Numerical Methodology for Detection and Analysis of Potential Cracks in an Existing Masonry Dam Structure

Mevcut Bir Yığma Baraj Yapısındaki Potansiyel Çatlakların Tespiti ve Analizi için Sayısal Metodoloji

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Öz

Son zamanlarda yapılan çok sayıda çalışma toprak dolgu ve beton baraj yapılarının sismik analizlerine odaklanırken, yığma barajlar üzerine yapılan araştırmalar sınırlı kalmıştır. Bu barajların sismik tepkisi öncelikle doğaları gereği süreksiz özelliklerinden etkilenmektedir. Bu nitelik, deprem olayları sırasında önemli reaksiyonlara neden olabilir. Bu makale, yığma bir barajda depremin neden olduğu hasar modellerini tahmin etmeyi amaçlayan sayısal bir metodoloji sunmaktadır. Bu sonuçların sonlu elemanlar yazılımına aktarılması ve ardından barajın yapısal bütünlüğünü gözlemlenen hasarlarla ilişkili olarak değerlendirmek ve bu hasarların zaman içinde nasıl geliştiğini araştırmak için analitik tekniklerin kullanılması süreci özetlenmektedir. Genişletilmiş Sonlu Elemanlar Yöntemi (XFEM) kullanan bir Sonlu Elemanlar (FE) modeli kullanılarak 2 boyutlu bir kabuk modeli ele alınarak sayısal bir simülasyon gerçekleştirilmiştir. FE modelleri doğrusal olmayan zaman tanım alanında analizleri kullanılarak gerçekleştirilmiştir. Hesaplanan sonuçları literatürdeki mevcut verilerle karşılaştırmak için bir değerlendirme yapılmıştır. Bu değerlendirme hem modal analizi hem de gerilme analizini kapsar ve yığma baraj içindeki çatlak ve hasar dağılımındaki değişimler de dahil olmak üzere birincil hasar mekanizmalarının incelenmesini içerir. Bu doğrulama adımı, simülasyonun gerçek hasar durumunu doğru bir şekilde temsil ettiğinden emin olmak için çok önemlidir.

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Anahtar Kelimeler: Yığma Baraj, Sismik Analizler, Sonlu Elemanlar, Hasar Örüntüleri.

Abstract

Many recent studies have focused on the seismic analysis of earthfill and concrete dam structures, while research on masonry dams remains limited. The seismic response of these dams is primarily influenced by their inherently discontinuous nature, which can lead to significant responses during earthquake events. This paper presents a numerical methodology aimed at predicting earthquake-induced damage patterns in a masonry dam. It outlines the process of importing these results into finite element software and then using analytical techniques to evaluate the structural integrity of the dam in relation to the observed damage, and to investigate how this damage evolves over time. A numerical simulation using a 2D shell model has been conducted using a Finite Element (FE) model with the Extended Finite Element Method (XFEM). The FE models have been developed using non-linear time history analysis. An evaluation will compare the calculated results with existing data from the literature, including both modal analysis and stress analysis. This evaluation involves investigating primary failure mechanisms, including variations in crack and damage distribution within the masonry dam. This verification step is crucial to ensure that the simulation accurately represents the real-world phenomenon.

Keywords: Masonry Dam, Seismic Analyses, Finite Element, Damage Patterns.

Concrete dams are susceptible to significant risks when subjected to powerful earthquakes, particularly damage and potential failure of the dam structure (Mridha and Maity 2014). This concern has prompted considerable interest among dam engineering researchers, leading to the development of numerical models capable of predicting and capturing cracks in the dam. Two primary categories of approaches have emerged for addressing this issue. The first category encompasses continuum cracking methods, such as the smeared crack approach and the plastic-damage constitutive model (Valamanesh et al. 2011; Wang et al. 2021; Haghani et al. 2022; Ouzandja and Berrabah 2023). These approaches provide an effective framework for characterizing initial damage and incorporating internal failure parameters to represent the stiffness reduction in solid materials without altering the FEM topology. They are well-suited for addressing complex engineering challenges. In contrast, the second category, which includes the fracture mechanics approach and the XFEM, is classified as a discrete crack approach (Guanglun et al. 2000). These approaches are particularly useful for modelling the later stages of damage and are characterized by their ability to represent crack propagation.

