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Mathematical Model of Cabin With Suspension System to Analyze its Oscillatory Stability During Vehicle Movement

Roman O. Maksimov[1, 3], Mikhail M. Zhileykin[1, 3], Aleksandr V. Klimov[1, 3], Andrey V. Keller[2, 3], Sergey S. Shadrin[2, 3], Daria A. Makarova[2, 3], Vladimir S. Ershov[3], Yury M. Furletov[3]

[1] *KAMAZ Innovation Center, 121205, Skolkovo Innovation Center, Bolshoi boulevard, 62, Moscow, Russia.*
[2] *Sociocenter, 123104, Tverskoy boulevard, 13, str. 1, Moscow, Russia*
[3] *Moscow Polytechnic University, 107023, Bolshaya Semyonovskaya street, 38, Moscow, Russia*

Abstract

This research focuses on the development of a methodology for the analysis of nonlinear oscillations of a vehicle cab with a suspension system. When designing vehicles with cab suspension systems, it is important to align their operation with other vehicle modules and systems that collectively ensure the required comfort and dynamic parameters and prevent resonant oscillations in the cab. A vehicle cab is a dynamic system with 6 degrees of freedom, therefore its oscillations are spatially complex and feature energy direction switching. Thus, the problems of suspended cab dynamics should be solved in a non-linear spatial setting that can account for oscillation energy redistribution between various spatial directions. The purpose of this work is to develop a mathematical model for the spatial oscillations of a suspended cab relative to the vehicle undercarriage that can help analyze the non-linear oscillations that occur in the cab to study their stability during vehicle movement. This study helped identify the adverse frequency ratios for the disturbing impact on the can suspension system that can reduce cab comfort and result in the instability of its oscillations. The authors developed methods to reduce the amplitudes of suspended cab sway when its spatial oscillations are unstable. The developed methods and mathematical model help identify and prevent resonant spatial phenomena in the cab at all stages of designing vehicle cab suspension systems. The authors proposed specific solutions to design cab suspension systems to improve cab comfort and reduce cab sway, including the usage of air bellows that can form progressive non-linear load characteristics and controllable hydraulic dampers that can increase the damping coefficient in the cab suspension system during cab sway.

Keywords: Mathematical model, Cab suspension system, Vehicle, Oscillation stability, Comfort.

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Contact
* Corresponding author
Yury M. Furletov
yury.furletov@gmail.com
Moscow Polytechnic University, 107023, Bolshaya Semyonovskaya street, 38, Moscow, Russia

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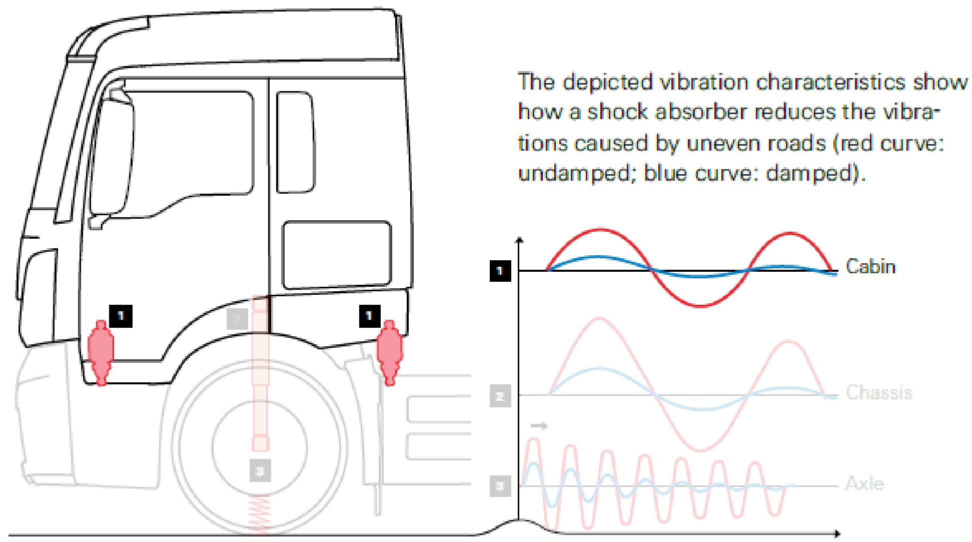
1. Introduction

Vehicle cab suspension systems started as auxiliary systems to automobile suspension aimed at improving the cab comfort during vehicle movement. This solution led to an increase in the cost of the automobile and required detailed theoretical, experimental, and practical research to assess the efficiency of using such systems [1].

Currently, the problems of studying, calculating, and designing cab suspension systems remain open as there are no straightforward and accurate theoretical approaches and practical solutions that could determine the research methods and engineering approaches to the design of such systems. Current approaches to vehicle cab suspension system development during the design stage are based on mathematical calculation and the simulation modeling of the dynamics [2] that act as the baseline for the development of structural and engineering solutions in cab suspension systems. When assessing the efficiency of vehicle cab suspension systems through the simulation modeling of the dynamics, researchers conventionally develop complex dynamic models of vehicles [3,4,5,6], although a more modern and faster approach stipulates using virtual cab setups [7].

When designing vehicles, it is important to align the operation of the cab suspension system with other vehicle systems that should jointly provide the required comfort and vehicle movement stability and prevent resonant phenomena [8]. The incorrect selection of elasticity and damping parameters in cab suspension systems may result in resonant phenomena in the vehicle cab, which may lead to a loss of comfort and movement stability and up to the destruction of the automobile as shown in Figure 1.



a

b

Fig. 1.a. The combined operation of the cab suspension system and other vehicle systems
b. The emergence of a resonant phenomenon in the truck cab.

During the initial stages of cab suspension system design, the majority of cab oscillation problems are considered basing upon the well-established theory of linear differential equations with constant coefficients [9]. However, the linear oscillation theory may not always reflect the reality as vehicle cab oscillations with 6 degrees of freedom in a 3D space can be described with a system of differential equations that contain various non-linear links between generalized coordinates that reflect the impacts of various forces (inertia, potential, dissipation, etc.). The presence of these non-linear links in certain scenarios creates conditions for the significant redistribution of oscillation energy between the generalized coordinates of the dynamic system of the suspended cab [7].

To analyze the compatibility of vehicle cab suspension system parameters with other systems for determining the probability of resonant phenomena and cab oscillation stability, it is necessary to develop a mathematical model of the suspended cab that can reflect the spatial oscillation of the cab and subsequently study its oscillations.

