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Van Hiele geometrik düşünme düzeyleri üzerine farklı ülkelerde yayınlanmış çalışmaların incelenmesi

Investigation of studies published in different countries on van Hiele geometric thinking levels

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Bu çalışma, Gül KALELİ YILMAZ danışmanlığında, Hülya SERT ÇELİK tarafından hazırlanan "van Hiele geometrik düşünme düzeyleri ile ilgili çalışmaların karşılaştırılmalı olarak incelenmesi" başlıklı doktora tezinden üretilmiştir.

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ÖZ

Bu çalışmada, uluslararası literatürde van Hiele geometrik düşünme düzeyleri üzerinde yapılan araştırmaların amaçları; yöntemleri, örneklem grupları, veri toplama araçları, sonuçlar ve öneriler açısından derinlemesine incelenmiş ve değerlendirilmiştir. Bu bağlamda, van Hiele geometrik düşünme düzeylerine yönelik 1982-2022 yılları arasında gerçekleştirilen 81 çalışmanın verileri analiz edilmiştir. İncelenen bu çalışmalara Bursa Uludağ Üniversitesi Kütüphanesi aracılığıyla (EBSCOhost, ProQuest ve ProQuest Dissertation and Theses veritabanlarından) ve Google Akademik üzerinden erişilmiştir. Araştırmaya dahil edilen her bir çalışma içerik analizi yöntemiyle değerlendirilmiş, elde edilen veriler frekans analiziyle yorumlanarak tablolar halinde sunulmuştur. Araştırmaların büyük bir bölümünde, öğrencilerin geometrik düşünme düzeylerinin değerlendirilmesi, çeşitli öğrenme ortamlarının bu düzeyler üzerindeki etkilerinin tespit edilmesi ve teorinin gelişiminin farklı açılardan incelendiği görülmektedir. Yöntem bakımından, niteliksel çalışmalar arasında vaka çalışmaları; niceliksel çalışmalar arasında ise deneysel yöntemler diğer yaklaşımlara göre daha fazla kullanılmıştır. Çalışmaların örneklemleri incelendiğinde, çoğunlukla öğretmen adayları ve lise öğrencileri örnekleme alınmış, az sayıda çalışmada ise okul öncesi öğrenciler tercih edilmiştir. Veri toplama araçları olarak ise çoğunlukla van Hiele Geometrik Testi ve mülakatlar tercih edilmiştir. Elde edilen bulgular, farklı öğrenme ortamı tasarımlarının öğrencilerin geometrik düşünme düzeylerini geliştirmede önemli bir etkiye sahip olduğunu ortaya koymaktadır. Son olarak, geometri eğitiminin öğrencilerin mevcut geometrik düşünme düzeylerinden başlaması ve eğitimin bu düzeye uygun şekilde tasarlanması gerektiği üzerine önerilerde bulunulmuştur.

Anahtar Sözcükler: van Hiele, betimsel analiz, uluslararası alandaki çalışmalar

ABSTRACT

In this study, the aims of the studies on van Hiele geometric thinking levels in the international literature were examined and evaluated in depth in terms of methods, sample groups, data collection tools, results and recommendations. In this context, the data of 81 studies conducted between 1982 and 2022 on van Hiele geometric thinking levels were analyzed. These studies were accessed through Bursa Uludağ University library (EBSCOhost, ProQuest and ProQuest Dissertation and Theses databases) and Google Scholar. Each study included in the research was evaluated by content analysis method, and the data obtained were interpreted by frequency analysis and presented in tables. The research indicates that a significant portion of the studies focused on evaluating students' geometric thinking levels, identifying the effects of various learning environments on these levels, and examining the development of theory from different perspectives. In terms of methodology, case studies among qualitative studies and experimental methods among quantitative studies were used more than other approaches. When the samples of the studies were analyzed, mostly pre-service teachers and high school students were included in the sample, while preschool students were preferred in a few studies. Van Hiele Geometric Test and interviews were mostly preferred as data collection tools. The findings reveal that different learning environment designs have a significant effect on improving students' geometric thinking levels. Finally, it was suggested that geometry education should start from students' current geometric thinking levels and education should be designed in accordance with this level.

Keywords: van Hiele, descriptive content analysis, studies in the international arena

INTRODUCTION

Mathematics, which is dynamic with the rapidly renewing and developing structure of today's world, goes beyond a way of thinking and becomes an indispensable part of society and individuals as a way of life. Geometry, which is used in understanding that mathematics discipline is a network of relationships and in making sense of mathematical thoughts and concepts, is one of the important building blocks of mathematics (National Council of Teachers of Mathematics-NCTM, 2000).

Geometry teaching is the most fun and enjoyable part of mathematics when it starts with games and continues with puzzles in preschool period and is developed with concepts, knowledge sets and predictions (Gür, 2005). In addition, another aim of geometry teaching is to develop students' spatial skills and visual reasoning skills, and to concretize the abstract concepts and structures of mathematics (Baki, 2019; NCTM, 2000; Özcan, 2012). This situation prepares the ground for students to analyze and solve daily life problems (Struchens, Harris & Martin, 2001). Based on the problems that students experience in learning geometry, it is necessary to organize and program the geometry teaching process in order to ensure the development of geometric thinking in students (Mistretta, 2000). First developed by NCTM in 1989, the influence of various approaches and theories was seen in the creation of geometry programs. With NCTM, "van Hiele Theory" was taken as a basis in the development of the geometry field (Choi-Koh, 1999; NCTM, 2000). For this purpose, van Hiele Theory comes to the forefront in determining the geometric thinking levels of students and developing these levels. Based on this importance of Van Hiele Theory, it is necessary to have a very good understanding of this theory, considering that geometry teaching is shaped around this theory.

In van Hiele Theory, students are found in five hierarchical levels including visual, analysis, inference based on experience, inference and the most advanced level (Kaleli-Yılmaz & Sert-Çelik, 2022).The progression between these levels develops in parallel with students' learning processes and geometry experience (Duatepe Paksu, 2016; Terzi, 2010).There have been previous studies on geometric thinking in many countries (Frank & Ablordeppey, 2020; Syamsuddin, 2019; İbili et al., 2020). However, the results of most studies examining students' geometric thinking levels show that students at each level of education are not at the desired level (Alex & Mammen, 2012; Fidan & Türnüklü 2010; Gökbulut, Sidekli & Yangın 2010; Halat, 2006; Kurtuluş & Akay, 2017). In order for students to be successful in geometry lessons, it is thought that the instruction should be structured in line with the levels of the students (Choi Koh, 1999). For this reason, it becomes important to determine the geometric thinking levels of students in order to increase success in mathematics and geometry teaching. In addition to the fact that the studies are subject-oriented, the teaching practices used in the research also vary greatly (Chang et al., 2007; İbili et al., 2020; Mdyunus & Hock, 2019).

