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Research Article

Effect of teaching with mathematical modeling on students' mathematics achievements: A meta-analysis

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Abstract: Considering its relationships with real life and mathematics, mathematical modeling is one of the most effective ways of learning and teaching mathematics. In this study, the effect of teaching with mathematical modeling on students' mathematics achievement was examined using the meta-analysis method. An effect size of 51 was achieved with 45 studies that met the inclusion criteria. Random effects model and Hedges's *g* value were used to calculate effect sizes, and the source of heterogeneity was tried to be determined through moderator analyses. Possible publication bias was examined with the funnel plot, Rosenthal's fail-safe N, Orwin's fail-safe N, Egger's linear regression analysis, Begg and Mazumdar's rank correlation, and Duval and Tweedie's trim-and-fill methods, and no findings of publication bias were found. According to the effect size results showed that the effect of teaching with mathematical modeling on students' mathematics achievement is high level (g=0.845, p<.001, 95% CI:0.712-0.978). In addition, the effect sizes show a heterogeneous distribution (Q=174.533, df=50, p<.001). While the effect of teaching with mathematical modeling on students' mathematics achievement differed statistically according to the moderators of research design (Q_B=11.894, df=1, p<.05) and the contents of mathematical modeling (i.e., learning area/subject) (Q_B=19.941, df=6, p<.05), it did not differ statistically according to the moderators of publication type (Q_B=3.666, df=2, p>.05), level of education (Q_B=3.497, df=3, p>.05), or implementation period (Q_B=3.200, df=2, p>.05). Some suggestions are presented at the end of the study for researchers and practitioners regarding teaching with mathematical modeling.

1. INTRODUCTION

One of the core issues that researchers focus on in mathematics education is problem-solving skills, which are considered to be among the crucial 21st-century skills of today's world (Lesh & Doer, 2003; Lesh & Lehrer, 2003; Partnership for 21st Century Skills, 2008; Pólya, 1954; Sevinç, 2022). Problem-solving processes centered solely on comprehending mathematical concepts are highly criticized because they cause mathematical concepts to remain abstract, unintelligible, and irrelevant to everyday life, resulting in students being unprepared for the real world. In line with these criticisms, mathematical modeling was developed based on the question of what might be needed beyond mathematical ideas in solving daily problems,

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moving students and teachers beyond the standard books, tests, and teaching practices used in schools (Lesh *et al.*, 1983; Lesh & Doer, 2003). Mathematical modeling is an approach for solving real-life problems with mathematical expressions. In this process, real-world problems are associated with abstract mathematical concepts (Greefrath *et al.*, 2022; Siller *et al.*, 2023; Stillman *et al.*, 2020). Unlike traditional problem-solving processes, which utilize mechanical solutions without incorporating cognitive or meta-cognitive processes (Schoenfeld, 1992), in problem-solving processes based on mathematical modeling, students may encounter diverse questions, issues, conflicts, solutions, and revisions (English & Doerr, 2004; Lesh & Sriraman, 2005; Mousoulides *et al.*, 2008). The problems addressed in such processes are referred to as model-eliciting activities (Lesh & English, 2005; Lesh & Sriraman, 2005). Mathematical modeling aims to provide a deep understanding of mathematics and improve students' skills. In other words, mathematical modeling is both a learning tool and a powerful approach that helps students understand mathematics more effectively and eloquently (Kaiser *et al.*, 2006; Lesh *et al.*, 2008; Lesh & Doer, 2003).

Mathematical modeling can play significant potential roles in increasing students' mathematics achievement (Blum, 2011). In the modeling process, students mathematicize real-life problems using mathematical symbols, allowing them to improve their skills of using mathematical language (DeBay, 2013; Doruk, 2010; Kurtuluş Kayan, 2019; Sandalcı, 2013). Researchers have shown that mathematical modeling equips students with the ability to approach more complex problems and improve their overall mathematics achievement by deepening their understanding of mathematical concepts and developing their thinking skills (English & Watters, 2005; Freeman, 2014; Lesh *et al.*, 2008; Sokolowski; 2015; Zihar, 2018). Thanks to mathematical modeling, abstract concepts are transformed into concrete ones, and mathematical concepts acquire a deeper meaning in students' minds. With this deeper meaning, mathematics courses become more appealing; thus, students are more motivated to take mathematics courses and actively participate in them. It was previously shown that the use of mathematical modeling in classrooms fostered positive attitudes and increased motivation, influencing mathematics achievement (Aktaş, 2019; Betanga, 2018; Novak *et al.*, 2018; Wethall, 2011; Yıldırım & Işık, 2015). Moreover, research has also shown that mathematical modeling has the potential to improve skills such as creativity (İlhan, 2021; Lu & Kaiser, 2022) and problem-solving (DeBay, 2013; Kertil, 2008). Students learn to think about and apply different solutions to real-life problems in various ways. In this process, they propose creative solutions. They also work in groups, which improves not only their communication and cooperation skills but also their sense of duty allocation and responsibility (Yıldırım & Işık, 2015). The necessity of working together in the modeling process can also be regarded as a potential way in which mathematical modeling improves mathematics achievement.

Based on the potential roles of mathematical modeling, it is seen that mathematical modeling is included in the mathematics curriculum of various countries (Australian Curriculum, Assessment and Reporting Authority [ACARA], 2017; Kultusministerkonferenz [KMK], 2003; Ministry of National Education [MoNE], 2024; National Council of Teachers of Mathematics [NCTM], 2014). In addition, mathematical modeling has a very important role in mathematics achievements, which is one of the main fields in international comparative exams such as the Programme for International Student Assessment (PISA) and Trends in International Mathematics and Science Study (TIMMS). The use of mathematical modeling is necessary in solving the questions related to daily life presented in these exams (Organisation for Economic Co-operation and Development [OECD], 2018). The PISA 2021 report highlighted the fact that mathematical modeling is the cornerstone of the PISA mathematics structure. The modeling process, which consists of steps of formulating, using, interpreting, and evaluating, is at the center of PISA's understanding of mathematical literacy (OECD, 2018).

Research has been conducted to synthesize national and international studies on mathematical modeling. These research syntheses were conducted as systematic literature reviews (Aztekin

& Taşpınar Şener, 2015; Birgin & Öztürk, 2021; Çevikbaş *et al.*, 2022; Yenilmez & Yıldız, 2019) and bibliometric analyses. Mathematical modeling has been used in mathematics education for decades (Lesh *et al.*, 1983). In recent years, many experimental studies have different research design (pre-experimental, quasi-experimental) have been conducted on the effect of mathematical modeling on mathematics achievement (Aztekin & Taşpınar Şener, 2015; Sokolowski, 2015). The effects of mathematical modeling on mathematics achievement (Armutcu & Bal, 2022; Aydoğdu & Tutak, 2018; Bakırcı, 2016; Cinislioğlu, 2017; Delikanlı, 2019; Dışbudak, 2014; Ellington, 2005; Kaya, 2019; Kurt, 2019; Kurtuluş Kayan, 2019; Muşlu & Çiltaş, 2016; Pazarcı Çelenk, 2019; Perk, 2019; Tezer & Cumhur, 2017; Yıldırım & Işık, 2015; Zihar, 2018), mathematics performance (Karacı Yaşa & Karataş, 2018), and learning quality (Kabadaş & Yavuz Mumcu, 2022) are also among the topics studied. Despite this increase in the number of experimental studies on mathematical modeling, syntheses of experimental research investigating the effect of mathematical modeling on mathematics achievement remain very limited (Sokolowski, 2015; Uysal, 2021). Therefore, an extensive meta-analysis study is required, constituting the starting point of the present research paper. Because of the results of these prior meta-analysis studies do not seem to be consistent with each other. There is a need for an updated meta-analysis study for various reasons, such as the fact that Sokolowski (2015) included only 13 studies published as articles between 2000 and 2013, a certain period of time has elapsed since that publication, and the number of related studies has increased in recent years. Furthermore, Uysal's (2021) study covered different disciplinary areas and did not specifically address mathematics achievement, and the results of these meta-analysis studies and other independent studies are not compatible with each other, differing according to many variables. Considering all these points, it is necessary to elucidate the effect of mathematical modeling on students' mathematics achievement, determine the direction and significance of that effect, and examine it according to various variables. In this context, it is thought that this meta-analysis study will provide ideas to teachers, researchers, curriculum developers, and politicians working in the field of mathematics education. Accordingly, the following questions were pursued in this study:

1. What is the effect of teaching with mathematical modeling on students' mathematics achievement?
2. Does the effect of teaching with mathematical modeling on students' mathematics achievement show significant differences according to the type of publication, level of education, research design, learning area/subject, or implementation period?

