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Risk evaluation of exchange rate portfolio based on the copula-GARCH approach

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Abstract

In this paper, risk estimation for the portfolio consisting of USD/TRY and JPY/TRY exchange rates is performed via the copula-GARCH approach. For this purpose, risk estimation models are created by means of alternative weighting techniques. The dependency between the related variables is modelled through copulas since they provide a flexible method for modelling various dependency structures such as tail dependency. It is aimed to obtain a better risk estimation model by combining the copula-GARCH approach with several weighting techniques. It is decided that the dependency between USD/TRY and JPY/TRY exchange rates is best modeled by Students' t copula among copulas tried in this study. The risk estimation models produced by the copula-GARCH approach outperform classical methods. Finally, it is concluded that the risk estimation model based on the copula-GARCH approach combined with the minimum variance weights gives better results than other weighting techniques in terms of both the performance of the risk measures and the backtesting outcomes.

***Keywords**: Copula approach, dependence modelling, exchange rate market, risk management, weighting techniques.*

Öz

Kopula-GARCH yaklaşımıyla döviz kurları portföyünün risk değerlendirmesi

Bu çalışmada, USD/TRY ve JPY/TRY döviz kurlarından oluşan portföyün kopula-GARCH yaklaşımıyla risk tahmini yapılmaktadır. Bu amaçla, alternatif ağırlıklandırma teknikleri kullanılarak risk tahmin modelleri oluşturulmaktadır. İlgili değişkenler arasındaki bağımlılık, kuyruk bağımlılığı gibi çeşitli bağımlılık yapılarının modellemede esnek bir yöntem sağladığı için kopulalar aracılığıyla modellenmektedir. Kopula-GARCH yaklaşımı çeşitli ağırlıklandırma teknikleri ile birleştirilerek daha iyi bir risk tahmin modeli elde edilmesi amaçlanmaktadır. USD/TRY ve JPY/TRY döviz kurları arasındaki bağımlılığın çalışmada denenen kopulalar arasında Student t kopula ile en iyi şekilde modellendiği belirlenmiştir. Kopula-GARCH yaklaşımı ile üretilen risk tahmin modellerinin klasik yöntemlere göre daha iyi performans göstermiştir. Son olarak, minimum varyans ağırlıkları ile birleştirilen kopula-GARCH yaklaşımına dayalı risk tahmin modelinin, hem risk ölçümlerinin performansı hem de geriye dönük test sonuçları açısından diğer ağırlıklandırma tekniklerine göre daha iyi sonuçlar verdiği sonucuna varılmıştır.

***Anahtar sözcükler**: Kopula yaklaşımı, bağımlılık modelleme, döviz kuru piyasası, risk yönetimi, ağırlıklandırma teknikleri.*

1. Introduction

Value at Risk (VaR) is a standard risk measure commonly used by financial institutions to determine the risks of assets. In other words, VaR is defined as the maximum possible loss of a financial position in a holding period and at a given confidence level. The estimation of VaR is not tough if any portfolio consists of a financial asset. However, if a portfolio consists of more than one asset, estimating accurately of VaR becomes difficult due to modelling the joint distribution functions of assets. There are several VaR estimation methods in the literature such as historical simulation and variance covariance. The second risk measure in the study is the expected shortfall (ES) which is called Conditional Value at Risk. ES that is evaluated based on loss exceeding of VaR is a consistent risk measure. In this paper, both VaR and ES estimation methods are generated by combining the copula-GARCH (Generalized Autoregressive Conditional Heteroscedasticity) approach with alternative weighting techniques. In traditional VaR estimation methods, the multivariate normal distribution is assumed for the joint distribution function of assets. However, empirical studies claim that the distribution of assets is asymmetrical and heavily tailed. Therefore, VaR estimation methods based on the assumption of multivariate normal distribution may give misleading results. The copula method is used to tackle such the problem. Sklar's theorem [1], which reveals the existence of copulas, introduces that the n-dimensional joint distribution function can be decomposed into n-marginal distributions and a copula that models the whole dependence structure between interested variables. This feature allows the margins and the copula to be selected from different distribution families in constructing the joint distribution function. Moreover, copulas are widely used in the financial area in particular since they are invariant under nonlinear strictly increasing transformations. They can model various dependency structures such as asymmetry and tail dependency.