When examining the literature, it is observed that there exists a systematic review by Trimurtini et al. (2022) that surveys van Hiele-related studies internationally. This review analyzed 36 studies conducted between 2017 and 2021, mapping the research trends on geometric thinking to 12 interrelated keywords over the last five years. However, there are some limitations to these findings. In this study, dimensional effect measurements were feasible only in quantitative research, while qualitative studies provided insights into the research focus. Consequently, there is a need for a more comprehensive and updated examination of studies on van Hiele's Geometric Thinking Levels. Thus, this article, which reviews the data from 81 studies on van Hiele geometric thinking levels spanning from 1982 to 2022, is believed to offer significant contributions to the literature. The variety and characteristics of each study on geometric thinking present both opportunities and challenges for future researchers in this field. The goal of this systematic review is to synthesize findings from valid empirical research to furnish a more extensive overview of studies concerning geometric thinking, thereby aiding future advancements. In this study, a general evaluation of how studies on van Hiele theory were handled (in terms of purpose, method, sample) and what conclusions were reached (in terms of results, recommendations) was made. As a matter of fact, when the relevant international literature is examined, it is noteworthy that the number of studies dealing with van Hiele theory has increased. Thus, a holistic evaluation of the studies was made and it was aimed to identify the deficiencies in the existing literature. In this context, the current research sought to address the following questions:

1. What are the objectives of international studies on van Hiele geometric thinking levels?
2. What are the methods used in international studies on van Hiele geometric thinking levels?
3. What are the sample characteristics preferred in international studies on van Hiele geometric thinking levels?
4. What are the data collection tools used in international studies on van Hiele geometric thinking levels?
5. What are the conclusions reached by international studies on van Hiele geometric thinking levels?
6. What are the recommendations of international studies on van Hiele geometric thinking levels?

METHOD

Research Design

In this research, descriptive content analysis technique, which is one of the non-interactive designs within qualitative research approaches in terms of process and subject matter, was applied. Descriptive content analysis is a systematic method that evaluates trends and research outcomes in a descriptive dimension (Çalık & Sözbilir, 2014, p.34).

In analyzing and interpreting the studies for this research, attention was given to several stages: naming, category development, frequency calculation, and interpretation. During the descriptive content analysis, particularly in the naming and category development phase, each article was meticulously reviewed. The methodology, instruments for data collection, participant samples, and data analysis techniques of each study were categorized into distinct groups.

Data Collection, Inclusion Criteria and Analysis

While gathering data for the research, care was taken to ensure that the data were sourced from specified databases, contained the relevant keywords, were published within the designated years, aimed to prevent data duplication, were appropriate for the research field, and were suitable for the sample. These features are explained below.

- Obtaining from Specified Databases: The studies analyzed within the scope of the research consist of studies accessed through Google Scholar search engine and Bursa Uludağ University library (EBSCOhost, ProQuest and ProQuest Dissertation and Theses databases).
- Keywords Used: The search terms employed for literature searches include "van Hiele", "geometric thinking", "van Hiele geometric thinking levels", and "instructional applications". Research specifically examining studies with titles and abstracts that mention van Hiele geometric thinking levels was conducted as part of this investigation.
- Being published in the specified years: A total of 81 studies published in internationally indexed journals between 1982 and 2021 were included in the research, especially the studies of scientists who are frequently cited in van Hiele studies, such as Usiskin.
- Aiming to Prevent Data Repetition: In this descriptive content analysis study, when a study was published simultaneously as a thesis and an article, only the versions published as articles were included to prevent data duplication.
- Being Suitable for the Research Area: Among the van Hiele studies examined in this study, only the studies published on mathematics education were included in the analysis.
- Appropriateness of the Sample: There are many studies on van Hiele geometric thinking levels in the international literature. For this reason, the studies were subjected to various screening procedures. It was ensured that the studies included primarily featured relevant keywords, that the research process involved teaching practices, and that studies conducted

by researchers with a high citation count and a significant presence in the field were incorporated into the scope of the research.

Validity and Reliability of the Study

Validity is categorized into two types: internal and external. Internal validity refers to the extent to which documents or data are accurately understood and whether the findings are influenced solely by the experimental variable. External validity concerns the authenticity of the documents or data (Çepni, 2018). In order to increase internal validity, triangulation, controlling the data by elements, extending observations over a long period of time, using participatory research models and reviewing the study by other researchers are preferred (Merriam, 1988). In this study, in order to increase internal validity, long-term interaction with the data obtained from the analyzed studies was ensured, sufficient time was allocated to the study, expert opinion was taken in case of disagreements between the researchers, and the interpretations made regarding the findings were faithful to the original research. In order to increase external validity, two methods are preferred; sample selection and comparison of the results obtained by conducting the same type of research in situations with similar characteristics (Çepni, 2018). In this thesis study, in order to increase external validity, sample selection was made and selection criteria were explained, the data collection and analysis process was systematically specified, the inclusion criteria of the studies used in the study were determined and explained in accordance with the research problem, and the reasons for the selection of the methods used in the study were presented in detail.

Reliability is the ability of independent researchers to repeat the research using the same methods and to reach similar data as a result. In short, reliability is a concept that is also expressed as the reproducibility of the findings (Çepni, 2018). To enhance reliability, it is crucial to choose a sufficiently large sample, involve multiple researchers to address the issue, consult various sources and opinions, securely store the obtained data, clearly specify the exact location of the environment and the researcher, and conduct the study impartially (Yıldırım & Şimşek, 2018). In this study, in order to increase reliability, technology was utilized in the coding process, the findings were presented without comment, a discussion was made in the conclusion part of the study to cover all the findings, and the consistency between the data and the literature was checked.

Research Ethics

All the rules stated in the "Higher Education Institutions Scientific Research and Publication Ethics Directive" were followed in the entire process from the planning, implementation, data collection to the analysis of the data. None of the actions specified under the second section of the Directive, "Scientific Research and Publication Ethics Actions" have been carried out.

During the writing process of this study, scientific, ethical and citation rules were followed; no falsification was made on the collected data and this study was not sent to any other academic media for evaluation.

Research Ethics Committee Approval Information

Since this study retrieved the data from an open-access databases and it is not included in the group of studies that require Ethics Committee Permission. Therefore, Ethics Committee Permission was not declared.

FINDINGS

In this section, the findings derived from the data analysis are presented in alignment with the research problems.

Objectives of the Studies Analyzed

Below are tables and explanations related to the objectives of the studies analyzed within the scope of the research.