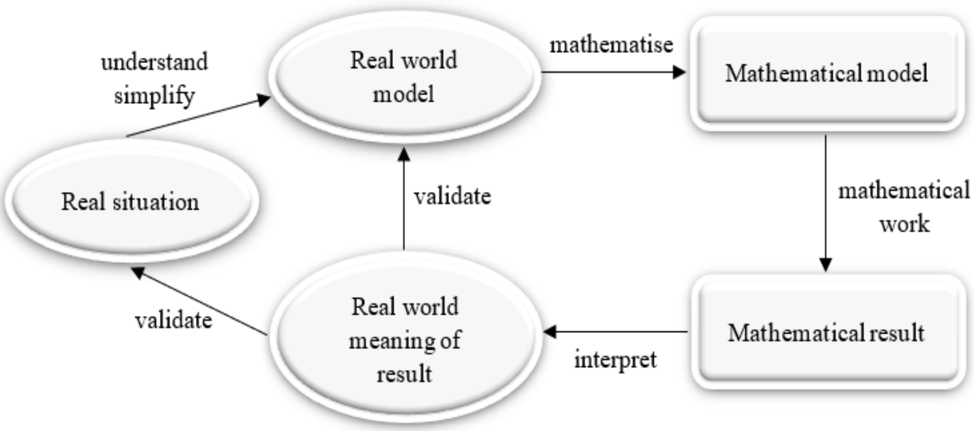
1.1. Mathematical Modeling

There are many definitions of mathematical modeling and some researchers have noted that there is no consensus on what mathematical modeling is (Altun, 2020; Aztekin & Taşpınar Şener, 2015; Tutak & Güder, 2014). Current definitions and approaches have emphasized the abstraction of situations and problems from everyday life (English, 2006; Haines & Crouch, 2007; Kertil, 2008; Lesh & Doerr, 2003; Niss & Blum, 2020). It is essential to understand the difference between a model and modeling to understand what mathematical modeling is. Mathematical models sometimes consist of abstract mathematical expressions such as equations, inequalities, and functions that show the relationship between two variables, and sometimes of concrete objects such as base ten blocks, fraction bars, and algebra tiles used to better understand mathematical concepts and operations (Crillo *et al.*, 2016). Mathematical modeling involves the process of expressing real-life problems through mathematical representations (Borromeo Ferri, 2006; NCTM, 1989). By one definition, mathematical modeling is a process that involves identifying, explaining, and conceptualizing existing situations and defining and interpreting complex systems (Sriraman & Lesh, 2006).

Mathematical modeling has a wide range of applications since it is utilized in solving problems encountered in biology, education, engineering, economics, and medicine in addition to pure and applied mathematics. Along these lines, various perspectives that highlight the power and flexibility of mathematical modeling have been introduced (Berry & Houston, 1995; Blum & Niss, 1989; Galbraith, 2012; Kaiser & Sriraman, 2006; Sevinç, 2022; Sriraman & Lesh, 2006). The starting point for the research that introduced these modeling perspectives constituted discussions on how there is no single epistemically homogeneous understanding of the topic (Borromeo Ferri & Lesh, 2013; Kaiser & Sriraman, 2006; Sevinç, 2022). The focus of these perspectives is the real-life problems encountered in natural settings rather than problems from textbooks or standardized tests, suggested based on research conducted on concept development (Lesh & English, 2005; Sriraman & Lesh, 2006). Modeling perspectives may differ in terms of pedagogical, psychological, and subject- and science-related goals and their emancipatory, pragmatic, and scientific assumptions (Sevinç, 2022). For example, Blum and Niss (1989) proposed five main types of approaches within mathematical modeling: developmental approaches, formal approaches, approaches focused on critical skills, approaches focused on usability, and approaches to enable learning. Modeling approaches were similarly categorized into four groups by Berry and Houston (1995), including experimental, theoretical, dimensional analysis, and simulation approaches. Kaiser and Sriraman (2006) classified mathematical modeling approaches under the following headings: realistic/applied modeling, conceptual/contextual modeling, educational modeling, socio-critical modeling, epistemological or theoretical modeling, and cognitive modeling. It can be seen that the current perspectives essentially differ according to whether mathematical modeling is viewed as an aim or as a tool in teaching mathematics (Blum & Niss, 1991; Galbraith, 2012).

Mathematical modelling is applied as a process in teaching environments. The literature contains both simple and simply defined modeling processes as well as complex modeling processes (Berry & Houston, 1995; Kaiser & Stender, 2013; Kapur, 1998; Maaß, 2010; Müller & Wittmann, 1984; Pollak, 2007). In the first studies in which mathematical modeling was used, basic steps consisting of model building, data processing, and interpretation were at the forefront, but subsequently the modeling components of real-life situations, mathematical models, mathematical solutions, and transfers to real life were also taken into consideration and cognitive activities were given importance (Hıdıroğlu & Bukova Güzel, 2013). On the simplest level, in the mathematical cycle, real-life problems are translated into the language of mathematics; that is, they are formulated and interpreted in the world of mathematics and then transferred to real life (Berry & Houston, 1995). Figure 1 shows an example of a mathematical modeling cycle (Kaiser & Stender, 2013, p. 279).

Figure 1. *Modeling cycle (Kaiser & Stender, 2013, p. 279).*



In general terms, the first step in this process is to transform the real-life problem into a mathematical problem. In this step, a real-life problem is understood, simplified, and presented mathematically. Problems concerning mathematical modeling must have a connection with daily life. These problems should be meaningful and valuable for students (Kaiser & Stender, 2013). In this sense, problems that require the use of the mathematical modeling process are not routine problems with a single answer. The main goal of this process is not to comprehend mathematical knowledge, but to understand real life and make connections between real life and mathematics (Lesh & Doerr, 2003). The second step is to solve the problem translated to mathematical language by using appropriate methods and tools. The third stage involves interpreting the mathematical solution in the context of a real-world problem. The final step involves reflecting the solution to the real-world problem and re-developing the model if necessary (Ministry of Education Singapore [MoES], 2013). In short, this process consists of producing a mathematical solution and adapting it to real life (Bukova Güzel *et al.*, 2021). The modeling process involves determining the variables in the problem, creating models from the variables, performing operations through the models, and interpreting the results (Common Core State Standards Initiative [CCSSI], 2010). However, it is important to remember that mathematical modeling processes are not always simple and straightforward processes that are easily understood. They might also be highly complex processes wherein different steps are intertwined.

1.2. The Rationale of Moderator Variables

Studies on mathematical modeling were still in the stage of preliminary research until the 1990s, and after 1990, they achieved uniqueness, diversified, and strengthened (Stillman *et al.*, 2015). The effect of mathematical modeling practices on cognitive outcomes is also a popular research topic. There are studies in the literature showing that mathematical modeling has a significant and positive effect on mathematics achievement (Aktaş, 2019; Bakırcı, 2016; Birinci Kara, 2020; Cinislioğlu, 2017; Czocher, 2017; Çelikkol, 2016; Çiltaş, 2011; DeBay, 2013; Delikanlı, 2019; Kertil, 2008; Kurt, 2019; Kurtuluş Kayan, 2019; Özturan Sağırlı, 2010; Yıldırım & Işık, 2015; Yılmaz, 2015; Zihar, 2018) as well as studies that show that the results are not significant (Büyükadıgüzel, 2019; Dışbudak, 2014; Freeman, 2014; Işık, 2016; İlhan, 2021; Nam, 2018; Pazarcı Çelenk, 2019; Perk, 2019; Tuluk, 2007; Türksever, 2019). These discrepancy findings may be due to factors such as differences in research methods, level of education, and contextual variables. The results of these studies have created the need to reach a solid conclusion by generalizing through synthesis about the effect of mathematical modeling on mathematics achievement. In this context, combining the results of experimental studies conducted so far can eliminate this uncertainty about the effect of mathematical modeling on academic achievement in mathematics.