In this paper, the VaR estimation for the portfolio consisting of USD/TRY and JPY/ TRY exchange rates is performed via the copula-GARCH approach. First, ARMA-GARCH models are used to capture the autocorrelation and heteroscedasticity of exchange rate returns. They can successfully model the characteristics of financial variables such as asymmetry and volatility clustering. Secondly, the opula method is employed to model the dependency structure between USD/TRY and JPY/TRY exchange rates. Risk estimation methods based on the copula-GARCH approach with several weighting techniques are constructed. After performing backtesting the out of sample data, the estimation performances of traditional VaR methods with the copula-GARCH methods are compared using alternative weighting techniques.

The rest of the paper is organized as follows. In section 2, related works are introduced. Section 3 presents the methodology used in the study. In section 4, experimental results are given and the main findings are emphasized. In section 5, the results of the paper are discussed.

2. Literature review

Patton [2] models the dependency structure between the Deutsche Mark and the Japanese Yen against the US dollar using the copula approach. He concluded that they are more correlated than appreciation when related exchange rates depreciate against the US dollar. Aloui et al. [3] investigates the dependency between crude oil and natural gas markets by the copula-GARCH method. The study suggested that crude oil and natural gas markets tend to comovement when general markets rise, but the tendency to comovemet in these markets decreases during decline of general markets. Yıldırım and Cengiz [4] discuss the dependency structure between exchange rate and gold prices by means of DCC (Dynamic Conditional Correlation)-GARCH (Generalized Autoregressive Conditional Heteroscedasticity)-Copula. They concluded that the dependency among the related variables changes over time and that Student-t copula outperforms other copula approaches in modelling the dependency. Roberedo et al. [5] study the dependency structure between the Czech Republic, Hungary, Poland and

Romania stock markets using elliptical and Archimedean copulas. Yıldırım and Cengiz [6] analyse the dependency between industry production and energy markets via stochastic copula approach and they determined that the dependency between industry production and energy markets evolves over time. Ignatieva and Trück [7] use the copula-GARCH approach to model the dependence between electricity spot prices in regional markets in Australia. They estimated Value at Risk for a portfolio composing of electricity spot prices. Lu et al. [8] estimate value at risk of an equal weighted portfolio consisting of crude oil and natural gas futures prices via the copula-GARCH method. Wu et al. [9] model the dependence between oil and exchange rates using the GARCH method based on the copula. Asset allocation was performed through this method and they found that the GARCH model combined with the Student t copula gives the best performance. Jin et al. [10] investigates the dependency structure between crypto, exchange rate, commodity and stock market markets with the GARCH-EVT-Copula approach. The related markets are analyzed separately before and after the Covid-19 outbreak. Before and after the outbreak, there has been a significant change in the dependency between the interested markets. Backtesting results suggest that the model offers accurate risk measures. Taleblou and Davoudi [11] estimate VaR and ES using the DCC-GARCH-Copula approach for a portfolio of 10 industrial indices on the Tehran stock exchange. They found that the DCC-GARCH model combined with Student's t copula provides reasonable results. Bruhn and Ernst [12] create a portfolio consisting of cryptocurrencies and estimate the risk measures of the portfolio via the GARCH-EVT-copula approach. They suggest that a portfolio of just one cryptocurrency or many cryptocurrencies have extreme risk of loss. He and Hamori [13] model the dependence between the oil market and the exchange rates of BRICS countries using the copula-GARCH approach. They found negative dependence and significant tail dependence between the related markets. Additionally, VaR and ES estimates are performed via the copula-GARCH approach for the portfolio created from the relevant markets.

There are several contributions to this paper. The copula-GARCH method does not require normality assumption and thus overcomes the limitations of conventional risk estimation methods. Moreover, it can model marginals with skewed distributions as well as asymmetric dependence structure. Additionally, this approach allows obtaining unknown multivariate distributions via a copula and marginals from different distribution families. Finally, risk estimation models based on the copula-GARCH method combined with alternative weighting techniques are constructed.

3. Methodology

3.1. ARMA-GARCH model

ARCH (Autoregressive Conditional Heteroscedasticity) model is introduced by Engle [14] to model time-varying volatility in financial time series. This model assumes that conditional heteroscedasticity is a linear combination of its lag errors. The shortcoming of the model requires a large number of parameter estimates to model the volatility of financial variables. To overcome this difficulty, Bollerslev [15] improved the GARCH model that is the generalized version of ARCH models. In this model, volatility is assumed to consist of both its own lagged values and its own lagged errors. Moreover, it does not require a large number of parameter estimates to model the volatilities of financial time series. ARMA-GARCH models are used since the returns of financial time series exhibit both autocorrelation and heteroscedasticity. These models are defined as in Eq. (1):

$$r_t = \mu + \sum_{i=1}^{p} \varphi_i \, r_{t-i} + \sum_{j=1}^{q} \theta_j \, \varepsilon_{t-j} + \varepsilon_t$$

$$\varepsilon_t = \sigma_t \eta_t \tag{1}$$

$$\sigma_t^2 = \omega + \sum_{i=1}^{k} \alpha_i \, \eta_{t-i}^2 + \sum_{j=1}^{l} \beta_j \, \sigma_{t-j}^2$$