Table 1
Data on the Purposes of the Studies Analyzed

Codes	Studies	f
Assessing the impact of various educational settings on the levels of geometric reasoning	A2, A3, A4, A5, A6, A9, A10, A11, A12, A13, A14, A16, A19, A20, A22, A23, A27, A28, A31, A33, A35, A36, A37, A40, A41, A45, A46, A47, A48, A49, A50, A52, A53, A56, A57, A59, A60, A61, A62, A65, A67, A68, A69, A70, A71, A73, A77, A79, A81	43
Determination of geometric thinking levels	A8, A15, A18, A26, A29, A42, A44, A43, A66, A72, A74, A80	12
Examination of the development of van Hiele Theory from different perspectives	A17, A18, A24, A25, A30, A34, A75	6
Describing students' problem solving levels of van Hiele Theory based learning	A1, A55	2
Investigating the application of van Hiele Theory in educational practices across different variables	A7	1
Investigation of psychometric properties of vn Hiele Geometry Test	A21	1
Investigating the validity of van Hiele geometry test in education system	A32	1
Investigating the learning and teaching process of geometry course for pre-service mathematics teachers	A38	1
Comparison of van Hiele Theory and SOLO taxonomy	A39	1
Examining the relationships between quadrilaterals within the framework of van Hiele Theory	A51	1
Examining the comprehension difficulties of van Hiele Theory based learning on functions	A54	1
Developing a multimedia geometry tool based on van Hiele Theory and assessing its effects on enhancing critical thinking skills	A58	1
Assessment of a non-Euclidean geometry course employing van Hiele Theory	A64	1
Investigating the correlation between van Hiele levels and proof proficiency levels	A63	1
Evaluating the effects of learning based on van Hiele Theory on the geometric performance and retention of pre-service teachers	A76	1
Exploring the relationship between van Hiele geometric thinking levels and geometry performance	A78	1

As shown in Table 1, many of the studies reviewed concentrated on evaluating the impact of various educational settings on geometric thinking levels, measuring geometric thinking levels, and exploring the evolution of van Hiele Theory from diverse angles. It is notable that there are relatively few studies conducted for other purposes.

Methods Used in the Studies Reviewed

Below are tables and explanations regarding the methods of the studies examined within the scope of the research.

Table 2
Data on the Methods of the Analyzed Studies

Codes		Studies	f
Quantitative	Experimental Method	A2, A3, A4, A6, A7, A9, A13, A16, A18, A19, A20, A22, A28, A35, A36, A45, A47, A49. A52, A56, A57, A58, A59, A62, A65, A68, A69, A70, A76, A77, A78, A81	32
Qualitative	Case Study	A12, A14, A23, A27, A41, A48	6
	Teaching Experiment	A37, A67	2
	Descriptive Research	A29, A33, A55	3
	Design-Based Research	A46	1
Action Research		A14	1
Mixed Method		A31, A50, A61, A74	4

As seen in Table 2, it is noteworthy that experimental method, which is one of the quantitative research methods, was preferred in a significant portion of the studies on van Hiele. Accordingly, it is seen that 32 of the studies were conducted with experimental design (quantitative); 1 study was conducted with quantitative methods (quantitative but no design was specified); 6 studies were conducted in accordance with case, 3 studies were descriptive, 2 studies were teaching experiments, 2 studies were qualitative (qualitative but no design was specified) and 1 study was conducted in accordance with design-based research design. In addition, 1 study was action research and 4 studies were mixed method. In the studies coded A5, A17, A21, A25, A32, A34, A39, A42, A43, A51, A53, A64, A66, A72, A73, A79, A80, there is no information about the method.

Most studies using experimental methods concentrated on determining the effects of different learning environments on students' geometric thinking levels. In the study identified as A8, which aimed to evaluate the van Hiele geometric thinking levels of students, the method section details a quantitative approach that employed a multiple-choice test. Additionally, in all case studies, the primary goal was to explore how different educational settings influence students' geometric thinking levels.

In the studies not included in the table, A1 stated that the "simultaneous triangulation" method, which is defined as a combination of qualitative and quantitative research, was used in the research. In the studies conducted by A10 and A11, "research and development (R&D)" method was preferred. A15 used "descriptive research design" to investigate students' geometric thinking levels and achievement scores. In the studies conducted by A24 and A30, although nothing is specified for the method, it is stated that the data were obtained through clinical interviews. A40 and A41 used "developmental research method" to produce the learning tool. A44 did not include any information in the method section, and it is stated that the data were obtained through a multiple-choice test developed by Usiskin (1982) and individual interviews with 64 students randomly selected from these students with interview questions prepared by Mayberry (1983) for 30-45 minutes to examine their reasoning in more depth. A60, on the other hand, is included in his research as a "development research" conducted to produce a product in the form of a module based on quadrilateral identification and classification at the higher education level. A75 used a standardized "pre-test-post-test design" that included four tests given to all students and one of three proof test forms given to some students. In the method section of the study conducted by A54, there is a statement written as distributing questionnaires to middle school students. In addition, it is seen that in some studies, the methods are given general names such as quantitative research (A8) and qualitative research (A26, A38), and the methods are not specifically specified.

Sample Group of the Analyzed Studies

Below are tables and explanations concerning the sample group of the studies analyzed within the scope of the research.

Table 3
Data on the Samples of the Analyzed Studies

Sample Type	Sample Level	Studies	f
Preschool	3-6 years	A24	1
	5-6 years old	A81	1
Primary School	2nd grade	A19	1
	3rd grade	A7, A65	2
	4th grade	A48, A67	2
	5th grade	A45, A58	2
	Mixed	A42, A47	2
Middle School	6th grade	A6, A23, A30, A43	3
	7th grade	A10, A11	2
	Grade 8	A59	1
	Mixed	A2, A36, A53, A54	4
	Gifted and talented	A44,	1
High School	9th grade	A4, A49, A55, A72	4
	Grade 10	A8, A9, A37, A61, A63, A75	6
	11th grade	A1, A22, A27	3
	12th grade	A40	1
	Mixed	A21, A52	2
	Unspecified	A16, A28, A56, A68, A77	5
Teacher Candidate	Classroom	A31, A50, A69, A79	4
	Mat Teacher.	A29, A33, A38, A46, A60, A62, A64, A70, A71, A76	10
	Unspecified	A13	1
Teacher	Classroom	A73	1
	Mathematics	A2, A10, A12, A14, A27	5
Other	12-18 age group students	A14	1
	From primary school to university	A18	1
	Management Information System student	A20	1
	Higher education student	A26	1
	15-17 years old	A32, A78	2
	Ages 8-11	A41	1
	9- 11 years old	A66	1
	11-16 age group	A51	1
	12 years old	A74	1

As can be seen from Table 3, it is seen that the studies analyzed were mainly conducted with high school students, followed by pre-service teachers. Among high school students, mostly tenth grade students, and among pre-service teachers, the largest share belongs to mathematics teachers. It is also noteworthy that studies were conducted with preschool students between the ages of 3-6. In the study in which van Hiele levels were tried to be determined (A80), seventy-eight students were studied and no information was given about the grade levels or ages of the students, but the number of students placed according to the levels was included.

Data Collection Tools Used in the Analyzed Studies

Below are tables and explanations concerning the data collection tools used in the studies analyzed within the scope of the research.

