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Research Article

Predictive analysis of urban wastewater capacity using ANN and ODE model: A case study

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ABSTRACT

The discharge of urban wastewater represents a significant aspect to be considered in the development and design of water and wastewater treatment projects. In this study, the annual urban wastewater discharge was estimated using an artificial neural network (ANN) and differential equations. In order to achieve this, data pertaining to the recorded wastewater per capita amount for the Kayseri province (located in Turkey) over a 17-year period between 2003 and 2020, the city's population, capacity, number of WWTPs, the amount of daily wastewater discharged per person, and the wastewater treated in WWTPs (y(t)) were collated. As the initial data set was insufficient, it was augmented using the ARIMA model and then normalised. The augmented and normalised data was trained with ANN on two occasions, thus demonstrating the impact of other variables on the y(t) variable. Additionally, mathematical ANN activation functions in the form of a tangent hyperbolic function were proposed for this variable. Subsequently, the arbitrary parameters employed in a linear system comprising differential equations representing the aforementioned five variables were estimated utilising the normalised original data, thereby facilitating the formulation of an Ordinary Differential Equation (ODE) model. The performance of two ANN and ODE models was evaluated on normalized real data, and the results were compared. Consequently, the estimation of the quantity of wastewater with the lowest error rate of 0.00001 MSE among the models incorporating four time-dependent variables as inputs was conducted using the ODE model. The model exhibited an R^2 value of 0.9363 and a MAPE value of 0.0231. The promising estimation results obtained demonstrate the potential utility of this approach for the efficient management of wastewater demand and the protection of valuable water resources.

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INTRODUCTION

Rapid population growth, economic growth and urbanization have brought about serious problems in terms of environment and water [1]. One of the most important challenges in sustaining aquatic environments in the future is the issue of water pollution we face, both current and future, and its prediction. Along with population growth, domestic sewage and agricultural activities have caused pollution in water resources to increase and become a serious problem [2]. Especially in environments with industrial development, more industrial enterprises mean more wastewater generation. Water-related environmental issues are particularly challenging in Kayseri because Kayseri has a developed industry and dense population. Regarding wastewater, analysing and understanding the differences in terms of the daily amount of wastewater discharged per person, the amount of treated water, and the number of people is an important issue in terms of developing the integrated wastewater management plan in the country.

Wastewater management [3] is of significant importance in residential areas for several reasons, encompassing environmental [4], public health [5], and economic aspects [6]. Here are some key points highlighting the importance of wastewater management in residential areas: Public Health Protection, Environmental Protection, Resource Conservation, Water Conservation, Compliance with Regulations, Prevention of Soil and Groundwater Contamination, Infrastructure Maintenance, Quality of Life and Economic Considerations. Several factors can affect wastewater generation in residential areas. The amount and composition of wastewater produced can vary based on a range of factors related to population, lifestyle, infrastructure, and local practices [7, 8, 9]. Municipalities play an important role in managing wastewater to ensure public health, protect the environment and comply with regulatory standards. Some of the activities and responsibilities undertaken by municipalities for wastewater management are: Wastewater Collection [10], Treatment Plant Operation [11], Infrastructure Maintenance [12], Stormwater management [13], Wastewater Reuse Programs [14], etc. Therefore, effective wastewater management requires a comprehensive and integrated approach involving various departments in municipalities. Collaboration between local government, residents and businesses is essential to the success of wastewater management programs.

The main purpose of this study is to make Artificial Neural Network (ANN) and Ordinary Differential Equation (ODE) model predictions in the future analysis of the amount of wastewater to be treated data for the Kayseri province of Turkey, received from the Turkish Statistical Institute (TUIK), and to examine each input parameter comparatively. Thus, the purpose of using the relevant models for this estimation is to measure the error values and evaluate the input parameters in terms of their effect on the output. There are some studies in the literature regarding this evaluation, both ANN-based and ODE-based. However, the majority of these studies are concerned with estimating the chemical properties of treated wastewater rather than the amount of treated wastewater in any facility. Moreover, The comparison of the results obtained with the ANN and ODE models is presented as an innovation in the literature.Summary information regarding the studies in literature is as given below.

Nasr et al. [15], used an ANN with Feed Forward Back Propagation to estimate the performance of EL-AGAMY WWTP-Alexandria in terms of Chemical Oxygen Demand (COD), Biochemical Oxygen Demand (BOD) and Total Suspended Solids. They focused on the implementation of the approach. To do this, they used data collected during a study conducted over a 1-year period. They concluded that ANN provides an effective analysis and diagnostic tool for understanding and simulating nonlinear plant behaviour and is used as a valuable performance evaluation tool for plant operators and decision makers. Wongburi et al. [16] used ANN for predicting water quality in wastewater treatment plants (WWTP). The research evaluates ANNs and LSTM models on WWTP data, identifying optimal settings (epoch 50, batch size 100) for minimized training time and root mean square error (RMSE). The precise predictions of effluent parameters showcase the applicability of these models in detecting potential upsets in WWTP operations. Yang et al. [17] presented a resilient and adaptive dynamic neural network model designed to forecast effluent quality and facilitate potential real-time adjustments in wastewater treatment operations. They introduced in this investigation a dynamic nonlinear autoregressive network with an exogenous input (NARX) model to predict effluent quality, fine-tuning performance through varied time-delay parameters and training algorithms. Moreover, a PCA-NARX hybrid model was implemented, surpassing the performance of two stationary ANN models. Among the four training algorithms for the NARX model, they proposed that the BR algorithm stands out as the most efficient.