Here, r_t and σ_t^2 represent the conditional mean variance of the returns at time t, in turn. μ, φ_i and θ_j in the mean equation are constant, AR and MA parameters, respectively. In the variance equation, ω, α_i and β_j are constant, ARCH and GARCH parameters, respectively. There are restictions for the parameters such as $\omega > 0$, $\alpha_i, \beta_j \geq 0$ and $\sum_{i=1}^{k} \alpha_i + \sum_{j=1}^{l} \beta_j \leq 1$. In addition, η_t is identically and independently distributed with zero mean and unit variance. Normal, Student t and skewed Student t distribution introduced by Theodossiov [16] are assumed for η_t standardized residuals.

3.2. Copula approach

Modelling the dependency structure between variables is crucial in the risk estimation of the portfolio. Since financial time series returns often exhibit asymmetry and kurtosis, modelling the dependency between these series using symmetric tools gives misleading results. To cope with this issue, the copula approach is proposed. Copulas gaining popularity especially in econometrics and finance can model asymmetric and tail dependencies as well as symmetric dependency between financial variables. Moreover, they do not require any assumptions to construct the joint distribution of the related financial variables. Therefore, the copula is a flexible approach for modelling the dependency structure between financial variables. According to Sklar's [1] theorem, any multivariate distribution function can be decomposed into marginal distributions and a copula that reflects the whole dependency structure between the relevant variables.

Let F, and $(F_1, \dots, F_n)$ be n-dimensional joint distribution function and the marginal distributions of the variables, respectively. Then, there is an n-dimensional copula for all x in R^n and it is defined as follows.

$$F(x_1, \dots, x_n) = C\big(F_1(x_1), \dots, F_n(x_n)\big) \tag{2}$$

Where, C is uniquely defined if $(F_1, \dots, F_n)$ are continuous. On the other hand, there is a n-dimensional F distribution function if $(F_1, \dots, F_n)$ marginal distribution functions and C is a copula. Any n-dimensional joint probability density function can be described with marginals and a copula as follows:

$$\begin{aligned} f(x_1, \dots, x_n) &= \frac{\partial F(x_1, \dots, x_n)}{\partial x_1, \dots, x_n} \\ &= \frac{\partial C(u_1, \dots, u_n)}{\partial u_1, \dots, u_n} x \prod_{i=1}^{n} \frac{\partial F(x_i)}{\partial x_i} \\ &= c(u_1, \dots, u_n) \; x \prod_{i=1}^{n} f_i(x_i) \end{aligned} \tag{3}$$

Here, c represents the copula density function. Eq. (3) means that selections of a copula and marginals do not depend on each other. Therefore, the copula approach enables to model the dependency flexibly. The parameter estimations of the copula are performed by Inference for Margins (IFM) method. For detailed information on theory of the copula, Nelsen [17] and Joe and Kurowicka [18] can be viewed.

3.3. Risk management

Risk estimation of financial assets is substantial for investors and financial institutions. For this purpose, there are two main measures commonly used in literature: VaR and ES. These measures refer to the maximum expected loss of the portfolio at a given level of confidence and over a holding period under normal market conditions. The VaR and ES of a financial position over t time period at a given p-probability are defined respectively as follows.

$$VaR_{1-p} = inf\{x|F_l(x) \geq 1-p\} \quad (4)$$

$$ES_{1-p} = E(X|X > VaR) = \frac{\int_{VaR}^{\infty} xf(x)dx}{P(X > VaR)} \quad (5)$$

Here $F_l(x)$ and $f(x)$ is the cumulative distribution function and probability density function of the portfolio, respectively. $F_l(x)$ can also be described as the loss function at time l and p is a given probability. As can be seen from Eq. (4) and Eq. (5), the selection of the distribution function plays essential role for VaR and ES.

