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A New EPQ Model for Imperfect Production, Screening, and Rework With Backorders Under Grey Uncertainty

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Abstract

The occurrence of imperfect products that are studied at a specific rate during production holds a significant and comprehensive position in the literature. The fundamental premises underlying the research on the EOQ and EPQ models are as follows: there exists a specific percentage of imperfect production; that the defective rate of production is in line with a certain probability distribution; and that defective products are in themselves defective, repairable (rework), and defective (selling at a low price) etc. The studies of EPQ models that consider the inspection rate and production rate hold significant significance in the literature. This work presents the development of a novel EPQ model to handle the issue of reduced production time to eliminate backorders. The model's functioning is described by including uncertainty modeling, specifically using grey values to represent production rate, costs and prices.

Keywords Economic production quantity model, Rework, Backorders, Grey system theory

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1. Introduction

In the field of inventory management, the Economic Production Quantity (EPQ) model has been widely researched and utilized across various industries to optimize production and reduce costs. The traditional EPQ model, which focuses on determining the optimal production lot size to balance setup and holding costs, assumes that all produced items are of perfect quality. However, real-world manufacturing processes often encounter defective items that need reworking before they can be sold. This necessitates extending the traditional EPQ model to include reworking costs and activities, providing a more accurate representation of production environments.

Numerous researchers have contributed to the development of EPQ models that account for imperfect quality items and the reworking process. For instance, Salameh & Jaber (2000) introduced a model that incorporates the proportion of defective items and rework costs, emphasizing the importance of considering quality issues in production planning. Further advancements in these models offer more comprehensive frameworks that address various practical considerations, including inspection, scrapping, and the cost implications of reworking activities.

The main objective of this paper is to extend the existing EPQ models by integrating a systematic approach to handling reworked items within the production cycle for imperfect production. Hayek & Salameh (2001) introduced an EPQ model to address the issue of extended production time, which was causing backorders. This work presents the development of a novel EPQ model to handle the issue of reduced production time to eliminate backorders. The model's functioning is described by including uncertainty modeling, specifically using grey values to represent production rate, costs and prices. So, we aim to provide a more robust model that can guide decision-makers in optimizing production lot sizing, backorder levels and costs reducing uncertainties associated with grey systems theory.

The remainder of this paper is organized as follows: Section 2 provides a detailed literature review, summarizing the key contributions and gaps in existing EPQ models with reworked items. Section 3 presents the proposed grey model and its mathematical formulation. Section 4 discusses the application of the model in a case study, followed by the results and analysis. Finally, Section 5 concludes the paper with a summary of the findings and suggestions for future research.

2. Related Literature

In terms of economic production quantity models, this section can be handled in three different ways. The first is the defective production process, the second is the rework situation and the third is probabilistic, fuzzy and grey uncertainty modeling. At each stage, the scientific literature will be summarized and evaluated for examples in terms of whether backordering is allowed or not.

Table 1. Related literature review

Author(s) and Year	Methodology/Approach
Schrady (1967); Richter (1997)	Basic EOQ/EPQ Modeling
Nahmiasj & Rivera (1979)	Inventory Classification
Hayek & Salameh (2001)	Rework with Stockouts
Chan et al. (2003)	Multi-Product Classification
Dobos & Richter (2003);Dobos & Richter (2004)	Reverse Logistics & Learning Effect
Eroğlu et al. (2008)	Defective Batch Scanning
Taleizadeh et al. (2014); Glock & Jaber (2013); Shah et al. (2016); Khalilpourazari et al. (2019); Taleizadeh et al. (2024)	Rework & Scrap Processes
Kozlovskaya et al. (2016); Eroğlu & Aydemir (2021); Eroğlu & Aydemir (2020); Eroğlu & Arslan (2022);Karagül & Eroğlu (2022)	Cost & Waste Optimization
Shah et al. (2016)	Cycle Time & Efficiency
Karagül & Eroğlu (2022);Eroğlu & Aydemir (2020)	Screening & Shortages
Widyadana & Wee (2009); Soni & Shah (2011); Aydemir et al. (2014); Aydemir et al. (2015); Mezei & Björk (2015); Wang & Tang (2009); Karbassi Yazdi et al. (2019); De & Bhattacharya (2022)	Stochastic/Fuzzy Systems
Taleizadeh et al. (2024); Gharaei et al. (2021)	Price Strategy & Scheduled Backorders

According to imperfect production, Rosenblatt & Lee (1986) examined the impact of the production of defective products on the optimal production cycle time, incorporating linear, exponential, and multi-state degradation processes. Building on this, Hyun Kim & Hong (1999) extended the study of defective products by considering past periods resembling racial distribution, similar to Rosenblatt and Lee's 1986 model. Chung & Hou (2003) developed a model addressing storage constraints and determining the optimal handling of defective products within the production system. Salameh & Jaber (2000) introduced an inventory model where products, whether purchased or produced, are imperfect, marking the first study to incorporate stock shortages into this context.

Eroglu & Ozdemir (2007) further advanced this field by developing an EPQ model that includes defective products for each ordered shipment. Sana (2010) conducted an EPQ study on a faulty production system in which control is randomly lost, transitioning from a controlled to an uncontrolled state.Wee et al. (2013) proposed an EPQ model and solution procedure for defective products, addressing deficiencies and scanning restrictions. Rezaei (2016) expanded EPQ models to encompass sampling planning and monitoring situations. De~Giovanni (2019) enhanced the model by incorporating the negative impact of product failures and developed an optimal control model. SSanjai & Periyasamy (2019) examined a planned inventory model where a single product is produced in one phase, incorporating defective products into the production system. Recently, Khara et al. (2020) explored two types of production, with the second product produced from a mixture dealing with defects in certain products recycled within the recycling cycle from used mixture products. Soleymanfar et al. (2021) proposed sustainable EOQ and EPQ models for supply chains with return policies. According to Rosmiati et al. (2022), the classical EPQ model was based on a reliable production process that has fixed setup costs. Dharmesh & Kunal (2022) focused on the application of these models by manufacturing industries to optimize various requirements so as to maximize total profit. Çelik Eroğlu & Şahin (2023) examined the viable role of fuzzy inventory models for recycling purpose.