It is possible to determine the factors affecting the relationship between mathematical modeling and mathematics achievement by examining potential moderator variables that may explain the differences in studies. It was determined that the moderator variables of publication type, level of education, research design, subject area, and implementation period were common in all studies included in this meta-analysis. For example, many studies have been conducted in different publication types such as master (Yazır, 2015) and Phd theses (DeBay, 2013), articles (Doerr *et al.*, 2014) on the potential role of modeling in mathematics education. These studies were conducted at primary (Kertil, 2008), middle (Kurt, 2019), high school (DeBay, 2013) and university (Nourallah & Farzad, 2012) levels; pre (Çelikkol, 2016) and quasi (Doerr *et al.*, 2014) experimental designs. Research on mathematical modeling has been conducted for different learning areas and topics such as problem-solving (Kim & Kim, 2010; Mousoulides *et al.*, 2008; Nourallah & Farzad, 2012; Voskoglou & Buckley, 2012), rational functions (Betanga, 2018), discrete mathematics (Greefrath *et al.*, 2022), linear and quadratic functions (Freeman, 2014), exponential growth (Siller *et al.*, 2023), and algebra (Ellington, 2005). On the other hand, the implementation period of these mentioned studies may be less than one period, one period, or more than one period (Doerr *et al.*, 2014; Kabadaş & Yavuz Mumcu, 2022; Karacı Yaşa & Karataş, 2018). Studies examining the effects of mathematical modeling on mathematics achievement differ according to specific variables such as publication type, level of education, research design, subject area, and implementation period. In order to reveal the reasons for this differentiation, combining independent studies conducted with quantitative methods with the meta-analysis method and synthesizing their results will facilitate the observation of the general results. Considering all these points, it is necessary to elucidate the effect of mathematical modeling on students' mathematics achievement, determine the direction and significance of that effect, and examine it according to the mentioned moderator variables. In this context, it is thought that this meta-analysis study will provide ideas to teachers, researchers, curriculum developers, and politicians working in the field of mathematics education.

2. METHOD

Some studies on mathematical modeling have focused on students' mathematics achievement. These studies differ from each other due to the different independent variables they include and their results do not seem to be consistent with each other. Meta-analysis is a method that combines and calculates many statistical findings using different techniques (Borenstein *et al.*, 2021). The present meta-analysis aims to determine the effect of teaching with mathematical modeling on students' mathematics achievement. In addition, the differences between studies were examined according to the moderating factors of publication type, level of education, research design, learning area/subject, and implementation period. In this study, Moher *et al*.'s (2009) PRISMA guidelines and the American Psychological Association (APA) Meta-Analysis Reporting Standards (MARS) modified from Cooper (2010) were followed throughout the meta-analysis process.

2.1. Inclusion and Exclusion Criteria

The inclusion criteria of this meta-analysis were as follows: (1) studies must investigate the effect of teaching with mathematical modeling on students' mathematics achievement; (2) studies must be full-text studies open to access and published between 2000 and 2022; (3) studies must be written in English or Turkish; (4) studies must be published as master's theses, doctoral dissertations, or articles in peer-reviewed journals; (5) studies must use an experimental design; and (6) studies must involve data that can be used to calculate the effect size value.

Since studies on mathematical modeling began growing more detailed in the 2000s, studies published between 2000 and 2022 were included in this meta-analysis. Publications based on these were included in the meta-analysis since they contained more detailed data, while other publications were excluded. Not all relevant proceedings and projects were included in the meta-analysis since problems such as lack of data were frequently encountered.

2.2. Data Sources and Keywords

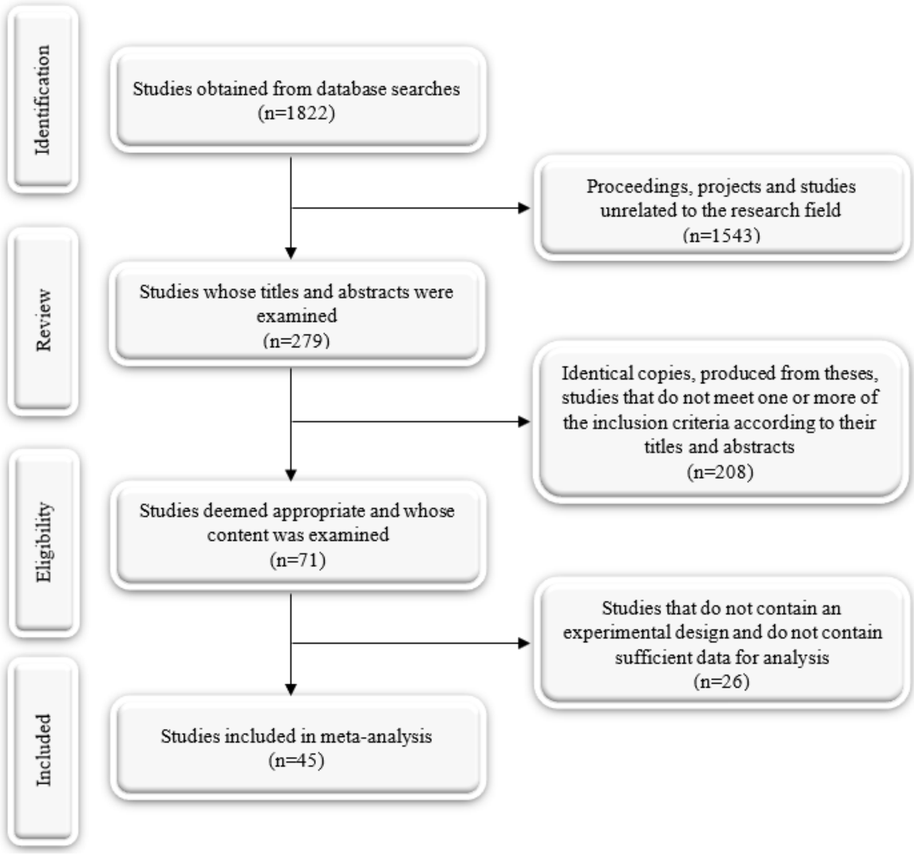
Different search sources were used to find as many studies as possible related to the research problems. The searches were conducted in the ProQuest Dissertations & Theses Global, Web of Science (WOS), Scopus, Education Resources Information Center (ERIC), the National Thesis Center of the Council of Higher Education, and DergiPark Academic in Türkiye databases and the Google Scholar search engine. Since the characteristics of each of the search sources were different, different combinations of keywords with Boolean operators were used. Searches were conducted in the databases and the search engine using the keywords formatted as ("math* model*" AND achievement). All searches were completed on 30.01.2023.

2.3. Search Process

No standardized search was used for any of the databases or the search engine. Some of the searches were conducted using different combinations and sometimes with Boolean operators. The aim of conducting the searches in this way was to find as many studies related to the research topic as possible and not overlook any studies related to the research topic.

During the search process, studies, proceedings, and projects that did not meet one or more of the inclusion criteria, were present as duplicates in more than one database, were published as articles based on theses, were not full-access or whose full text could not be accessed, or did not contain sufficient data to calculate the effect size were excluded. A detailed description of the search process is important to maintain the transparency and clarity of the research. The PRISMA flow chart of the screening process is shown in Figure 2.

Figure 2. *PRISMA flow chart of study screening process (Moher et al., 2009).*



As can be seen in Figure 2, 1822 studies were initially found as a result of the searches performed in the databases with the keywords given above. First, proceedings, projects, and 1543 studies that were not related to the research topic were excluded. After that exclusion process, the remaining 279 studies were examined according to their titles and abstracts; 208 studies were excluded due to being duplicates, being based on theses, not offering full access, or not meeting one or more of the inclusion criteria. The contents of the remaining 71 studies that were considered acceptable for analysis according to their titles and abstracts were examined in detail, especially in terms of their purposes, problems, methods, and results. In the end, 45 studies were included in the meta-analysis after 26 studies were excluded for not involving an experimental design or not having sufficient data for calculating effect size.

2.4. Coding Process and Reliability

In meta-analyses, studies examined using a coding form and explaining the coding process in detail are important for reliability (Card, 2012). A coding form was prepared by the researchers to be used in the coding process. This form involved information about the characteristics of the studies: (1) study number, (2) author(s) of the study, (3) year of publication, (4) title of the study, (5) type of publication, (6) research design, (7) level of education, (8) content of the mathematical modeling (learning area/subject), (9) duration of implementation, and (10) sample numbers, arithmetic means, standard deviations of the experimental/control groups, and other data to calculate the effect size.

For the present study, coding was carried out by two individuals, one of whom was a researcher and the other an academic who had knowledge on meta-analysis. The coders first read the titles and abstracts of the studies they identified, and then they examined the contents of the studies. Each of the studies analyzed using this method was recorded separately by the coders using this form. Inconsistencies in studies in which there was disagreement were resolved by mutual agreement. The coding process continued until there was full agreement between the coders.

Different methods such as percent agreement and the correlation coefficient are used to determine inter-coder reliability (Jonsson & Svingby, 2007). The coding reliability of this study was first calculated using the percentage of inter-coder agreement and then Cohen's kappa coefficient since it incorporates coincidental agreement. Of the 71 studies whose contents were analyzed, there was disagreement about 8 of them and consensus for 63. It was decided to exclude 5 of the 8 studies about which the coders disagreed because they did not contain the necessary data for analysis and to include the remaining 3 in the meta-analysis.