In recent years, a variety of modeling techniques, including ANN and ODE, have been employed to enhance the prediction accuracy and operational efficiency of wastewater treatment processes. These models offer valuable insights into the non-linear behaviors of such systems. Guo et al. [18] investigated the use of ANN models to forecast effluent quality parameters in municipal wastewater treatment plants. Their study emphasized the role of various input features and configurations in improving predictive performance. The results suggested that ANN models not only predicted effluent quality with a high degree of accuracy but also contributed to optimizing the operation costs of the plant without compromising the treatment efficacy. Waqas et al. [19] utilized ANN and SVM to optimize membrane permeability in a Membrane Rotating Biological Contactor (MRBC), focusing on operational parameters like disk rotational speed, hydraulic retention time (HRT), and sludge retention time (SRT). Their findings showed that ANN with 13 hidden layers achieved an R^2 value greater than 0.99, while SVM, using the Bayesian optimizer approach, also performed exceptionally well, indicating the effectiveness of machine learning in enhancing wastewater treatment. Godini et al. [20] modeled an industrial aerated lagoon system for wastewater treatment, focusing on key operational parameters to predict system performance under various conditions. The primary objective was to identify potential bottlenecks and improvements in the system's operation. To achieve this, they used a neuro-evolutive approach that combined ANN with the Differential Evolution (DE) algorithm. Tarun et al. [21] explored the use of a Neural Networks-based Model Predictive Controller (MPC) for controlling Dissolved Oxygen (DO) in a wastewater treatment plant. The study addressed the complexity of wastewater treatment, which is influenced by biological, physical, and chemical factors. The regulation of DO is critical for maintaining the plant's efficiency. Zhao et al. [22] developed a parallel hybrid model combining ODE and ANN to simulate biological phosphorus removal (BPR) with greater accuracy and interpretability. The model incorporates the Activated Sludge Model No. 2d (ASM2d), a standard model in wastewater treatment, with an ANN to enhance the estimation of phosphate removal in anaerobic/aerobic sequencing batch reactors.

On the other hand, Differential Equation, is a mathematical tool used to model systems that involve rates of change with respect to a variable. Mathematical modeling using Differential Equations is prevalent in various fields, including Physics [23], Biology [24], Engineering [25], Medicine [26], Economics [27], Epidemic diseases [28, 29] and more. More complex systems involve systems of coupled ODEs, which might represent interactions between multiple variables. The specific form of the ODEs depending parameters depends varies according to the nature of the modelled system. Some studies in the literature on treated wastewater in WWTPs through differential equations are given below.

Diehl and Farås [30] modelled biological reactions in WWTP with two nonlinear ordinary differential equations and two hyperbolic partial differential equations (PDEs) with coefficients that are discontinuous functions in space due to the continuous sedimentation process, inputs, and outputs. They proved that the most desired steady-state solutions can be parameterized with a natural control variable. They supported their theoretical results with simulations. Zlateva and Dimitrova [31] defined a simplified mathematical model for the process of waste water treatment with active sludge in three nonlinear differential equation. They stated that their results may be useful in designing actual control strategies of active sludge waste water treatment processes.

The main contributions of this research study:

•The amount of wastewater treated through WWTP was estimated by modelling with ANN and ODE, based on real data.

•A comparative study on wastewater analysis was carried out in Turkey and an applicable simulation was provided for all regions in the world.

•With the ODE model, which can make predictions with a very low error rate (RMSE=0.0001), an analysis has been done that has the potential to contribute to future planning for sustainability in local government policies.

•The effects of four different input parameters on the amount of wastewater to be treated for the future were comparatively analyzed and the results were discussed.

The remainder of the article is organized in two sections broadly. One .section presents the proposed methodology and its details, and another section discusses the comparative analysis results and performance evaluations of both ANN and ODE models. In conclusion, a general evaluation of the study and suggestions for future studies is presented.

METHODOLOGY

The scope of this study includes analysis of wastewater data obtained from the Turkish Statistical institute (TUIK) of Kayseri province between 2003 and2020. For this analysis, mathematically presented and analyzed ANN activation functions and differential equation system were used. The process steps for the training, testing and analysis processes of the proposed models are as shown in Figure 1. The process starts with compiling the data from TUIK and augmenting the data with ARIMA modelling since 10 data from 18 years are not sufficient for ANN modelling. The augmented data was used only in the training processes of the ANN models, and only the original data obtained from TUIK was used in both the ODE model analysis and testing processes. In the next step, the data was normalized. The training processes of two different ANN models given as tangent hyperbolic (tanh) functions were carried out with the normalized data, and the parameters determination of the tanh activation function was carried out, as detailed in this section. After this process, the normalized original data was used to suggest two different ANN models and One (1) ODE model. With the data resulting from the estimation, MSE values were calculated separately for each model. Thus, the process of revealing the model was carried out with a comparative analysis, that predicts the amount of wastewater to be treated data with the lowest error rate.

Dataset

Kayseri (in Turkey) is a large city located in the central Anatolian region of Turkey. As of 2024, it is a crowded city with a population of approximately 1.5 million. There are more than 500 industrial establishments in the city and it is ranked 18th in Turkey according to industrial development. In Kayseri, which is a very large and developed city, it is very important to analyze both industrial and domestic wastewater production and the factors affecting it, and this study has addressed this issue.