The study also presents a new fuzzy inventory model in recycling which highlights how important effective inventory management is in eco-friendly production applications.

About the rework process, when examining repair (rework) processes in EPQ and EOQ models, the first model attempting to determine the optimal production quantity and the amount of reworkable products was developed by Schrady (1967). Nahmiasj & Rivera (1979) explored an inventory model stocking two types of items: repaired and unrepaired. In his study, Richter (1997) added waste disposal to the EOQ model with variable setup numbers for both production and repair. Hayek & Salameh (2001) developed an EPQ model that allows stockouts and can fully restore defective items to a new-like condition. Chan et al. (2003) considered three types of products in their study and demonstrated their sellability at different prices as repairable, flawless, and scrap products. Dobos and Richter (2003, 2004) proposed a model involving reverse logistics with repair/production and waste materials in their recent studies and later incorporated the learning effect into this new EPQ model. Eroğlu et al. (2008) developed an EOQ model where each ordered batch contains defects and deficiencies by scanning both good and defective products. Taleizadeh et al. (2014) analyzed and developed an economic production quantity inventory model incorporating scrap and rework processes with interruptions. Glock & Jaber (2013) presented a study where suboptimal large-batch goods can be produced, introducing defective items into the supply chain, and developed a new mathematical model. Kozlovskaya et al. (2016) provided a net solution for rework costs, waste/scrap disposal costs, and replacement costs. In their study, Shah et al. (2016) considered the cycle time by focusing on the quick production of defective items after repair to maximize profit. Khalilpourazari et al. (2019) aimed to develop a multi-component EPQ formulation that includes rework and defective items. Eroglu and Aydemir (2020) and (2021) developed a new model with reworked items for recovery products in the re-production process. Eroğlu & Arslan (2022) introduced expansions in economic production quantity models to accommodate defective products, considering scenarios where defective product rates follow uniform distributions. Karagül & Eroğlu (2022) presented a model for screening activities were conducted during the production period to ensure the availability of perfect items to meet demand. Their model was developed where all defective products are identified randomly and reworked, and shortages are permitted. Gharaei et al. (2021) presented the EPQ) model with partial backorders and re-workable products considering linear and fixed backordering costs. Taleizadeh et al. (2024) integrate price strategy, rework, and scheduled backorders with the well-known EPQ inventory model.

An in depth review of the literature on stochastic, fuzzy, and grey uncertainties in Economic Production Quantity (EPQ) models reveals a diverse landscape of research dedicated to optimizing production processes under various forms of uncertainty. The EPQ model has been extensively examined in deterministic, stochastic, and fuzzy demand and cost scenarios (Soni & Shah, 2011). Researchers have explored practical applications of EPQ models, taking into account factors such as machine breakdowns and stochastic repair times (Widyadana & Wee, 2009).

Additionally, studies have investigated the integration of imperfect items using interval grey numbers to enhance the traditional EPQ model (Aydemir et al., 2014; 2015). The utilization of fuzzy logic in EPQ models has garnered attention, with research addressing concerns like finite production rates, backorders, and fuzzy cycle times (Björk, 2012; Mezei & Björk, 2015). Furthermore, models incorporating rework processes and service level constraints have been examined to enhance production efficiency (Chiu et al., 2006). Recent research efforts have focused on optimizing multi-product EPQ models by considering scrap items and implementing enhanced delivery policies (Wu et al., 2014). Fuzzy EPQ inventory models that account for backorders and characterize costs as fuzzy variables have been developed to address uncertainties in production settings (Wang & Tang, 2009). Incorporating grey relational analysis and space constraints into multi-item EPQ strategies has been studied to optimize production under varying environmental conditions (Karbassi Yazdi et al., 2019). Models considering deterioration under pollution effects and fuzzy systems have been developed to enhance the resilience of EPQ models (De & Bhattacharya, 2022).

In conclusion, the literature on stochastic, fuzzy, and grey uncertainties in EPQ models presents a wide array of studies aimed at improving production efficiency, reducing costs, and managing uncertainties in manufacturing processes. This work presents the development of a novel EPQ model to handle the issue of reduced production time in order to eliminate backorders. The model's functioning is described by including uncertainty modeling, specifically using grey values to represent production rate, costs and prices. Researchers have investigated various factors such as imperfect items, rework processes, preventive maintenance, and environmental considerations to develop robust EPQ models capable of adapting to dynamic production environments.

3. The Proposed Model

The main objective of this paper is to extend existing EPQ models by incorporating a systematic approach to managing reworked items within the production cycle for imperfect production. Hayek & Salameh (2001) introduced an EPQ model to address the problem of extended production time leading to backlogs. This paper presents the development of a novel EPQ model designed to address the problem of reduced production time to eliminate backlogs. The model is described by incorporating uncertainty modelling, specifically the use of grey values to represent production rate, costs and prices. All notations are given as follows:

3.1. Crisp Model

In this study, an EPQ model is developed for the scenario where the production time is less than the time required to eliminate backorders, and the functioning of the model is explained with examples. First of all, the crisp model is given with the required formulation as following the Figure 1. A single-item product is produced with a production rate of α and is inspected at a rate of x to distinguish between defective products and good products. The inspection rate is assumed to be greater than the production rate ($x > \alpha$). During the period t_1, a backorder build up of w quantity with demand rate β occurs (backorder build-up). Here,

$$t_1 = \frac{w}{\beta} \tag{1}$$

Table 2. Notations of the model

β	Demand rate of good quality products per unit time (items/week)
α	Production rate per unit time (items/week)
α_1	Rework rate of defective items per unit time (items/week)
x	Screening rate per unit time (items/week)
h	Unit holding cost per unit time ($/item/unit time)
h_1	Unit holding cost of defective items per unit time ($/item/unit time)
b	Unit shortage cost per unit time ($/item/unit time)
c	Unit production cost ($/item)
c_r	Unit rework cost ($/item)
d	Unit screening cost ($/item)
K	Production setup cost per cycle ($/cycle)
p	Random proportion of defective items with probability density function $f(p)$
z_1	Inventory level at the end of the rework period (items)
z_2	Inventory level at the end of the production & screening period (items)
z_3	Maximum defective items inventory level at the end of the production & screening period (items)
t_1	Backordered build-up time length (week)
t_2	Production time length (week)
$t_3 + t_4$	Rework time length (week)
t_5	Inventory (z_1) consumption time length (week)
t	Production cycle length $t = \sum_{i=1}^{5} t_i$
ETCU	Expected total cost per unit cycle time
w	Maximum backorder level allowed in a cycle
q	Total items produced during a cycle

Since q quantity of products will be produced with production rate α during the period t_2

$$t_2 = \frac{q}{\alpha} \tag{2}$$

and at the same time from Figure 1, another equation is obtained for the period t_2

$$t_2 = \frac{w - z_2}{\alpha A} \tag{3}$$

Here, $A = 1 - p - \left(\frac{\beta}{\alpha}\right)$ and as Eq. 2 and Eq. 3 are solved together,

$$z_2 = w - Aq \tag{4}$$

For the functioning of the model, $\alpha A > 0$ or $\alpha > \frac{\beta}{1-p}$ condition must be met. On the other hand, since pq quantity of defective products will be produced with production rate $p\alpha$ during t_2;

$$z_3 = pq \tag{5}$$

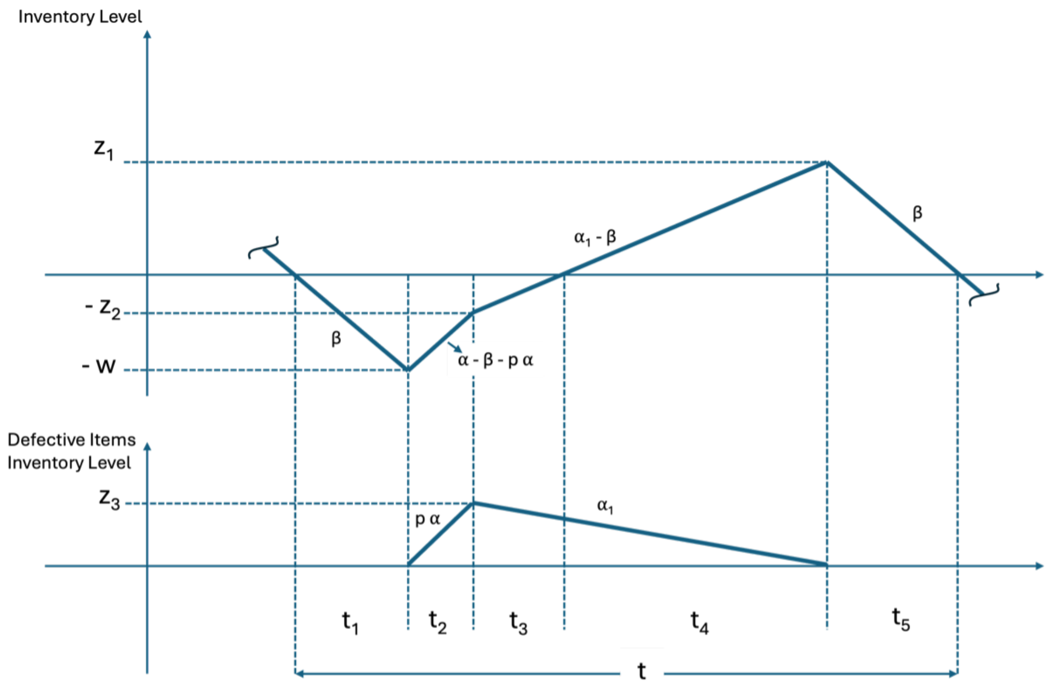

Figure 1. Inventory levels over cycle time

During $(t_3 + t_4)$, pq quantity of defective products are repaired at a rate of α_1 and become sound products and added to the sound product inventory. Hence,

$$t_3 + t_4 = \frac{pq}{\alpha_1} \tag{6}$$

Again, when Figure 1 is taken into consideration for the functioning of the model, the condition $\alpha_1 - \beta > 0$ must be met. Considering the figures and the equations obtained, the following equations are obtained.

$$t_3 = \frac{w - Aq}{\alpha_1 - \beta} \tag{7}$$

$$t_4 = \left(\frac{p}{\alpha_1} + \frac{A}{\alpha_1 - \beta}\right) q - \frac{w}{\alpha_1 - \beta} \tag{8}$$

$$z_1 = \left(1 - \frac{\beta}{\alpha} - \frac{\beta p}{\alpha_1}\right) q - w \tag{9}$$

$$t_5 = \left(\frac{1}{\beta} - \frac{1}{\alpha} - \frac{p}{\alpha_1}\right) q - \frac{w}{\beta} \tag{10}$$

Thus, the total cost (TC) in a cycle is obtained by the following equation.