3.4. Copula-GARCH method

Autocorrelation and heteroskedasticity often exist in financial time series. Therefore, these series are not independent and identically distributed. To overcome this problem, the ARMA-GARCH approach can be used, thus obtaining independent and identically distributed series required for dependency modeling. Various dependency structures, such as asymmetric and tail dependency, may exist between new series. Therefore, flexible dependency modeling tools are needed, and copulas offer significant advantages in this context. Thanks to the copulas, the dependency structures required for risk estimation are obtained. Combining these two methods is called the copula-GARCH approach in the literature. With this approach, estimations are performed for future periods and risk measures such as VaR and ES are computed.

In this study, VaR and ES estimations of the portfolio are performed by means of the copula-GARCH approach. VaR and ES estimations based on the copula-GARCH model are obtained via the steps in Table 1.

Table 1. Algorithm for VaR and ES estimation based on the copula-GARCH approach

Step 1	ARMA-GARCH models are estimated through fitted marginal distribution for each financial asset return.
Step 2	For the T + 1 time, one-step ahead conditional means and volatilities of returns are forecasted.
Step 3	N times data is simulated from the bivariate distribution modelled by the copula-GARCH approach for T + 1 time and standardized (simulated) residuals are obtained by applying inverse transformation function to (simulated) values.
Step 4	The logarithmic return series of the assets are obtained using the conditional means and volatilities forecasted in Step 2 and the standardized residuals produced in Step 3.
Step 5	Step 3 and Step 4 are repeated for N times and then the logarithmic returns are multiplied by the weighting vector and then sorted ascending. The 95% and 99% VaR and ES estimates are calculated as follows.

	i. 95 % VaR $= N * (1 - 0.95)$ observation in sorted series. ii. 99 % VaR $= N * (1 - 0.99)$ observation in sorted series. iii. 95 % ES = mean of the first $N * (1 - 0.95)$ observations in sorted series. iv. 99 % ES = mean of the first $N * (1 - 0.99)$ observations in sorted series.

When VaR and ES are estimated by the copula-GARCH approach, the selection of N simulations is very critical. The higher the N number, the more accurate the VaR and ES estimates. However, increasing the number of simulations is time consuming. Therefore, 100000 simulation data proposed in the literature are used in this paper.

3.4.1. Portfolio weighting techniques

Portfolio weighting is one of the most essential elements of portfolio selection problems. In this paper, equal weighting, inverse volatility weighting and minimum variance weighting techniques are applied. Let n be the number of assets in a portfolio and weighting techniques are summarized in Table 2.

Table 2. Portfolio weighting techniques

Weighting techniques	Formula
Equal weighting	$w_i = \frac{1}{n}$
Inverse volatility weighting	$w_i = \frac{1/\sigma_i}{\sum_{i=1}^{n} 1/\sigma_i}$
Minimum variance weighting	$w = \min_w w^T \sum w$ $1^T w = 1$ $w \geq 0$ $c < 0$

3.4.2. Evaluation of VaR estimation

Risk estimation performances of the portfolio can be evaluated by using out of sample data set. In this sense, unconditional coverage test (KT) of Kupiec [19] and conditional coverage test (CT) of Christoffersen [20] are backtesting methods commonly used in literature. The former is based on the number of losses exceeding the VaR while the latter takes account of both the number of losses exceeding VaR and the dependency between losses. In both test, acceptance of null hypothesis indicates that proposed model is accurate. For more detailed information on these tests, Kupiec [19] and Christoffersen [20] can be viewed. The methods mentioned above select appropriate model by considering the frequency of losses exceeding the VaR and the independence of the losses. However, the information contained in these losses is limited. The loss function introduced by Lopez [21] takes into account the magnitude of the losses rather than the number of losses. Let L_t be loss at t time. So, the loss function of Lopez is defined as follows.

$$C_t^L = \begin{cases} 1 + (|L_t| - VaR_t)^2 & L_t < -VaR_t \\ 0 & L_t > -VaR_t \end{cases} \tag{6}$$

Here, C_t^L is the loss function in t time. Backtesting is performed by meaning the loss functions and estimated model with the lowest value is selected as the best fitted model.

$$\hat{C}^L = \frac{1}{T}\sum_{t=1}^{T} C_t^L \quad (7)$$

It is noted that firstly fitted model should pass the statistical tests of Kupiec [19] and Christoffersen [20]. Then, the best model is determined by comparing candidate models with the help of the loss function.