A number of studies have examined the response to earthquakes and the collapse behaviour of typical dams using a variety of methods (Rezaiee-Pajand et al. 2024; Galván et al. 2022; Huang and Han 2023; Peramuna et al. 2024; Mirzabozorg 2024). Among these, the smeared crack model (SCM) and the discrete crack model (DCM) are two common FE models. Fenves and Vargas-Loli (1988) conducted a study investigating the seismic behavior of Pine Flat dam. Utilizing a smeared crack model and actual earthquake data, their findings suggested that dams could withstand earthquakes with low peak accelerations, underscoring their structural resilience. This research has had a significant impact on the development of fundamental dam safety guidelines, influencing engineering approaches and emergency preparedness in earthquake-prone regions. In their analysis of the seismic response of the Koyna dam, Ayari and Saouma (1990) employed the DCM model. Their research revealed the presence of cracks on both the upstream and downstream faces of the dam, particularly in the vicinity of a sharp change in downstream slope elevation. This discovery provides crucial insights into the stability of dams when subjected to seismic events. In a study conducted

by Omidi et al. (2012), a failure analysis of the Koyna dam was performed using a combination of the FE method and the nonlinear stress-strain curve. The results revealed that crack propagation within the dam initially showed minimal progression. However, as the analysis continued, cracks eventually propagated vertically. This type of analysis is significant as it assists researchers in predicting potential points of failure and enables the implementation of preventive measures before catastrophic failures can occur. In a study by Wang et al. (2015), the influence of initial crack location on damage propagation within a concrete gravity dam was examined. To conduct this research, the XFEM-based cohesive segment method was employed, which involved incorporating phantom nodes to replicate crack initiation and progression along arbitrary paths. This innovative approach offers valuable insights into the behavior of dam structures, contributing to a deeper comprehension of crack development processes. In a study by Haghani et al. (2022), a model that combines the XFEM with the a-method was developed. This model was applied to gravity dams to obtain dynamic fracture results. In this study, numerical analyses were conducted to identify the modal characteristics of the F dam. To validate the model, an investigation was performed using a model containing a single initial crack to assess the seismic crack propagation in a masonry dam. The study also examined the influence of crack length on crack propagation in the FE model subjected to seismic loading. The distribution of crack damage patterns was analyzed. The primary objective of this study was to develop and validate a FE modelling approach using ABAQUS (2017) software.

The aim of this study is to predict earthquake-induced damage patterns in masonry dams and to evaluate the effect of these damages on structural integrity. To this end, crack initiation and propagation are analyzed using different numerical modelling approaches and the results are compared with the existing literature. The study aims to develop and validate modelling methods to improve the seismic safety of masonry dams.

NUMERICAL CAMPAIGN

The masonry gravity dam situated in Seia, Portugal. It features an M-shaped plan, stands at a height of 28 meters from its foundation, has a crest elevation of 1600m, and spans a length of 1200m. An overall view of the dam model is depicted in Figure 1. Construction of the dam commenced in 1912 and reached a height of 6m by 1914. Subsequently, the structure underwent multiple upgrades. The thickness of this concrete slab varies, ranging from 0.5m near the crest to 0.7m near the foundation. Additionally, above the original crest of the dam at 1597 meters elevation, a new concrete block with dimensions of 2m by 3m was added, and it is connected to the concrete slab Feitosa e Castro et al. (1994).