Table 4

Data on the Data Collection Tools of the Analyzed Studies

Data Collection Type	Data Collection Tool	Studies	f
Test	van Hiele Geometric Thinking Test (VHGT)	A1, A2, A5, A7, A8, A9, A13, A15, A16, A21, A26, A28, A29, A31, A32, A35, A36, A48, A49, A50, A52, A53, A55, A57, A62, A63, A68, A69, A70, A72, A73, A75, A76, A78, A80	35
	Wu's Geometry Test (WGT)	A6, A19, A42, A47	4
	Geometry Test	A4, A8, A21, A22, A33, A40, A45, A48, A50, A52, A61, A68, A73, A77, A81	15
	Spatial Ability Test	A59	1
	Geometry Proof Test	A62, A63, A69	3
	Mathematical Reasoning Ability	A60	1
	Google Form test	A14	1
	Geometry Readiness Test	A28	1
	Geometric Diagnostic Test	A20	1
	Torrance's Test of Figural Creative Thinking (TTCT)	A65	1
	Learning transfer test	A7	1
	Critical Thinking Skills Test	A58	1
	Purdue Spatial Visualization Test	A31, A52	2
	Geometric Vocabulary Test	A31	1
	Problem Solving aptitude test	A1	1
	Algebraic Thinking Test	A26, A72	2
	Polygon Sorting Test	A50	1
	CDASSG Proof Test	A63	1
	Geometric shapes test	A66	1
	Card Rotation Test	A68	1
	Concept Understanding test	A71	1
	Plane Geometry National Achievement Test	A78	1
Scale/Survey	Geometry Attitude Scale	A7, A36	2
	Mathematics Learning Motivation Scale	A49, A77	2
	Student survey paper	A10	1
	Spatial Thinking Attitude Questionnaire	A31	1
	Evidence Beliefs Survey	A62	1
	Mathematics/Teaching Attitude Questionnaire	A50	1
	Survey	A22, A54, A60, A69, A74	4
	Metacognitive Questionnaire	A50	1

Data Collection Type	Data Collection Tool	Studies	f
Interview/Interview	Interview(Interview)/	A1, A2, A3, A16, A18, A23, A24, A26, A27, A29, A30, A31, A33, A40, A46, A62, A81	17
Observation/	Interview Form	A1, A10, A12, A23, A26, A27, A38, A41, A58, A60, A67, A73	12
Observation Form Document	Observation	A16, A24, A35, A45, A46, A50, A51, A71	8
	Worksheets/Activity	A5, A14, A23, A26, A37, A61, A62	7
	Screen recordings/ Video shooting	A16, A31, A50, A62	4
	Daily	A27	1
	Document analysis	A31, A38	2
	Field notes	A35, A52	2
	Mathematics textbook	A16, A38, A61	3
	Notebook/lecture notes	A23, A37	2
	Voice recording	A65, A69	2
	Open-ended Questions	A60	1
	Product Practicality Assessment Sheets (PPAS)	A60	1
	Validity Assessment Sheets (VAS)	A58	1
	Learning Implementation Plan	A4, A6	2
	Module	A4,	1
	Hard Disk	A11	1
	Verification page	A11, A37, A62	3
	Student comments	A14	1
	Artwork analysis	A14, A31, A38, A58	4
	Written Responses	A19	1
	Multimedia Learning Software	A20	1
	Bayesian training data	A38, A67	2
	Learning materials	A73	1

Table 4 shows that tests were used in a significant number of the studies analyzed. The biggest share among these is the test developed by Usiskin (1982) to measure van Hiele geometric thinking levels. The other important share among the tests belongs to the geometry test, which was used in 14 studies in total. For example, in the geometry test used in the study coded E8, depending on the purpose of the study, the content consisted of basic geometric concepts and topics such as classification and properties of triangles and quadrilaterals, while in the study coded E81, it included geometric shapes such as rectangle, square, triangle and circle. Another frequently preferred data collection tool for these studies is interviews. The reason why interviews are preferred may be that it is desired to examine the students' ways of reasoning about the subject being studied in depth and in detail. Because only in this way can the most robust and reliable information be obtained. Another frequently preferred data collection tool is observation, worksheets/activities and screen recordings. (A17, A25, A34 and A39) did not use any data collection tool in their studies. Since these studies contributed to the development of van Hiele Theory, they contain the thoughts and ideas of the researchers. It is noteworthy that the studies conducted with other methods are fewer in number and show diversity.

Results Obtained from the Studies Reviewed

The results from the analyzed studies were examined in detail, and those directly related to van Hiele theory are presented in the table below.

Table 5
Results Obtained from the Studies

Codes	Studies	f
Various learning environments are effective in enhancing students' geometric thinking levels	A2, A3, A4, A5, A6, A9, A10, A11, A13, A16, A19, A23, A31, A35, A36, A37, A45, A47, A48, A49, A50, A53, A56, A57, A60, A62, A68, A69, A73, A78	30
Different learning environments are effective on students' geometry achievement.	A7, A33, A36, A61, A67, A76, A77, A79, A81	9
Students' geometric thinking levels fall below the anticipated standard	A8, A15, A29, A39, A63, A70, A74, A75, A80	9
Various learning environments are effective in enhancing students' spatial skills and creativity	A31, A37, A40, A59, A65	5
Learning based on van Hiele Theory is effective in helping students acquire different types of skills	A1, A10, A55, A58	4
van Hiele levels are hierarchical	A9, A21, A39, A44	4
Various learning environments did not prove effective in improving students' geometric thinking levels	A5, A52, A70	3
The findings indicated that students with low geometric thinking levels also displayed low algebraic thinking skills, whereas students with high geometric thinking levels exhibited high algebraic thinking skills	A26, A52	2
It is an effective tool to design curricula/ plan and deliver lessons based on van Hiele Theory	A27, A28	2
According to van Hiele Theory, there is a significant relationship between learning and attitude	A7	1
The strategies used by pre-service mathematics teachers in teaching and learning geometry are compatible with the basic levels of van Hiele Theory, but limited at advanced levels	A12	1
Supports the importance of developing insight for students, as embodied in van Hiele Theory	A14	1
Teaching based on van Hiele Teaching Stages is effective in increasing algebra achievement	A16	1
Van Hiele levels adequately explain students' geometric thinking levels	A18	1
Students with different van Hiele levels exhibit different behaviors according to their levels.	A18	1
Van Hiele geometric thinking levels are continuous	A18	1
An innovative pedagogical tool integrating van Hiele Theory, cognitive models, and Bayesian networks was developed to create a web-based intelligent van Hiele Problem Solver	A20	1
The van Hiele Geometry Test was updated, and modifications were made to the classification criteria	A21	1
While significant differences were found at the basic levels of van Hiele Theory in different learning environments, no significant differences were found at the advanced levels.	A22	1