The inter-coder agreement was found to be 88.73% as a result of the calculation according to the number of studies with consensus and disagreement (Miles & Huberman, 1994). Similarly, Cohen's kappa coefficient was calculated using IBM SPSS Statistics 26 and found to be .749. A Cohen kappa coefficient greater than .600 (.749) indicates a sufficient level of agreement between coders (Wood, 2007). This is also supported by the percentage of agreement calculated according to the formula proposed by Miles and Huberman (1994).

2.5. Characteristics of the Included Studies

Six of 45 studies (Büyükadıgüzel, 2019; Doerr *et al.*, 2014; Doruk, 2010; Emlek, 2007; Johnson & Galluzzo, 2014; Perk, 2019) were each evaluated as two independent studies with consideration of their numbers of experimental/control groups and applications. As a result, 51 studies were included in the meta-analysis. The characteristics of the 51 independent studies included in the meta-analysis are given in Table 1. The publication year of these studies varied between 2005 and 2022, despite 2000-2022 having been taken as the range for this variable. It was seen that the number of studies had increased in recent years and studies were concentrated in some years. The studies with experimental designs were categorized as true, quasi, and pre-experimental experimental designs, but no studies with true experimental designs were found. The levels of education of the participants ranged from 3rd grade to university. Considering that the number of groups would be too large when classified according to individual grade levels, the participants in the studies were grouped according to their levels of education as primary school (grades 1-4), middle school (grades 5-8), high school (grades 9-12), and university. In some of the studies (Armutcu & Bal, 2022; Koç, 2022), the subject, learning domain, or outcome related to mathematical modeling was not clearly stated. Those studies' subject areas were coded as "problems" since they addressed mathematical modeling problems and were not associated with any specific learning domain/subject/outcome. It was seen that the curricula of countries changed over time and the same subjects or outcomes could be addressed using different learning areas. Mathematical modeling contents were coded as numbers and operations, geometry and measurement, analysis, algebra, differential equations, mixed (i.e., more than one learning area/subject), and problems, considering the students' levels of education in parallel with the content standards specified by the NCTM (2000). The implementation periods of the studies were expressed in units such as years, months, weeks, semesters, sessions, lesson hours, and minutes. Numerical equivalents could not be established for times of the same unit. To decrease the group number of time units and because the approximate duration of 1 semester at different levels of education is known, implementation periods were classified into 3 groups as less than 1 semester, 1 semester, and more than 1 semester. Although most studies had small samples, the sample sizes of the studies varied between 12 and 502. The total sample size of all combined studies was 3105.

Table 1. *Characteristics of the studies included in the meta-analysis.*

ID	Author(s), year	Publication type	Research design	Level of education	Learning area/subject	Implementation period
1	Aktaş, 2019	Master	Quasi	Middle school	Numbers & operations	Less
2	Armutcu & Bal, 2022	Article	Quasi	Middle school	Problems	Less
3	Bakırcı, 2016	Master	Quasi	Middle school	Numbers & operations	Less
4	Betanga, 2018	Phd	Quasi	University	Algebra	Less
5	Birinci Kara, 2020	Master	Quasi	Middle school	Geometry & measurement	Less
6	Büyükadıgüzel, 2019_1	Master	Quasi	Middle school	Mixed	Less
7	Büyükadıgüzel, 2019_2	Master	Quasi	Middle school	Mixed	Less
8	Cinislioğlu, 2017	Master	Quasi	Middle school	Algebra	Less
9	Czocher, 2017	Article	Quasi	University	Differential equations	Semester
10	Çelikkol, 2016	Master	Pre	Middle school	Mixed	Less
11	Çiltaş, 2011	Phd	Quasi	University	Analysis	Semester
12	DeBay, 2013	Phd	Pre	High school	Mixed	More
13	Delikanlı, 2019	Master	Quasi	Primary school	Numbers & operations	Less
14	Dışbudak, 2014	Master	Quasi	Middle school	Mixed	Less
15	Doerr *et al.*, 2014_1	Article	Quasi	University	Analysis	More
16	Doerr *et al.*, 2014_2	Article	Quasi	University	Analysis	More
17	Doruk, 2010_1	Phd	Quasi	Middle school	Mixed	Semester
18	Doruk, 2010_2	Phd	Quasi	Middle school	Mixed	Semester
19	Ellington, 2005	Article	Quasi	University	Algebra	Semester
20	Emlek, 2007_1	Master	Quasi	High school	Analysis	Less
21	Emlek, 2007_2	Master	Quasi	University	Analysis	Less
22	Ergene, 2019	Phd	Pre	University	Analysis	Less
23	Erol, 2015	Phd	Quasi	High school	Algebra	Less
24	Freeman, 2014	Phd	Quasi	University	Algebra	Less
25	Işık, 2016	Phd	Quasi	Primary school	Numbers & operations	Less
26	İlhan, 2021	Article	Quasi	Primary school	Geometry & measurement	Less
27	Johnson & Galluzzo., 2014_1	Article	Quasi	University	Analysis	Semester
28	Johnson & Galluzzo., 2014_2	Article	Quasi	University	Analysis	Semester
29	Kabadaş & Y. Mumcu, 2022	Article	Quasi	Middle school	Algebra	Less
30	Karabörk, 2016	Master	Quasi	Middle school	Geometry & measurement	Less
31	Karacı Yaşa & Karataş, 2018	Article	Pre	University	Mixed	Semester
32	Kaya, 2019	Master	Quasi	Middle school	Numbers & operations	Less
33	Kertil, 2008	Master	Pre	Primary school	Mixed	Less
34	Koç, 2022	Master	Pre	University	Problems	Less
35	Kurt, 2019	Master	Quasi	Middle school	Geometry & measurement	Less
36	Kurtuluş Kayan, 2019	Master	Quasi	Middle school	Numbers & operations	Less
37	Nam, 2018	Master	Quasi	Middle school	Algebra	Less
38	Nourallah & Farzad, 2012	Article	Quasi	University	Mixed	Semester
39	Özer Demir, 2022	Master	Quasi	Middle school	Mixed	Less
40	Özturan Sağırlı, 2010	Phd	Quasi	High school	Analysis	Less
41	Pazarcı Çelenk, 2019	Master	Quasi	Middle school	Numbers & operations	Less
42	Perk, 2019_1	Master	Quasi	High school	Algebra	Less
43	Perk, 2019_2	Master	Quasi	High school	Algebra	Less
44	Sandalcı, 2013	Master	Quasi	Middle school	Algebra	Less
45	Tezer & Cumhur, 2017	Article	Pre	Middle school	Geometry & measurement	Less
46	Tuluk, 2007	Phd	Quasi	University	Algebra	Less
47	Türksever, 2019	Master	Quasi	Middle school	Algebra	Less
48	Yazır, 2015	Master	Quasi	High school	Algebra	Less
49	Yıldırım & Işık, 2015	Article	Quasi	Middle school	Geometry & measurement	Less
50	Yılmaz, 2015	Master	Pre	University	Analysis	Less
51	Zihar, 2018	Master	Pre	Middle school	Numbers & operations	Less

*Master: master's thesis, Phd: doctoral dissertation, Pre: pre-experimental research design, Quasi: quasi-experimental research design, Less: less than a semester, Semester: a semester, More: more than a semester.

2.6. Calculation of the Effect Sizes

A free trial version of the Comprehensive Meta-Analysis (CMA) program was used for publication bias control, calculation of effect size values, heterogeneity testing, and moderator analysis. This program was chosen because it can combine different categories of data from studies and perform calculations easily. In this study, effect size values based on standardized mean difference were used because they provide comparable values between studies. As stated in the coding form, the necessary and sufficient data obtained from the studies (experimental/control groups means, means difference, standard deviation, sample size, variance, correlation, *p* and *t* values, moderator variables, etc.) were meticulously recorded in the CMA program, and all calculations were performed on this program. Additionally, all data were interpreted at the .05 significance level.

Effect sizes of studies with a sample size smaller than 20 included in meta-analysis can be calculated using Hedges's *g* instead of Cohen's *d* (Borenstein *et al.*, 2010). In this study, Hedges's *g* was preferred since there were studies with sample sizes smaller than 20 (Çelikkol, 2016; Koç, 2022).

