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Performance Comparison of Recent Metaheuristic Algorithms on Engineering Design Optimization Problems

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Abstract
Metaheuristic algorithms have been extensively applied in a variety of complex engineering design optimization problems (EDOPs) due to their capability of yielding near-optimal solutions without excessive computational times. The aim of this study is to investigate the performance comparison among seven novel metaheuristic optimization algorithms: Artificial Hummingbird Algorithm (AHA), Artificial Protozoa Optimizer (APO), African Vultures Optimization Algorithm (AVOA), Electric Eel Foraging Optimization (EEFO), Mountain Gazelle Optimizer (MGO), Pied Kingfisher Optimizer (PKO), and Quadratic Interpolation Optimization (QIO). This comparison is performed with twelve engineering design optimization problems evaluating the best, worst, mean, and standard deviation of their results. We also use non-parametric statistical tests such as the Friedman rank test and Wilcoxon signed rank test to finally compare the performance of algorithms. The results show the merits and demerits of each algorithm, which give us clues on their suitability for different engineering design problems. According to Friedman rank test, EEFO surpasses the other algorithms in these EDOPs. In addition, it performs statistically better than AVOA and QIO according to Wilcoxon signed rank test.

1. Introduction

Optimization is a process carried out to find the values of the variables that maximize or minimize the value of a function within problem-specific restrictions. In optimization problems, the variables are called decision variables, this function is called the objective function, and the restrictions are referred to as constraints. Once the problem is defined, an optimization method is selected based on the problem's characteristics to solve it.

Many researchers have utilized the metaheuristic algorithms for solving optimization problems. As metaheuristic algorithms are stochastic by nature, they cannot guarantee the achievement of the optimal solution. Although the structures of metaheuristic algorithms do not guarantee finding the global optimum, they are used to find the global optimum value or a close value in a timely manner. New metaheuristic algorithms are continually introduced into the literature by researchers. These metaheuristic algorithms, which are quite numerous, show different performances in different optimization problems. Problem sets have been created in the literature to compare the performances of the algorithms. One of problem sets is engineering design optimization problems (EDOPs). Most EDOPs are highly complex, with many variables, constraints, and objectives. These problems are so expensive from the computational point of view that they are not easy to solve in any classical way.

There are several studies showing the effectiveness of metaheuristics in this field. For instance, the hierarchical surrogate-assisted memetic

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algorithm (HSAMA) has been proven effective in reducing computational costs and efficient on Competitions on Evolutionary Computation (CEC) benchmark problems as well as a series of real-world engineering design tasks by Zhou [1]. The Artificial Bee Colony (ABC) algorithm improved handling large-scale and constrained optimization problems, which was successful in the solution of CEC benchmark functions and EDOPs [2]. Besides, rank-iMDDE, as an enhanced constrained differential evolution method, has demonstrated competitive results in enhancing solution quality and convergence rates [3]. Additionally, the IEEE CEC'2013 benchmark suite is constructed with characteristics representative of real-world problems, and hence, it aids in benchmarking most evolutionary techniques [4]. Cooperative co-evolution frameworks have effectively dealt with complex design problems that exist in concurrent engineering by decomposing them into smaller subproblems [5]. Furthermore, a Novel Differential Evolution Algorithm (NDE) using triangular mutation rules has also outperformed the list of CEC benchmark functions and EDOPs [6]. The improved Butterfly Optimization Algorithm (BOA) with the use of the cross-entropy method enhances global search abilities to become capable of successfully solving classical EDOPs and the CEC benchmark functions [7]. The Search and Rescue optimization algorithm (SAR) has been successfully applied to solve constrained EDOPs and CEC [8]. Besides, Atomic Orbital Search (AOS) algorithm has also proved effective on constrained EDOPs, including those benchmarked by CEC 2020, using principles from quantum mechanics [9]. Lastly, the convergence rates and performance of the Accelerated Arithmetic Optimization Algorithm by Cuckoo Search (AOACS) algorithm are better when tested using both CEC 2019 functions and EDOPs [10]. These are all studies collectively demonstrating the fact that metaheuristic algorithms offer robust and practical solutions to a large set of benchmarks as well as real-world problems, like CEC competitions.

The aim of this study is to compare the performance of recent metaheuristic optimization algorithms on selected twelve EDOPs. For this purpose, seven metaheuristic optimization algorithms developed in the literature in recent years namely Artificial Hummingbird Algorithm (AHA), Artificial Protozoa Optimizer (APO), African Vultures Optimization Algorithm (AVOA), Electric eel foraging optimization (EEFO), Mountain Gazelle Optimizer (MGO), Pied Kingfisher Optimizer (PKO) and Quadratic Interpolation Optimization (QIO), are used in this study. Despite these algorithms are utilized to solve some EDOPs, it is aimed to compare these algorithms' performance under the same conditions (the same number of function evaluations (FEs) and the same population size) in this study. These algorithms' performance is compared through the best, the worst, standard deviation and mean values. Also, non-parametric statistical tests (Friedman rank test and Wilcoxon signed rank test) are organized to compare the performance further.

2. Material and Method

2.1. Optimization Algorithms

Seven metaheuristic optimization algorithms were used in the study. Brief information about algorithms is given in this section. Detailed information about the algorithms and the source codes can be reached from the references and code columns given in Table 1.

Table 1. Information about optimization algorithms

Algorithm	Inspiration	Reference	Code
Artificial Hummingbird Algorithm (AHA)	Flight skills and foraging strategies of hummingbirds	[11]	[12]
Artificial Protozoa Optimizer (APO)	Foraging, dormancy, and reproductive behaviors of protozoa	[13]	[14]
African Vultures Optimization Algorithm (AVOA)	Foraging and navigation behavior of African vultures	[15]	[16]
Electric Eel Foraging Optimization (EEFO)	Intelligent group foraging behavior of electric eels	[17]	[18]
Mountain Gazelle Optimizer (MGO)	Social life and hierarchy of wild mountain gazelles	[19]	[20]
Pied Kingfisher Optimizer (PKO)	Hunting behavior and symbiotic relationships of pied kingfishers	[21]	[22]
Quadratic Interpolation Optimization (QIO)	Derived from generalized quadratic interpolation method	[23]	[24]

2.2. Engineering Optimization Problems

Twelve EDOPs were used in this study. Detailed information about the EDOPs is given in this section.

The first of the EDOPs is the Cantilever Beam, which focuses on the objective of minimizing the weight of a cantilever beam with a square cross-section. It is shown in Figure 1 that one end of the beam is fixed rigidly, and at the other end, a vertical force is applied. The beam has five hollow square blocks with uniform thickness, whereas the heights (or widths) of these blocks are taken as the decision variables, each having a fixed thickness of 2/3. A general mathematical formulation of the problem may be stated analytically as follows [25]:

Minimize:

$$f(X) = 0.0624(x_1 + x_2 + x_3 + x_4 + x_5) \qquad (1)$$

Subject to:

$$g(X) = \frac{61}{x_1^3} + \frac{37}{x_2^3} + \frac{19}{x_3^3} + \frac{7}{x_4^3} + \frac{1}{x_5^3} - 1 \leq 0 \qquad (2)$$