Some of the factors affecting the amount of treated wastewater in a settlement can be given as a) population, b) capacity of WWTPs, c) number of WWTPs, and d) amount of daily wastewater discharged per person in municipalities. Therefore, the data obtained from TUIK for these five variables for Kayseri province, Turkey are given in Table 1 below.

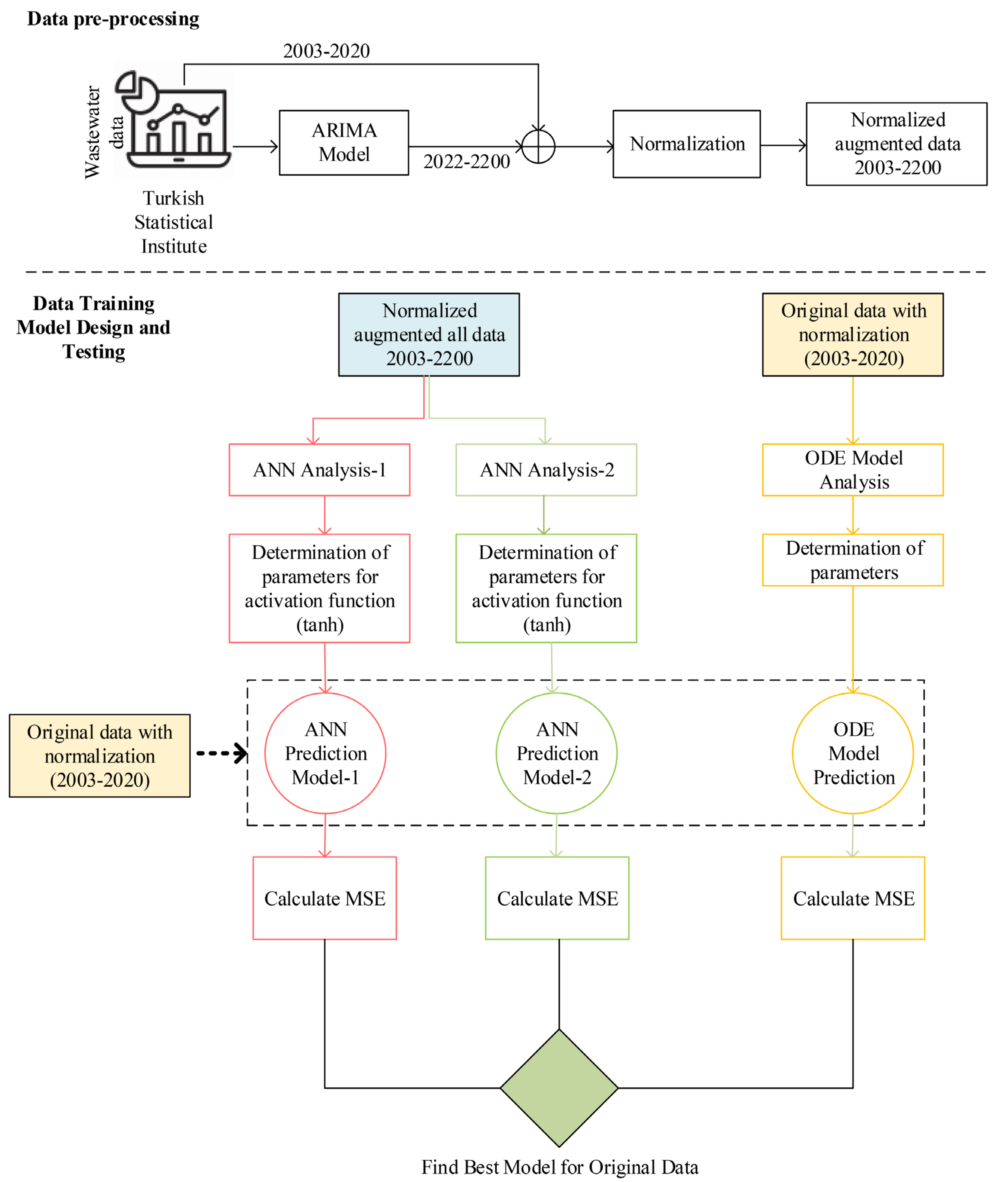

Figure 1. Methodology Diagram of Proposed Models through ANN activation functions and ODE

The analysis procedures in the remainder of the study were carried out based on the data in Table 1.

Data Augmentation with Autoregressive Integrated Moving Average (ARIMA)

ARIMA model is a traditional approach used for various time series forecasting and is used for applications such as economic forecasting and experimental parameter prediction [33]. However, ARIMA model is not suitable for non-stationary data [34]. For non-stationary data, model complexity increases and may lead to overfitting. Therefore, it is essential to investigate the stationarity of the data before using ARIMA model.

In order to test the stationarity of the data, the Augmented Dickey-Fuller (ADF) test table and p value calculation were performed within the scope of this study and it was investigated whether it was necessary to take a difference to make the series stationary. The test results were interpreted using the p value, ADF statistics and ADF critical values (1%, 5% or 10%). If the p value is typically lower than 0.05, the non-stationary null hypothesis is rejected and the stationarity of the series is proven [35] [36] In addition, if the ADF test statistics value is < ADF critical values, evidence for the stationarity of the series is obtained. A high p value is a situation that creates doubts about the stationarity of the series. In this case, the series should be made stationary by taking a derivative [37].