There are many classifications of effect size and significance. In this study, the classification of Thalheimer and Cook (2002) was used because it is more detailed compared to other classifications and can also be used with Hedges's *g*. The ranges and significance classifications for Hedges's *g* are as follows: $0.00 \leq g < 0.15$, trivial; $0.15 \leq g < 0.40$, low; $0.40 \leq g < 0.75$, moderate; $0.75 \leq g < 1.10$, high; $1.10 \leq g < 1.45$, very high; $1.45 \leq g$, excellent.

2.7. Selection of the Model

The fixed-effect model and random-effects model are two models used to combine effect sizes obtained from independent studies (Cooper *et al.*, 2009). In meta-analysis, the mean effect size varies according to the model. The model used in meta-analysis studies of high quality is specified in advance (Borenstein *et al.*, 2021). It is recommended to use the random-effects model for meta-analysis studies (Field & Gillett, 2010). In this study, the mean effect size was calculated according to the random-effects model for reasons such as the populations of different studies not being the same, the use of different data collection tools, the presence of uncontrollable variables that indirectly or directly affected the implementation beyond sampling errors, and the heterogeneous distribution of the effect sizes of the studies. This selection was also supported by heterogeneity test results.

2.8. Heterogeneity

Cochran's *Q* test (Hedges & Olkin, 1985) was used to determine the difference between effect sizes and the I^2 statistic (Higgins *et al.*, 2003) was used to determine the significance. When the significance *p* value is less than .05 and the Cochran *Q* value is greater than the value in the chi-square table (i.e., the critical value), it is concluded that the studies show heterogeneous distribution. An I^2 statistic value above 50% indicates a sufficient level of heterogeneity, while a value above 75% indicates a high level of heterogeneity.

2.9. Publication Bias

Publication bias in meta-analysis is related to the low willingness to publish studies that are not statistically significant or that have no effect in the expected direction. Publication bias means that significant results are more likely to be published than statistically insignificant ones (Petitti, 2000). Published studies cannot represent all studies in a field. This is also called the file-drawer problem. In order to eliminate the publication bias that may occur due to this problem, comprehensive research aimed at accessing all relevant studies should be conducted (Rosenthal, 1979). Different methods are used to determine whether publication bias is present in a meta-analysis study. The publication bias methods used in the current study were as follows: (1) the funnel plot method, (2) Rosenthal's fail-safe N method, (3) Orwin's fail-safe N method, (4) Egger's linear regression analysis, (5) Begg and Mazumdar's rank correlation, and (6) Duval and Tweedie's trim-and-fill method.

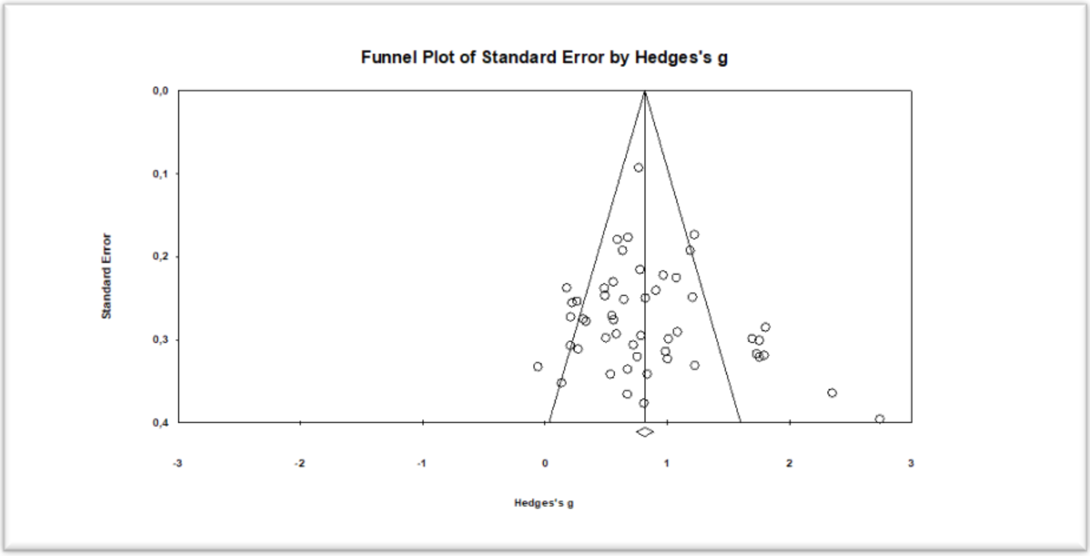
3. FINDINGS

3.1. Publication Bias

The funnel plot method, Rosenthal's fail-safe N method, Orwin's fail-safe N method, Egger's linear regression analysis, Begg and Mazumdar's rank correlation, and finally Duval and Tweedie's trim-and-fill method were used to determine the presence of publication bias.

In Figure 3, it can be said that most of the effect sizes are located within the funnel plot and they are symmetrically distributed. However, it may be argued that the studies on the right side disrupt the symmetrical structure, albeit to a limited extent. On the other hand, it can also be said that the lower left side of the graph is not empty (Borenstein *et al.*, 2021). This subjective interpretation of the funnel plot necessitated using other methods in determining publication bias.

Figure 3. *Funnel plot of standard error by Hedges's g.*



In Rosenthal's analysis, the number of unpublished studies that would need to be added to the analysis for the significance to change is specified. This number, called the fail-safe N (FSN), was found to be 6866 at a significance level of α=.05 in the present meta-analysis. Considering that 51 studies were included, this number is significantly higher than the number of 265 (5.51+10) obtained from the rule of 5k+10 (k represents the number of studies included in the meta-analysis) suggested by Rosenthal (1979). This is interpreted as signifying a low probability of publication bias (Rosenthal, 1991). Similarly, Orwin's FSN number was found to be 4111 for the present meta-analysis. This value represents the number of studies required to reduce Hedges's *g* to a specified value (.01). The fact that this number is also large and it would be difficult to reach 4111 studies supports the conclusion that the publication bias of this study is low.

The null hypothesis of "there is no publication bias" was tested using Egger's linear regression analysis (Egger *et al.*, 1997). According to the obtained *p* value, the result was not significant (*t*=1.007, *df*=49, *p*=.159, *p*>.05). This result is similar to the results of Begg and Mazumdar's rank correlation (tau *b*=0.120, *p*=.211, *p*>.05). Accordingly, it can be said that this study has no publication bias.

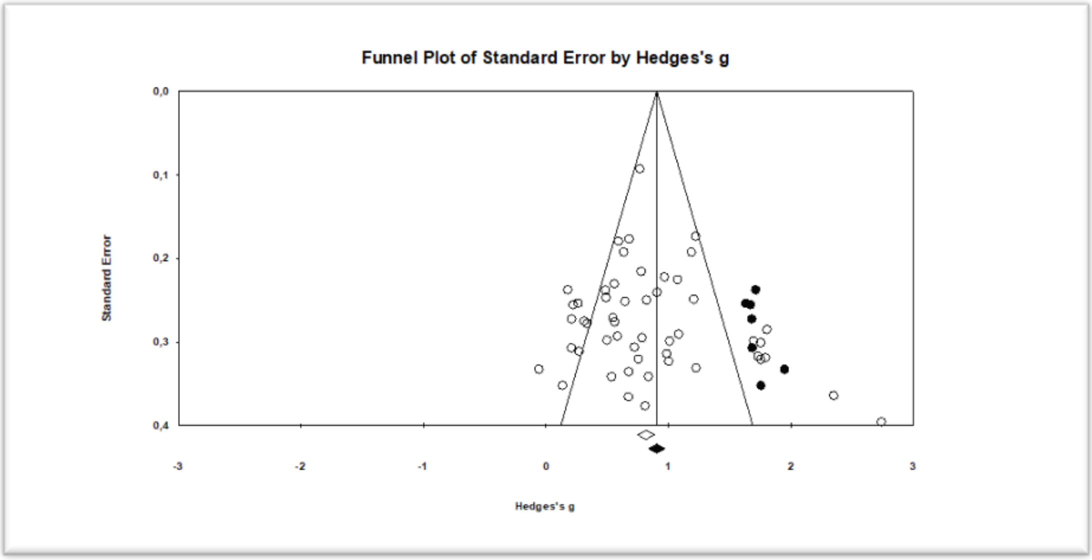
Duval and Tweedie's trim-and-fill method is also recommended for use in situations where there are doubts about publication bias (Yıldırım & Şen, 2021). In addition to the previous findings, findings from Duval and Tweedie's method (Table 2) were analyzed.