Variable Range:

$$0.01 \leq x_i \leq 100, i = 1, \ldots, 5 \qquad (3)$$

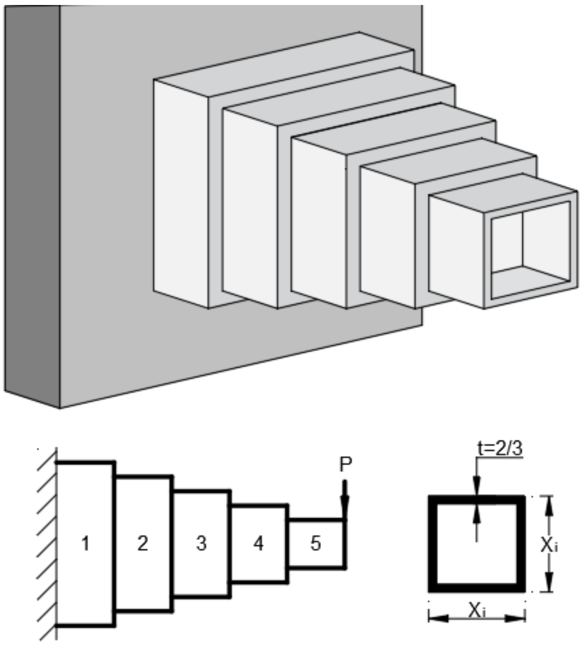

Figure 1. Schematic representation of cantilever beam design problem

Another fascinating problem in EDOPs is the I-beam design problem. It is a crucial benchmark problem; the formulation to be solved is finding the minimum vertical deflection of a beam as shown in Figure 2 considered loads, with constraints on the cross-sectional area and stress. The design variables are the flange width b (x_1), section height h (x_2), web thickness t_w (x_3), and flange thickness t_f (x_4). The maximum deflection y of the beam is given by y = PL³ / 48EI, where L= 5200 cm is the length of the beam and E = 523.104 kN/cm² is the modulus of elasticity. This problem is stated as [26]:

Minimize:

$$f(X) = \frac{5000}{\frac{x_3(x_2-2x_4)^3}{12} + (x_1 x_4^3/6) + 2bx_4(x_2 - x_4/2)^2} \qquad (4)$$

Subject to:

$$g_1(X) = 2x_1x_3 + x_3(x_2 - 2x_4) \leq 300 \qquad (5)$$

$$g_2(X) = \frac{18x_2 \times 10^4}{x_3(x_2-2x_4)^3 + 2x_1x_3\left(4x_4^2 + 3x_2(x_2-2x_4)\right)} + \frac{15x_1 \times 10^3}{(x_2-2x_4)x_3^2 + 2x_3x_1^3} \leq 56 \qquad (6)$$

Variable Range:

$$10 \leq x_1 \leq 50 \qquad (7)$$
$$10 \leq x_2 \leq 80 \qquad (8)$$
$$0.9 \leq x_3 \leq 5 \qquad (9)$$
$$0.9 \leq x_4 \leq 5 \qquad (10)$$

The third EDOPs aims to minimize the volume of a statically loaded three-bar truss design with stress (σ) constraints on its members as shown in Figure 3. The optimization is done to determine the cross-sectional areas A_1 (x_1) and A_2 (x_2). The constants in the formulas are l=100 cm, $= 2\ kN/cm^2$, $\sigma = 2\ kN/cm^3$. Mathematically, the formulation for this problem can be represented as [27]:

Minimize:

$$f(X) = \left(2\sqrt{2}x_1 + x_2\right) \times l \qquad (11)$$

Subject to:

$$g_1(X) = \frac{\sqrt{2}x_1 + x_2}{\sqrt{2}x_1^2 + 2x_1x_2} P - \sigma \leq 0 \qquad (12)$$

$$g_2(X) = \frac{x_2}{\sqrt{2}x_1^2 + 2x_1x_2} P - \sigma \leq 0 \qquad (13)$$

$$g_3(X) = \frac{1}{x_1 + \sqrt{2}x_2} P - \sigma \leq 0 \qquad (14)$$

Variable Range:

$$0 \leq x_1, x_2 \leq 1 \qquad (15)$$

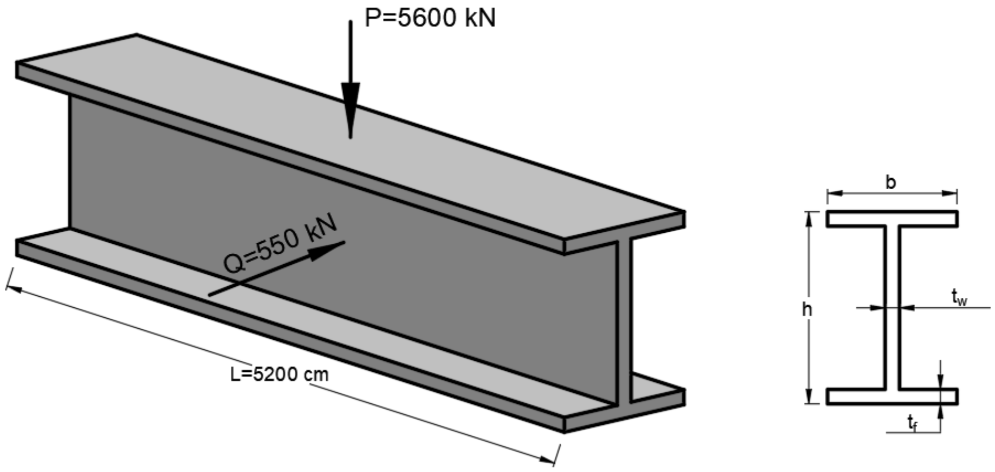

Figure 2. Schematic representation of I shaped beam design problem

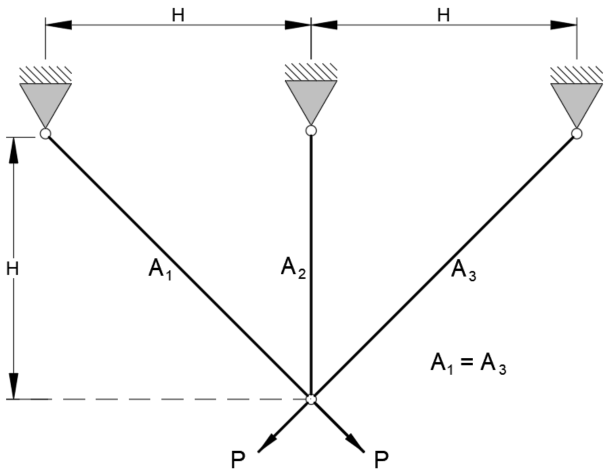

Figure 3. Schematic representation of three-bar truss design problem

Tubular column design problem aims to design a uniform column in the tubular section subjected to a compressive load with minimum cost. The two design variables considered for this problem are the mean diameter of the column, d (x_1), and the thickness of the tube, t (x_2). The cross-sectional geometry and the critical dimensions of the column are shown in Figure 4. The material used to fabricate the column has a yield stress of σ_y = 500 kgf/cm² and a modulus of elasticity of E = 0.85×10⁶ kgf/cm². The optimization model for this problem is formulated as follows [28]:

Minimize:

$$f(X) = 9.82x_1x_2 + 2x_1 \quad (16)$$

Subject to:

$$g_1(X) = \frac{2500}{\pi x_1 x_2} - \sigma_y \leq 0 \quad (17)$$

$$g_2(X) = \frac{2500}{\pi x_1 x_2} - \frac{\pi^2 E\left(x_1^2 + x_2^2\right)}{8(250)^2} \leq 0 \quad (18)$$

$$g_3(X) = 2 - x_1 \leq 0 \quad (19)$$

$$g_4(X) = x_1 - 14 \leq 0 \quad (20)$$

$$g_5(X) = -x_2 + 0.2 \leq 0 \quad (21)$$

$$g_6(X) = x_2 - 0.8 \leq 0 \quad (22)$$

Variable Range:

$$2 \leq x_1 \leq 14 \quad (23)$$

$$0.2 \leq x_2 \leq 0.8 \quad (24)$$

The constraints g_1 and g_2 ensure that the column-induced stress is less than the buckling and yield stresses, respectively. Furthermore, other constraints namely, g_3, g_4, g_5, and g_6, confine the design variables to their allowable ranges.