Codes	Studies	f
Different learning environments were not effective on students' geometry achievement	A28	1
Geometry teaching should be structured in accordance with the levels of van Hiele Theory	A32	1
Geometric thinking levels and geometry skills are effective in the acquisition of geometry understanding	A34	1
Employing a deductive approach in the teaching process is effective in enhancing students' geometric thinking according to van Hiele Theory	A38	1
Van Hiele and SOLO levels are similar	A39	1
Van Hiele Theory is ineffective in outlining thinking processes that transition from concrete to abstract	A41	1
Students' geometric thinking levels meet the expected standards	A42	1
Teaching based on van Hiele Theory is effective for students to see geometric relationships	A43	1
Geometric thinking levels are not age dependent, may vary according to geometry content objectives	A44	1
There is no significant correlation between spatial visualization ability and van Hiele levels	A68	1
Van Hiele-based transformation geometry worksheets supported by GeoGebra should be developed to improve students' understanding of the concepts	A71	1
It was determined that students at van Hiele level 3 solved algebraic geometry questions appropriately	A72	1

When examining Table 5, it is evident that different learning environments have significantly enhanced students' geometric thinking levels in a substantial portion of the results obtained from the studies. The results of various studies were obtained, showing the effectiveness of different learning environments on students' geometry achievement (studies A7, A33, A36, A61, A67, A76, A77, A79, A81), and indicating that students' geometric thinking levels were below the expected level (studies A8, A15, A29, A39, A63, A70, A74, A75, A80). Additionally, when examining studies (A1, A10, A55, A58) where students acquired different types of skills through van Hiele Theory-based learning, it was found that studies A1 and A55 led to improvements in problem-solving skills, A10 enhanced mathematical operation skills, and A58 boosted critical thinking skills. Studies A9, A21, A39, and A44 supported the hierarchical nature of van Hiele Theory and levels. In studies A26 and A52, it was revealed that students with low geometric thinking levels also had low algebraic thinking skills, whereas those with high levels exhibited high algebraic thinking skills. The study A16 concluded that teaching based on van Hiele Teaching Stages effectively increased algebra achievement. Studies A27 and A28 demonstrated that van Hiele Theory is a suitable tool in teaching, bridging the gap between students and educators in geometry. Study A7 identified a significant relationship between learning according to van Hiele Theory and attitudes. In study A12, researchers concluded that selected mathematics teachers from the faculty of education showed a good conceptual understanding of geometry, corresponding to Van Hiele levels 1 and 2, which facilitated the teaching and learning of geometry. However, the teaching strategies of these mathematics teachers were not structured to support the development of geometric thinking as described by van Hiele levels 3 and 4. Study A14 underscored the importance of developing insight in students as suggested by van Hiele in 1986.

Upon reviewing the studies, several findings were noted: van Hiele Theory adequately describes students' geometric thinking levels, which are structured continuously, and students at different levels exhibit behaviors consistent with their respective levels (A18). Utilizing a deductive approach in the teaching process is effective for developing students' geometric thinking as per van Hiele Theory (A38). Van Hiele levels and SOLO (Structure of the Observed Learning Outcome) levels display similarities (A39). Instruction based on Van Hiele Theory effectively helps students perceive geometric relationships (A43). There is no significant correlation between spatial visualization abilities and van Hiele levels (A68). Geometric thinking levels are not age-dependent but may vary according to the objectives of the geometry content (A44).

Upon examining the table, it was noted that study A20 investigates the innovative development of a van Hiele-based intelligent tutoring system for computer programming, which incorporates Bayesian technology. Study A21's findings indicate that although some questions in the van Hiele test posed unexpected challenges and exhibited low item discrimination, utilizing diverse selection criteria across different levels enhanced item discrimination. This was particularly true for questions that initially had low discrimination index estimates. These findings led to the identification of questions in the van Hiele Geometry Test that could be modified, and adjustments to the classification criteria were suggested. These changes aim to increase the accuracy in assigning students to a general level of geometric thinking as per the theory. In addition, there are studies showing that while there are significant differences in the basic levels of van Hiele Theory in different learning environments, there is no significant difference in the advanced levels (A22) and different learning environments are not effective on students' geometry achievement (A28). As a matter of fact, there are also studies that conclude that geometry teaching should be structured in accordance with the levels of van Hiele Theory (A32) and that geometric thinking levels and geometry skills are effective in gaining understanding of geometry (A34). And finally, it is seen that van Hiele Theory is not effective in defining thinking processes that move thinking from concrete to abstract (A41), students' geometric thinking levels are at the expected level (A42), van Hiele-based transformation geometry worksheets supported by GeoGebra should be developed to improve students' understanding of concepts (A71), and students at van Hiele level 3 solve algebraic geometry questions appropriately (A72).

Findings and Suggestions from the Studies Reviewed

The findings and suggestions derived from the reviewed studies were thoroughly examined, and those directly related to van Hiele Theory are presented in the table below.

Table 6

Findings and Suggestions Obtained from the Studies

Codes	Studies	f
It is recommended that teaching in geometry should start from the thinking level of the learners and the teaching should be structured accordingly.	A2, A7, A9, A15, A44, A68	6
Different teaching practices can be used to improve students' van Hiele geometric thinking levels	A6, A47, A77	3
The principles underlying van Hiele Theory can be incorporated into curriculum design	A8, A9, A78	3
More work can be done to develop the principles of van Hiele Theory and apply them to other branches of mathematics education	A28, A61, A76	3
Instructors are recommended to consider the development of students' geometric thinking in preparing and planning activities.	A29, A33, A43	3
Van Hiele Theory can be used in a useful way in organizing content and implementing activities related to learning geometry	A3, A9	2
Van Hiele Theory should be included in the undergraduate education program	A12, A13	2
Investigate the teaching of other geometry areas using van Hiele Theory	A12, A13	2
More interactive and hands-on learning activities can be organized for geometry learning at the secondary school level	A35, A36	2

Codes	Studies	f
Different teaching practices can be included in future studies as a research plan in the context of van Hiele Theory	A49, A81	2
Teaching based on van Hiele Theory can improve students' geometric thinking levels and students' mathematical communication skills	A10	1
Seminars and workshops can also be organized to evaluate the use of VHPI.	A13	1
In studies examining the effect of technology on geometry learning, programs that address van Hiele-based goals for student achievement can be used	A16	1
Van Hiele Geometry Test should be revised	A21	1
Instructors are advised to implement learning strategies that can help and encourage students to develop their geometric thinking.	A29	1
Longitudinal studies can be conducted to explore how spatial and geometric thinking develops or does not develop	A31	1
Geometry teaching can be done at different levels in the context of van Hiele Theory	A32	1
It is recommended that students gain visual, verbal, drawing, logic and application skills in geometry teaching	A34	1
Further research could uncover the connection between van Hiele geometric thinking levels and spatial abilities.	A37	1
The C-A-C learning model can be incorporated into primary school math curricula	A41	1
The effect of van Hiele on geometric thinking based on different geometric systems can be examined with Crocodile Mathematics Software	A49	1
It is recommended that VH-iSTEM application can be used in geometry learning and its effectiveness together with other approaches should be investigated.	A53	1
The application of Van Hiele's theory can be applied to the development of visualization of three-dimensional objects	A56	1
Develop learning tools that can help develop students' spatial abilities in the context of van Hiele Theory	A59	1
Research can be conducted using dynamic geometry software to improve students' mathematical and geometric skills, proof-structure performance and van Hiele levels.	A62	1
A new geometric thinking test that is more comprehensive (analytic geometry, transformation geometry, etc.) can be developed	A70	1
Teachers should guide class investigations and discussions to develop students' geometric thinking	A74	1
Using the van Hiele and ARCS models in the classroom can benefit both teachers and students by presenting the curriculum in a way that best meets the needs of individual students and motivates them	A77	1

When Table 6 is examined, the suggestions are directly related to the subject and are determined to be structured. The suggestions of the analyzed studies will be examined in detail by dividing them into two as related to the learning process and related to future studies.