Figure 1. Cross-section geometry

Two distinct materials were employed to model the weight structures of concrete and masonry. These materials share a uniform density of 2500kg/m³ and a Poisson ratio of 0.2. The modulus of elasticity, however, differs according to the material type: 30GPa for concrete and 11GPa for masonry. The compressive and tensile strengths of concrete are 3MPa and 0.3MPa, respectively, as referenced by Bretas et al. (2012). The hydrostatic pressure applied is 3.06MPa.

For the numerical analysis, Rayleigh damping coefficients were established by referencing the first and fourth vibration modes of the dam, aiming for a critical damping ratio of 5%. An artificial acceleration record, created in accordance with EC8 (2010) recommendations, was employed for the analysis. This record had a peak ground acceleration of 0.15g and a duration of 20 s, as illustrated in Figure 2.



Figure 2. Processed acceleration history of ground motion

In the dynamic analysis of FE dam model, three distinct loads were taken into consideration. Firstly, the dead load represents the dam's own weight. Secondly, a hydrostatic water pressure load of 3.06 MPa was applied to the upstream face and dynamic implicit method-based solver was used. The applied loadings are visualized in Figure 3a. The concrete dam is discretized using hourglass control and 2-dimensional shell element (CPE4R). In the modeling, the dam body is represented as a continuous mesh. The FE mesh used in the analysis of the FE model consists of 4989 nodes and 4816 elements. The mesh generation was gradually refined to achieve convergence. The appropriate mesh size in this study was found to be 0.25m. The FE model of the dam is given in Figure 3b.



Figure 3. FE model of the Lagoa Comprida gravity dam

Results

Modal analysis was applied as the first step. The aim is to obtain the natural frequency values and mode shapes of the FE modelled dam model. The comparison of the mode shapes obtained in ABAQUS (2017) software with the results from the discrete element method (DEM) obtained by Bretas et al. (2015) is presented in Figure 4.



(b)

Figure 4. Comparison of the mode shape results (a) FE model and (b) DEM model (Bretas et al. 2015)

The comparison of the numerical and DEM model of the first four natural frequency values obtained from Lagoa Comprida dam is given in Table 1.

Mode	Natural Frequencies (Hz)		Error Data (%)
	DEM model	FEM model	Error Rate (%)
1	5.70	5.77	1.21
2	13.3	12.93	2.65
3	18.9	18.71	1.02
4	24.5	25.21	2.81

 Table 1. Comparison of natural vibration frequencies results

As shown in Table 1, the largest calculated difference between numerical and experimental natural frequency values is 2.81%. Since this difference is less than 5%, The FE model does not need to update. Also, it is seen that mode shapes obtained from the numerical FE model and DEM model are compatible (Figure 4). In order to determine the initial crack location in the dam body, the earthquake analysis of the dam was performed first. The maximum principal stress contour diagrams obtained from the nonlinear time history analysis of the model dam are given in Figure 5.



Figure 5. (a) Tensile stress and (b) Third stress invariant (I3)

The highest stress regions were determined after dynamic analysis using a nonlinear time history analysis. The presence of cracks increases the stress locally. On the other hand, the stress distribution around the fractures cau-

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ses crack propagation (Wang et al., 2021). Predicting the crack propagation path of a model is extremely difficult because stress and damage are highly interdependent.

As shown in Figure 5, the tensile stress and the third stress invariant (I3) occur around 0.37MPa and 0.39MPa at the upstream face of the FE dam, respectively. Stress distribution is also considered for the two-dimensional model. In order to determine the height position of the maximum tensile stress concentration node on the upstream surface, which can be the crack initiation, ABAQUS (2017) software determines the maximum value using the contour limits options. The principal stress contours show the value at the node in the FE dam in Figure 6.