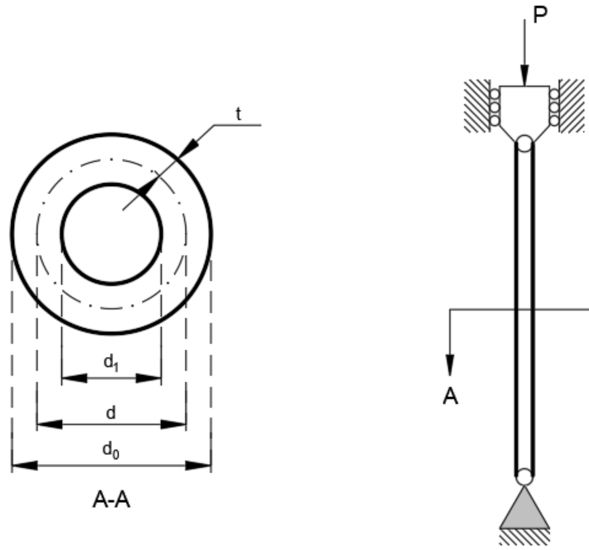

Figure 4. Schematic representation of tubular column design problem

The Welded Beam design problem is one of the EDOPs introduced by Coello [29]. A vertical force loads the beam, as shown in Figure 5. The task is to find the best possible design for the welded beam concerning minimizing manufacturing cost under seven constraints regarding stress, deflection, welding, and geometry. The design variables are h (x_1), l (x_2), t (x_3), and b (x_4). The constants are $P = 6000\, lb,\ L = 14\, in\, lb,\ \delta_{max} = 0.25\, in,\ E = 30 \times 10^6\, psi,\ G = 12 \times 10^6\, p,\ \tau_{max} = 13{,}600\, psi$, $\sigma_{max} = 30{,}000\, psi$. This objective function can mathematically be given as [29]:

Minimize:

$$f(X) = 1.10471x_1^2x_2 + 0.04811x_3x_4(14.0 + x_2) \tag{25}$$

Subject to:

$$g_1(X) = \tau(X) - \tau_{max} \leq 0 \tag{26}$$
$$g_2(X) = \sigma(X) - \sigma_{max} \leq 0 \tag{27}$$
$$g_3(X) = \delta(X) - \delta_{max} \leq 0 \tag{28}$$
$$g_4(X) = x_1 - x_4 \leq 0 \tag{29}$$
$$g_5(X) = P - P_c(X) \leq 0 \tag{30}$$
$$g_6(X) = 0.125 - x_1 \leq 0 \tag{31}$$
$$g_7(X) = 0.10471x_1^2 + 0.04811x_3x_4(14.0 + x_2) - 5.0 \leq 0 \tag{32}$$
$$\tau(X) = \sqrt{(\tau')^2 + 2\tau'\tau''\frac{x_2}{2R} + (\tau'')^2} \tag{33}$$
$$\tau' = \frac{P}{\sqrt{2}x_1x_2} \tag{34}$$
$$\tau'' = \frac{MR}{J} \tag{35}$$
$$M = P\left(L + \frac{x_2}{2}\right) \tag{36}$$
$$R = \sqrt{\frac{x_2^2}{4} + \left(\frac{x_1 + x_3}{2}\right)^2} \tag{37}$$
$$J = 2\left\{\sqrt{2}x_1x_2\left[\frac{x_2^2}{12} + \left(\frac{x_1 + x_3}{2}\right)^2\right]\right\} \tag{38}$$
$$\sigma(X) = \frac{6PL}{x_4x_3^2} \tag{39}$$
$$\delta(X) = \frac{4PL^3}{Ex_3^3x_4} \tag{40}$$
$$P_c(X) = \frac{4.013E\sqrt{x_3^2x_4^6/36}}{L^2}\left(1 - \frac{x_3}{2L}\sqrt{\frac{E}{4G}}\right) \tag{41}$$

Variable Range:

$$0.1 \leq x_1 \leq 2 \tag{42}$$
$$0.1 \leq x_2 \leq 10 \tag{43}$$
$$0.1 \leq x_3 \leq 10 \tag{44}$$
$$0.1 \leq x_4 \leq 2 \tag{45}$$

Amir and Hasegawa [30] formulated an EDOP for designing a reinforced concrete beam, presented in Figure 6. The beam is simply supported with a span equal to 30 feet and subjected to a live loading of 2000 lbs and a dead loading of 1000 lbs, which includes the weight of the beam. The strength of the concrete is σ_c = 5 ksi and the strength of the reinforcing steel is equal to σ_y = 50 ksi. The cost for concrete is \$0.02/in²/ft, while the price for steel is \$1.0/in²/ft. The subject of EDOP is determined to satisfy the structural requirements of the ACI building code 318-77, in which the area of reinforcement A_s (x_1), the width of the beam b (x_2), and the depth of the beam h (x_3) are the design variables such that the total cost of the structure can be minimized.

Minimize:

$$f(X) = 2.9x_1 + 0.6x_2x_3 \tag{46}$$

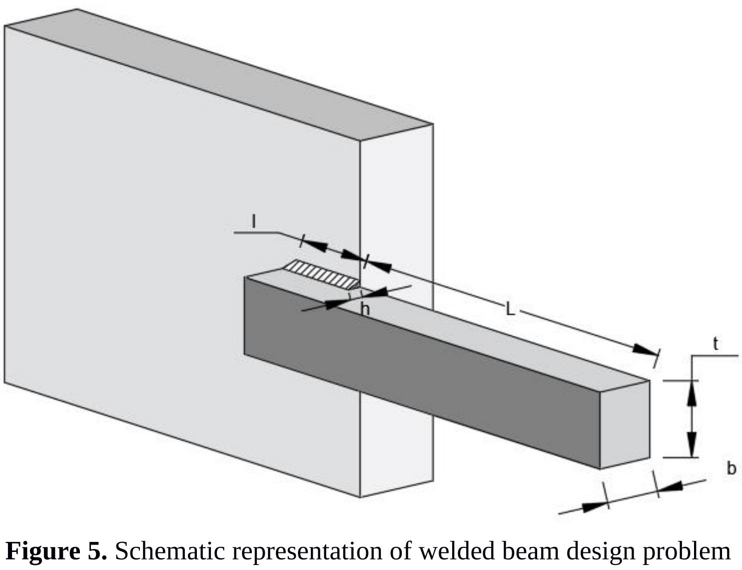

Figure 5. Schematic representation of welded beam design problem

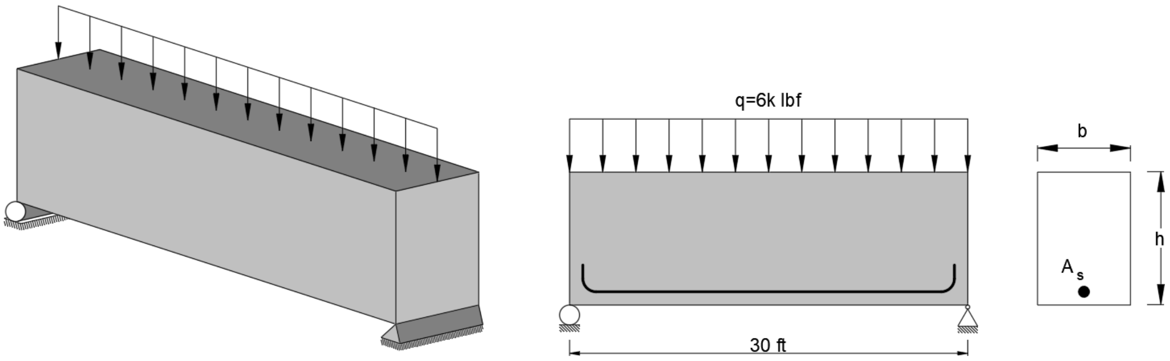

Figure 6. Schematic representation of reinforced concrete beam design problem

Subject to:

$$g_1(X) = \frac{x_2}{x_3} - 4 \leq 0 \quad (47)$$

$$g_2(X) = 180 + 7.375\frac{x_1^2}{x_3} - x_1 x_2 \leq 0 \quad (48)$$

Variable Range:

$$x_1 \in \{6, 6.16, 6.32, 6.6, 7, 7.11, 7.2, 7.8, 7.9, 8, 8.4\} \quad (49)$$

$$x_2 \in \{28, 29, 30, \ldots, 40\} \quad (50)$$

$$5 \leq x_3 \leq 105 \quad (51)$$

The corrugated bulkhead EDOP shown in Figure 7, concerns the minimization of a bulkhead weight in a chemical tanker with a corrugated bulkhead, where its design variables are the width (x_1), depth (x_2), length (x_3), and plate thickness (x_4) of the bulkhead. The mathematical model for this optimization problem is described as follows [31]:

Minimize:

$$f(X) = \frac{5.885 x_4 (x_1 + x_3)}{x_1 + \sqrt{|x_3^2 - x_2^2|}} \quad (52)$$

Subject to:

$$g_1(X) = -x_4 x_2 \left(0.4 x_1 + \frac{x_3}{6}\right) + 8.94\left(x_1 + \sqrt{|x_3^2 - x_2^2|}\right) \leq 0 \quad (53)$$

$$g_2(X) = -x_4 x_2^2 \left(0.2 x_1 + \frac{x_3}{12}\right) + 2.2\left(8.94\left(x_1 + \sqrt{|x_3^2 - x_2^2|}\right)\right)^{4/3} \leq 0 \quad (54)$$

$$g_3(X) = -x_4 + 0.0156 x_1 + 0.15 \leq 0 \quad (55)$$

$$g_4(X) = -x_4 + 0.0156 x_3 + 0.15 \leq 0 \quad (56)$$

$$g_5(X) = -x_4 + 1.05 \leq 0 \quad (57)$$

$$g_6(X) = -x_3 + x_2 \leq 0 \quad (58)$$

Variable Range:

$$0 \leq x_1, x_2, x_3 \leq 100 \quad (59)$$

$$0 \leq x_4 \leq 5 \quad (60)$$

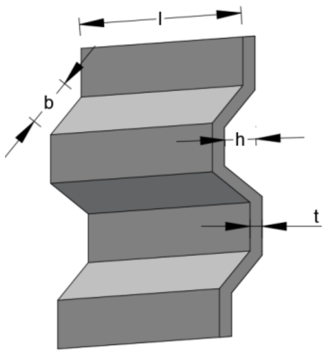

Figure 7. Schematic representation of corrugated bulkhead design problem

The tension/compression spring design problem introduced in [32] is aimed at weight minimization of a tension/compression spring, as shown in Figure 8. The design problem consists of constraints on minimum deflection, shear stress, surge frequency, outside diameter limits, and design variables, which are the wire diameter d (x_1), the mean coil diameter D (x_2), and the number of active coils N (x_3). The mathematical formulation is written as follows:

Minimize:

$$f(X) = (x_3 + 2) x_2 x_1^2 \quad (61)$$

Subject to:

$$g_1(X) = 1 - \frac{x_2^3 x_3}{71785 x_1^4} \leq 0 \quad (62)$$

$$g_2(X) = \frac{4x_2^2 - x_1 x_2}{12566(x_2 x_1^3 - x_1^4)} + \frac{1}{5108 x_1^2} \leq 0 \quad (63)$$

$$g_3(X) = 1 - \frac{140.45 x_1}{71785 x_1^4} \leq 0 \quad (64)$$

$$g_4(X) = \frac{x_1 + x_2}{1.5} - 1 \leq 0 \quad (65)$$

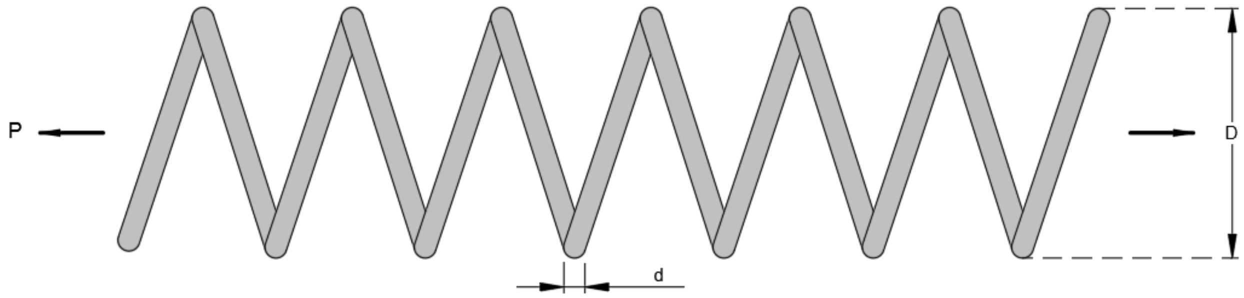

Figure 8. Schematic representation of spring design problem

Variable Range:

$$0.05 \leq x_1 \leq 2 \quad (66)$$
$$0.25 \leq x_2 \leq 1.3 \quad (67)$$
$$2 \leq x_3 \leq 15 \quad (68)$$

The optimization of the design of a cylindrical pressure vessel contained between two hemispherical heads is concerning minimum total cost that is comprising material, forming, and welding costs plotted in Figure 9. The problem consists of four design variables the thickness of the shell *Ts* (x_1), the thickness of the head *Th* (x_2), the inner radius *R* (x_3), and the length of the cylindrical section of the vessel, excluding the head *L* (x_4). Additionally, x_1 and x_2 are integer multiples of 0.0625 inches, and other variables are continuous. The resulting optimization problem can be cast as follows [33]:

Minimize:

$$f(X) = 0.6224x_1x_3x_4 + 1.7781x_2x_3^2 + 3.1661x_1^2x_4 + 19.84x_1^2x_3 \quad (69)$$

Subject to:

$$g_1(X) = -x_1 + 0.0193x_3 \leq 0 \quad (70)$$
$$g_2(X) = -x_2 + 0.00954x_3 \leq 0 \quad (71)$$
$$g_3(X) = -\pi x_3^2x_4 + \frac{4}{3}\pi x_3^3 + 1{,}296{,}000 \leq 0 \quad (72)$$
$$g_4(X) = x_4 - 240 \leq 0 \quad (73)$$

Variable Range:

$$x_1, x_2 \in \{1 \times 0.0625, 2 \times 0.0625, 3 \times 0.0625, \ldots, 99 \times 0.0625\} \quad (74)$$
$$0 \leq x_3, x_4 \leq 200 \quad (75)$$

In mechanical systems, the speed reducer is a crucial component of the gearbox and is utilized in various applications [33]. This optimization problem which is illustrated in Figure 10 aims to minimize the weight of the speed reducer, subject to eleven constraints. The problem involves seven design variables: face width *b* (x_1), module of teeth *m* (x_2), number of teeth in the pinion *z* (x_3) length of the first shaft between bearings l_1 (x_4), length of the second shaft between bearings l_2 (x_5), diameter of the first shaft d_1 (x_6), and diameter of the second shaft d_2 (x_7). The mathematical formulation of this problem is as follows:

Minimize:

$$f(X) = 0.7854x_1x_2^2(3.3333x_3^2 + 14.9334x_3 - 43.0934) - 1.508x_1(x_6^2 + x_7^2) + 7.4777(x_6^3 + x_7^3) + 0.7854(x_4x_6^2 + x_5x_7^2) \quad (76)$$

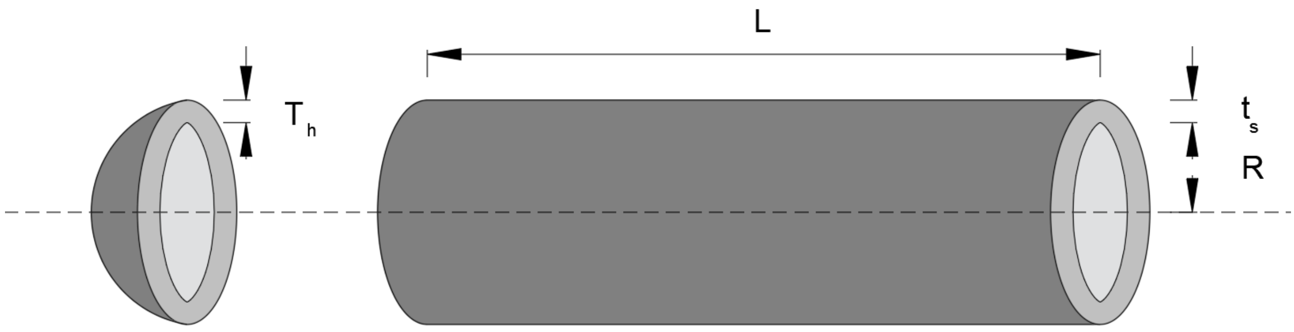

Figure 9. Schematic representation of pressure vessel design problem

Subject to:

$$g_1(X) = \frac{27}{x_1 x_2^2 x_3} - 1 \leq 0 \quad (77)$$
$$g_2(X) = \frac{397.5}{x_1 x_2^2 x_3^2} - 1 \leq 0 \quad (78)$$
$$g_3(X) = \frac{1.93x_4^3}{x_2 x_6^4 x_3} - 1 \leq 0 \quad (79)$$
$$g_4(X) = \frac{1.93x_5^2}{x_2 x_7^4 x_3} - 1 \leq 0 \quad (80)$$
$$g_5(X) = \frac{\sqrt{(745x_4/x_2x_3)^2 + 16.9\times10^6}}{110x_6^3} - 1 \leq 0 \quad (81)$$
$$g_6(X) = \frac{\sqrt{(745x_5/x_2x_3)^2 + 157.5\times10^6}}{85x_7^3} - 1 \leq 0 \quad (82)$$
$$g_7(X) = \frac{x_2 x_3}{40} - 1 \leq 0 \quad (83)$$
$$g_8(X) = \frac{5x_2}{x_1} - 1 \leq 0 \quad (84)$$
$$g_9(X) = \frac{x_1}{12x_2} - 1 \leq 0 \quad (85)$$
$$g_{10}(X) = \frac{1.5x_6 + 1.9}{x_4} - 1 \leq 0 \quad (86)$$
$$g_{11}(X) = \frac{1.1x_7 + 1.9}{x_5} - 1 \leq 0 \quad (87)$$

Variable Range:

$$2.6 \leq x_1 \leq 3.6 \quad (88)$$
$$0.7 \leq x_2 \leq 0.8 \quad (89)$$
$$17 \leq x_3 \leq 28 \quad (90)$$
$$7.3 \leq x_4, x_5 \leq 8.3 \quad (91)$$
$$2.9 \leq x_6 \leq 3.9 \quad (92)$$
$$5 \leq x_7 \leq 5.5 \quad (93)$$

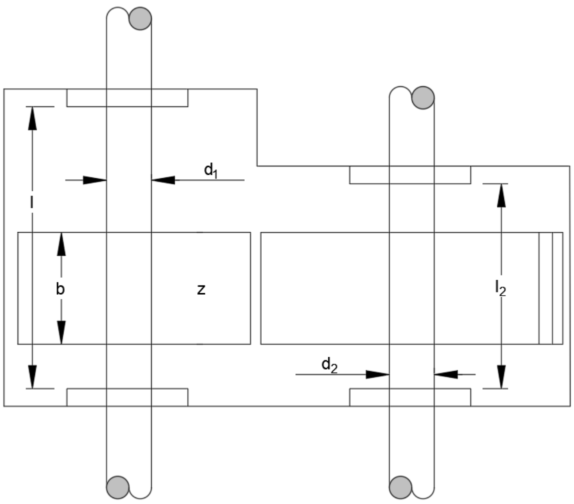

Figure 10. Schematic representation of speed reducer design problem

The gear train design problem, introduced by Sandgren [34], is an unconstrained discrete EDOP in mechanical engineering. The objective of this problem is to minimize the gear ratio, defined as the ratio of the angular velocity of the output shaft to the angular velocity of the input shaft. The design variables are the number of teeth of gears n_A (x_1), n_B (x_2), n_C (x_3), and n_D (x_4). Figure 11 illustrates the model of this problem. The mathematical formulation is as follows:

Minimize:

$$f(X) = \left(\frac{1}{6.931} - \frac{x_3 x_2}{x_1 x_4}\right)^2 \quad (94)$$

Variable Range:

$$12 \leq x_1, x_2, x_3, x_4 \leq 60 \quad (95)$$

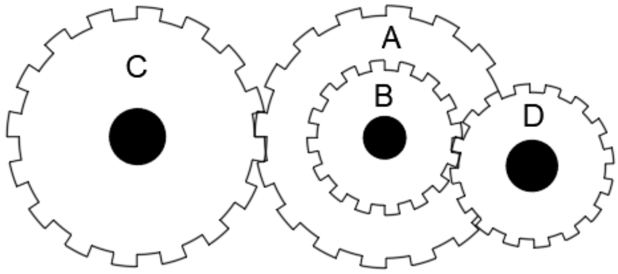

Figure 11. Schematic representation of gear train design problem

In the European Enhanced Vehicle-Safety Committee testing procedure, this EDOP relates to subjecting a car to a side impact to minimize the door's weight. The problem considers eleven decision variables which are the thicknesses of the B-pillar inner (x_1), B-pillar reinforcement (x_2), floor side inner (x_3), cross members (x_4), door beam (x_5), door beltline reinforcement (x_6), roof rail (x_7), materials of the B-pillar inner (x_8), floor side inner (x_9), barrier height (x_{10}), and hitting position (x_{11}). In this EDOP, the formulation that Youn et al. [35] proposed is used as follows:

Minimize:

$$f(X) = 1.98 + 4.90x_1 + 6.67x_2 + 6.98x_3 + 4.01x_4 + 1.78x_5 + 2.73x_7 \quad (96)$$

Subject to:

$$g_1(X) = 1.16 - 0.3717x_2x_4 - 0.00931x_2x_{10} - 0.484x_3x_9 + 0.01343x_6x_{10} - 1 \leq 0 \quad (97)$$

$$g_2(X) = 46.36 - 9.9x_2 - 12.9x_1x_8 - 0.1107x_3x_{10} - 32 \leq 0 \quad (98)$$

$$g_3(X) = 33.86 + 2.95x_3 + 0.1792x_{10} - 5.057x_1x_2 - 11.0x_2x_8 - 0.0215x_5x_{10} - 9.98x_7x_8 + 22.0x_8x_9 - 32 \leq 0 \quad (99)$$

$$g_4(X) = 28.98 + 3.818x_3 - 4.2x_1x_2 - 0.0207x_5x_{10} + 6.63x_6x_9 - 7.7x_7x_8 + 0.32x_9x_{10} - 32 \leq 0 \quad (100)$$

$$g_5(X) = 0.261 - 0.0159x_1x_2 - 0.188x_1x_8 - 0.019x_2x_7 + 0.0144x_3x_5 + 0.0008757x_5x_{10} + 0.08045x_6x_9 + 0.00139x_8x_{11} + 0.00001575x_{10}x_{11} - 0.32 \leq 0 \quad (101)$$

$$g_6(X) = 0.214 + 0.00817x_5 - 0.131x_1x_8 - 0.0704x_1x_9 + 0.03099x_2x_6 + 0.018x_2x_7 + 0.0208x_3x_8 + 0.121x_3x_9 - 0.00364x_5x_6 + 0.0007715x_5x_{10} - 0.0005354x_6x_{10} + 0.00121x_8x_{11} + 0.00184x_9x_{10} - 0.02x_2^2 - 0.32 \leq 0 \quad (102)$$

$$g_7(X) = 0.74 - 0.61x_2 - 0.163x_3x_8 + 0.001232x_3x_{10} - 0.166x_7x_9 + 0.227x_2^2 - 0.32 \leq 0 \quad (103)$$

2.3. Constraint Handling Mechanism

A parameter free constraint handling approach known as the Inverse-Tangent-Constraint-Handling (ITCH) method is used to address EDOPs in this study. The ITCH technique which is initially developed by Kim et al. [36] assesses each solution vector using Equation (104):

$$f(\vec{x}) = \begin{cases} \hat{g}(\vec{x}) = g_{max}(\vec{x}) & if\ g_{max}(\vec{x}) > 0 \\ \hat{f}(\vec{x}) = atan[f(\vec{x})] - \pi/2 & otherwise \end{cases} \quad (104)$$

Here $g_{max}(\vec{x})$ is defined as max $g_i(\vec{x})$ are the constraint functions. The inverse tangent function is represented by $atan[.]$. It is important to note that $\hat{g}(\vec{x}) < 0$ for any $\vec{x}$, which ensures $\hat{f}(\vec{x}) < \hat{g}(\vec{x})$.