Regarding the learning process, it is seen that a significant number of them suggest that in order to realize effective teaching in geometry, teaching should start from the thinking level of the learners and the teaching should be structured accordingly (A2, A7, A9, A15, A44, A68). In addition, it was stated that the stages of geometry learning based on van Hiele Theory can be used in a useful way in organizing the content and implementing activities related to geometry learning (A3, A9) and that it can facilitate students' geometric thinking levels and improve students' mathematical communication skills (A10). Upon examining the table, it is evident that various teaching practices can enhance students' geometric thinking levels. Specific suggestions include integrating Google SketchUp software into lessons to aid primary school students in advancing through van Hiele's geometric thinking levels (A6), utilizing VH-GSU and VH-PL modules to develop activities tailored for teachers according to van Hiele's levels (A47), and incorporating van Hiele and ARCS models in the classroom. This approach benefits both teachers and students by presenting the curriculum in a manner that optimally meets individual students' needs and boosts motivation (A77). Furthermore, it is recommended that instructors consider the development of students' geometric thinking when planning and preparing geometry lessons, implementing learning strategies that foster and support the progression of students' geometric thinking levels (A29, A33, A43). It is suggested that more interactive and applied learning activities can be applied for geometry learning at the secondary school level (A35, A36). It is also suggested that in the classroom setting, teachers should select relevant materials and make hands-on explorations to develop students' geometric thinking, make conjectures and conduct geometry projects, and even engage in class investigations and discussions (A74) to provide opportunities to deepen students' understanding of geometry.

Regarding the suggestions for future studies, it was suggested that curriculum developers and textbook authors could incorporate the principles of van Hiele Theory into instruction and curriculum design (A8, A9, A78) and seminars and workshops could be organized to evaluate the use of VHPI (A13). It was expressed that more work could be done to develop the principles of Van Hiele Theory and apply them to other branches of mathematics education (A28, A61, A76). Several studies highlight the potential to incorporate van Hiele Theory into the undergraduate curriculum and suggest exploring its application in teaching other geometry areas such as three-dimensional shapes, circles, and coordinate geometry (A12, A13). These studies advocate for the inclusion of different teaching practices in future research within the context of van Hiele Theory (A49, A81). Specific suggestions include conducting research on the impact of Crocodile Mathematics Software on students' van Hiele geometric thinking levels concerning three-dimensional geometric shapes, and its effectiveness across various geometric systems like Euclidean and non-Euclidean geometries (A49). Additionally, it is proposed that Realistic Mathematics Education (RME) and a van Hiele Theory-based learning method should be incorporated into relevant mathematics subjects as a research approach (A81). In the studies examining the use of technology in geometry teaching and learning, it is stated that programs based on van Hiele-based goals can be used to increase student achievement (A16); VH-Istem teaching application can be used in school geometry and can be used in future research to investigate its effectiveness in relation to different approaches (A53). It is recommended that longitudinal studies be carried out to investigate the development or lack thereof in geometric and spatial thinking (A31), and to clarify the relationship between Van Hiele geometric thinking levels and spatial skills (A37). They also suggested that van Hiele Theory may be particularly suitable for educational practitioners involved in mathematics learning to provide more opportunities to develop students' spatial abilities and also to develop learning tools that can help students develop their spatial abilities (A59). Regarding the van Hiele geometric thinking levels test developed by Usiskin (1982), there were also suggestions that the test should be revised (A21) and that a new geometric thinking test that is more comprehensive (analytic geometry, transformation geometry, etc.) could be developed (A70). In addition, it was stated that geometry teaching can be done at different levels in the context of van Hiele Theory (A32) and that students should gain visual, verbal, drawing, logic and application skills (A34). The C-A-C learning model is recognized as an effective framework for facilitating the transition of geometric thinking from concrete to abstract levels, and it is suggested that this model be integrated into the primary school mathematics curriculum (A41). Furthermore, studies could examine how dynamic geometry software might improve the mathematical and geometric skills, proof-structure performance, and Van Hiele levels of pre-service mathematics teachers (A62). While Van Hiele Theory was initially crafted for geometry, its use can also be extended to enhancing visualization skills with 3D objects (A56).

DISCUSSION, CONCLUSION and RECOMMENDATIONS

When examining the results from international literature, it is noted that research typically concentrates on evaluating the impact of various educational settings on geometric thinking levels, measuring geometric thinking levels, and investigating the evolution of Van Hiele Theory from multiple viewpoints. Researchers who have contributed to the development of the theory have actively participated in defining or broadening the scope of the theory. In addition, there are many studies such as comparing van Hiele Theory with different models (determining similarities and differences), examining the place and psychometric properties of van Hiele Geometric Test, which is frequently preferred by researchers in this field, in the education system.

When the studies analyzed within the scope of the research were examined, it was determined that experimental studies among quantitative studies and case studies among qualitative studies were preferred more than others. The fact that the researchers preferred to conduct studies examining the effect and development of students' van Hiele geometric thinking levels by using different teaching practices and that the sample was deliberately selected from studies involving teaching practices may have increased the number of studies conducted with experimental design. In addition, in the studies conducted with this method, pre-tests and post-tests were used, and situations suitable for the qualitative paradigm were also used. The purpose of these studies may be to examine the process in depth and to ensure the validity and reliability of the data obtained. In addition, it is seen that the case study method was used in other studies with the same purpose of using the experimental method. This situation brings with it the thought that it may be due to the fact that studies in the international literature have made flexible choices in the method section. In addition, methods such as literature review, meta-synthesis and meta-analysis were not found. Conducting studies including these methods is important for determining the trends in the field and providing resources for new research. In addition, the fact that different methods such as research and development and design-based research have been used internationally has added richness to the field.

In the majority of the studies, high school students and pre-service teachers were included in the sample, whereas in a small number of studies, students aged 3-6 years and preschool students aged 5-6 years were included in the sample. There may be different reasons for this situation: First, the van Hiele Geometric Test in Usiskin's (1982) study was applied to students in the 10th grade and above in the US sample, and second, pre-service teachers may be an easily accessible study group for researchers. When the findings are analyzed, it is seen that a wide range of age groups were included. The reason for specifying age groups is remarkable and may have been done to avoid confusion in grade levels for different countries.