Figure 6. Determination of the nodal point of the maximum tensile stress value (Unit: Pascal)

As shown in Figure 6a, the contour command option was used to determine the point of greatest tensile stress value. Cracks in the upstream face of the dam started between 14m and 22m from the heel, where the tensile stress intensity was high, as shown in Figure 6b. The tensile stress in this critical zone is typically represented by the red color. In the DEM study, the crack started to propagate at a height of 15m at 1587m above the heel of the dam. Also, in the FE model, the largest tensile stress value on the upstream surface of the dam was located at 15.1m at the nodal point (Figure 6b).

By comparing the DEM and FE model studies, the difference between the heights of the crack locations was approximately 1%. In this case, the value

of the maximum tensile stress of the dam after the earthquake was found to be about 23% higher than the value of 0.3MPa (tensile strength). Therefore, cracks may occur in different critical zones on the dam body above the upstream surface, and significant damage may occur at these locations. The present study focuses on semi-brittle materials. Therefore, the failure criterion was chosen according to the third stress invariant, I3. Papadopoulos (1987) concluded that fracture will occur when the determinant of the stress tensor in his study reaches a critical value. As shown in Figure 7, a model with a horizontal initial crack length named (a) was built on the nodal point located on the upstream surface. XFEM discretization is shown in Figure 7.



Figure 7. Initial embedded crack location of FE dam

The FE model based on the XFEM method with initial cracks in the upstream face at a height of 15.1m above the heel is investigated with four values: 0.2m, 0.8m, 1.4m, and 2.0m. named as M1-Scenario, M1-Scenario, M1-Scenario and M1-Scenario. Figure 8, Figure 10, Figure 12, and Figure 14 illustrate the time evolution of different crack propagation paths. The ABAQUS (2017) software provided the functionality of crack surface visualization (PHILSM). This parameter was used to define and/or show the location of the crack within the enriched model part and is non-zero only for the enriched model part. STATUSXFEM is a scalar parameter indicating the degree of damage or "cracking" in the enrichment zone of the model part. For a completely cracked model part, the value of 0. Values between 0 and 1

indicate partial damage or cracking in the enrichment zone of the model. The final crack propagation paths depending on the length of the crack in the FE dam body (Figure 9, Figure 11, Figure 13, and Figure 15).



(a) (b) **Figure 9.** *XFEM damage contour of M1-Scenario (a) STATUSXFEM and (b) PHILSM (t=10s)*

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M2-Scenario

Figure 10. Crack propagation of M2-Scenario (a=0.8 m)



(a) (b) **Figure 11.** *XFEM damage contour of M2-Scenario case (a) STATUSXFEM and (b) PHILSM*







Figure 13. XFEM damage contour of M3-Scenario case (a) STATUSXFEM and (b) PHILSM



M4-Scenario

Figure 14. Crack propagation of M4-Scenario (a=2 m)



Figure 15. *XFEM damage contour of M4-Scenario (a) STATUSXFEM and (b) PHILSM*

Discussion

This section illustrates the crack propagation in the Lagoa Comprida dam during the earthquake at different initial crack length. As seen in the figures, distribution of tensile damage variable (DAMAGET) damage limits in the contour diagrams use red and blue colors to represent the damaged and undamaged regions of the dam models, respectively.

The final damage-crack propagations of the FE model with four scenarios are illustrated in Figures 8–15. The effect of crack lengths on the damage-crack propagation trajectory can be examined by comparing Figure 8, Figure 10, Figure 12, and Figure 14. It can be observed that the initial length of the crack has a certain impact on the propagation of crack intensities. Whenever the initial cracking lengths at the upstream face are 0.2m and 0.4m, the upstream crack does not extend to the downstream face. There are no penetrating cracks on the downstream face. With the rise in initial crack length to 1.2m, the upstream crack penetrates completely to the downstream face, penetrating the whole body of the dam. However, with 2 m crack lengths, there are significant cracks on different parts of the downstream face and toe region, as illustrated in Figure 15.