Table 2. *Publication bias findings according to Duval and Tweedie's trim-and-fill method.*

	Studies trimmed	$\overline{ES}$	Lower limit	Upper limit	*Q*
Observed values		0.845	0.711	0.977	174.532
Adjusted values	7	0.951	0.814	1.089	239.910

Additionally, the funnel plot obtained according to this method is shown in Figure 4. The mean effect size value obtained according to the random-effects model was 0.845, and the mean effect size value obtained with the 7 reported missing studies was 0.951. If the difference between the observed and adjusted mean effect sizes (11.15%) is "absent or negligible," it is interpreted as signifying no publication bias (Vevea *et al.*, 2019).

Figure 4. *Funnel plot according to Duval and Tweedie's trim-and-fill method.*



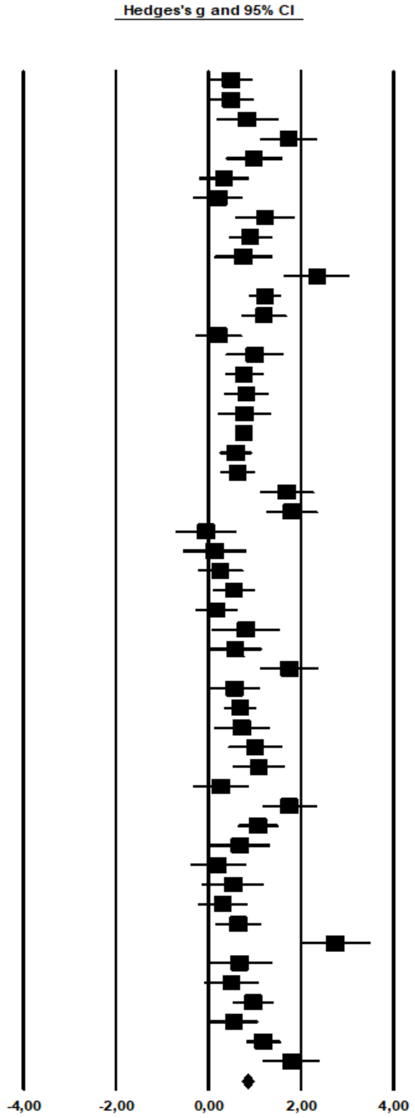
To detect possible publication bias, the findings of all methods described above were independently analyzed. According to those findings, it can be said that there is no publication bias that may affect the validity of this study. In other words, publication bias is not an important factor affecting the mean effect size.

3.2. Mean Effect Size and Heterogeneity Test

The effect size values of the studies range between -0.056 and 2.740 (Figure 5). The highest Hedges's *g* value was obtained for the study of Tezer and Cumhur (2017) at 2.740, and the lowest was obtained for Freeman (2014) at -0.056. The effect size *p* values of the studies by Büyükadıgüzel (2019), Dışbudak (2014), Freeman (2014), İlhan (2021), Işık (2016), Johnson and Galluzzo (2014), Nam (2018), Pazarcı Çelenk (2019), Perk (2019), Tuluk (2007), and Türksever (2019) are not significant (p>.05). There are 50 (98%) positive effect size values and 1 (2%) negative effect size value. According to Thalheimer and Cook's (2002) classification, there are 2 (3.9%) insignificant, 8 (15.7%) low, 15 (29.4%) moderate, 14 (27.5%) high, 4 (7.8%) very high, and 8 (15.7%) excellent effect sizes. Effect sizes of medium and high significance are predominant.

Figure 5. *Forest plot of studies included in meta-analysis.*

Study name	Hedges's g	Standard error	Variance	Lower limit	Upper limit	Z-Value	p-Value
Aktas, 2019	0,486	0,238	0,057	0,019	0,953	2,041	0,041
Armutcu & Bal, 2022	0,491	0,247	0,061	0,007	0,975	1,986	0,047
Bakirci, 2016	0,837	0,342	0,117	0,168	1,506	2,451	0,014
Betanga, 2018	1,732	0,317	0,101	1,110	2,353	5,460	0,000
Birinci Kara, 2020	0,988	0,314	0,099	0,372	1,603	3,142	0,002
Buyukadiguzel, 2019_1	0,337	0,278	0,077	-0,209	0,882	1,210	0,226
Buyukadiguzel, 2019_2	0,211	0,273	0,074	-0,323	0,746	0,776	0,438
Celikkol, 2016	1,225	0,331	0,110	0,576	1,874	3,701	0,000
Ciltas, 2011	0,908	0,241	0,058	0,436	1,380	3,770	0,000
Cinislioglu, 2017	0,755	0,321	0,103	0,126	1,383	2,354	0,019
Czocher, 2017	2,350	0,364	0,133	1,636	3,064	6,453	0,000
DeBay, 2013	1,223	0,174	0,030	0,882	1,564	7,038	0,000
Delikanli, 2019	1,208	0,249	0,062	0,720	1,696	4,849	0,000
Disbudak, 2014	0,223	0,256	0,065	-0,278	0,724	0,871	0,384
Doerr et al., 2014_1	1,002	0,323	0,104	0,369	1,636	3,100	0,002
Doerr et al., 2014_2	0,778	0,216	0,047	0,355	1,201	3,606	0,000
Doruk, 2010_1	0,823	0,250	0,062	0,333	1,313	3,293	0,001
Doruk, 2010_2	0,785	0,295	0,087	0,207	1,363	2,661	0,008
Ellington, 2005	0,768	0,093	0,009	0,585	0,950	8,240	0,000
Emlek, 2007_1	0,592	0,180	0,032	0,240	0,944	3,297	0,001
Emlek, 2007_2	0,636	0,193	0,037	0,258	1,013	3,298	0,001
Ergene, 2019	1,697	0,299	0,089	1,111	2,283	5,675	0,000
Erol, 2015	1,805	0,285	0,081	1,246	2,364	6,325	0,000
Freeman, 2014	-0,056	0,333	0,111	-0,708	0,596	-0,168	0,867
Ilhan, 2021	0,136	0,352	0,124	-0,554	0,827	0,387	0,699
Isik, 2016	0,262	0,254	0,064	-0,235	0,760	1,033	0,302
Johnson & Galluzzo, 2014_1	0,558	0,231	0,053	0,107	1,010	2,422	0,015
Johnson & Galluzzo, 2014_2	0,179	0,238	0,057	-0,287	0,645	0,753	0,451
Kabadas & Yavuz Mumcu, 2022	0,811	0,377	0,142	0,073	1,549	2,153	0,031
Karabork, 2016	0,584	0,293	0,086	0,009	1,158	1,991	0,046
Karaci Yasa & Karatas, 2018	1,754	0,321	0,103	1,125	2,383	5,463	0,000
Kaya, 2019	0,564	0,276	0,076	0,022	1,105	2,040	0,041
Kertil, 2008	0,680	0,177	0,031	0,333	1,027	3,842	0,000
Koc, 2022	0,723	0,306	0,094	0,122	1,323	2,359	0,018
Kurt, 2019	1,010	0,299	0,089	0,424	1,596	3,376	0,001
Kurtulus Kayan, 2019	1,085	0,291	0,085	0,515	1,655	3,730	0,000
Nam, 2018	0,272	0,311	0,097	-0,338	0,883	0,874	0,382
Nourallah & Farzad, 2012	1,753	0,301	0,091	1,164	2,343	5,826	0,000
Ozer Demir, 2022	1,075	0,226	0,051	0,633	1,517	4,766	0,000
Ozturan Sagirli, 2010	0,677	0,336	0,113	0,019	1,334	2,017	0,044
Pazarci Celenk, 2019	0,208	0,307	0,094	-0,394	0,810	0,677	0,498
Perk, 2019_1	0,536	0,342	0,117	-0,134	1,206	1,569	0,117
Perk, 2019_2	0,313	0,275	0,076	-0,227	0,852	1,136	0,256
Sandalci, 2013	0,646	0,252	0,063	0,153	1,139	2,567	0,010
Tezer & Cumhur, 2017	2,740	0,396	0,157	1,964	3,515	6,921	0,000
Tuluk, 2007	0,675	0,366	0,134	-0,042	1,392	1,846	0,065
Turksever, 2019	0,498	0,298	0,089	-0,086	1,082	1,672	0,095
Yazir, 2015	0,968	0,222	0,049	0,532	1,404	4,353	0,000
Yildirim ve Isik, 2015	0,547	0,271	0,073	0,016	1,078	2,019	0,043
Yilmaz, 2015	1,188	0,193	0,037	0,811	1,566	6,165	0,000
Zihar, 2018	1,793	0,319	0,102	1,167	2,418	5,619	0,000
	0,845	0,068	0,005	0,712	0,978	12,456	0,000



As seen in Table 3, according to the fixed-effect model, the mean effect size is 0.816, the standard error is 0.035, the lower limit of the 95% confidence interval is 0.748, and the upper limit is 0.884 (z=23.400, p<.001). According to the random-effects model, the mean effect size is 0.845, the standard error is 0.068, the lower limit of the 95% confidence interval is 0.712, and the upper limit is 0.978 (z=12.456, p<.001). According to the random-effects model, the average effect size value is high according to the classification of Thalheimer and Cook (2002). This finding shows that teaching with mathematical modeling has a strong impact on students' mathematics achievement.