It was determined that van Hiele Geometric Test and then interviews were mostly preferred as data collection tools in the analyzed studies. In 33 of the 81 studies (A1, A2, A5, A7, A8, A9, A13, A15, A16, A21, A26, A28, A29, A31, A32, A35, A36, A49, A50, A52, A53, A55, A57, A62, A63, A68, A69, A70, A72, A73, A75, A76, A78) used the test developed by Usiskin (1982), while A48 and E80 used the geometric thinking test developed by other researchers. In the studies coded E6, A19, A42, A47, a specially adapted test called WGT was used to measure the geometric thinking levels of primary school students. Therefore, developing and using alternative tests for geometric thinking levels is a pleasing situation. The fact that the other frequently preferred data collection tool is the interview may be due to the desire to examine the students' reasoning about the subject being studied in depth and in detail. Because only in this way can the most robust and reliable information be obtained. It is also seen that achievement tests are mostly preferred in the studies. The reason for the use of these tests may be that experimental studies are predominant, data collection is easy and more data is desired to be reached in less time (Günay & Aydın, 2015). In addition, observation, the use of worksheets and screen recordings/video recordings are other preferred data collection tools. Collecting data through observation and monitoring the process with video recordings are seen as equivalent. The difference between the two is that video recordings give the chance to be watched repeatedly according to observations (Bağ & Çalık, 2017). The use of artifact analysis, multiple learning software, lesson plans and most of the test types in the tests section as data collection tools was limited.

When examining the results of international studies, it is found that different learning environments significantly enhance students' geometric thinking levels in many cases. This improvement is often attributed to the adoption of diverse, modern teaching practices and experimental approaches that positively impact classroom dynamics, as supported by research from Aydeniz et al. (2012), Herrenkohl & Cornelius (2013), Memiş (2014), and McNeill (2011). However, there are also studies indicating that different learning environments do not effectively increase students' geometric thinking levels (E5, E52, E70). This lack of effectiveness may be due to the teaching practices not aligning well with the nature of the study or the students' geometric thinking levels being below the anticipated standard. In the E74 study, it was concluded that the students' low level was global in a way that exceeded the results of teaching practices and curriculum. In addition, the results of the study confirmed the hierarchical structure of the levels (A9, A21, A39, A44), the importance of developing insight for students (A14), the continuous structure (A18), the fact that the levels of geometric thinking do not depend on age and can vary according to geometry content objectives (A44). In addition, there were some studies showing that van Hiele Theory is not effective in defining thinking processes from concrete to abstract and that there is no significant relationship between levels and spatial visualization ability. In addition, the problematic items in the Usiskin (1982) test in A21 were identified and changes in the classification criteria were proposed to increase the number of students who could be assigned a general geometric thinking level according to the theory.

It was seen that there were many suggestions emphasized within the scope of the studies and some of them were listed. When the suggestions were examined, the most common suggestion was that geometry teaching should start from the thinking level of the learners and the teaching should be structured accordingly. Additionally, various teaching practices can be employed to enhance students' van Hiele geometric thinking levels. Principles derived from van Hiele Theory can be incorporated into curriculum design, encouraging further research to expand these principles and apply them to other areas of mathematics education. Instructors should also take into account the development of students' geometric thinking when preparing and planning educational activities.

Limitations of the Study

- The international studies included in the thematic analysis within the scope of the research are limited to 81 studies published until February 02, 2022, which can be accessed through open access and given in the bibliography. Among these studies, the studies that based the teaching processes on any teaching practice and addressed these processes in a clear and detailed manner were taken as basis.
- The studies reviewed in this research are limited to data directly related to van Hiele geometric thinking levels.
- Within the scope of this research, only studies published as articles were used, despite the availability of similar studies published both as articles and theses.
- The scope of the research is limited to studies that contain the specified keywords in their content.
- The research is limited to the findings obtained from the analyzed studies.

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Statement of Contribution Rate

The authors of the study contributed equally to all processes of the study.

Declaration of Conflict of Interest

As the study authors, we declare that we do not have any declaration of interest/conflict.

Statement of Publication Ethics

All the rules stated in the "Higher Education Institutions Scientific Research and Publication Ethics Directive" were followed in the entire process, from the planning, implementation, data collection to the analysis of the data. None of the actions specified under the second section of the Directive, "Scientific Research and Publication Ethics Actions" have been carried out.

During the writing process of this study, scientific, ethical, and citation rules were followed; no falsification was made on the collected data, and this study was not sent to any other academic media for evaluation.

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GENİŞLETİLMİŞ ÖZ

Giriş

Geometri, matematik dünyasını ve evreni anlama, yorumlama ve keşfetme imkanı sunan önemli bir matematik dalıdır. Aynı zamanda gerçekle ilişkili ve sezgiye dayalı olarak somut özellikleri de içinde barındırmaktadır (National Council of Mathematics Teachers [NCTM], 2000).

Geometri öğretimi, okul öncesi dönemde oyunla başlayan ve bulmacalarla devam eden bir süreçtir. Bu süreç, kavramların, bilgi kümesinin ve tahminlerin geliştirilmesiyle matematiğin en keyifli ve zevkli bölümünü oluşturur (Gür, 2005). Öğrencilerin geometri öğrenme sürecinde yaşadıkları zorluklar göz önüne alındığında, öğrencilerdeki geometrik düşüncenin geliştirilmesi için geometri öğretim sürecinin düzenlenmesi ve planlanması gereklidir (Mistretta, 2000). Geometri programlarının geliştirilmesinde çeşitli teoriler ve yaklaşımların etkisi gözlemlenmiş olup, bu bağlamda 1989'da NCTM tarafından geliştirilen bir metodoloji öne çıkmaktadır. Bu yüzden, geometri alanının geliştirilmesi ve yapılandırılması sürecinde "van Hiele Teorisi"nin temel alındığı ifade edilmektedir (Choi-Koh, 1999; NCTM, 2000).

Bu sebeple, 1982-2022 yılları arasında van Hiele geometrik düşünme düzeyleri üzerine yapılmış 81 çalışmanın verilerini analiz eden bu makalenin, literatüre önemli katkılarda bulunacağı öngörülmektedir. Bu bağlamda bu sistematik incelemenin amacı, gelecekteki ilerlemeler için geometrik düşünme üzerine ve onunla ilgili çalışmaların daha geniş tanımlarını sunmak için geçerli ampirik araştırmalardan elde edilen sonuçları sentezlemektir. Bu araştırmada van Hiele teorisi ile ilgili yapılan çalışmaların nasıl ele alındığının (amaç, yöntem, örneklem bağlamında) ve genel olarak nasıl sonuçlara ulaşıldığının (sonuçlar, öneriler bağlamında) genel bir değerlendirilmesi yapılmıştır.