All these situations were evaluated together and the ADF test results performed on the original data were as shown in the Tablo-2. Accordingly, since the ADF_statistics value is lower than the ADF critical value, it shows that the original data is stationary. When the 1st degree derivative is taken, the p value also drops below 0.05 and supports stationarity.

Table 1. Some wastewater data for the years 2003-2020 obtained from TUIK for Kayseri province [32]

	Year	Population of the Municipality Served by the WWTP	WWTPs Capacity (Thousand M3/Year)	Number of WWTPs	Amount of aily Wastewater Discharged per Person in Municipalities (Litre per Person perDay)	Amount of Wastewater Treated in WWTPs (Thousand M3/ Year)
Order	**t**	**x(t)**	**k(t)**	**z(t)**	**h(t)**	**y(t)**
1	2003	576543	78475	1	172	42068
2	2004	595116	78475	1	195	50102
3	2006	872313	83835	2	165	51500
4	2008	861146	83835	2	161	51892
5	2010	986358	83835	2	160	58826
6	2012	1064662	87425	7	157	61416
7	2014	1164617	89836	10	155	65799
8	2016	1175068	89201	11	150	64349
9	2018	1321717	89603	14	126	60676
10	2020	1134304	51150	14	157	65008

Table 2. ADF Test Results

		ADF_statistic	ADF Critical Value (5%)	p-value	ADF_Result
Feature -1	Original	-1.64	-3.64	0.45	Stationary
	Diff_1	-4.22	-3.36	0.0	
	Diff_2	-3.48	-3.92	0.0	
Feature-2	Original	-0.74	-3.64	0.83	Stationary
	Diff_1	2.24	-3.64	0.99	
	Diff_2	-1.07	-3.47	0.72	
Feature-3	Original	0.1	-3.28	0.96	Stationary
	Diff_1	-2.46	-3.36	0.12	
	Diff_2	-3.14	-3.47	0.02	
Feature-4	Original	-1.13	-3.47	0.69	Stationary
	Diff_1	-5.23	-3.36	0.0	
	Diff_2	-4.04	-3.47	0.0	
Feature-5	Original	-2.14	-3.28	0.22	Stationary
	Diff_1	-3.07	-3.36	0.02	
	Diff_2	-2.81	-3.47	0.05	
Feature-4	Original	-1.13	-3.47	0.69	Stationary
	Diff_1	-5.23	-3.36	0.0	
	Diff_2	-4.04	-3.47	0.0	
Feature-5	Original	-2.14	-3.28	0.22	Stationary
	Diff_1	-3.07	-3.36	0.02	
	Diff_2	-2.81	-3.47	0.05	

The results prove that the original data is stationary in terms of time series and is suitable for ARIMA estimation. Data augmentation is a usable approach for problems where data is not sufficient [38], and in this study, data augmentation was performed with an ARIMA-based model.

Within the scope of this study, a time series forecasting based on Box-Jenkins models was carried out to augment the data taken from Turkish Statistical Institute (TUIK) to be used in the analysis and forecasting model of WWTP data. For this estimation, the ARIMA model structure was used within the designed algorithm as shown in Figure 2. The processing steps shown in Figure 1 were applied to select the p,d,q combination with the lowest RMSE value for each attribute. The p value indicates the number of autoregressive terms, the d value indicates the number of non-seasonal differences to maintain stationarity, and the q value indicates the number of lagged forecast errors in the forecast equation.

The general expression of the ARIMA (p,d,q) model can be shown as follows [39, 40]:

$$(w_t = \theta_1 w_{t-1} + \theta_2 w_{t-2} + \ldots + \theta_3 w_{t-p} + a_t - \theta_1 a_{t-1} - \theta_1 a_{t-2} - \ldots - \theta_q a_{t-q} \quad (1)$$

The moving average parameters (θ) are defined as negative in the equation following the Box-Jenkins convention. If the first differences make the series stationary, the d=1 difference equation will emerge, and, if the second differences make the series stationary, the d=2 difference equation will emerge, as shown in Eq.2 below:

$$\begin{aligned} w_t &= W_t \; for \; d = 0 \\ w_t &= W_t - W_{t-1} \; for \; d = 1 \\ w_t &= (W_t - W_{t-1}) - (W_{t-1} - W_{t-2}) \; for \; d = 2 \end{aligned} \quad (2)$$

Development of the model was implemented by Python version 3.10.6 and Matplotlib, Pandas, Seaborn, Numpy, Imblearn statsmodels. The performance evaluation criterion was taken into account with the calculation of Absolute Error ($\in_{AE}$) in Eq. 3. The original value and estimated value are symbolized as y_o and y_t, respectively.