2. Mathematical Model of Spatial Cab Oscillations

When developing the mathematical model of suspended cab oscillations, make the following assumptions:

The cab is a perfectly rigid body, and its elastic oscillations are not considered;

The elastic elements of the cab suspension system have a linear loading characteristic within their working stroke;

The cab suspension system has viscous-friction damping elements whose force is linearly dependent on their deformation rate;

The attachment points of the elastic damping elements of the cab suspension system are located in the same horizontal plane when the cab is in the static balance state.

2.1 Kinematic Relations

The spatial positioning of the cab's rigid body can be determined as shown in Figure 2, using two rectangular coordinate systems: the fixed inertial coordinate system O2×2Y2Z2 and the moving system OXYZ bound to the cab. The positioning of the moving coordinate system relative to the fixed one is determined by the coordinates of the radius vector $\boldsymbol{R_0} = [x_2^0, y_2^0, z_2^0]$ of its pole O and the angles between the axes of the fixed and moving coordinate systems. The mutual orientation of the fixed and moving coordinate system axes can be determined by the precession θ, nutation ψ, and self-rotation φ angles, referred to as Euler angles, using trigonometric functions that express all of the direction cosines that form a direction cosine matrix A. The Euler angles, along with the coordinates of the vector $\boldsymbol{R_0}$, are the six generalized coordinates that determine the spatial position of the vehicle cab.

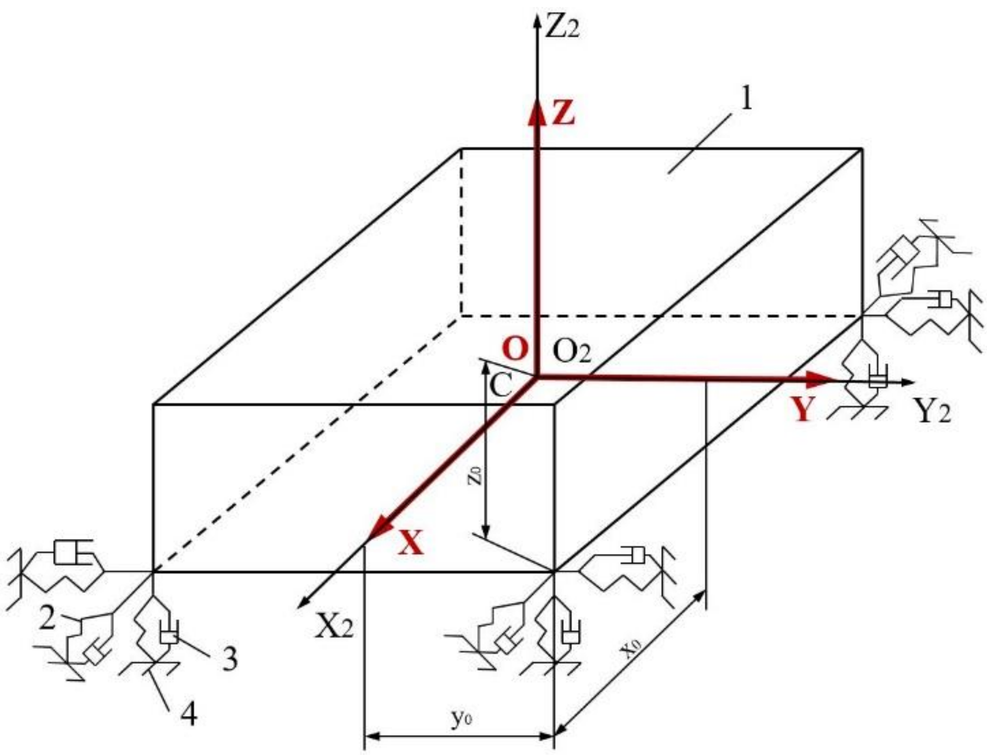


Figure 2. The calculation model for the suspended cab

1 – cab; 2 – elastic element; 3 – damping element; 4 – vehicle undercarriage

The coordinates $\boldsymbol{R_2} = [x_2, y_2, z_2]$ of a random cab point in the fixed coordinate system are linked to its coordinates $\boldsymbol{R} = [x, y, z]$ in the moving coordinate system with vector relation in Eq. (1):

$$\boldsymbol{R_2^{\mathrm{T}}} = \boldsymbol{R_2^0} + \boldsymbol{AR}^{\mathrm{T}}, \tag{1}$$

where T is the transposition symbol.

The angular velocity vector ω of the cab equals vector sum in Eq. (2):

$$\boldsymbol{\omega} = \dot{\boldsymbol{\varphi}} + \dot{\boldsymbol{\psi}} + \dot{\boldsymbol{\theta}}. \tag{2}$$

2.2 Differential Equations of the Cab Dynamics

Cab movement can be described with vector equations in Eq. (3):

$$M\frac{\mathrm{d}\boldsymbol{V}_c}{\mathrm{d}t} = \boldsymbol{H} = \boldsymbol{H}^{\mathrm{y}} + \boldsymbol{H}^{\mathrm{c}};$$
$$\frac{\mathrm{d}\boldsymbol{\sigma}_c}{\mathrm{d}t} = \boldsymbol{M}_0 = \boldsymbol{M}_0^{\mathrm{y}} + \boldsymbol{M}_0^{\mathrm{c}}, \tag{3}$$

where M is the cab weight, kg; $\boldsymbol{V}_c$ is the velocity vector of the cab mass center, m/sec; $\boldsymbol{\sigma}_c$ is the angular momentum relative to the mass center, m²·kg/sec; $\boldsymbol{H}$, $\boldsymbol{M}_0$ are the principal vector and moment of forces affecting the cab respectively, N, N·m; $\boldsymbol{H}^{\mathrm{y}}$, $\boldsymbol{M}_0^{\mathrm{y}}$ are the principal vector and moment of elastic forces affecting the cab respectively, N, N·m; $\boldsymbol{H}^{\mathrm{c}}$, $\boldsymbol{M}_0^{\mathrm{c}}$ is the principal vector and moment of resistance forces affecting the cab respectively, N, N·m;

The vehicle cab is connected to the undercarriage with 4 elastic and 4 viscous-friction damping elements that form the cab's oscillation system with six degrees of freedom. In the vertical direction, elastic elements are represented by springs with stiffness c_u where the u index denotes the deformation along the elastic element axis. Vertical damping elements are represented by hydraulic dampers with the damping coefficient k_u where the u index denotes the deformation along the damping element axis. In the longitudinal and transverse directions, the elastic and damping elements are represented by the rubber bushings of the cab suspension system guide featuring the stiffness c_{xy} and the damping coefficient k_{xy}, respectively.

