced that change of the material gradient index has remarkable influence on the response of AFG circular rods in case of forced vibration. Results of the presented method can be used as benchmark for future researches.

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