$$\in_{AE} = |y_t - y_o \quad (3)$$

As stated in this section, it is important to select the p, d and q parameters in the ARIMA model structure, and the error values of the estimated values vary significantly depending on the selected values. Within the scope of this study, p,d,q parameters were selected separately for a total of 4 input and 1 output value within the flow diagram shown in Figure 1. For this selection, in the first stage, data was predicted for the years 2022-2200 using the original data (measured sensor data) containing records between 2003-2020 and the ARIMA model, which works with randomly selected p,d,q parameters. However, since there is no exact measurement value for the forecast period, it is not possible to measure the errors of the predicted values. For this reason, the same process was applied in reverse. Forecast data between 2022-2200 was fed to the same ARIMA model for training purposes, and data forecasting was performed for the years 2003-2020. We now have the ARIMA model forecast data and the original final data. Thus, the RMSE error value between both data was calculated. This process was repeated in 125 different combinations for all choices in the [0-5] range for p,d and q, and the ARIMA model parameter with the lowest RMSE value was selected. In this way, time-dependent data estimation was achieved with the lowest RMSE value and data that approximated the original data with the least error value was produced. The main purpose of the algorithm given in this section is to increase data with ARIMA and to ensure that these synthetic data are as close to the original data as possible. As a result, data augmentation was achieved with the ARIMA model, which uses the best p,d,q value for each attribute. The reason for this data increase is that it is not possible to train the ANN model with the smaller original data between 2003 and 2020, the details of which are explained in the next section,. This problem was intended to be overcome by using the ARIMA model structure to increase the data based on prediction.

Table 3 shows sample data regarding data predictions made within the flow chart given in Figure1. x(t),k(t),z(t),h(t) refer to the attributes given as input to the ANN and y(t) refers to the value received as output. The ARIMA model augmentation process was applied separately for each input and output value. This dataset, which includes original and predicted data, was used within the scope of this study to perform analysis with the model given in the form of ANN and differential equation system to estimate the amount of wastewater treated in WWTPs. In the last stage, the analysis results were compared with the original and real measurement data and the error values were compared.