$$TC = cq + dq + c_r pq + K + h\left[\frac{(t_4+t_5)z_1}{2} + \frac{t_2 z_3}{2}\right] + h_1\left[\frac{(t_3+t_4)z_3}{2}\right] + b\left[\frac{t_1 w}{2} + \frac{t_2(w+z_2)}{2} + \frac{t_3 z_2}{2}\right]$$
$$= (c+d+c_r p)q + K + \left\{\frac{\left(1-\frac{\beta}{\alpha}\right)^2}{2}\left(1-\frac{\beta}{\alpha_1}\right)\left(\frac{h}{\beta}+\frac{b}{\alpha_1}\right) - \frac{b\left(1-\frac{\beta}{\alpha}\right)}{2}\alpha + \frac{(h+b)}{2}\left(1-\frac{\beta}{\alpha_1}\right)\right.$$
$$\left.\left(\frac{\left(1+\frac{\beta}{\alpha_1}\right)}{\alpha} - \frac{2}{\alpha_1}\right)p + \left[h_1 + \frac{\left(b+h\frac{\beta}{\alpha_1}\right)}{\left(1-\frac{\beta}{\alpha_1}\right)}\right]\frac{p^2}{2\alpha_1}\right\}q^2 \tag{11}$$
$$-\left\{\frac{\frac{h}{\beta}+\frac{b}{\alpha_1}-(h+b)\left(\frac{1}{\alpha}+\frac{p}{\alpha_1}\right)}{\left(1-\frac{\beta}{\alpha_1}\right)}\right\}qw + \left\{b + \frac{\left(h+b\frac{\beta}{\alpha_1}\right)}{\left(1-\frac{\beta}{\alpha_1}\right)}\right\}\frac{w^2}{2}\beta$$

The expected value of the total cost per cycle, ETC,

$$\mathrm{ETC} = m_1 q + K + m_2 q^2 - m_3 qw + m_4 w^2 \tag{12}$$

Here,

$$m_1 = c + d + c_r E[p] \tag{13}$$

$$m_2 = \frac{\left(1-\frac{\beta}{\alpha}\right)^2}{2}\left(1-\frac{\beta}{\alpha_1}\right)\left(\frac{h}{\beta}+\frac{b}{\alpha_1}\right) - \frac{b\left(1-\frac{\beta}{\alpha}\right)}{2}\alpha$$
$$+\frac{(h+b)}{2}\left(1-\frac{\beta}{\alpha_1}\right)\left(\frac{\left(1+\frac{\beta}{\alpha_1}\right)}{\alpha} - \frac{2}{\alpha_1}\right)E[p] \tag{14}$$
$$+\left[h_1 + \frac{\left(b+h\frac{\beta}{\alpha_1}\right)}{\left(1-\frac{\beta}{\alpha_1}\right)}\right]\frac{E[p^2]}{2\alpha_1}$$

$$m_3 = \frac{\frac{h}{\beta}+\frac{b}{\alpha_1}-(h+b)\left(\frac{1}{\alpha}+\frac{E[p]}{\alpha_1}\right)}{\left(1-\frac{\beta}{\alpha_1}\right)} \tag{15}$$

$$m_4 = \left\{b + \frac{\left(h+b\frac{\beta}{\alpha_1}\right)}{\left(1-\frac{\beta}{\alpha_1}\right)}\right\}\frac{1}{2}\beta \tag{16}$$

The expected value of the total cost per unit time, ETCU , is obtained by the ratio of the expected value of the total cost per cycle to the cycle time as follows.

$$\mathrm{ETCU} = \beta m_1 + K\frac{\beta}{q} + \beta m_2 q - \beta m_3 w + \frac{\beta m_4 w^2}{q} \tag{17}$$

n order to find the optimum production and backorder quantities (decision variables), the partial derivatives of the ETCU function concerning the decision variables must be taken and set equal to zero.

$$\frac{\partial \mathrm{ETCU}}{\partial w} = -\beta m_3 + \frac{2\beta m_4 w}{q} = 0 \tag{18}$$

$$\frac{\partial \mathrm{ETCU}}{\partial q} = -K\frac{\beta}{q^2} + \beta m_2 - \frac{\beta m_4 w^2}{q^2} = 0 \tag{19}$$

The following optimum production quantity and backorder quantity are obtained.

$$q = \sqrt{\frac{K}{\left(m_2 - \frac{m_3^2}{4m_4}\right)}} \tag{20}$$

$$w = \frac{m_3 q}{2m_4} \tag{21}$$

3.2. Grey Systems and Degree of Greyness

Grey System Theory (GST), introduced by Deng in 1982, is a powerful method for handling uncertainty and discrete data in various academic fields. It has been widely applied in social, economic, and technical systems, and has proven effective in diverse areas such as applied sciences, technology, industry, and natural sciences. GST is particularly useful in situations where information is incomplete or insufficient, experimental data is lacking, sample sizes are small, or the data does not follow any known distribution (Aydemir et al., 2015; Liu & Forrest, 2010; Xie, 2017).

GST offers a unique approach to modeling systems, classifying them as black box, white box, or grey (box) systems based on the level of understanding of their behaviour. Grey numbers, a key concept in GST, represent uncertain values within an interval and are particularly valuable for modelling diverse and uncertain information. Due to its versatility, GST has become an important alternative to fuzzy logic in areas such as algorithm development, prediction, decision-making, and correlation analysis (Liu et al., 2017; Xie & Liu, 2011). The degree of greyness for an interval grey number can be easily calculated using its lower and upper limits. The absolute degree of greyness is tied to the quantity and units of measurement, while the relative degree of greyness is dimensionless, often represented as a percentage. When operations like addition or subtraction are performed on interval grey numbers with a real number, the absolute degree of greyness is used because it typically remains unchanged. Similarly, in multiplication or division with a real number, the relative degree of greyness is preferred, as it also generally remains constant.However, mixed situations can occur in practical applications. Numerous studies in the scientific literature have explored concepts such as the “kernel,” the absolute degree of greyness, and the relative degree of greyness (Yang & Liu, 2011). The degree of greyness is derived from interval grey numbers and can be illustrated using typical possibility functions.

The degree of greyness is obtained from interval grey numbers easily and is defined as follows in Figure 2 with typical possibility functions.