Yöntem

Bu araştırmada, nitel araştırma yaklaşımları içinde süreç ve konu bakımından yer alan etkileşimsiz desenlerden biri olan betimsel içerik analizi tekniği kullanılmıştır. Betimsel içerik analizi belirli bir konu alanında yapılan çalışmaların belirlenen kriterlere göre ele alınıp, araştırma sonuçlarının ve eğilimlerinin de tanımlayıcı boyutta değerlendirilmesinin yapıldığı sistematik bir çalışma biçimidir (Çalık & Sözbilir, 2014, s. 34). Dolayısıyla bu araştırmada incelenen çalışmaların analiz ve yorumlama sürecinde adlandırma, kategori geliştirme, frekansların hesaplanması ve yorumlama aşamalarına dikkat edilmiştir. Yapılan betimsel içerik analizinde adlandırma ve kategori geliştirme aşamalarında, her bir makale çok dikkatli bir şekilde incelenmiş ve yöntem, veri toplama araçları, örneklem ve veri analiz yöntemleri gibi faktörler kategoriler altında sınıflandırılmaya tabi tutulmuştur.

Van Hiele geometrik düşünme düzeyleri üzerine uluslararası alanda yazılmış birçok çalışma bulunmaktadır. Bu nedenle, bu çalışmaları seçerken çeşitli eleme işlemlerine başvurulmuştur. İlgili anahtar kelimeleri içeren İngilizce çalışmalar öncelikli olarak tercih edilmiş, ayrıca herhangi bir öğretim uygulaması ile gerçekleştirilmiş ve bu alanda tanınmış araştırmacıların yüksek atıf alan çalışmaları dikkate alınmıştır.

Bulgular

Uluslararası literatürde yapılan çalışmaların amaçları incelendiğinde, öğrencilerin geometrik düşünme düzeylerini tespit etme, farklı öğrenme ortamlarının bu düzeylere etkilerini belirleme ve van Hiele Teorisi'nin çeşitli yönlerini araştırma gibi konuların ağırlıklı olduğu görülmektedir. Değerlendirilen çalışmalarda genellikle nicel yöntemlerle deneysel araştırmalar, nitel yöntemlerle ise durum çalışmaları yapılmıştır. Çalışmaların örneklemleri çoğunlukla lise düzeyindeki öğrenciler ve öğretmen adaylarından oluşurken, az sayıda çalışmada 3-6 ve 5-6 yaş arası okul öncesi çocuklar da incelenmiştir. Veri toplama yöntemleri olarak en sık van Hiele düzey belirleme testleri ve mülakatlar kullanılmıştır. Uluslararası literatür sonuçları, farklı öğrenme ortamlarının öğrencilerin geometrik düşünme düzeylerini geliştirmede etkili olduğunu ortaya koymuştur.

Tartışma ve Sonuç

Uluslararası literatürde yapılan incelemeler, van Hiele Teorisi'nin sınırlarını tanımlamak veya genişletmek amacıyla teorinin evrimini ele alan birçok çalışmanın yapıldığını göstermektedir. Ayrıca, van Hiele Teorisi'ni farklı modellerle karşılaştırarak benzerliklerini ve farklılıklarını belirlemeye yönelik çalışmalar da yaygındır. Bu alanda faaliyet gösteren araştırmacılar arasında, van Hiele Geometrik Testi'nin psikometrik özelliklerini incelemek ve eğitim sistemlerindeki yerini belirlemek gibi konulara odaklanan birçok çalışma bulunmaktadır.

Araştırma kapsamında incelenen çalışmalar göz önüne alındığında, deneysel desen kullanılarak yapılan çalışmaların diğer yöntemlere göre daha fazla tercih edildiği gözlemlenmiştir. Bu durum, araştırmacıların farklı öğretim uygulamalarını kullanarak öğrencilerin van Hiele geometrik düşünme düzeylerinin etkisini ve gelişimini daha detaylı incelemeyi tercih etmeleri ile ilişkilendirilebilir. Bu eğilim, öğretim yöntemlerinin geometrik düşünme üzerindeki potansiyel etkilerini anlamak amacıyla kontrollü ve sistematik bir yaklaşımı vurgulamaktadır.

Yapılan çalışmaların çoğunluğunda tercih edilen örneklem grubuna bakılacak olursa, lise düzeyindeki öğrenciler ve öğretmen adaylarının olduğu tespit edilmiştir. Çalışmaların küçük bir bölümünde ise 3-6 yaş ve 5-6 yaş aralığındaki okul öncesi düzeyindeki öğrenciler örnekleme dahil edilmiştir. Lise düzeyindeki öğrencilerin daha çok tercih edilme sebebi; van Hiele Geometrik Testi'nin, Usiskin (1982) tarafından Amerika örnekleminde 10. sınıf ve üstü öğrencilere uygulanmasının yaygın olması düşüncesinden kaynaklanıyor olabilir. Ayrıca bu durumun akabinde; öğretmen adaylarının da çoğu çalışmada yer almasının nedeni, araştırmacılar için erişimi kolay bir çalışma grubu olmalarından kaynaklanıyor olabilir.

İncelenen çalışmalarda veri toplama araçları olarak özellikle Van Hiele Geometrik Testi ve onu izleyen mülakatlar sıkça kullanılmıştır. Usiskin (1982) tarafından geliştirilen testin yanı sıra, diğer araştırmacılar tarafından geliştirilen testler de öğrencilerin geometrik düşünme düzeylerini belirlemek için tercih edilmiştir. Bu, geometrik düşünme düzeylerini ölçmek için alternatif araçların geliştirilmesinin ve kullanılmasının olumlu bir gelişme olduğunu gösterir. Çalışmalarda başarı testlerinin de sıklıkla kullanıldığı gözlemlenmiştir. Deneysel çalışmaların yaygın olarak tercih edilmesi ve bu testlerin veri toplamayı hızlı ve kolay bir şekilde gerçekleştirmeye olanak tanıması, kullanım sıklıklarını artıran faktörler arasındadır (Günay & Aydın, 2015). Ayrıca, çalışma kağıtları, ekran kayıtları ve video çekimleri gibi veri toplama araçları da kullanılmaktadır. Gözlem yoluyla veri toplama ile video kayıtlarının süreci izleme açısından eşdeğer olduğu kabul edilse de; video kayıtlarının tekrar izlenebilir olması, bu yöntemi avantajlı kılan bir özellik olarak belirtilmiştir (Bağ & Çalık, 2017). Bu çeşitlilik, araştırmaların kapsamlılığını ve veri toplama süreçlerinin esnekliğini artırmaktadır.

Son olarak, uluslararası literatürde yer alan çalışmaların sonuçlarına göre, öğrencilerin geometrik düşünme düzeylerine yönelik farklı öğrenme ortamlarının olumlu etkisi olduğu görülmektedir. Bu durum, geleneksel yöntemlere kıyasla daha çeşitli ve güncel öğretim uygulamalarının ve sınıf içi deneysel faaliyetlerin yoğun bir şekilde kullanılmasından kaynaklanmaktadır (Aydeniz vd., 2012; Herrenkohl & Cornelius, 2013; Memiş, 2014; McNeill, 2011). Bu sonuçlar, eğitimde yenilikçi ve çeşitlendirilmiş uygulamaların önemini ön plana çıkarmaktadır.

Appx. 1. Studies Analyzed within the Scope of the Research

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