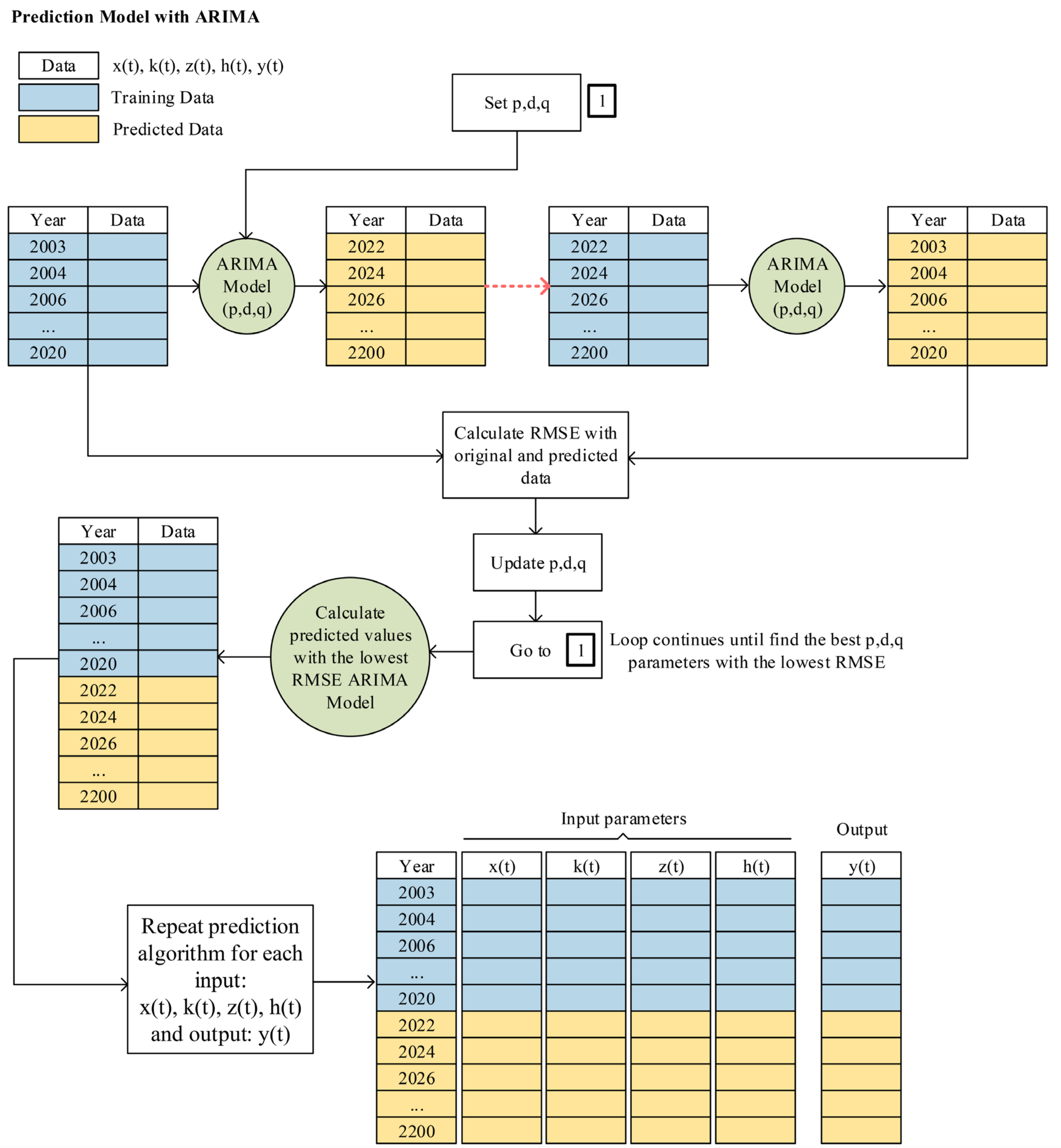

Figure 2. ARIMA data augmentation application flow diagram

Table 3. Increasing the Data in Table 1 to the Years 2022-2200 with ARIMA

Order	t	x(t)	k(t)	z(t)	h(t)	y(t)
11	2022	1227368	77580	15	155	67549
12	2024	1329078	76855	17	154	69309
…	…	…	…	…	…	…
26	2052	2756863	71279	37	130	91763
27	2054	2858848	70970	37	129	93311
…	…	…	…	…	…	…
41	2082	4286642	66640	58	106	114245
42	2084	4388627	66331	59	104	115687
…	…	…	…	…	…	…
56	2112	5816420	62001	79	81	135167
57	2114	5918405	61692	81	79	136509
…	…	…	…	…	…	…
71	2142	7346198	57363	101	56	154637
72	2144	7448184	57053	102	54	155885
…	…	…	…	…	…	…
86	2172	8875977	52724	122	31	172755
87	2174	8977963	52415	124	29	173917
…	…	…	…	…	…	…
100	2200	10303770	48394	142	8	188529

Normalization of Augmented Data

Here, the data in Table 2 are normalized with ARIMA sklearn.preprocessing.Normalizer (norm=l2). The purpose of doing this is to perform analysis with smaller error values. At the end of the analysis processes, normalized data will be denormalized and predictions will be made. The explanation is as follows:

Let $\omega=\omega_{orijinal}=(\omega_1,\omega_2,\ldots,\omega_n)$, $n\in Z^+$ be elements of the data matrix. The normalized set of these data is given as $\omega_{normalized}=(\omega_1/\|\omega\|_2, \omega_2/\|\omega\|_2, \ldots, \omega_n/\|\omega\|_2)$, where $\|\omega\|_2$ is the Euclidean norm in $\mathbb{R}^n$ such that $\|\omega\|_2 = \sqrt{(\omega_1{}^2 + \omega_2{}^2 + \cdots + \omega_n{}^2)}$. Equation (4) is used to obtain original data from normalized data.

$$\omega_{orijinal} = (\|\omega\|_2) * (\omega_{normalized}) \tag{4}$$

For the data in Table 2, the norm of each variable is found to be

$$\|t\|_2 \cong 21018,\ \|x\|_2 \cong 60278211,\ \|k\|_2 \cong 649511,\ \|z\|_2 \cong 826,\ \|h\|_2 \cong 1012,\ \|y\|_2 \cong 1311678 \tag{5}$$

Therefore, the normalized set of values of the variables is given in Table 4.

ANN Analysis

The purpose of ANNs is essentially to enable machines to learn involve and acquire knowledge from data, discern and identify patterns, generate predictive insights, and perform tasks that traditionally required explicit programming [41, 42, 43, 44]. Their versatility and ability to handle complex relationships in data make them valuable tools across various domains. ANNs are a versatile technique used in many disciplines and industrial practice. Some branches and application areas where ANN is widely used are: Computer Science [45], Electrical and Electronics Engineering [46], Statistics [47] and Data Science [48], Chemistry and Biology [49], Medicine and Health Sciences [50], Finance [51], Transport and Logistics [52], Energy Systems [53], Agriculture and Environmental Sciences [54], Robotics and automation [55], and training and pedagogy [56]. ANNs have a wide range of applications in data analysis, modelling and predictability.

ANNs can be used for wastewater amount estimation by training the network on historical data that relates input features to the corresponding wastewater amounts. In an ANN, an activation function refers to a mathematical operation applied to the output of every neuron (or node) within a layer of the network. The activation function introduces non-linearity to the network, facilitating the acquisition of intricate patterns and relationships present in the data. The activation function determines whether a neuron should be activated (i.e., contribute to the network's output) based on its input.

There are several types of activation functions, each with its own characteristics. Here are some common activation functions used in ANNs [57, 58, 59, 60, 61, 62]:

- Sigmoid Activation Function,
- Hyperbolic Tangent (tanh) Activation Function,
- Rectified Linear Unit (ReLU) Activation Function,
- Leaky ReLU Activation Function,

Table 4. Increasing the Data in Table 1 to the Years 2022-2200 with ARIMA

Order	t	x(t)	k(t)	z(t)	h(t)	y(t)
1	**0.09530**	**0.009565**	**0.12082175**	**0.0012112**	**0.1700134**	**0.0320719**
2	**0.09535**	**0.009873**	**0.12082175**	**0.0012112**	**0.1927477**	**0.03819688**
3	**0.09544**	**0.014471**	**0.12907412**	**0.0024224**	**0.1630942**	**0.03926269**
4	**0.09554**	**0.014286**	**0.12907412**	**0.0024224**	**0.1591404**	**0.03956154**
5	**0.09563**	**0.016363**	**0.12907412**	**0.0024224**	**0.158152**	**0.0448479**
6	**0.09573**	**0.017662**	**0.13460135**	**0.0084782**	**0.1551866**	**0.04682247**
7	**0.09582**	**0.019321**	**0.13831338**	**0.0121118**	**0.1532097**	**0.05016399**
8	**0.09592**	**0.019494**	**0.13733572**	**0.0133229**	**0.1482675**	**0.04905854**
9	**0.09601**	**0.021927**	**0.13795465**	**0.0169564**	**0.1245447**	**0.04625831**
10	**0.09611**	**0.018818**	**0.07875161**	**0.0169564**	**0.1551866**	**0.04956095**
…	**…**	**…**	**…**	**…**	**…**	**…**
56	**0.10049**	**0.096493**	**0.09545876**	**0.0961713**	**0.0797777**	**0.10304878**
57	**0.10058**	**0.098185**	**0.09498262**	**0.0977679**	**0.0781384**	**0.10407191**
…	**…**	**…**	**…**	**…**	**…**	**…**
86	**0.10334**	**0.14725**	**0.08117461**	**0.1481703**	**0.030598**	**0.13170536**
87	**0.10344**	**0.148942**	**0.08069847**	**0.1498891**	**0.0289587**	**0.13259138**
…	**…**	**…**	**…**	**…**	**…**	**…**
100	**0.10467**	**0.170937**	**0.07450867**	**0.1724634**	**0.0076475**	**0.14373123**