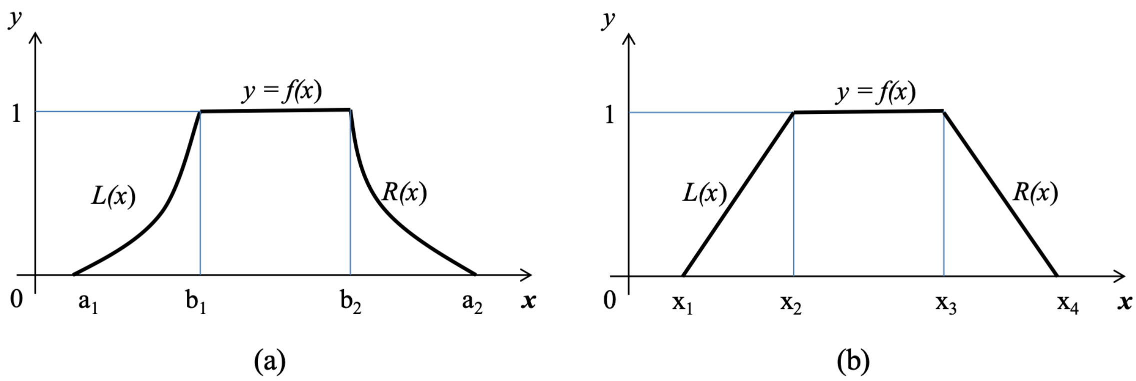

Figure 2. The typical possibility functions for grey numbers (Liu & Forrest, 2010)

A degree of greyness g^0 is obtained from the typical possibility function $f(x)$ in Figure 2 (a) and given as follows (Aydemir et al., 2015; Liu & Forrest, 2010):

$$g^0 = \frac{2|b_1 - b_2\ |}{b_1 + b_2} + \max\left\{\frac{|a_1 - b_1|}{b_1}, \frac{|a_2 - b_2\ |}{b_2}\right\} \tag{22}$$

The equation g^0 is the sum of two parts. The first one is the peak area and the second one is $L(x)$ and $R(x)$ functions. If

$$\max\left\{\frac{|a_1 - b_1|}{b_1}, \frac{|a_2 - b_2|}{b_2}\right\} = 0 \text{ and } g^0 = \frac{2|b_1 - b_2|}{b_1 + b_2} \tag{23}$$

provided that;

$$g^0 = \frac{2|b_1 - b_2|}{b_1 + b_2} = 0 \tag{24}$$

Grey number $\otimes = b = b_1 = b_2$ has a base value. Besides, if $g^0 = 0$, $\otimes$ is a white number. Using the degree of greyness is firstly presented by Aydemir et al. (2015) for a whitenization coefficient (α). Also, in this paper, the degree of greyness approach is extended to the grey parameter values of the degree of greyness by using Eq. 24 for a whitenization coefficient (α).

3.3. Grey Model

About the grey model; the model parameters have been designed with the grey values such as demand rate, production rate, inspection rate and also cost values. The proposed model is given as Figure 3 under grey uncertainties.

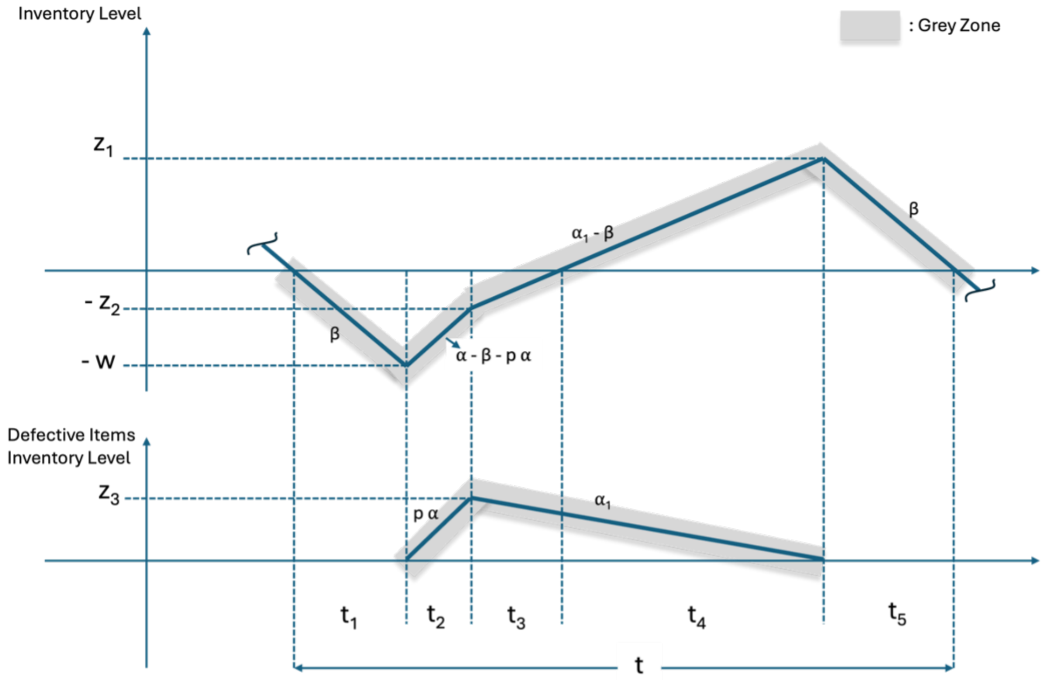

Figure 3. Grey inventory levels on time-horizon

All periods are recalculated to grey parameters. During the period t_1, a backorder build up of w quantity with demand rate β occurs (backorder build up) in grey values. Here,

$$t_1 = \frac{\otimes w}{\otimes \beta} \tag{25}$$

Since q quantity of products will be produced with production rate α during the period t_2

$$t_2 = \frac{\otimes q}{\otimes \alpha} \tag{26}$$

and at the same time from Figure 3, another equation is obtained for the period t_2

$$t_2 = \frac{\otimes w - z_2}{\otimes \alpha . \otimes A} \tag{27}$$

As Eq. 26 and Eq. 27 are solved together,

$$z_2 = \otimes w - \otimes A . \otimes q \tag{28}$$

For the functioning of the model, $\alpha A > 0$ or $\alpha > \frac{\beta}{1-p}$ condition must be met. On the other hand, since pq quantity of defective products will be produced with production rate $p\alpha$ during t_2;

$$z_3 = \otimes p . \otimes q \tag{29}$$

During $(t_3 + t_4)$, pq quantity of defective products are repaired at a rate of α_1 and become sound products and added to the sound product inventory. Hence,

$$t_3 + t_4 = \frac{\otimes p . \otimes q}{\otimes \alpha_1} \tag{30}$$

Again, when Figure 3 is taken into consideration for the functioning of the model, the condition $\alpha_1 - \beta > 0$ must be met. Considering the figures and the equations obtained, the following equations are obtained.