3. Results and Discussion

Seven recent metaheuristic algorithms performance is evaluated through twelve EDOPs in this study. The algorithms are implemented in MATLAB 2024a, and all experiments are performed on a PC with 2.3 GHz CPU and 16 GB RAM running under a Windows 11 operating system. To make a fair comparison, the number of function evaluation is determined as 50000 for all algorithms. The parameters for all compared algorithms from the literature are summarized below in Table 2.

Table 2. Algorithm parameters

Algorithm	Parameter settings
AHA	Migration coefficient M
APO	Number of neighbor pairs np=1, pfmax =0.1, maximum proportion fraction
AVOA	L1=0.8, L2=0.2, w=2.5, P1=0.6, P2=0.4, P3=0.6
EEFO	Parameter Free
MGO	Parameter Free
PKO	Beating factor (BF) =0.8, PEmax=0.5, PEmin=0
QIO	Exploration weight (W1), Exploitation weight (W2)

The best, the worst, mean and standard deviation of the results are reported for each problem in the Table 3-Table 14 respectively. Also, two non-parametric statistical tests (Friedman rank test and Wilcoxon signed rank test) are organized to evaluate algorithms performance further in Table 15.

The results of the cantilever beam problem are given in Table 3. It is determined that the performance of AHA, APO, EEFO, PKO, and QIO algorithms is indifferent since there is a negligible difference between the best values. There exist some variations between the worst values, especially with AVOA and MGO present a little bit higher value. The mean values are very close with AHA and APO in the lowest means. The standard deviations are low, showing that they are very precise and have a very little variability in their performance for this problem.

Table 3. Computational results of cantilever beam problem

Performance Metric	AHA	APO	AVOA	EEFO	MGO	PKO	QIO
Best	**1.339912**	**1.339912**	1.339924	**1.339912**	1.339921	**1.339912**	**1.339912**
Worst	1.339913	1.339913	1.340140	1.339932	1.34027	1.339936	1.339924
Mean	**1.339912**	**1.339912**	1.339966	1.339917	1.340016	1.339923	1.339914
Std. Dev	**3.06E-07**	3.01E-07	4.7E-05	5.58E-06	8.07E-05	6.95E-06	2.38E-06

The best, worst, and mean values for the results of the I-shaped beam problem given in Table 4, are amazingly identical for all algorithms and value of 0.013074. This shows that every one of the algorithms performs under the best and the uniform conditions in this problem. The extremely low standard deviations confirm the high precision and reliability of these algorithms when applied to the I-Shaped Beam Problem.

Table 4. Computational results of I-shaped beam problem

Performance Metric	AHA	APO	AVOA	EEFO	MGO	PKO	QIO
Best	**0.013074**	**0.013074**	**0.013074**	**0.013074**	**0.013074**	**0.013074**	**0.013074**
Worst	0.013074	0.013074	0.013074	0.013074	0.013074	0.013074	0.013074
Mean	0.013074	0.013074	0.013074	0.013074	0.013074	0.013074	0.013074
Std. Dev	1.47E-11	3.68E-13	1.68E-09	2.75E-09	8.06E-09	**2.73E-17**	3.44E-08

In the three bar truss problem given in Table 5, there are slight differences in the best and mean values among the algorithms; in fact, they are all very close to each other. The best and means of AHA, APO, and EEFO are almost the same, which reflects their best performance and consistency. Only the worst values are slightly more in AVOA and MGO, denoting variability. The AHA, APO, EEFO, and QIO standard deviations are very low, meaning very great precision; the AVOA and MGO standards are relatively a bit higher.

Table 5. Computational results of three bar truss problem

Performance Metric	AHA	APO	AVOA	EEFO	MGO	PKO	QIO
Best	**263.8826**	**263.8826**	263.8827	**263.8826**	263.8827	**263.8826**	263.8827
Worst	263.8826	263.8826	263.8995	263.8826	264.0332	263.8828	263.8827
Mean	**263.8826**	**263.8826**	263.8858	**263.8826**	263.8963	263.8827	263.8827
Std. Dev	**5.84E-12**	5.41E-12	0.004709	5.09E-12	0.028167	3.18E-05	1.49E-05

The results of the tubular column design problem given in Table 6, indicate that all algorithms are behaving in quite the same manner, having the best and mean values around 26.53132. The worst values show minimal differences, with QIO showing a slightly higher worst value. The very low standard deviations for all algorithms suggest that the performance of these algorithms is exact and consistent for this design problem.

Table 6. Computational results of tubular column design problem

Performance Metric	AHA	APO	AVOA	EEFO	MGO	PKO	QIO
Best	**26.53132**	**26.53132**	**26.53132**	**26.53132**	**26.53132**	**26.53132**	26.53133
Worst	26.53132	26.53132	26.53133	26.53132	26.53132	26.53132	26.53135
Mean	**26.53132**	**26.53132**	**26.53132**	**26.53132**	**26.53132**	**26.53132**	26.53134
Std. Dev	6.58E-14	6.09E-14	6.7E-07	5.95E-14	9.91E-12	4.78E-14	7.18E-06

For the welded beam design problem given in Table 7, the best values are quite the same amongst the algorithms, although the AVOA attains a slightly higher best value. The worst values present a more significant variation because both are higher for the AVOA and MGO. The mean values are very close, with the lowest means being for the AHA, APO, and EEFO. The standard deviations are low for the AHA, APO, and EEFO, which show high precision, but the AVOA and MGO show more variability in their results.

Table 7. Computational results of welded beam design problem

Performance Metric	AHA	APO	AVOA	EEFO	MGO	PKO	QIO
Best	**1.724717**	**1.724717**	1.724895	**1.724717**	1.724728	**1.724717**	1.72472
Worst	1.724717	1.724717	1.799024	1.724717	2.050502	1.724724	1.72474
Mean	**1.724717**	**1.724717**	1.744075	**1.724717**	1.746229	1.724718	1.724725
Std. Dev	1.19E-12	1.79E-09	0.021077	4.3E-11	0.059604	1.7E-06	4.18E-06

The results of the concrete beam design problem given in Table 8, show that the best and mean values are all clustered around 359.2037. The means only differ at the second decimal for some of the algorithms. The AVOA and MGO have worse values than the rest, which means that their results are more inconsistent. The AHA, APO, EEFO, PKO, and QIO have very low standard deviations, which tell about very high levels of precision. At the same time, the AVOA and MGO are much more variable in their results.

Table 8. Computational results of concrete beam design problem

Performance Metric	AHA	APO	AVOA	EEFO	MGO	PKO	QIO
Best	**359.2037**	**359.2037**	**359.2037**	**359.2037**	**359.2037**	**359.2037**	**359.2037**
Worst	359.2037	362.2494	362.6302	359.2037	362.2494	359.2037	359.2038
Mean	**359.2037**	359.3052	360.5108	**359.2037**	359.3052	**359.2037**	**359.2037**
Std. Dev	1.26E-12	0.556069	1.632746	1.14E-12	0.556069	1.01E-12	2.05E-05

For the problem of corrugated bulkhead design given in Table 9, all the algorithms have the best and mean value that is very close and has negligible variations. The worst values are near to each other with slight differences, with the AVOA and QIO being slightly higher. Standard deviations are extremely low in the AHA, APO, EEFO, and PKO, which means high precision and reliability, whereas the AVOA and QIO have higher variability in their results.

Table 9. Computational results of corrugated bulkhead design problem

Performance Metric	AHA	APO	AVOA	EEFO	MGO	PKO	QIO
Best	**6.842374**	**6.842374**	6.842375	**6.842374**	**6.842374**	**6.842374**	6.842382
Worst	6.842374	6.842374	6.848544	6.842374	6.842374	6.842374	6.842457
Mean	**6.842374**	**6.842374**	6.843027	**6.842374**	**6.842374**	**6.842374**	6.842412
Std. Dev	4.24E-15	1.34E-12	0.001603	5.5E-15	2.84E-11	5.78E-12	1.8E-05

In the spring design problem given Table 10, the best and mean values of all algorithms are close to 0.012662, with a slight difference in means for the AVOA and MGO. The worst values are very different, indicating higher values for the AVOA and MGO. Standard deviations are low for the AHA, APO, EEFO, PKO, and QIO, which presents them as high precisions compared to the other methods; the AVOA and MGO have high variability in their performance.