4. Empirical application

In this section, the risk estimation of a portfolio consisting of USD/TRY and JPY/TRY exchange rates is investigated. The data set consisting of 1402 daily closing prices ranges from January 1, 2015 to May 15, 2020. The data is extracted from Yahoo Finance database. For the analysis, exchange rates with daily frequency are used due to some reasons. First, it is extremely challenging to make accurate risk estimations because there are many fluctuations in daily price series. Secondly, there are various dependency structures such as asymmetric and tail dependency among daily price series, and there is a need to overcome such dependencies. It is aimed to indicate that the copula-GARCH approach is available for modelling such data. This paper is carried out with R software and some R packages are utilized for statistical tests. The price series of the USD/TRY and JPY/TRY exchange rates are displayed in Fig 1.

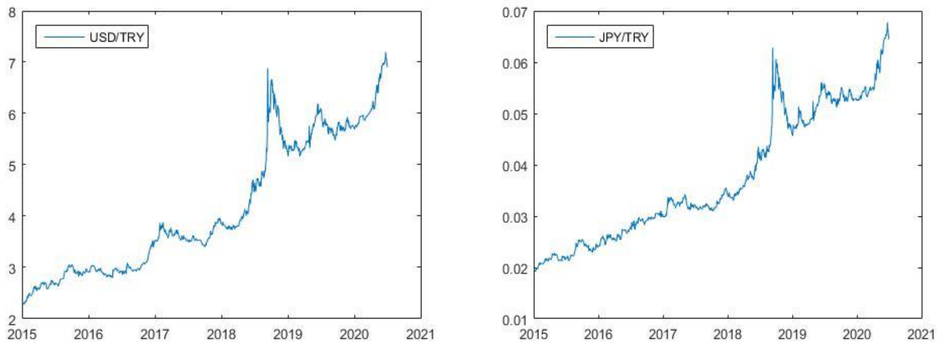

Figure 1. The price series of the USD/TRY and JPY/TRY exchange rates, respectively

In this paper, the log-returns from the original price series are analysed and they are calculated as given in Eq. (8).

$$r_t = ln\left(\frac{p_t}{p_{t-1}}\right) \quad (8)$$

Where, p_t represents the price of a financial asset in t time. The data set is divided into two parts to check the accuracy of the risk estimation: train sample and test sample. The train sample covers between January 1, 2015 and December 31, 2018 and consists of 1042 observations. The test sample ranges from January 1, 2019 to May 15, 2020, and comprise of 359 daily closing prices. The test sample equals to approximately 25% of the full sample. While estimation model is constructed using the train sample, performances of the estimated models are evaluated by means of test sample. Descriptive statistics for return series are presented in Table 3.

Table 3. Descriptive statistics for daily exchange rate returns

	Full Sample		Train Sample		Test Sample	
	USD/TRY	JPY/TRY	USD/TRY	JPY/TRY	USD/TRY	JPY/TRY
Mean	0.00077	0.00085	0.00078	0.00086	0.00074	0.00081
Std. Dev.	0.01050	0.01207	0.01110	0.01261	0.00867	0.01036
Skewness	1.96382	1.33120	2.23655	1.42049	0.11403	0.77600
Kurtosis	32.83721	19.54702	34.70541	20.80580	7.51758	7.44898
JB Stats	64044	22793	53384	19228	859.42	879.41
P-Value	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000

It is evident that the means of USD/TRY and JPY/TRY returns are positive. Besides, return of JPY/TRY is higher than the return of USD/TRY and same result is valid for their volatilities. This confirms the conclusion that assets with high return have high risk. On the other hand, the standard deviations of returns are greater than their means and this indicates that the relevant exchange rates are highly volatile. The skewness values are positive for both the exchange rates. This refers that there is a high probability of observing large positive returns at related exchange rates. Moreover, the values of kurtosis show that the exchange rates are distributed leptokurtic. These results imply that the return series are not normally distributed. Jarque Bera statistics reject significantly the null hypothesis of unconditional normality for both returns series at 1% significance level. All results seemed to be almost the same for both the train sample and the test sample. Thus, the test sample can reflect the features of all data and more reasonable results are produced.

Table 4. Stationary and autocorrelation results for daily exchange rate returns, respectively

	ADF	P-Value	Ljung-Box Q	P-Value	Ljung-Box Q^2	P-Value
USD/TRY	-28.4	0.00000	50.873	0.00000	285.63	0.00000
JPY/TRY	-30.9	0.00000	19.309	0.00727	384.31	0.00000

Table 4 presents the stationary and autocorrelation test results of exchange rate returns. The ADF (Augmented Dickey-Fuller) test rejects the null hypothesis of the non-stationary time series for both return series at 1% level of significance. Stationary time series models can be used for the conditional mean of USD/TRY and JPY/TRY exchange rates. Ljung-Box Q statistics for both returns series demonstrate that there is an autocorrelation in these series. In this case, ARMA models can be employed for modelling conditional means of the return series. Ljung-Box Q test is applied to squared return series and it is determined that these series are dependent. Lagrange multiplier test demonstrates that there is an ARCH effect in the return series. Therefore, ARMA-GARCH approach is needed for modelling the return series of USD/TRY and JPY/TRY.