•Parametric ReLU (PReLU) Activation Function,

•Exponential Linear Unit (ELU) Activation Function

The incorporation of these activation functions imparts non-linear characteristics to the network, facilitating its ability to learn and approximate intricate functions. The selection of an activation function is contingent upon the problem's inherent nature, the data's characteristics, and considerations for ensuring training stability. The Hyperbolic Tangent (tanh) activation function is a mathematical function commonly used in neural networks, especially in the hidden layers. The tanh activation function is often used in the hidden layers of neural networks, especially when the data is standardized or normalized. It is a common choice when the goal is to model relationships with complex and nonlinear patterns.

The structure used for ANN analysis of the data in Table 3 is given schematically in Figure 3.

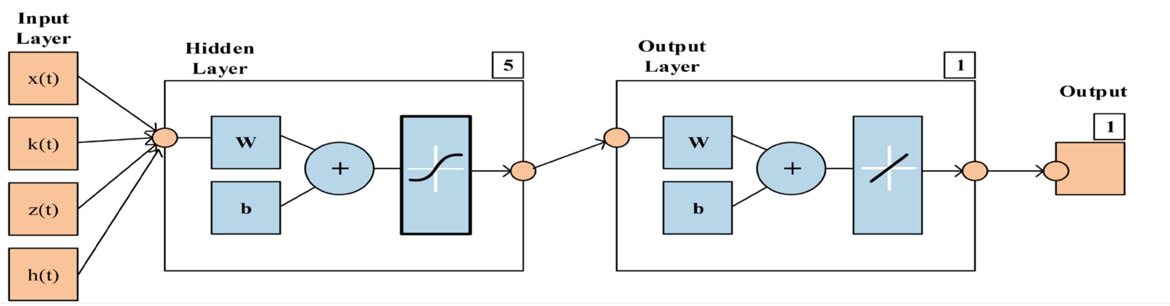

Figure 3. Structure of the used ANN Model

The ANN used in this study is a feedforward network with one hidden layer. The architecture of the ANN is designed to predict the y(t) variable using four input variables as described in Table 3. The details of the architecture and the hyperparameters selected for the ANN model are as follows:

Input Layer: The input layer consists of 4 neurons, each representing one of the input variables.

Hidden Layer: The hidden layer contains 5 neurons. The selection of 5 neurons was based on preliminary experiments aiming to balance model complexity and performance. Too many neurons could lead to overfitting, while too few neurons could lead to underfitting.

Output Layer: The output layer consists of 1 neuron, corresponding to the predicted y(t) variable.

Activation Function: In the hidden layer, the tanh activation function is used. The choice of the tanh function was made because it ensures non-linearity and better performance for normalized data by outputting values between -1 and 1.

Training Function: The Levenberg-Marquardt backpropagation algorithm (trainlm) was selected as the training function due to its efficiency in handling moderate-sized datasets and faster convergence compared to other methods like gradient descent.

Hyperparameters:

Number of epochs: 10,000 epochs were selected to ensure sufficient training time for convergence.

Learning rate: The learning rate is implicitly managed by the trainlm function, and no manual tuning of the learning rate was required.

Performance function: The mean squared error (MSE) was used as the performance function to measure the network's prediction error during training.

In this study, the y(t) variable was estimated by ANN with the help of other variables. In order to better demonstrate the performance of ANN, two different estimation processes were carried out.

ANALYSIS AND RESULTS

Within the scope of this study, three models based on ANN and ODE were proposed to perform wastewater analysis for Kayseri province and were analysed with data received from TUIK. Based on the result of the analysis, the aim was to find the best model for future prediction of the amount of wastewater to be treated and to present the estimate closest to the original data with the lowest MSE rate. In this section, the results and selected parameters for two (2) ANN models and one (1) ODE model are explained in detail.

Design of ANN Prediction Models and Results

With the two (2) ANN models proposed for this study, two (2) separate model designs were made. Training was carried out with different parameters, and they were tested with the normalized versions of the original data collected between 2003-2020.

Parameter selection of ANN

Different model parameters for the two (2) ANN analysis are shown in Table 5. Model parameter selections were determined by considering the situations in which the best results were obtained.