$$t_3 = \frac{\otimes w - \otimes A \otimes q}{\otimes \alpha_1 - \otimes \beta} \tag{31}$$

$$t_4 = \left(\frac{\otimes p}{\otimes \alpha_1} + \frac{\otimes A}{\otimes \alpha_1 - \otimes \beta}\right) \otimes q - \frac{\otimes w}{\otimes \alpha_1 - \otimes \beta} \tag{32}$$

$$z_1 = \left(1 - \frac{\otimes \beta}{\otimes \alpha} - \frac{\otimes \beta \otimes p}{\otimes} \alpha_1\right) \otimes q - \otimes w \tag{33}$$

$$t_5 = \left(\frac{1}{\otimes \beta} - \frac{1}{\otimes \alpha} - \frac{\otimes p}{\otimes} \alpha_1\right) \otimes q - \frac{\otimes w}{\otimes \beta} \tag{34}$$

Thus, the total cost (TC) in a cycle is obtained by the following equation.

$$\begin{aligned}
TC &= \otimes c \otimes q + \otimes d \otimes q + \otimes c_{r\otimes} p \otimes q + \otimes K + \otimes h\left[\frac{(t_4 + t_5) z_1}{2} + \frac{t_2 z_3}{2}\right] \\
&+ \otimes h_1 \left[\frac{(t_3 + t_4) z_3}{2}\right] + \otimes b\left[\frac{t_1 \otimes w}{2} + \frac{t_2(\otimes w + z_2)}{2} + \frac{t_3 z_2}{2}\right] \\
&= (\otimes c + \otimes d + \otimes c_r \otimes p) \otimes q + \otimes K \\
&+ \left\{ \frac{\left(1 - \otimes \frac{\beta}{\otimes} \alpha\right)^2}{2} \left(1 - \otimes \frac{\beta}{\otimes} \alpha_1\right) \left(\frac{\otimes h}{\otimes \beta} + \frac{\otimes b}{\otimes} \alpha_1\right) - \frac{\otimes b\left(1 - \frac{\otimes \beta}{\otimes \alpha}\right)}{2 \otimes \alpha} \right. \\
&+ \frac{(\otimes h + \otimes b)}{2} \left(1 - \frac{\otimes \beta}{\otimes} \alpha_1\right) \left(\frac{\left(1 + \frac{\otimes \beta}{\otimes \alpha_1}\right)}{\otimes \alpha} - \frac{2}{\otimes} \alpha_1 \right) \otimes p \\
&+ \left. \left[\otimes h_1 + \frac{\left(\otimes b + \otimes h \otimes \frac{\beta}{\otimes} \alpha_1\right)}{\left(1 - \otimes \frac{\beta}{\otimes} \alpha_1\right)}\right] \frac{\otimes p^2}{2 \otimes \alpha_1} \right\} \otimes q^2 \\
&- \left\{ \frac{\frac{\otimes h}{\otimes \beta} + \frac{\otimes b}{\otimes} \alpha_1 - (\otimes h + \otimes b)\left(\frac{1}{\otimes \alpha} + \frac{\otimes p}{\otimes} \alpha_1\right)}{\left(1 - \frac{\otimes \beta}{\otimes} \alpha_1\right)} \right\} \otimes q \otimes w \\
&+ \left\{ \otimes b + \frac{\left(\otimes h + \otimes b \otimes \frac{\beta}{\otimes} \alpha_1\right)}{\left(1 - \otimes \frac{\beta}{\otimes} \alpha_1\right)} \right\} \otimes \frac{w^2}{2 \otimes \beta}
\end{aligned} \tag{35}$$

The expected value of the total cost per cycle, ETC,

$$\mathrm{ETC} = m_1 \otimes q + \otimes K + m_2 \otimes q^2 - m_3 \otimes q \otimes w + m_4 \otimes w^2 \tag{36}$$

Here,

$$m_1 = \otimes c + \otimes d + \otimes c_r E[\otimes p] \tag{37}$$

$$m_2 = \frac{\left(1-\otimes\frac{\beta}{\otimes}\alpha\right)^2}{2}\left(1-\otimes\frac{\beta}{\otimes}\alpha_1\right)\left(\frac{\otimes h}{\otimes \beta}+\frac{\otimes b}{\otimes}\alpha_1\right)-\frac{\otimes b\left(1-\frac{\otimes\beta}{\otimes\alpha}\right)}{2\otimes\alpha}$$
$$+\frac{(\otimes h+\otimes b)}{2}\left(1-\frac{\otimes \beta}{\otimes}\alpha_1\right)\left(\frac{\left(1+\frac{\beta}{\alpha_1}\right)}{\otimes\alpha}-\frac{2}{\otimes}\alpha_1\right)E[\otimes p] \tag{38}$$
$$+\left[\otimes h_1+\frac{\left(\otimes b+\otimes h\otimes\frac{\beta}{\otimes}\alpha_1\right)}{\left(1-\otimes\frac{\beta}{\otimes}\alpha_1\right)}\right]\frac{E[\otimes p^2]}{2\otimes\alpha_1}$$

$$m_3 = \frac{\frac{\otimes h}{\otimes\beta}+\frac{\otimes b}{\otimes\alpha_1}-(\otimes h+\otimes b)\left(\frac{1}{\otimes\alpha}+\frac{E[\otimes p]}{\otimes\alpha_1}\right)}{\left(1-\frac{\otimes\beta}{\otimes\alpha_1}\right)} \tag{39}$$

$$m_4 = \left\{\otimes b+\frac{\left(\otimes h+\otimes b\otimes\frac{\beta}{\otimes}\alpha_1\right)}{\left(1-\otimes\frac{\beta}{\otimes}\alpha_1\right)}\right\}\frac{1}{2\otimes\beta} \tag{40}$$