In practice, vehicle cab suspension systems use complex devices that combine both the elastic and the damping elements as shown in Figure 3. Thus, the mathematical model uses the deformation of such device as the u parameter.

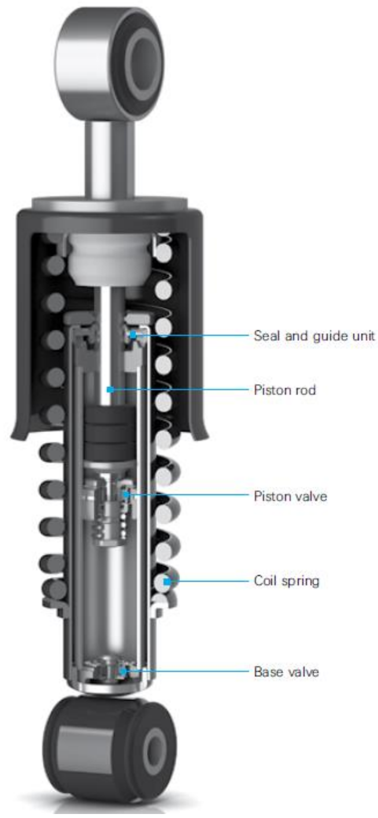


Fig. 3. The elastic and damping device for the cab suspension system

It is presumed that initially, the axes of the fixed and moving coordinate systems have the same origin of coordinates which is located in the cab mass center. For each elastic and damping element i, we used the vector coordinates $\boldsymbol{E}_i = [x_i, y_i, z_i], i = 1, \ldots, 4$ of its cab attachment point in the moving coordinate system. In this case, the coordinates of the same points in the fixed coordinate system $\boldsymbol{E}_{2i} = [x_{2i}, y_{2i}, z_{2i}], i = 1, \ldots, 4$ can be obtained using expression (1). The coordinates of the undercarriage attachment point vector of the elastic and damping element i are denoted in the fixed coordinate system through $\boldsymbol{G}_{2i} = [x_{2i}^0, y_{2i}^0, z_{2i}^0], i = 1, \ldots, 4$. Assume that the elastic and damping forces of each element are directed along the axis of this element. The deformation of elastic and damping element i is denoted as $u_i, i = 1, \ldots, 4$. The deformation value in Eq. (4) is determined in the fixed coordinate system:

$$u_i = \sqrt{(x_{2i} - x_{2i}^0)^2 + (y_{2i} - y_{2i}^0)^2 + (z_{2i} - z_{2i}^0)^2}. \qquad (4)$$

Then the elastic restoring force vector of elastic element i applied in the direction of its deformation can be calculated as $\boldsymbol{H}_i^y = c_u u_i, i = 1, \ldots, 4$.. To determine the projections of $\boldsymbol{H}^y$ on the axis of the fixed coordinate system, we used the calculation model shown in Figure 4.

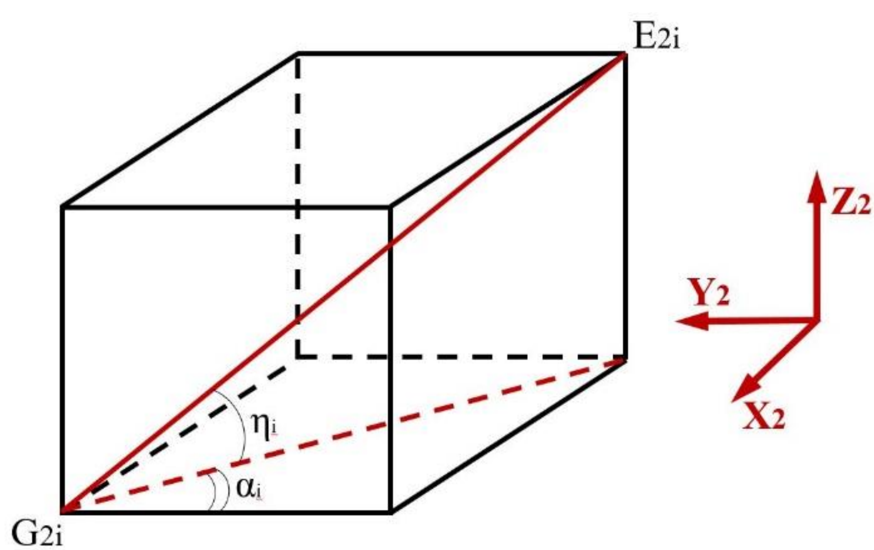


Fig. 4. The calculation model to determine the direction of forces in elastic and damping element i of the cab suspension system

The elastic force projections of elastic element i on the fixed coordinate system axis look as follows in Eq. (5):

$$H_{ix2}^{y}=c_u u_i \cos\eta_i \sin\alpha_i;$$
$$H_{iy2}^{y}=c_u u_i \cos\eta_i \cos\alpha_i;$$
$$H_{iz2}^{y}=c_u u_i \sin\eta_i;$$
$$\eta_i=\arcsin\frac{z_{2i}-z_{2i}^0}{u_i};$$
$$\alpha_i=\operatorname{arctg}\frac{x_{2i}-x_{2i}^0}{y_{2i}-y_{2i}^0}. \quad (5)$$

The coordinates of the velocity vector of the undercarriage attachment points $\boldsymbol{V}_{G_2}$ can be determined by solving the differential equations of vehicle movement. The coordinates of the vector $\boldsymbol{V}_{E_2}$ can be determined as $\boldsymbol{V}_{E_2}=\boldsymbol{V}_{C2}+\boldsymbol{\omega}\times\boldsymbol{R}$, where $\boldsymbol{V}_{C2}$ is the velocity vector of the cab mass center in the fixed coordinate system, m/sec. Thus, the deformation rate of damping element i can be calculated using expression in Eq. (6):

$$\dot{u}_i=\frac{V_{z2_i}-V_{z2_i}^0}{\sin\eta_i}. \quad (6)$$

The resistance force projections of damping element i on the fixed coordinate system axis can be determined with expressions in Eq. (7):

$$H_{ix2}^{c}=k_u\dot{u}_i\cos\eta_i\sin\alpha_i;$$
$$H_{iy2}^{c}=k_u\dot{u}_i\cos\eta_i\cos\alpha_i; \quad (7)$$
$$H_{iz2}^{c}=k_u\dot{u}_i\sin\eta_i.$$