The state of Model-C1, Model-C2, and Model-C3 at 4 seconds and 4.75 seconds shows that the crack path in the dam body changes and progresses vertically towards the center of the dam base, eventually reaching the surface. For Model-C4, significant damage occurs in the dam body compared to the other models. Damage extends from the downstream face to the dam's toe. In all these cases, no new damage is observed after 5.60 seconds, and the damaged gravity dam remains the same until the end of the analysis. The final damaged dam models for Model-C1, Model-C2, and Model-C3 are

similar. In different scenarios, the crack distribution in its final form leads to significant damage or failure in the dam body. Therefore, it can be observed that the crack propagation consists of one main damage zone at the upstream face of the dam. It is noted that the red-colored damaged regions have caused a reduction in stiffness. This crack distribution and damage model align with existing results in the literature (Feltrin et al. 1992; Bhattacharjee and Leger 1993; Wang et al. 2015; Wang et al. 2021; Parvathi et al. 2022).

Considering the FE model results presented in Figure 16, similar results were obtained for the crack patterns predicted by the DEM model. It can be seen that the XFEM method can effectively capture the crack propagation patterns and trajectories in the M1-Scenario (0.2m). For this scenario, it is assumed that the proposed XFEM model can simulate the expected damage-crack patterns in the dam structure. The final crack patterns in these different cases ultimately lead to the partial destruction or complete collapse of the dam. Nevertheless, some cracks exist in the XFEM model but do not exist in the DEM model. These might be due to different factors, like material properties (damage parameters, tensile strength value), or mesh size.



Figure 16. Comparison between the final crack propagation in (a) the DEM model (adopted from Bretas et al. 2015) and (b) the FE model

Conclusion

This paper presents a comprehensive numerical methodology for assessing the seismic response and predicting earthquake-induced damage patterns in masonry dams. The research addresses a critical gap in the literature, where the focus has predominantly been on masonry dam structures. The inherent discontinuous characteristics of masonry dams contribute significantly to their seismic vulnerability, and understanding their response is crucial for ensuring their structural integrity in earthquake-prone regions.

The proposed numerical methodology involves a 2D shell model utilizing the Extended Finite Element Method (XFEM) for a detailed analysis of earthquake-induced damage. The process includes importing results into finite element software, conducting nonlinear time-history analyses, and employing numerical analysis to evaluate the observed damages. Modal analysis and stress analysis are carried out to comprehensively assess the structural behavior of the masonry dam.

The effect of the initial cracks with varying lengths at the upstream face is also investigated. The results indicate that the M1-Scenario with a 0.2m length of crack has trajectories comparatively comparable to those of the damaged dam depicted by the DEM model.

In conclusion, the presented results demonstrate the XFEM method's capability in predicting damage-crack patterns in masonry dams under different scenarios. The insights gained from this analysis contribute to a better understanding of the relationship between initial crack lengths and the ultimate structural response, guiding future efforts in dam engineering and seismic risk assessment.

Experimental validation, such as shake table tests, is essential for ensuring the accuracy of numerical models like the Extended Finite Element Method (XFEM) used to analyze masonry dams. These models rely on assumptions, so validation against real-world data is crucial. Shake table tests use scaled models to simulate earthquake forces, allowing observation of damage patterns and crack formation. Comparing these results with XFEM simulations helps refine the models, improving their predictive capabilities. This process enhances confidence in the reliability of numerical methods, ensuring they can accurately assess seismic risk and guide safer dam design in earthquake-prone areas. These additions would guide future research efforts in improving the seismic resilience of masonry dams.

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Author Contributions

All authors conceived the idea and were responsible for the literature review. The methodology was planned collaboratively. Boudjamaa Roudane conducted the numerical analyses, while Ali Kaya interpreted the results. Boudjamaa Roudane and Ali Kaya wrote the manuscript. All authors discussed the findings, provided critical feedback, and contributed to the final version of the paper.

Conflict of Interest

The Authors declare that there is no conflict of interest.

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Ethical Statement

Ethics committee permission is not required for the study.