The expected value of the total cost per unit time, ETCU , is obtained by the ratio of the expected value of the total cost per cycle to the cycle time as follows.

$$\mathrm{ETCU} = \otimes\beta m_1+\frac{\otimes K\beta}{\otimes q}+\otimes\beta m_2\otimes q-\otimes\beta m_3\otimes w+\frac{\otimes\beta m_4\otimes w^2}{\otimes q} \tag{41}$$

In order to find the optimum production and backorder quantities (decision variables), the partial derivatives of the ETCU function concerning the decision variables must be taken and set equal to zero.

$$\frac{\partial\mathrm{ETCU}}{\partial w} = -\otimes\beta m_3+\frac{2\otimes\beta m_4\otimes w}{\otimes q}=0 \tag{42}$$

$$\frac{\partial\mathrm{ETCU}}{\partial q} = -\frac{\otimes K\otimes\beta}{\otimes}q^2+\otimes\beta m_2-\frac{\otimes\beta m_4\otimes w^2}{\otimes q^2}=0 \tag{43}$$

The following optimum production quantity and backorder quantity are obtained.

$$\otimes q = \sqrt{\frac{\otimes K}{\left(m_2-\frac{m_3^2}{4m_4}\right)}} \tag{44}$$

$$w = \frac{m_3\otimes q}{2m_4} \tag{45}$$

4. Numerical Analysis

The instance of the model for numerical analysis is given in Table 3 for uniform distribution, normal distribution and grey uncertainty. In addition, for the grey uncertainty, the whitenization approaches and their solutions are also given in Table 4.

Table 3. Dataset for numerical analysis

Parameters	Stochastic Models		Grey Models				
	Uniform Dist. Model	Normal Dist. Model	Grey Values		Mean Whitenization (DoG = 0.5)	Degree of Greyness Whitenization	
			Min	Max		DoG	Value
β	1000	1000	900	1100	1000	0.2	1060
α	4000	4000	3600	4400	4000	0.2	4240
α_1	2000	2000	1800	2200	2000	0.2	2120
x	5000	5000	4500	5500	5000	0.2	5300
h	0.8	0.8	0.7	0.9	0.8	0.25	0.85
h_1	1	1	0.9	1.1	1	0.2	1.06
b	0.4	0.4	0.3	0.5	0.4	0.5	0.4
c	100	100	90	110	100	0.2	106
c_r	20	20	18	22	20	0.2	21.2
d	1.4	1.4	1.2	1.6	1.4	0.2857	1.486
K	1700	1700	1500	1900	1700	0.2353	1805.882

About the defective items rate p random variable, for both uniform and normal distribution models in stochastic models, the following assumptions are taken as distribution density functions:

$$f(p) = \begin{cases} 10 \text{ if } 0.35 \leq p \leq 0.45 \\ 0 \quad \text{otherwise} \end{cases} \tag{46}$$

So, the main model is designed for the production system to have a high defection rate, and according to Eq. 43, for the uniform distribution, the values of $E[p]$ and $E[p^2]$ is 0.40 and 0.1608 respectively. Then, for the normal distribution, the values are obtained as 0.38 and 0.1451. About the grey models, in a theoretic view no distribution is required, however, the defection rate is assumed as $\otimes \in [0.35, 0.45]$ an interval grey number. So, for the mean whitenization dataset, the model is established as equal-weighted and the defection rate p is 0.40 and the solution is the same with the uniform distribution model.

About the degree of greyness model, the degree of greyness level is obtained as 0.25 for the grey defection rate $\otimes \in [0.35, 0.45]$ and then, $E[\otimes p]$ is 0.425 and $E[\otimes p^2]$ is 0.1806 to the grey model. The computational results are gathered from MATLAB® software with M2 CPU and 16 GB RAM in the reasonable times and given in Table 4 and illustrated in Figure 4.

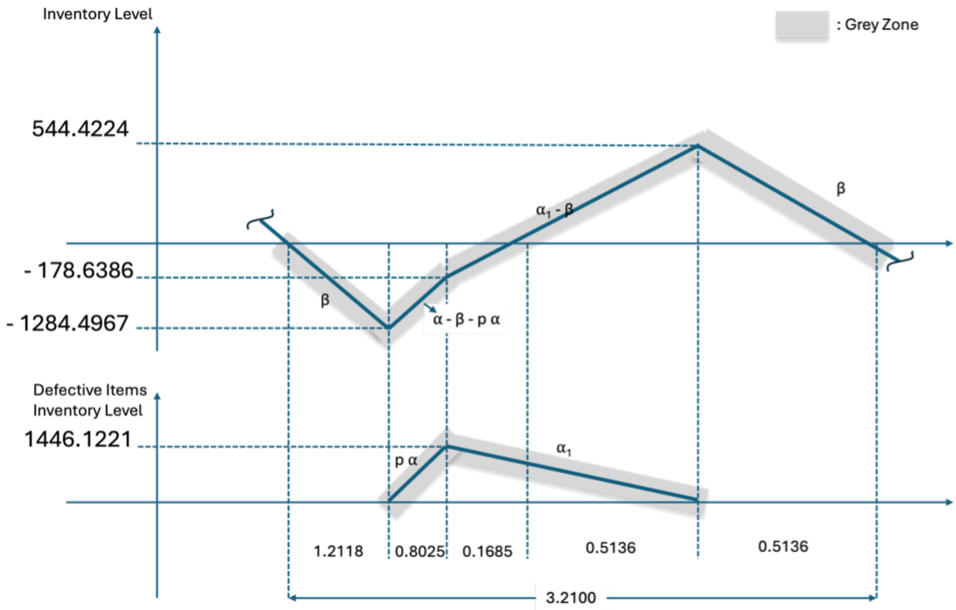

Figure 4. Solution of grey model over time

It is shown that while the Grey Model provides results comparable to the stochastic models for many parameters, there are some differences, particularly in ETCU and z_2. The cycle is time 3.3331 weeks and 3.3816 weeks for uniform and normal distribution models but it's 3.2100 weeks for the model that is much shorter than the stochastic models since the much more EPQ level. Then, Figure 5 is shown that the differences on q,w and ETCU are presented statistical distributions versus grey uncertainty.