The moment equations from the projections of forces affecting the cab in the moving coordinate system can be written down as follows in Eq. (8):

$$M_{0x}=\sum_{i=1}^{4}(H_{iy}^{y}+H_{iy}^{c})z_i-\sum_{i=1}^{4}(H_{iz}^{y}+H_{iz}^{c})y_i;$$
$$M_{0y}=\sum_{i=1}^{4}(H_{iz}^{y}+H_{iz}^{c})x_i-\sum_{i=1}^{4}(H_{ix}^{y}+H_{ix}^{c})z_i; \quad (8)$$
$$M_{0z}=\sum_{i=1}^{4}(H_{ix}^{y}+H_{ix}^{c})y_i-\sum_{i=1}^{4}(H_{iy}^{y}+H_{iy}^{c})x_i,$$

where xi, yi, zi are the coordinates of vehicle cab attachment points of elastic and damping element i, m.

2.3 Transforming Mathematical Model of Suspended Cab Oscillations into a Quasi-Linear Form

The structure of non-linear forces in the equation system (3) is very complex. Expressing the left parts of these equations through Euler angles and their derivatives, we get cumbersome trigonometric relations between six coordinates and their derivatives that do not allow for the separation of variables. The most rational way to analyze non-linear systems is based on the usage of approximate analysis methods for systems with low non-linearity. These systems were termed quasi-linear. Movement equations in Eq. (3) become quasi-linear if the analysis is limited to the oscillations whose angles do not exceed $\pm 30°$. This limitation allows for the approximate replacement of trigonometric functions of the Euler angles with the first two terms of their power series expansion in Eq. (9), i.e.:

$$\sin\theta=\theta-\frac{\theta^3}{6};$$
$$\cos\theta=1-\frac{\theta^2}{2}. \quad (9)$$

This approach helps research the key effects of non-linear oscillations of systems. It is also important that the majority of non-linear terms in equations in Eq. (3) are represented by the products of two or more coordinates and their derivatives. The existing mathematical methods [10] are suitable for the analysis of systems of non-linear differential equations of rigid body oscillations while being limited to coordinate values that allow for the consideration of movement equations as quasi-linear.

Having transformed equation in Eq. (2) using proportions in Eq. (9), the generalized form of the equation system in Eq. (3) after the reduction of similar terms and only retaining small first-order terms in the projections on the moving coordinate system axis can be presented as a system of differential equations in Eq. (10):

$$\dot{x}-V_x=0;$$
$$\dot{V}_x+\lambda_1^2x+\frac{4k_{xy}}{M}V_x-\frac{4c_{xy}z_0}{M}\psi-\frac{4k_{xy}z_0}{M}\omega_\psi=H_x(t);$$
$$\dot{y}-V_y=0;$$
$$\dot{V}_y+\lambda_2^2y+\frac{4k_{xy}}{M}V_y+\frac{4c_{xy}z_0}{M}\theta+\frac{4k_{xy}z_0}{M}\omega_\theta=H_y(t);$$
$$\dot{z}-V_z=0;$$
$$\dot{V}_z+\lambda_3^2z+\frac{4k_z}{M}V_z=H_z(t);$$
$$\dot{\theta}-\omega_\theta=0;$$
$$\dot{\omega}_\theta+\lambda_4^2\theta+\frac{4k_{xy}z_0^2+4k_zy_0^2}{J_x}\omega_\theta+\frac{4c_{xy}z_0}{J_x}y+\frac{4k_{xy}z_0}{J_x}V_y=M_x(t); \quad (10)$$
$$\dot{\psi}-\omega_\psi=0;$$
$$\dot{\omega}_\psi+\lambda_5^2\psi+\frac{4k_{xy}z_0^2+4k_zx_0^2}{J_y}\omega_\psi-\frac{4c_{xy}z_0}{J_y}x-\frac{4k_{xy}z_0}{J_y}V_x=M_y(t);$$
$$\dot{\varphi}-\omega_\varphi=0;$$
$$\dot{\omega}_\varphi+\lambda_6^2\varphi+\frac{4k_{xy}(x_0^2+y_0^2)}{J_z}\omega_\varphi=M_z(t).$$

The analysis of equations in Eq. (10) shows that variables x_2, y_2, z_2 can be separated if this is allowed by elastic forces $H_{x2}^{y}, H_{y2}^{y}, H_{z2}^{y}$ and resistance forces $H_{x2}^{c}, H_{y2}^{c}, H_{z2}^{c}$. However, the separation of angular variables θ, ψ, φ is impossible in any circumstances. The inertial links between angular coordinates prove more solid than the links between linear coordinates.

The natural oscillation frequencies of the suspended cab can be determined using proportions in Eq. (11):

$$\lambda_1^2=\frac{4c_{xy}}{M};\ \lambda_2^2=\frac{4c_{xy}}{M};\ \lambda_3^2=\frac{4c_z}{M};$$
$$\lambda_4^2=\frac{4c_{xy}z_0^2+4c_zy_0^2}{J_x};\ \lambda_5^2=\frac{4c_{xy}z_0^2+4c_zx_0^2}{J_y};\ \lambda_6^2=\frac{4c_{xy}(x_0^2+y_0^2)}{J_z}. \tag{11}$$

The analysis of the obtained equations in Eq. (10) shows that the vertical and angular (relative to the vertical Z axis) oscillations of the suspended cab are independent and can be described with linear differential equations, while oscillatory phenomena relative to these phase variables are subject to the known resonance laws in linear systems. The longitudinal-angular cab oscillations (relative to the Y axis) are associated with the longitudinal oscillations relative to the X axis. The transverse cab oscillations (relative to the X axis) are associated with the transverse oscillations relative to the Y axis.

In practice, situations where oscillations start in the direction of one of the coordinates and automatically cause cab oscillations in the directions of its other coordinates leading to spatial resonance in the cab and cab oscillation instability are common during vehicle movement. In this case, the structure of oscillatory movement gets some components with frequencies that are multiples of the natural cab oscillations.