Table 10. Computational results of spring design problem

Performance Metric	AHA	APO	AVOA	EEFO	MGO	PKO	QIO
Best	**0.012662**	**0.012662**	**0.012662**	**0.012662**	**0.012662**	**0.012662**	**0.012662**
Worst	0.012662	0.012669	0.014593	0.012663	0.016077	0.012715	0.012663
Mean	**0.012662**	**0.012662**	0.013009	**0.012662**	0.013075	0.012677	**0.012662**
Std. Dev	1.17E-07	1.79E-06	0.000466	3.4E-07	0.000849	1.95E-05	3.07E-07

The results of the pressure vessel design problem given in Table 11, indicates that the best values slightly differ among algorithms, while a slight difference is seen; the worst values differ significantly with higher values on the AVOA. The mean varies for all; the AHA and EEFO present the lowest values for it. For the standard deviation, the AHA and EEFO have the lowest values, meaning more precision than the other algorithms. The other algorithms have shown more variability in their results.

Table 11. Computational results of pressure vessel design problem

Performance Metric	AHA	APO	AVOA	EEFO	MGO	PKO	QIO
Best	**6059.083**	**6059.083**	**6059.083**	**6059.083**	6059.084	**6059.083**	6059.09
Worst	6820.026	7332.543	7544.493	6370.352	7333.142	6820.027	6820.08
Mean	6202.341	6389.97	6366.711	**6083.951**	6600.921	6085.477	6107.482
Std. Dev	256.6121	378.3935	434.1332	78.56102	501.2787	138.8485	148.8167

In the speed reducer design problem given in Table 12, the best and mean values are found to be around 2996.274 for most algorithms, with a little difference in the means for the AVOA. Its high worst values reveal a poorer behavior of the AVOA, showing less consistency. The minor standard deviations of the AHA, APO, EEFO, PKO, and QIO reveal good precision and reliability. On the other hand, the AVOA is much more variable in its performance.

Table 12. Computational results of speed reducer design problem

Performance Metric	AHA	APO	AVOA	EEFO	MGO	PKO	QIO
Best	**2996.274**	**2996.274**	2996.275	**2996.274**	**2996.274**	**2996.274**	2996.668
Worst	2996.274	2996.274	3014.038	2996.274	2996.274	2996.274	2997.32
Mean	**2996.274**	**2996.274**	2998.756	**2996.274**	**2996.274**	**2996.274**	2996.954
Std. Dev	5.18E-08	3.06E-10	3.726336	3.44E-08	3.66E-10	2.04E-10	0.163654

The mean and best values present approximately 2.7E-12 for most of the algorithms in the Gear Train Problem given in Table 13, the means are not being wildly divergent for the AVOA. The worst values are considerably higher for the AHA and AVOA, which means that these two methods have a lower level of consistency. The corresponding standard deviations are low for the AHA, APO, EEFO, PKO, and QIO, proving that they have high precision and reliability in their performance.

Table 13. Computational results of gear train problem

Performance Metric	AHA	APO	AVOA	EEFO	MGO	PKO	QIO
Best	**2.7E-12**	**2.7E-12**	**2.7E-12**	**2.7E-12**	**2.7E-12**	**2.7E-12**	**2.7E-12**
Worst	2.36E-09	9.94E-11	2.36E-09	1.18E-09	9.92E-10	9.92E-10	9.92E-10
Mean	3.22E-10	**1.2E-11**	8.15E-10	1.68E-10	1.35E-10	3.62E-10	6.4E-11
Std. Dev	6E-10	1.9E-11	8.16E-10	3.78E-10	2.92E-10	4.33E-10	1.86E-10

The results for the car side impact design problem given in Table 14, show the best and mean values to be around 22.841, with very slight variations for most of the algorithms. The worst values show colossal difference, with higher values in the AVOA. The low values for standard deviations are revealed for the AHA, APO, EEFO, PKO, and QIO, rendering them of the high precision and reliability. In contrast, for the AVOA, the standard deviations are not relatively so low.

Table 14. Computational results of car side impact design problem

Performance Metric	AHA	APO	AVOA	EEFO	MGO	PKO	QIO
Best	22.84151	**22.84138**	22.8432	22.84148	22.84144	22.84139	22.84163
Worst	23.47903	22.84143	24.0513	23.33362	23.30264	22.8463	23.26352
Mean	23.04541	**22.84138**	23.16012	22.90698	23.00187	22.8424	22.89669
Std. Dev	0.18936	8.74E-06	0.29405	0.124723	0.156148	0.001181	0.122991

The statistical comparison given in Table 15, based on the Friedman test ranks the EEFO first, thereby showing its overall the best performance with problems of different types. The Wilcoxon signed-rank test indicates the p-values showing statistical differences between the EEFO and other algorithms. The EEFO statistically performs better than the AVOA and QIO as p <0.1.

Table 15. Statistical test results

Friedman test average rankings		Wilcoxon signed-rank test between EEFO and the other algorithms	
Algorithms	Sum of Ranks	EEFO vs.	p-value
AHA	2.91667 (2)	AHA	0.70020
APO	3.00000 (3)	APO	0.85010
AVOA	6.16667 (7)	AVOA	0.00146
EEFO	2.66667 (1)	MGO	0.01221
MGO	5.41667 (6)	PKO	0.27832
PKO	3.33333 (4)	QIO	0.06396
QIO	4.50000 (5)		

4. Conclusion

In this study, it is presented that a comparative analysis of the performance assessment of seven latest metaheuristic algorithms over twelve EDOPs. The results prove that all algorithms can find close to optimal solutions, but the algorithms significantly vary in terms of consistency and accuracy. The Friedmans test showed that the Electric Eel Foraging Optimization (EEFO) algorithm outperforms all other tested optimization algorithms overall. Furthermore, the Wilcoxon signed-rank test confirms EEFO is significantly better than AVOA and QIO. Although performances vary with the type of problem, this study has concluded that these metaheuristic algorithms have good potential to deal with a wide range of complex engineering optimization problems. Future work will further develop the ability of these algorithms to adapt and solve a larger class of optimization problems and investigate hybrid approaches, which attempt to leverage multiple strengths from various algorithms.

Contributions of the authors

Mümin Emre Şenol Tülin Çetin, and Mustafa Erkan Turan contributed to the study conception and design. Mümin Emre Şenol, Tülin Çetin, and Mustafa Erkan Turan read and approved the draft/final manuscripts.

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Conflict of Interest Statement

There is no conflict of interest between the authors.

Statement of Research and Publication Ethics

The study is complied with research and publication ethics.

References

[1] Z. Zhou, “Memetic algorithm using multiple surrogates for complex engineering design optimization”, Doctoral thesis, Nanyang Technological University, Singapore, 2008, https://doi.org/10.32657/10356/13587.

[2] B. Akay and D. Karaboğa, “Artificial bee colony algorithm for large-scale problems and engineering design optimization” *Journal of Intelligent Manufacturing*, 23, pp. 1001-1014, 2012, https://doi.org/10.1007/s10845-010-0393-4.

[3] W. Gong, Z. Cai and D. Liang, “Engineering optimization by means of an improved constrained differential evolution”, *Computer Methods in Applied Mechanics and Engineering*, 268, pp. 884-904, 2014, https://doi.org/10.1016/J.CMA.2013.10.019.

[4] M. Omidvar, X. Li and K. Tang, “Designing benchmark problems for large-scale continuous optimization”, *Information Sciences*, 316, pp. 419-436, 2015, https://doi.org/10.1016/j.ins.2014.12.062.

[5] X. Lu, S. Menzel, K. Tang and X. Yao, “Cooperative Co-Evolution-Based Design Optimization: A Concurrent Engineering Perspective”, *IEEE Transactions on Evolutionary Computation*, 22, pp. 173-188, 2018, https://doi.org/10.1109/TEVC.2017.2713949.

[6] A. Mohamed, "A novel differential evolution algorithm for solving constrained engineering optimization problems", *Journal of Intelligent Manufacturing*, vol. 29, pp. 659-692, 2018, https://doi.org/10.1007/s10845-017-1294-6.