Table 5: Parameter estimations for marginal distributions and statistical tests

	USD/TRY: GARCH-ST			JPY/TRY: GARCH-ST (Skewed)		
Parameter	Value	SE	P-Value	Value	SE	P-Value
μ	0.00033	0.00021	0.12096	0.00079	0.00030	0.00895
φ_1	0.59244	0.01625	0.00000	1.31142	0.00143	0.00000
φ_2	-0.99356	0.01172	0.00000	-0.99320	0.00525	0.00000
θ_1	-0.58772	0.00359	0.00000	-1.32846	0.00181	0.00000
θ_2	1.00151	0.00199	0.00000	1.00159	0.00036	0.00000
ω	0.00001	0.00001	0.42420	0.00000	0.00001	0.00000
α_1	0.13388	0.02302	0.00000	0.09866	0.01206	0.00000
β_1	0.81075	0.04617	0.00000	0.83502	0.02123	0.00000
ν	5.32426	1.62294	0.00103	1.13123	0.04799	0.00000
ξ	/	/	/	5.35866	0.80896	0.00000

	Statistics	P-Value	Statistics	P-Value
Ljung Box Q	4.94874	0.66621	1.56861	0.97994
Engle test	8.24863	0.31122	9.45190	0.22187

In addition to the GARCH model, alternative GARCH models such as GJRGARCH, TGARCH and NGARCH are tried. The best model is selected based on AIC and BIC criteria and it is evaluated whether the estimated model is suitable via diagnostic tests. It is decided that ARMA (2,2) -GARCH (1,1) Student t distribution and ARMA (2,2) -GARCH (1,1) skewed Student t distribution are convenient for USD/TRY and JPY/TRY, respectively. Table 5 displays the parameter estimations and model sufficiency for marginal models. For USD/TRY returns, all parameters except constant term in the model are found to be significant at 1% significance level while all parameters of the model estimated for JPY/TRY are significant at 1% significance level. For JPY/TRY, ξ skewness parameter is discovered to be significant and this result shows that skewed Student t distribution is required for the exchange rate. It is determined that the residuals obtained from estimated models do not demonstrate autocorrelation and ARCH effects. Hence, estimated models are adequate for both USD/TRY and JPY/TRY exchange rates. In other respects, the large estimations of $(\alpha_i + \beta_i)$ parameters in GARCH models mean that volatility is persistent. Standardized residuals obtained from ARMA-GARCH models are transformed into uniform inputs needed for the copula via probability integral transform. The joint distribution function of the portfolio is constructed based on the copula that can model the whole dependency structure between the variables.

Table 6. Selection of the copula and parameter estimations

Model	Parameter	LogL	AIC	BIC	Upper tail	Lower Tail
Normal	0.69	285.60	-569.21	-564.26	/	/
Student t	0.70;3.22	325.06	-646.13	-636.23	0.436	0.436
Clayton	1.35	216.77	-431.53	-426.58	/	0.598
Gumbel	1.94	304.45	-606.90	-601.95	0.571	/

Frank	5.87	286.63	-571.27	-566.32	/	/
Joe	2.24	255.38	-508.76	-503.81	0.638	/

Table 6 presents the estimations of parameters, which are the correlation coefficient for the Normal copula and correlation coefficient as well as degree of freedom for the Student copula, and goodness of fit for the copula models. It is deduced that Student t copula is the best fitted model since it has the smallest values of AIC and BIC. Student t copula enables to model linear and symmetrical tail dependency. Correlation coefficient and degree of freedom for Student t copula are estimated to be 0.70 and 3.22, respectively. Furthermore, estimation of tail dependency is obtained as 0.445. These results reveal that dependence structure between USD/TRY and JPY/TRY returns demonstrate a reasonably symmetrical and tail dependency.

The copula-GARCH method with historical simulation and variance-covariance approaches are analysed using various weighting techniques. Risk estimation results of equally weighted, inverse volatility weighted and minimum variance weighted portfolios are presented in Table 7, Table 8 and Table 9, respectively.