Table 4. Computational results for numerical analysis

Parameters	Stochastic Models		Grey Models
	Uniform Dist. Model	Normal Dist. Model	Degree of Greyness Whitenization
q	3331.0665	3381.6212	3402.6403
w	1276.9088	1330.1044	1284.4967
ETCU	110420.6941	110005.4349	124610.9068
z_1	555.1777	563.6035	544.4224
z_2	111.0355	78.9045	178.6386
z_3	1332.4266	1285.0161	1446.1221
t_1	1.2769	1.3301	1.2118
t_2	0.8328	0.8454	0.8025
t_3	0.111	0.0789	0.1685
t_4	0.5552	0.5636	0.5136
t_5	0.5552	0.5636	0.5136
t	3.3311	3.3816	3.21

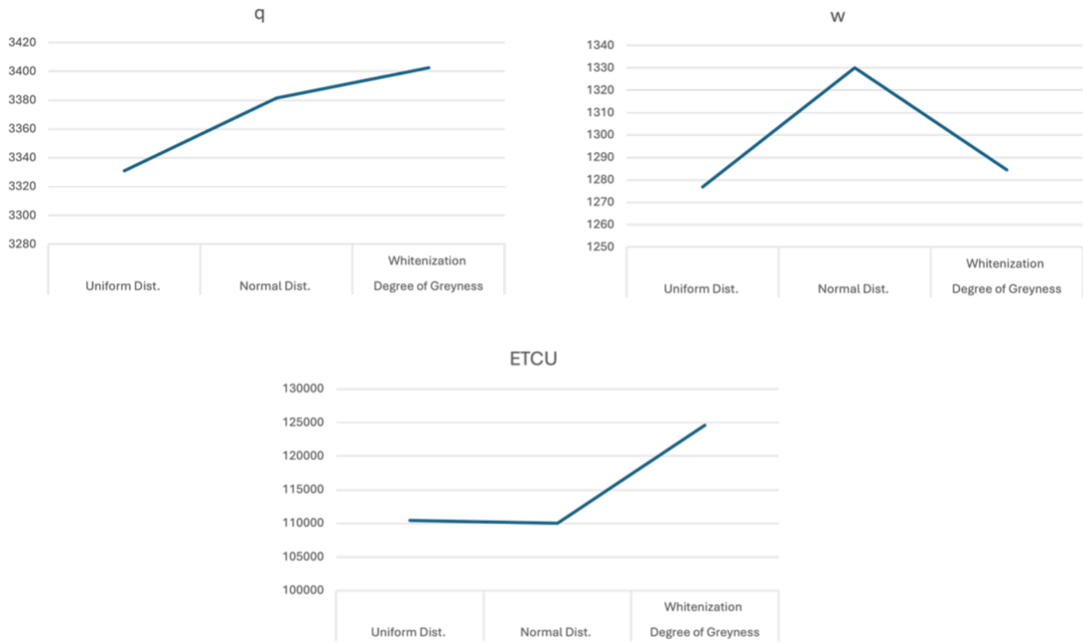

Figure 5. The differences on statistical distributions versus grey uncertainty

Figure 5 aims to illustrate the potential impact of different distribution assumptions (Uniform, Normal, and Degree of Greyness) on key parameters of an EPQ model: production quantity (q), backorder level (w), and total cost per unit time (ETCU). As uncertainty increases, especially under whitenization, production quantities rise while backorders decrease, indicating a strategy to mitigate risk by producing more. However, this comes at the cost of significantly higher total costs, as seen in the sharp increase in ETCU. This suggests that managing uncertainty in production systems requires careful consideration of inventory and service levels, with the potential for increased operational costs.

5. Conclusions and Further Research

This work introduces a novel Economic Production Quantity (EPQ) model designed to address the challenge of minimizing production time to prevent backorders. The model incorporates uncertainty modeling by utilizing grey values to represent variables such as production rate, costs, and prices. The goal is to create a more robust model that aids decision-makers in optimizing production lot sizes, managing backorder levels, and reducing costs by addressing uncertainties inherent in grey systems theory. For the model proposed in this study to be effective in use, the rework ability and capacity of the production company must be greater than the customer demand rate.

According to the computational results, the grey model generally produces results comparable to the stochastic models, though notable differences are observed in certain parameters. For instance, the grey model yields the highest values for q and z_3, and significantly higher for ETCU compared to the uniform and normal distribution models, indicating potential variations in cost-related outcomes. Meanwhile, time-related parameters t_1 to t_5 show only slight variations across all models, with the grey model often producing slightly lower values. These observations suggest that while the grey model aligns with the stochastic models in many aspects, it may be capturing different system characteristics or reacting differently to input uncertainties.So, the grey model could be more sensitive to certain inputs or could be capturing different aspects of the system that the stochastic models might not fully address.

As further research, it is suggested that the model can be developed by eliminating the assumptions on the EPQ model, developing models on products that can be recycled rather than reworked, and models based on rare operations or orders, and developing models by eliminating uncertainties such as grey and fuzzy models and/or clarifying its effects.

Declarations

Conflict of interest The authors have no competing interests to declare that are relevant to the content of this article.

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