Table 1.The adverse frequency ratios for suspended vehicle cab oscillation stability

#	Adverse frequency ratio	External disturbance frequency ωs :
	External resonance	
	$\lambda_3\approx\omega_s$	-eigen vertical oscillation frequency of the suspended mass; -eigen vertical oscillation frequency of the vehicle on tires;
	$\lambda_4\approx\omega_s;\ \lambda_5\approx\omega_s;$ $\lambda_4\approx2\omega_s;\ \lambda_5\approx2\omega_s;$ $\lambda_4+\lambda_5\approx\omega_s;$ $\lambda_4+\lambda_5\approx2\omega_s;$ $\lvert\lambda_4-\lambda_5\rvert\approx\omega_s;$ $\lvert\lambda_4-\lambda_5\rvert\approx2\omega_s;$ $\lambda_4+2\lambda_5\approx\omega_s;$ $2\lambda_4+\lambda_5\approx\omega_s;$ $\lvert2\lambda_4-\lambda_5\rvert\approx\omega_s;$ $\lvert\lambda_4-2\lambda_5\rvert\approx\omega_s.$	-eigen longitudinal-angular oscillation frequency of the suspended mass; -eigen transverse-angular oscillation frequency of the suspended mass; -eigen torsional oscillation frequencies of the vehicle undercarriage; -eigen bending (vertical plane) oscillation frequencies of the vehicle undercarriage;
	Internal resonance	
	$\lambda_4\approx2\lambda_2;\ \lambda_5\approx2\lambda_1$	-

3. Results and Discussion

3.1 Suspended Cab Oscillation Stability Analysis

Considering the equation system in Eq. (10) and the specifics of vehicle oscillation excitation, which is more probable in the vertical, longitudinal-angular, and transverse-angular direction, we can write down the adverse ratios for the quasi-linear system of a cabin with a suspension system as shown in Table 1.

The equation system in Eq. (10) is broken into four independent systems: vertical oscillations, angular oscillations relative to the vertical axis, longitudinal-angular oscillations (relative to the Y axis) and the associated longitudinal oscillations relative to the X axis; transverse-angular oscillations (relative to the X axis) and the associated transverse oscillations relative to the Y axis. The methods of combating unstable resonant oscillations in the vertical directions are known, and the emergence of resonance in the direction of angular oscillations relative to the vertical Z axis is very unlikely. Thus, we shall consider the remaining two cases.

To analyze the stability of suspended cab oscillations, we used the second Lyapunov method which states that any system can be stable if its total time derivatives of Lyapunov functions must not be positive. Lyapunov functions and their derivatives for suspended cab oscillations for the third (V_3) and the fourth (V_4) cases as the squared form of phase variables (12):

$$V_3=x^2+V_x^2+\psi^2+\omega_\psi^2>0;\ V_4=y^2+V_y^2+\theta^2+\omega_\theta^2>0.$$
$$\dot V_3=2xV_x+2V_x\dot V_x+2\psi\omega_\psi+2\omega_\psi\dot\omega_\psi\le0; \tag{12}$$
$$\dot V_4=2yV_y+2V_y\dot V_y+2\theta\omega_\theta+2\omega_\theta\dot\omega_\theta\le0.$$

The selected total derivatives are shown in expressions (13) and (14):

$$\dot V_3=4k_{xy}\left[\frac{z_0}{J_y}V_x\omega_\psi-\frac{1}{M}V_x^2+\frac{z_0}{M}V_x\omega_\psi-\frac{z_0^2}{J_y}\omega_\psi^2\right]+4c_{xy}\left[\frac{1}{M}V_x\psi+\frac{z_0}{J_y}x\omega_\psi\right]-\frac{4k_zx_0^2}{J_y}\omega_\psi^2+xV_x+\psi\omega_\psi-\lambda_1^2xV_x-\lambda_5^2\psi\omega_\psi; \tag{13}$$

$$\dot V_4=-4k_{xy}\left[\frac{z_0}{J_x}V_y\omega_\theta+\frac{1}{M}V_y^2+\frac{z_0}{M}V_y\omega_\theta+\frac{z_0^2}{J_x}\omega_\theta^2\right]-4c_{xy}\left[\frac{1}{M}V_y\theta+\frac{z_0}{J_x}y\omega_\theta\right]-\frac{4k_zy_0^2}{J_x}\omega_\theta^2+yV_y+\theta\omega_\theta-\lambda_2^2yV_y-\lambda_4^2\theta\omega_\theta. \tag{14}$$

The obtained expressions in Eq. (13) and in Eq. (14) show that conditions in Eq. (12) may not be fulfilled for all adverse frequency ratios shown in Table 1. This may result in short-term increases of oscillation amplitudes when the frequency of external impacts changes as well. In practice, these phenomena can occur during vehicle acceleration, braking, or turning.

In expression in Eq. (13), the component $-\frac{4k_zx_0^2}{J_y}\omega_\psi^2<0$, while in expression in Eq. (14), the component $-\frac{4k_zy_0^2}{J_x}\omega_\theta^2<0$. We can increase the absolute values of these components to keep functions $\dot V_3$ and $\dot V_4$ in the non-positive domain. This can be achieved in two ways:

1)Increasing the values of x_0 and y_0. In practice, this means that the attachment points of the elastic and damping elements of the cab suspension system should be located as far from the cab mass center as possible in the horizontal plane;

2)If unstable cab oscillations occur during vehicle movement, it is necessary to increase the damping coefficient k_z of the cab suspension system damping elements.

It is impossible to completely avoid unstable cab oscillations as there can be various coordinate combinations and disturbances. However, it is possible to reduce their frequency during vehicle movement.

3.2Ways to suppress unstable oscillations of a cab with the suspension system

The first suppression method for the vertical unstable oscillations of a suspended cab requires to increase the damping coefficient k_z of the cab suspension system hydraulic dampers. To justify the second suppression method, we considered the manifestations of unstable modes in a non-linear system with one degree of freedom where oscillations are described with the Duffing equation [11]. Strictly speaking, this phenomenon does not occur in a non-linear system of unstable oscillations. However, it may have modes that are close to resonant or near-resonant. These modes occur under a specific combination of initial system movement conditions and external impact intensity. We can say that the resonant frequency of a non-linear oscillatory system is one of its limit parameters that the system can tend to but cannot reach due to its phase instability. A non-linear system with one degree of freedom can only have one near-resonant mode at frequencies close to its resonant frequency if the phase instability of the elasticity, inertia, and resistance forces is insignificant. The emergence of other resonant modes (subharmonic, superharmonic, and combined) is impossible [11, 12].