Table 3. *Average effect size and relevant values of the studies included in the meta-analysis, as well as heterogeneity test findings.*

Model	*k*	$\overline{ES}$	*z*	*p*	*SE*	95% CI Lower limit	95% CI Upper limit	*df*	*Q*	*p*	I^2
Fixed	51	0.816	23.400	.000*	0.035	0.748	0.884	50	174.533	.000*	71.352
Random	51	0.845	12.456	.000*	0.068	0.712	0.978				

*p<.001

As can also be seen in Table 3, when the heterogeneity test findings are analyzed, the p value is <.001 and the Q value is 174.533. It is significant that the calculated p value is less than .05 and the Q value is greater than the chi-square table value (67.505) at 50 degrees of freedom and a significance level of .05 ($Q>\chi^2$, $p<.05$). Accordingly, the effect sizes show a heterogeneous distribution. Considering the I^2 statistic value (71.352), this value being close to 75% indicates a sufficient and high degree of heterogeneity (Higgins *et al.*, 2003). It can be said that 71.36% of the variance between effect sizes is due to actual differences in effect sizes and 28.64% is due to error (Li *et al.*, 2021). Moderator analyses are required to explain those differences (Lipsey & Wilson, 2001).

3.3. Moderator Analyses

The mean effect size and heterogeneity test analysis results for the moderators of publication type, level of education, research design, learning area/subject, and implementation period are given in Table 4.

Table 4. *Moderator analysis findings.*

Variable	Category	*k*	$\overline{ES}$	*SE*	95% CI		*z*	*p*	Q_B	*df*	*p*
					Lower limit	Upper limit					
Publication type	Article	13	1.022	0.174	0.681	1.362	5.881	.000*	3.666	2	.160
	Master	27	0.726	0.070	0.589	0.864	10.359	.000*			
	Phd	11	0.966	0.170	0.633	1.299	5.686	.000*			
	Overall	51	0.793	0.061	0.674	0.912	13.060	.000*			
Level of education	Primary school	4	0.605	0.222	0.170	1.039	2.726	.006	3.497	3	.321
	Middle school	24	0.762	0.098	0.570	0.954	7.797	.000*			
	High school	7	0.885	0.180	0.533	1.238	4.920	.000*			
	University	16	1.011	0.132	0.753	1.269	7.675	.000*			
	Overall	51	0.832	0.068	0.698	0.966	12.159	.000*			
Research design	Pre	9	1.391	0.033	1.037	1.745	7.697	.000*	11.894	1	.001
	Quasi	42	0.728	0.065	0.600	0.856	11.145	.000*			
	Overall	51	0.805	0.061	0.684	0.925	13.099	.000*			
Learning area/subject	Analysis	10	0.802	0.121	0.565	1.038	6.641	.000*	19.941	6	.003
	Algebra	13	0.757	0.127	0.507	1.007	5.941	.000*			
	Differential equations	1	2.350	0.364	1.636	3.064	6.453	.000*			
	Geometry & measurement	6	0.975	0.313	0.362	1.589	3.116	.002			
	Mixed	11	0.903	0.150	0.609	1.198	6.007	.000*			
	Problems	2	0.582	0.192	0.205	0.959	3.027	.002			
	Numbers & operations	8	0.794	0.184	0.434	1.154	4.327	.000*			
	Overall	51	0.838	0.063	0.714	0.962	13.220	.000*			
Implementation period	Less	39	0.786	0.079	0.631	0.942	9.908	.000*	3.200	2	.202
	Semester	9	1.044	0.182	0.688	1.401	5.740	.000*			
	More	3	1.029	0.147	0.741	1.317	7.000	.000*			
	Overall	51	0.867	0.065	0.739	0.995	13.301	.000*			

*p<.001, Master: master's thesis, Phd: doctoral dissertation, Pre: pre-experimental research design, Quasi: quasi-experimental research design, Less: less than a semester, Semester: a semester, More: more than a semester.

The heterogeneity test for publication type resulted in a p value of .160 and a Q_B value of 3.666. The fact that the calculated p value is greater than .05 and the Q_B value is lower than the critical value (5.991) indicates that there is no significant difference between the groups ($Q_B<\chi^2$, $p>.05$). This finding supports the findings related to publication bias. The heterogeneity test for level of education resulted in a p value of .321 and a Q_B value of 3.497. The fact that the calculated *p* value is greater than .05 and the Q_B value is lower than the critical value (7.815) indicates that there is no significant difference between the groups ($Q_B<\chi^2$, $p>.05$). Accordingly, it can be said that the moderator of level of education does not affect students' mathematics achievement in teaching with mathematical modeling. The mean effect size values are 0.605, 0.762, 0.885, and 1.011 for primary school, middle school, high school, and university, respectively. Although level of education was not a significant moderator, teaching with mathematical modeling affected the mathematics achievement of primary school students at a moderate level according to Thalheimer and Cook's (2002) classification, while it affected the mathematics achievement of others at a high level. The heterogeneity test for research design resulted in a *p* value of .001 and a Q_B value of 11.894. The fact that the calculated *p* value is lower than .05 and the Q_B value is greater than the critical value (3.841) indicates that there is a significant difference between the groups ($Q_B>\chi^2$, $p<.05$). Therefore, the moderator of research design significantly affects students' mathematics achievement in teaching with mathematical modeling. The mean effect size values are 1.391 and 0.728 for pre and quasi-experimental designs, respectively. Teaching with mathematical modeling with a pre-experimental design had a very high effect on students' achievement according to one classification and a moderate effect according to the other. The heterogeneity test for learning area/subject resulted in a *p* value of .003 and a Q_B value of 19.941. The fact that the calculated *p* value is lower than .05 and the Q_B value is greater than the critical value (12.592) indicates that there is a significant difference between the groups ($Q_B>\chi^2$, $p<.05$). Therefore, the moderator of learning area/subject significantly affects students' mathematics achievement in teaching with mathematical modeling. The mean effect size values are 0.802, 0.757, 2.350, 0.975, 0.903, 0.582, and 0.794 for analysis, algebra, differential equations, geometry and measurement, mixed subjects, problems, and numbers and operations, respectively, as learning areas or subjects. Teaching with mathematical modeling using problems had a moderate effect on students' mathematics achievement according to this classification, while an excellent effect was observed for differential equations and high effects were seen for other learning areas/topics. The heterogeneity test for implementation period resulted in a *p* value of .202 and a Q_B value of 3.200. The fact that the calculated *p* value is greater than .05 and the Q_B value is lower than the critical value (3.200) indicates that there is no significant difference between the groups ($Q_B<\chi^2$, $p>.05$). Accordingly, it can be said that the moderator of implementation period does not affect students' mathematics achievement in teaching with mathematical modeling. The mean effect sizes for durations of less than 1 semester, 1 semester, and more than 1 semester are 0.786, 1.044, and 1.029, respectively. Although implementation period was not a significant moderator, teaching with mathematical modeling had a high effect on students' mathematics achievement in all groups according to this classification.

4. DISCUSSION and CONCLUSION

In line with the aim of this study, the 51 independent studies included in the meta-analysis were examined in detail and it was seen that they had different effect sizes, significance levels, and effect directions. For example, the effect of Freeman's (2014) study was found to be negative and insignificant according to Thalheimer and Cook's (2002) classification. Although the effect directions of the other studies were found to be positive, they had different significance levels according to the aforementioned classification. In this study, the mean effect size was calculated as 0.845 (Hedges's *g*) according to the random-effects model and this value was highly significant according to the classification used. In Sokolowski's (2015) study, the mean effect size of 13 studies on mathematical modeling was found to be 0.690. That value is moderately significant according to the classification used. In an interdisciplinary study (Uysal, 2021), the mean effect size of 23 studies on mathematical modeling was found to be 0.976. This value is highly significant according to the classification used. In this respect, the effect of teaching with mathematical modeling on students' mathematics achievement as calculated in the present study is close to that reported by Sokolowski (2015) and also similar to the value obtained by

Uysal (2021). This study concluded that mathematical modeling has a significant contribution to mathematical achievement. This result may be due to the goals of modeling approaches and the stages in mathematical modeling cycles. Because in mathematical modeling approaches and cycles, emphasis is placed on student-centered learning environments, real-life problems, teaching concepts by making sense of them, and developing thinking skills (Bukova Güzel *et al.*, 2021; Doruk, 2010; Dost, 2019; Kaiser & Sriraman, 2006; Ural, 2018). In a recent study, mathematics teachers and educators who adopted epistemological, pedagogical, applied and pedagogical and conceptual mathematical modeling approaches participated. The participants stated that mathematical modeling makes teaching and learning mathematics easier (Xu *et al.*, 2022).