Table 7. Backtesting of risk estimations for equally weighted portfolio

Estimation Methods	Copula-GARCH	Historical simulation	Variance-Covariance
Statistical tests			
Rate of VaR exceeds at 95 %	0.03342	0.03064	0.02228
Rate of ES exceeds at 95 %	0.01114	0.01392	0.01392
KT at 95 %	2.33880	3.26738	7.25712
p-value	0.12618	0.07066	0.00706
CT at 95 %	3.02848	4.19812	9.23877
p-value	0.21997	0.12257	0.00985
Loss functions			
Lopez at 95 %	0.19722	0.26909	0.55154
Statistical tests			
Rate of VaR exceeds at 99 %	0.00557	0.01392	0.01392
Rate of ES exceeds at 99 %	0.00278	0.00278	0.00835
KT at 99 %	0.84708	0.49845	0.49845
p-value	0.35737	0.48017	0.48017
CT at 99 %	0.86955	4.33933	4.33933
p-value	0.64740	0.11421	0.11421
Loss functions			
Lopez at 99 %	0.01408	0.01107	0.01107

Table 8. Backtesting of risk estimations for inverse volatility weighted portfolio

Estimation Methods	Copula-GARCH	Historical simulation	Variance-Covariance
Statistical tests			
Rate of VaR exceeds at 95 %	0.03342	0.03064	0.02228
Rate of ES exceeds at 95 %	0.01114	0.01392	0.01392
KT at 95 %	2.33880	3.26738	7.25712
p-value	0.12618	0.07066	0.00706
CT at 95 %	3.02848	4.19812	9.23877
p-value	0.21997	0.12257	0.00985
Loss functions			
Lopez at 95 %	0.19722	0.26909	0.55154
Statistical tests			
Rate of VaR exceeds at 99 %	0.00557	0.01114	0.01392
Rate of ES exceeds at 99 %	0.00278	0.00278	0.00835
KT at 99 %	0.84708	0.04561	0.49845
p-value	0.35737	0.83088	0.48017
CT at 99 %	0.86955	4.85736	4.33933
p-value	0.64740	0.08152	0.11421
Loss functions			
Lopez at 99 %	0.01408	0.00093	0.01107

Table 9. Backtesting of risk estimations for minimum variance portfolio

Estimation Methods	Copula-GARCH	Historical simulation	Variance-Covariance

Statistical tests			
Rate of VaR exceeds at 95 %	0.03342	0.03342	0.02228
Rate of ES exceeds at 95 %	0.00835	0.01392	0.01392
KT at 95 %	2.33880	2.33880	7.25712
p-value	0.12618	0.12618	0.00706
CT at 95 %	3.02848	3.02848	9.23877
p-value	0.21997	0.21997	0.00985
Loss functions			
Lopez at 95 %	0.19722	0.19722	0.55154
Statistical tests			
Rate of VaR exceeds at 99 %	0.00557	0.01114	0.01392
Rate of ES exceeds at 99 %	0.00000	0.00557	0.00835
KT at 99 %	0.84708	0.04561	0.49845
p-value	0.35737	0.83088	0.48017
CT at 99 %	0.86955	4.85736	4.33933
p-value	0.64740	0.08815	0.11421
Loss functions			
Lopez at 99 %	0.01408	0.00093	0.01107

It is concluded that risk estimation models based on the copula-GARCH approach created with alternative weighting techniques are more reasonable models at both 5% and 1% significance levels in terms of Kupiec and Christoffersen tests. In comparison with rates of VaR and ES exceeding, it is discovered that VaR and ES exceeding rates of models based on the copula-GARCH approach are smaller than other approaches at 1% significance level while they are almost the same at 1% level of significance. In addition, except of portfolios formed with minimum variance weights, values of the loss function obtained from the copula-GARCH methods are smaller than other approaches at 5% significance level even though they are close to each other at 1% significance level. All results reveal that risk estimation models based on the copula-GARCH approach created by alternative weighting techniques give better results. On the other hand, it is deduced that the risk estimation model based on the copula-GARCH approach created with minimum variance weights outperforms those combined with other weighting techniques.

5. Conclusion

In this paper, the risk estimation of the exchange rate portfolio consisting of USD/TRY and JPY/TRY exchange rates is investigated using variance-covariance, historical simulation and the copula-GARCH approach combined with alternative weighting techniques. The study consists of three stages. First, ARMA-GARCH type models are used to model the marginals of USD/TRY and JPY/TRY exchange rate returns. For the returns of USD/TRY and JPY/TRY, it is decided based on both information criteria and diagnostic tests that the best fitted models are ARMA (2,2) -GARCH (1,1) with Student t distribution and ARMA (2,2) -GARCH (1,1) with skewed Student t distribution, respectively.