The value of the damping coefficient k_z that characterizes the dissipation losses in the vehicle cab suspension system can only affect the nature of oscillation attenuation in the system. If there are dissipation losses, the non-linear system has a resonance failure illustrated in Figure 5.a. The unstable BE branch represents the movements that are always impossible. Fitting the cab suspension system with an elastic element with a non-linear progressive loading characteristic shown in Figure 5.b results in increased comfort due to the reduced amplitudes of oscillations during resonant phenomena.

In practice, the first and the second suppression methods for the vertical unstable oscillations of a suspended cab are implemented in the vehicle cab suspension designs through the introduction of complex structures (see Figure 5.c) featuring an air bellow with non-linear progressive loading characteristic and a controllable hydraulic damper that can increase the damping coefficient k_z in the cab suspension system if unstable cab oscillations occur.

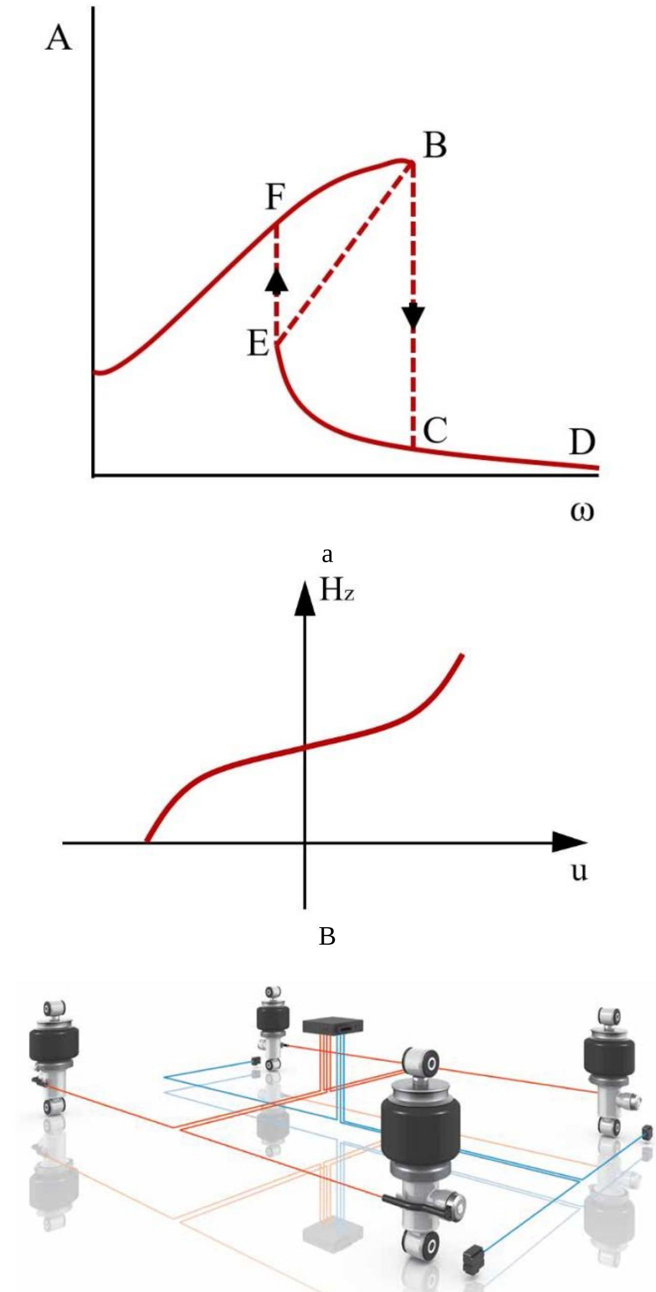


Fig. 5. a. The resonance failure in systems with non-linear elastic characteristics b. The non-linear loading characteristic of an elastic element of the suspension system c. The complex elastic/damping device design to implement unstable oscillation suppression methods in a suspended cab

3.3 Experimental Study into the Emergence of Resonant Oscillations in the Suspended Vehicle Cab

During the bench tests of a cab with a suspension system featuring spring elastic elements and hydraulic dampers using the probes and equipment shown in Figure 6, we identified various types and forms of oscillations that occur in a suspended vehicle cab.

We revealed 3 basic types of disposition toward spatial resonance in a suspended cab:

The classic resonance shown in Figure 7. It is manifested in low-frequency cab sway with a large and constantly increasing amplitude;