When the effect sizes of the studies included in this meta-analysis were examined according to the type of publication, it was found that articles and doctoral dissertations reported that teaching with mathematical modeling affected students' mathematics achievement at high levels, while master's theses reported a medium effect. The effect sizes of publication types appear to be close to each other, however. When the effect sizes of the studies were analyzed statistically, no significant difference was found between the groups according to the type of publication. This shows that the effect of teaching with mathematical modeling on students' mathematics achievement is similar regardless of the type of publication.

The studies included in this meta-analysis were also examined according to level of education. The effect sizes of studies in which the sample consisted of primary school students were found to be medium, while the effect sizes of studies on middle school, high school, and university students were found to be high. Heterogeneity test results showed that effect sizes did not differ significantly according to this moderator. In other words, the fact that studies were conducted at different levels of education did not significantly change the effect of teaching with mathematical modeling on students' mathematics achievement. This is likely because mathematical modeling is based on real-life problems and these problems exist at all levels of education (Stillman *et al.*, 2020). However, these results do not coincide with the findings of Uysal (2021) and Sokolowski (2015). Sokolowski (2015) compared the effect of mathematical modeling on students' achievement according to two different levels of education, namely high school and university, and found a significant difference in favor of high school students, which was attributed to the difficulties university students experience in moving beyond their old ways of thinking. Similarly, Uysal (2021) compared the effect of mathematical modeling on students' academic achievement according to three levels of education, namely middle school, high school, and university, and found a significant difference in favor of university students, which was attributed to the new mathematical modeling practices introduced in school courses and to changes in the exam system. Furthermore, in the present study, when the effect of teaching with mathematical modeling on students' mathematics achievement is examined in this respect, it is seen that the effect size increases as the level of education increases. Although this finding is not significant, it can be concluded that teaching with mathematical modeling positively affects students' mathematics achievement in terms of both significance and size. The reason for this may be that mathematical modeling is used to a greater extent as the level of education increases (MoNE, 2018; NCTM, 2014), which leads to a larger effect on achievements. On the other hand, researchers have stated that there is no need to wait until high school to use mathematical modeling and suggested a framework for use in primary school. This framework consists of developing and anticipating, enacting and revisiting stages (Carlson *et al.*, 2016).

The studies included in this meta-analysis were also examined according to research design. The effect sizes of studies with pre-experimental designs were found to be very high and those of studies with quasi-experimental designs were moderate. Heterogeneity test results showed that effect sizes differed significantly according to this moderator. In other words, studies having pre-experimental designs or quasi-experimental designs significantly impacted the effect of teaching with mathematical modeling on students' mathematics achievement.

Considering that the pre-experimental studies included in this meta-analysis were single-group studies, mathematical modeling may have been more effective in those studies than others due to the lack of control of exogenous variables. This finding does not coincide with the study conducted by Uysal (2021), who compared the mean effect sizes of pre and quasi-experimental studies and found no significant difference.

The studies included in this meta-analysis were further examined according to learning area/subject. The effect size of studies on the learning area/subject of "problems" was found to be moderate; the effect sizes of studies on analysis, algebra, geometry and measurement, mixed subjects, and numbers and operations were found to be high; and the effect size of one study on differential equations was found to be excellent. Heterogeneity test results showed that effect sizes differed significantly according to this moderator. In other words, the fact that the studies included in this meta-analysis addressed different learning areas/subjects significantly changed the effect of teaching with mathematical modeling on students' mathematics achievement. When the learning areas/subjects were analyzed, it was seen that this difference, in terms of both significance and size, was caused by the learning area/subject of differential equations. Excluding the study that addressed the learning area/subject of differential equations, the effect sizes of the other studies involving different learning areas/subjects were close to each other in terms of both significance and size. Since differential equations courses are more procedural in nature, the fact that the real-life problems examined in those courses give meaning to the aforementioned operations for students may explain the excellent significance achieved in that particular study (Alnaser & Hoffmeier, 2023; Apkarian *et al.*, 2023). Regardless of whether the study on differential equations is included in the meta-analysis, teaching with mathematical modeling significantly affects students' mathematics achievement in analysis, algebra, geometry and measurement, mixed subjects, problems, and numbers and operations. The findings of this study are similar to those of Sokolowski (2015) in this respect. Sokolowski (2015) compared the effect of mathematical modeling on students' achievement in the learning areas of algebra, analysis, statistics and probability, and geometry and found a significant difference, which was caused by the learning area of statistics and probability with excellent significance.

The studies included in this meta-analysis were examined according to implementation period. The effect sizes of all considered implementation periods were found to be high heterogeneity test results showed that effect sizes did not differ significantly according to this moderator. In other words, the fact that the studies were conducted with different implementation periods did not significantly change the effect of teaching with mathematical modeling on students' mathematics achievement. Therefore, it can be said that the effect of teaching with mathematical modeling on students' mathematics achievement is independent of the implementation period. This may be explained by the possibility that the authors of the studies included here planned and carried out their implementations of teaching with mathematical modeling effectively. Independently of the implementation period, the meticulous execution of all steps of the implementation process may have increased students' mathematics achievement to the same degree (Betanga, 2018; DeBay, 2013; İlhan, 2021). This result is similar to the results obtained by Uysal (2021), who conducted a meta-regression analysis for implementation period and effect size and reported no linear relationship. This means that implementation period was not a significant predictor of effect size. However, the results of this study do not coincide with those of Sokolowski (2015), who compared the effect of mathematical modeling on students' achievement between two groups, one including studies with implementation periods of 1 semester and the other including studies with implementation periods of less than 1 semester. A significant difference was found between the groups in favor of short-term implementations, which was explained by the effect sizes of the experimental and control groups. Despite these results, it should be considered that long-term implementations will facilitate the comprehension and application of mathematical modeling (Mousoulides *et al.*, 2008; Zubi *et al.*, 2019).

4.1. Implications

In this study, it was concluded that the effect of teaching with mathematical modeling on students' mathematics achievement is high. Accordingly, it is recommended to use mathematical modeling in education and training to increase mathematics achievement. The effect of teaching with mathematical modeling on mathematics achievement increases as the level of education increases, although moderator analyses showed that this finding was not statistically significant. Accordingly, it is recommended that teachers at higher levels of education should include more mathematical modeling in their teaching practices. It was furthermore observed that the effect of teaching with mathematical modeling on mathematics achievement in the learning area/subject of differential equations was higher compared to other subjects. Therefore, mathematical modeling can be used especially at the university level in the teaching of areas such as differential equations, which are predominantly computational subject areas. Although it was not a statistically significant finding, the effect of teaching with mathematical modeling on mathematics achievement decreased as the implementation period decreased. Thus, it is suggested to include non-short-term mathematical modeling practices in teaching practices.

4.2. Limitations and Suggestions for Future Studies

Only studies written in English or Turkish were included in the current meta-analysis. This is a limitation of this research. Studies published in different languages could be analyzed with the help of a translation team and included in the analysis. This would help in investigating the effect of teaching with mathematical modeling on students' mathematics achievement more comprehensively. The differences between the studies were explored in terms of the moderators of publication type, research design, level of education, learning area/subject, and implementation period. There are many variables that may explain the differences between studies included in meta-analysis studies. Differences between studies could be further explained by analog ANOVA and meta-regression analyses using different moderators. Although not examined in this study, analyses showed that the reviewed works had in-group differences in some cases. These differences were not analyzed, which is another limitation of the present study. Other moderators could be identified and analyzed in detail, revealing the in-group differences between studies.

Considering that the number of studies on this research topic is increasing day by day, this meta-analysis could be repeated at certain time intervals, changes could be examined, studies could be analyzed according to different moderators, and the results could be updated.

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Declaration of Conflicting Interests and Ethics

The authors declare no conflict of interest. This study complies with research publishing ethics. The scientific and legal responsibility for manuscripts published in IJATE belongs to the authors. This study does not involve humans and/or animals; for this reason, no ethics approval was involved in the writing of this paper.

Contribution of Authors

Veli Ünlü: Investigation, Resources, Visualization, Software, Analysis, and Writing-original draft. **Erhan Ertekin**: Methodology, Resources, Supervision, Critical review, and Validation.

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