In the second stage, the copula approach is used to model the dependency between related exchange rates. The dependency between USD/TRY and JPY/TRY exchange rates is found out to be best modelled via Student t copula depending on the information criteria. In the third stage, various risk estimation models are created through several weighting techniques and they are compared by backtesting methods. It is concluded that the copula-GARCH approach is more reasonable model compared to historical simulation and variance-covariance methods. Furthermore, risk estimation models based on the copula-GARCH approach combined with minimum variance weights give better results than those created with other weighting techniques. In this study, risk estimation models are constructed for exchange rate markets. For future research, different markets such as stock and energy can be studied by means of the copula-GARCH approach formed by alternative weighting techniques.

References

[1] Sklar, M. (1959). Fonctions de repartition an dimensions et leurs marges. *Publ. inst. statist. univ. Paris*, *8*, 229-231.

[2] Patton, A. J. (2006). Modelling asymmetric exchange rate dependence. *International economic review*, *47*(2), 527-556.

[3] Aloui, R., Aïssa, M. S. B., Hammoudeh, S., and Nguyen, D. K. (2014). Dependence and extreme dependence of crude oil and natural gas prices with applications to risk management. *Energy Economics*, *42*, 332-342.

[4] Yıldırım, E., and Cengiz, M. A. (2018). Dependency between exchange rate and gold price via copula-DCC-GARCH approach. *International Journal for Scientific Research and Development*, 6(5), 974-978.

[5] Reboredo, J. C., Tiwari, A. K., and Albulescu, C. T. (2015). An analysis of dependence between Central and Eastern European stock markets. *Economic systems*, *39*(3), 474-490.

[6] Yıldırım, E., and Cengiz, M. A. (2022). Modeling dependency between industry production and energy market via stochastic copula approach. *Communications in Statistics-Simulation and Computation*, 1-14.

[7] Ignatieva, K., and Trück, S. (2016). Modeling spot price dependence in Australian electricity markets with applications to risk management. *Computers & Operations Research*, *66*, 415-433.

[8] Lu, X. F., Lai, K. K., and Liang, L. (2014). Portfolio value-at-risk estimation in energy futures markets with time-varying copula-GARCH model. *Annals of operations research*, *219*(1), 333-357.

[9] Wu, C. C., Chung, H., and Chang, Y. H. (2012). The economic value of co-movement between oil price and exchange rate using copula-based GARCH models. *Energy Economics*, *34*(1), 270-282.

[10] Jin, F., Li, J., & Li, G. (2022). Modeling the linkages between Bitcoin, gold, dollar, crude oil, and stock markets: A GARCH-EVT-copula approach. *Discrete Dynamics in Nature and Society*, *2022*.

[11] Taleblou, R., & Davoudi, M. M. (2020). Calculating Value at Risk: DCC-GARCH-Copula Approach. *Iranian Journal of Economic Research*, *25*(82), 43-82.

[12] Bruhn, P., & Ernst, D. (2022). Assessing the Risk Characteristics of the Cryptocurrency Market: A GARCH-EVT-Copula Approach. *Journal of Risk and Financial Management*, *15*(8), 346.

[13] He, Y., & Hamori, S. (2019). Conditional dependence between oil prices and exchange rates in BRICS countries: An application of the copula-GARCH model. *Journal of Risk and Financial Management*, *12*(2), 99.

[14] Engle, R. F. (1982). A general approach to Lagrange multiplier model diagnostics. *Journal of Econometrics*, *20*(1), 83-104.

[15] Bollerslev, T. (1986). Generalized autoregressive conditional heteroskedasticity. *Journal of econometrics*, *31*(3), 307-327.

[16] Theodossiov, P. (1998). Financial data and the skewed generalized t distribution. *Management Science*, *44*(12-part-1), 1650-1661.

[17] Nelsen, R. B. (2007). *An introduction to copulas*. Springer Science & Business Media.

[18] Joe, H., and Kurowicka, D. (Eds.). (2011). *Dependence modeling: vine copula handbook*. World Scientific.

[19] Kupiec, P. H. (1995). *Techniques for verifying the accuracy of risk measurement models* (Vol. 95, No. 24). Division of Research and Statistics, Division of Monetary Affairs, Federal Reserve Board.

[20] Christoffersen, P. F. (1998). Evaluating interval forecasts. *International economic review*, 841-862.

[21] Lopez, J. A. (1998). Methods for evaluating value-at-risk estimates. *Economic Policy Review*, *4*(3)