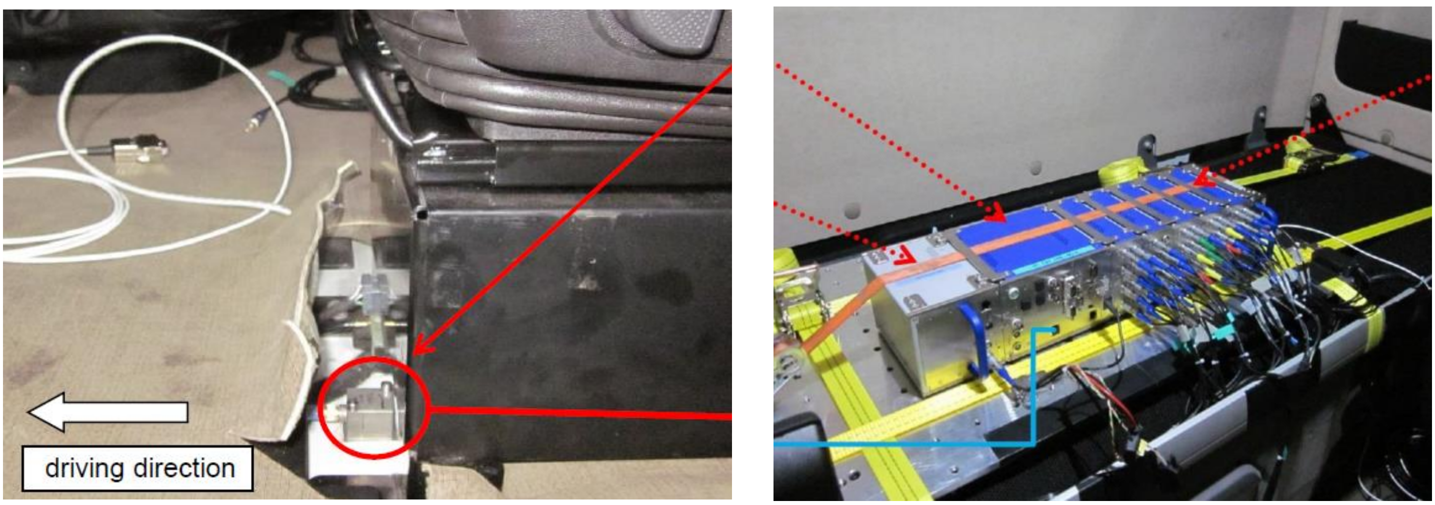


Fig. 6 . Probes and equipment for cab bench tests

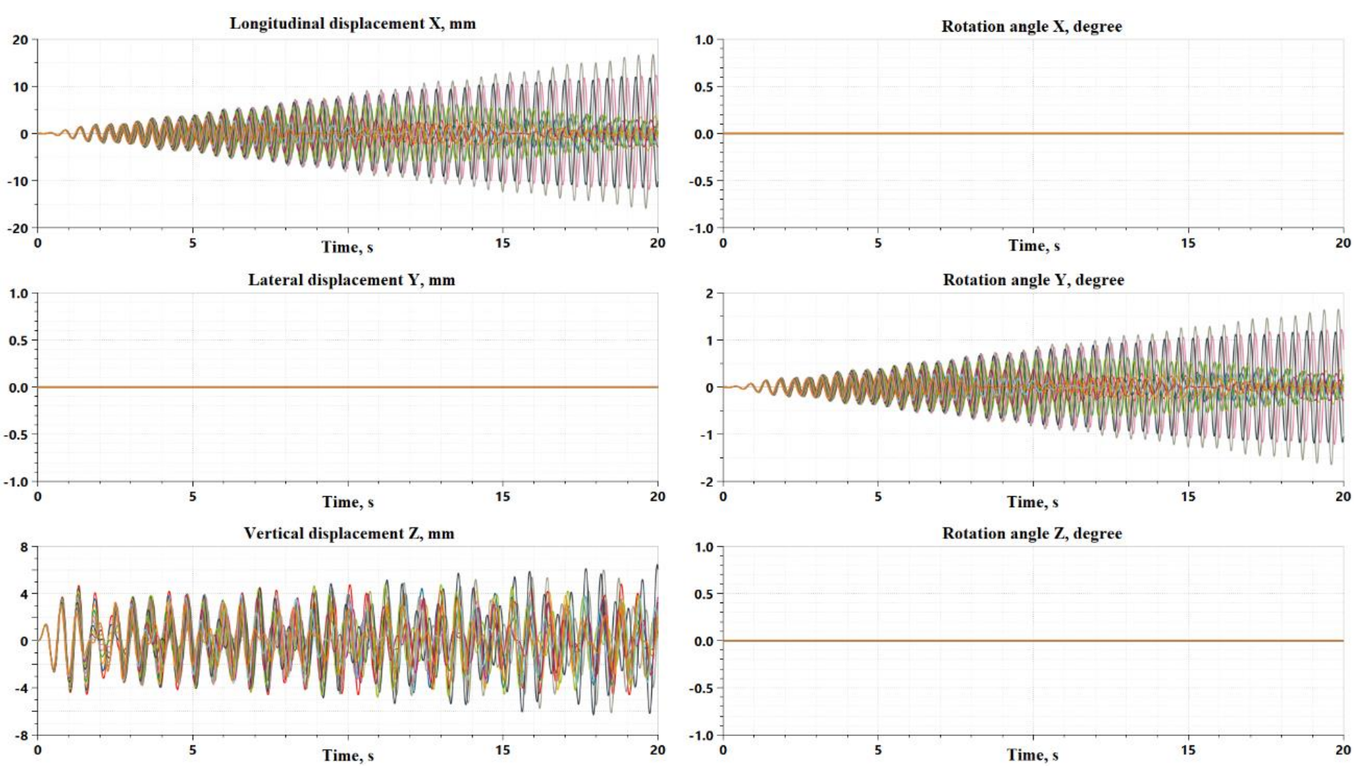


Fig. 7. The emergence of a classical spatial resonance in the vehicle cab with a suspension system