[7] G. Li, F. Shuang, P. Zhao and C. Le, "An Improved Butterfly Optimization Algorithm for Engineering Design Problems Using the Cross-Entropy Method", *Symmetry*, vol. 11, no. 1049, 2019, https://doi.org/10.3390/sym11081049.

[8] A. Shabani, B. Asgarian, M. Salido and S. Gharebaghi, "Search and rescue optimization algorithm: A new optimization method for solving constrained engineering optimization problems", *Expert Systems with Applications*, 161, 113698, 2020, https://doi.org/10.1016/j.eswa.2020.113698.

[9] M. Azizi, S. Talatahari and A. Giaralis, "Optimization of Engineering Design Problems Using Atomic Orbital Search Algorithm", *IEEE Access*, 9, pp.102497-102519, 2021, https://doi.org/10.1109/ACCESS.2021.3096726.

[10] M. Hijjawi, M. Alshinwan, O. Khashan, M. Alshdaifat, W. Almanaseer, W. Alomoush, H. Garg, and L. Abualigah, "Accelerated Arithmetic Optimization Algorithm by Cuckoo Search for Solving Engineering Design Problems", *Processes* 11, no.5, 1380, 2023, https://doi.org/10.3390/pr11051380.

[11] W. Zhao, L. Wang and S. Mirjalili, "Artificial hummingbird algorithm: A new bio-inspired optimizer with its engineering applications", *Computer Methods in Applied Mechanics and Engineering*, vol. 388, no. 114194, 2022, https://doi.org/10.1016/j.cma.2021.114194.

[12] W. Zhao, "Artificial Hummingbird Algorithm", 2024, [Online] Available: https://www.mathworks.com/matlabcentral/fileexchange/101133-artificial-hummingbird-algorithm, MATLAB Central File Exchange. [Accessed: May, 9, 2024].

[13] X. Wang, V. Snášel, S. Mirjalili, J.-S. Pan, L. Kong, and A. S. Hisham, "Artificial Protozoa Optimizer (APO): A novel bio-inspired metaheuristic algorithm for engineering optimization", *Knowledge-Based Systems*, Volume 295, 111737, 2024, https://doi.org/10.1016/j.knosys.2024.111737.

[14] W. Xiaopeng, "Artificial Protozoa Optimizer", [Online] Available: https://www.mathworks.com/matlabcentral/fileexchange/162656-artificial-protozoa-optimizer , MATLAB Central File Exchange. [Accessed: May, 9, 2024].

[15] B. Abdollahzadeh, F. S. Gharehchopogh, and S. Mirjalili, "African vultures optimization algorithm: A new nature-inspired metaheuristic algorithm for global optimization problems", *Computers & Industrial Engineering*, Volume 158, 107408, 2021, https://doi.org/10.1016/j.cie.2021.107408.

[16] B. Abdollahzadeh, F. S. Gharehchopogh, and S. Mirjalili, "Artificial Protozoa Optimizer", [Online] Available: https://www.mathworks.com/matlabcentral/fileexchange/94820-african-vultures-optimization-algorithm, MATLAB Central File Exchange. [Accessed: May, 9, 2024].

[17] W. Zhao, L. Wang, Z. Zhang, H. Fan, J. Zhang, S. Mirjalili, N. Khodadadi and Q. Cao, "Electric eel foraging optimization: A new bio-inspired optimizer for engineering applications", *Expert Systems with Applications*, 238, 122200, 2024, https://doi.org/10.1016/j.eswa.2023.122200.

[18] W. Zhao, L. Wang, Z. Zhang, H. Fan, J. Zhang, S. Mirjalili, N. Khodadadi and Q. Cao, "Electric Eel Foraging Optimization", [Online] Available: https://ww2.mathworks.cn/matlabcentral/fileexchange/153461-electric-eel-foraging-optimization-eefo, MATLAB Central File Exchange. [Accessed: May, 9, 2024].

[19] B. Abdollahzadeh, F. S. Gharehchopogh, N. Khodadadi and S. Mirjalili, "Mountain Gazelle Optimizer: A new Nature-inspired Metaheuristic Algorithm for Global Optimization Problems", Advances in Engineering Software, Volume 174, 103282, 2022, https://doi.org/10.1016/j.advengsoft.2022.103282.

[20] W B. Abdollahzadeh, "Mountain Gazelle Optimizer", [Online] Available: https://www.mathworks.com/matlabcentral/fileexchange/118680-mountain-gazelle-optimizer, MATLAB Central File Exchange. [Accessed: May, 9, 2024].

[21] A. Bouaouda, F. A. Hashim, Y. Sayouti and A. G. Hussien, "Pied kingfisher optimizer: a new bio-inspired algorithm for solving numerical optimization and industrial engineering problems", *Neural Computing and Applications*, 2024, https://doi.org/10.1007/s00521-024-09879-5

[22] A. Hussien, "Pied Kingfisher Optimizer (PKO) [Online] Available: (https://www.mathworks.com/matlabcentral/fileexchange/160043-pied-kingfisher-optimizer-pko), MATLAB Central File Exchange. [Accessed: May, 9, 2024].

[23] W. Zhao, L. Wang, Z. Zhang, S. Mirjalili, N. Khodadadi and Q. Ge, "Quadratic Interpolation Optimization (QIO): A new optimization algorithm based on generalized quadratic interpolation and its applications to real-world engineering problems", *Computer Methods in Applied Mechanics and Engineering*, 116446, 2023, https://doi.org/10.1016/j.cma.2023.116446.

[24] W. Zhao, "Quadratic Interpolation Optimization (QIO) [Online] Available: (https://ww2.mathworks.cn/matlabcentral/fileexchange/135627-quadratic-interpolation-optimization-qio), MATLAB Central File Exchange. [Accessed: May, 9, 2024].

[25] H. Chickermane and H. C. Gea, "Structural optimization using a new local approximation method", *International Journal for Numerical Methods in Engineering*, vol. 39, no. 5, pp. 829–846, 1996.

[26] S. Gold and S. Krishnamurty, "Trade-offs in Robust Engineering Design", *Proceedings of the 1997 ASME Design Engineering Technical Conferences*, DETC97/DAC3757, September 14-17, Saramento, California, 1997.

[27] T. Ray and P. Saini, "Engineering design optimization using a swarm with an intelligent information sharing among individuals", *Engineering Optimization*, vol. 33, no. 6, pp. 735-748, 2001.

[28] S. S. Rao, *Engineering optimization: Theory and practice*, 5th ed. Nashville, TN: John Wiley & Sons, 2019.

[29] C. A. C. Coello, "Use of a self-adaptive penalty approach for engineering optimization problems", *Computers in Industry*, vol. 41, no. 2, pp. 113-127, 2000.

[30] H. M. Amir and T. Hasegawa, "Nonlinear mixed-discrete structural optimization"," *Journal of Structural Engineering"*, vol. 115, no. 3, pp. 626–646, 1989

[31] A. H. Gandomi, X. S. Yang and A. H. Alavi, "Cuckoo search algorithm: a metaheuristic approach to solve structural optimization problems", *Engineering with Computers*, 29, pp. 17-35, 2013.

[32] J. S. Arora, *Introduction to Optimum Design*, 2nd ed. San Diego, CA: Academic Press, 2004.

[33] D. Sattar and R. Salim, "A smart metaheuristic algorithm for solving engineering problems," *Eng. Comput.*, vol. 37, no. 3, pp. 2389–2417, 2021.

[34] E. Sandgren, "Nonlinear integer and discrete programming in mechanical design optimization," *J. Mech. Des. N. Y.*, vol. 112, no. 2, pp. 223–229, 1990. https://doi.org/10.1115/1.2912596

[35] B. D. Youn and K. K. Choi, “A new response surface methodology for reliability-based design optimization,” *Comput. Struct.*, vol. 82, no. 2–3, pp. 241–256, 2004.

[36] T.-H. Kim, I. Maruta, and T. Sugie, “A simple and efficient constrained particle swarm optimization and its application to engineering design problems,” *Proc Inst Mech Eng Part C,* vol. 224, no. 2, pp. 389–400, 2010.