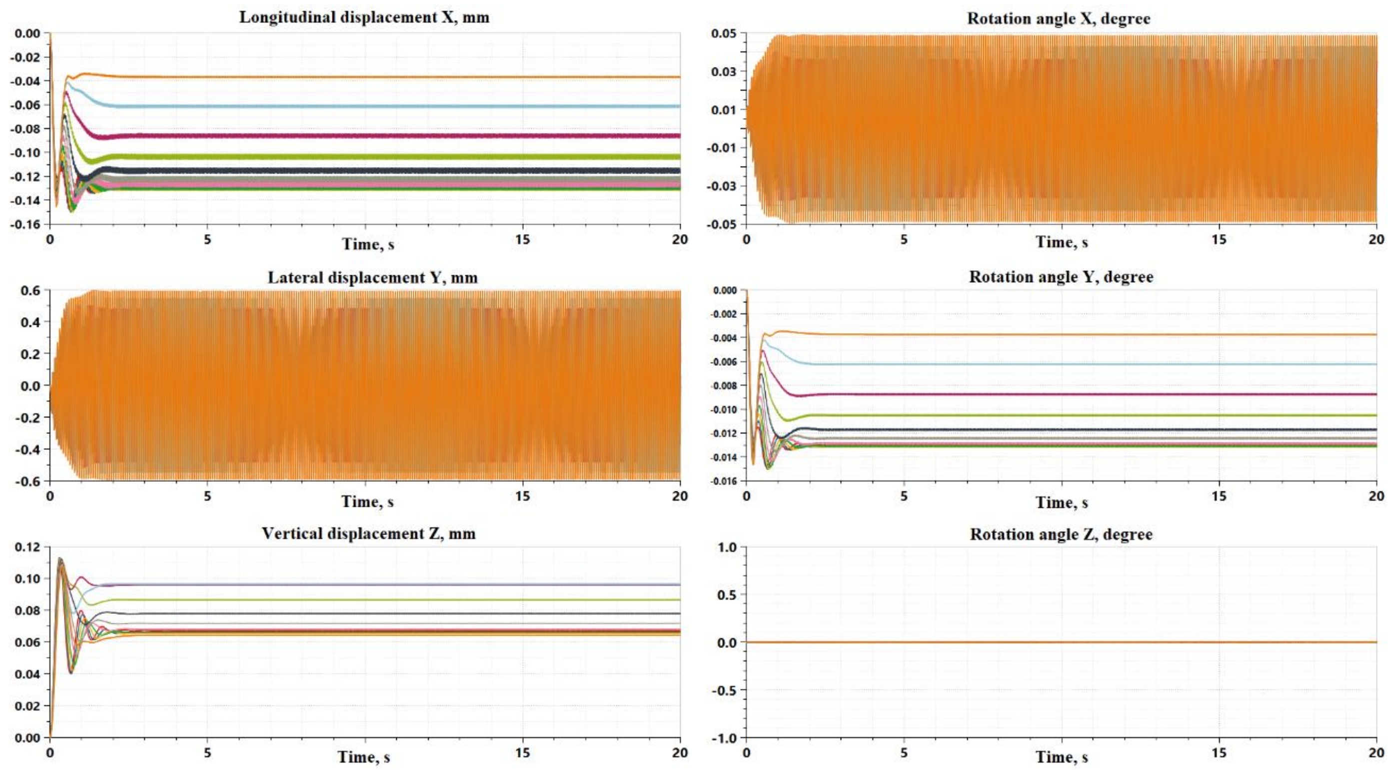


Fig. 8. The emergence of a spatial rattle in the vehicle cab with suspension

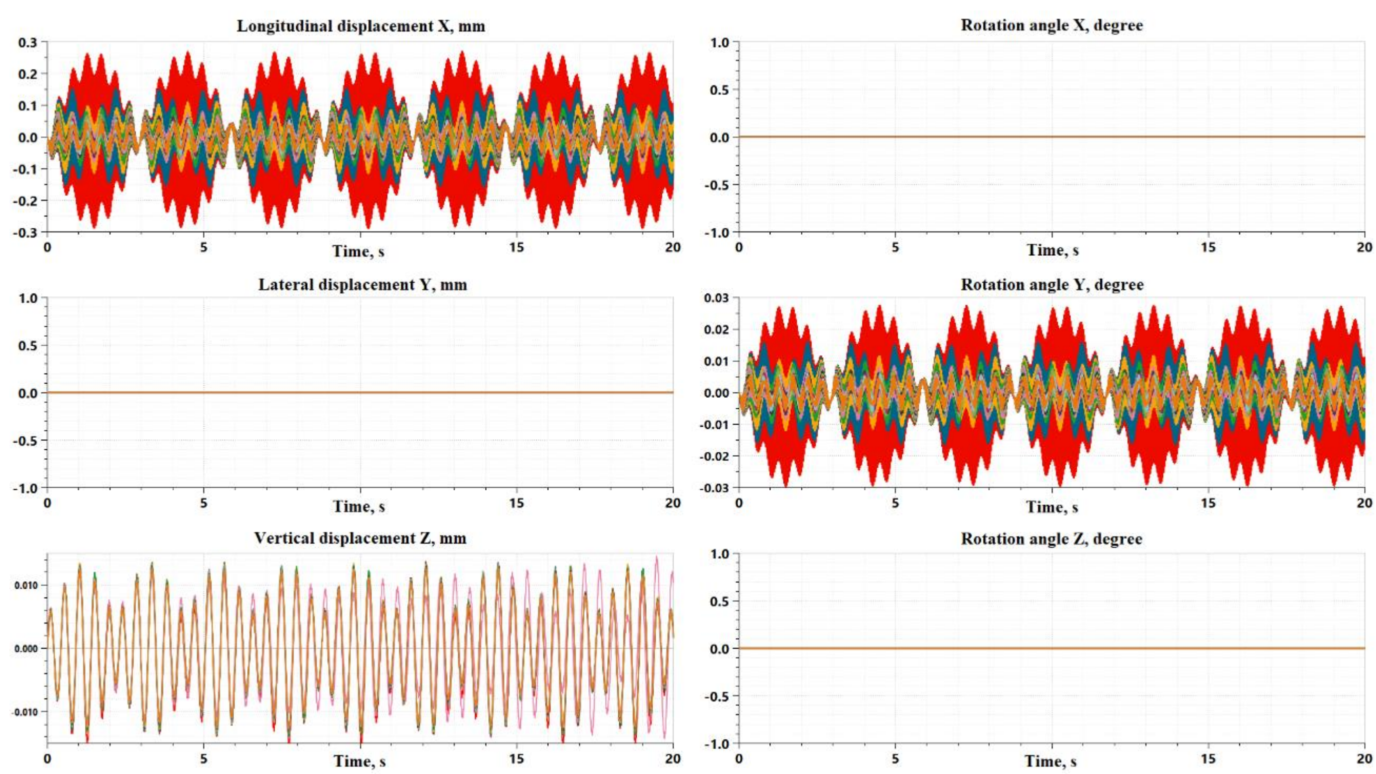


Fig. 9. The emergence of beating in the vehicle cab with suspension

The rattle shown in Figure 8. It is manifested in high-frequency cab oscillations with slowly increasing and decreasing amplitudes.

The beating shown in Figure 9. It is manifested in high-frequency cab oscillations with rapidly increasing and decreasing amplitudes.

The presented spatial resonance prevention methods for suspended vehicle cabs whose design is shown in Figure 5.c can help combat the types of resonance shown in Figure 7–9, which has been proved in experiments.

Thus, the results of the experiments confirm that the phenomena that can be analytically calculated using the developed mathematical model of a suspended cab can occur. This model can be used to improve the cab oscillation stability during vehicle movement.

4. Conclusions

In this research, we established that the vertical and angular oscillations of a suspended cab relative to the vertical Z axis are independent. The longitudinal-angular cab oscillations (relative to the Y axis) are associated with the longitudinal oscillations relative to the X axis. The transverse cab oscillations (relative to the X axis) are associated with the transverse oscillations relative to the Y axis. We identified the adverse natural frequency ratios of the cab suspension system that can lead to unstable oscillations. In practice, the identified adverse frequency ratios can help analyze the parameters of the cab suspension system components and take action to prevent unstable vehicle cab oscillations during the design stage.

The research helped identify the methods to reduce the amplitudes of suspended cab sway when its spatial oscillations are unstable:

The designed attachment points of the elastic and damping elements of the cab suspension system should be located as far from the cab mass center as possible in the horizontal plane.

If unstable cab oscillations occur in any spatial direction, it is necessary to increase the damping coefficient of the damping elements of the cab suspension system, which can be implemented by fitting the cab suspension system with controllable damping elements;

Fitting the cab suspension system with elastic elements with a non-linear progressive loading characteristic. This can be implemented by fitting the cab suspension system with air bellows, which can significantly improve the cab comfort during vehicle movement.

The developed mathematical model of the suspended cab to analyze its oscillation stability during vehicle movement can be used to select the required stiffness and damping parameters in vehicle cab suspension systems to ensure high cab comfort and prevent unstable oscillations.

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