When Eq. 5 and the coefficients in Table 5 are considered together, two different ANN mathematical models are given below. Therefore, we have

$$yANN\ Prediction_1(t) = (0.3924) - 0.0767\cdot\tanh(2.8205 - 9.5779\cdot x(t) + 18.4063\cdot k(t) + 1.1281\cdot z(t) - 20.0762\cdot h(t)) + 0.0698\cdot\tanh(-0.3346 - 15.4020\cdot x(t) + 4.3192\cdot k(t) + 19.2210\cdot z(t) - 3.4523\cdot h(t)) + 0.0224\cdot\tanh(-2.1522 + 21.5775\cdot x(t) + 17.3640\cdot k(t) - 11.9887\cdot z(t) - 2.5399\cdot h(t)) - 0.2423\cdot\tanh(3.9206 + 8.7037\cdot x(t) + 6.6566\cdot k(t) - 21.5598\cdot z(t) - 6.7050\cdot h(t)) - 0.0089\cdot\tanh(-4.6525 - 0.9762\cdot x(t) + 27.8592\cdot k(t) - 18.2762\cdot z(t) + 11.3490\cdot h(t))$$ and

$$yANN\ Prediction_2(t) = (0.0396) + 0.0086\cdot\tanh(2.1338 + 10.7134\cdot x(t) - 38.3338\cdot k(t) + 1.9339\cdot z(t) - 15.7047\cdot h(t)) - 0.0817\cdot\tanh(-8.4515 + 5.3516\cdot x(t) + 60.4351\cdot k(t) + 8.0447\cdot z(t) - 0.5551\cdot h(t)) - 0.0240\cdot\tanh(1.2683 + 15.7308\cdot x(t) - 1.4211\cdot k(t) - 16.2572\cdot z(t) - 9.8105\cdot h(t)) + 0.0674\cdot\tanh(-3.2027 + 3.9454\cdot x(t) + 42.6491\cdot k(t) + 5.3263\cdot z(t) - 16.1992\cdot h(t)) + 0.0248\cdot\tanh(4.1221 - 19.2592\cdot x(t) - 29.3135\cdot k(t) - 5.1683\cdot z(t) - 10.2564\cdot h(t))$$

Activation function of ANN

The resulting model with activation function (tanh) of ANN is shown in Eq. 5.

■(y(t)=b_2+LW tanh(b_1+IWx)&(5))

Thus, the obtained coefficients of Eq. 5 are given in Table 6.

For ANN analysis, 15 of the 100 Normalized Data for the years 2003-2200 were used for testing and 15 for verification. Regarding the results obtained, the training, validation and test graphics of the 1st and 2nd ANN models are shown in Figure 4 and Figure 5, respectively. As seen in Figure 4 and Figure 5, the lowest training R value was 0.99 for ANN training and 0.98 for validation. High R values obtained during the training process generally indicate how well the model has learned the relationship between input data and target output. As the R value approaches 1, it is stated that the variations in the model's training data are more strongly associated with the variations in the target output. This indicates that the models are well adapted to the training data. A high R value on the training set (0.99) generally indicates that the model has learned the patterns in the training data very well. However, a validation set is often used to evaluate the model's ability to apply the same success to new, unseen data. The R value (0.98) obtained in the validation set shows that the model can be successfully generalized to data outside the training set. This shows that the models do not have overfitting problems and have a good performance in general.

Table 5. Training Progress for ANN

	ANN Prediction_1			ANN Prediction_2		
	Initial Value	Stopped Value	Target Value	Initial Value	Stopped Value	Target Value
Epoch	**0**	**18**	**10000**	**0**	**12**	**10000**
Elapsed Time	**-**	**00:00:03**	**-**	**-**	**00:00:03**	**-**
Performance	**0.773**	**3.53e-07**	**0**	**2.31**	**1.64e-06**	**0**
Gradient	**3.52**	**0.000171**	**1e-07**	**5.44**	**7.6e-05**	**1e-07**
Mu	**0.001**	**1e-07**	**1e+10**	**0.001**	**1e-06**	**1e+10**
Validation Checks	**0**	**6**	**6**	**0**	**6**	**6**

Table 6. Table coefficient of tangent hyperbolic model in Equation (2).

	ANN Prediction_1	ANN Prediction_2
b_1	$\begin{pmatrix} 2.8205 \\ -0.3346 \\ -2.1522 \\ 3.9206 \\ -4.6525 \end{pmatrix}_{5x1}$	$\begin{pmatrix} 2.1338 \\ -8.4515 \\ 1.2683 \\ -3.2027 \\ 4.1221 \end{pmatrix}_{5x1}$
b_2	$(0.3924)_{1x1}$	$(0.0396)_{1x1}$
IW	$\begin{pmatrix} -9.5779 & 18.4063 & 1.1281 & -20.0762 \\ -15.4020 & 4.3192 & 19.2210 & -3.4523 \\ 21.5775 & 17.3640 & -11.9887 & -2.5399 \\ 8.7037 & 6.6566 & -21.5598 & -6.7050 \\ -0.9762 & 27.8592 & -18.2762 & 11.3490 \end{pmatrix}_{5x4}$	$\begin{pmatrix} 10.7134 & -38.3338 & 1.9339 & -15.7047 \\ 5.3516 & 60.4351 & 8.0447 & -0.5551 \\ 15.7308 & -1.4211 & -16.2572 & -9.8105 \\ 3.9454 & 42.6491 & 5.3263 & -16.1992 \\ -19.2592 & -29.3135 & -5.1683 & -10.2564 \end{pmatrix}_{5x4}$
LW	$(-0.0767 \quad 0.0698 \quad 0.0224 \quad -0.2423 \quad -0.0089)_{1x5}$	$(0.0086 \quad -0.0817 \quad -0.0240 \quad 0.0674 \quad 0.0248)_{1x5}$

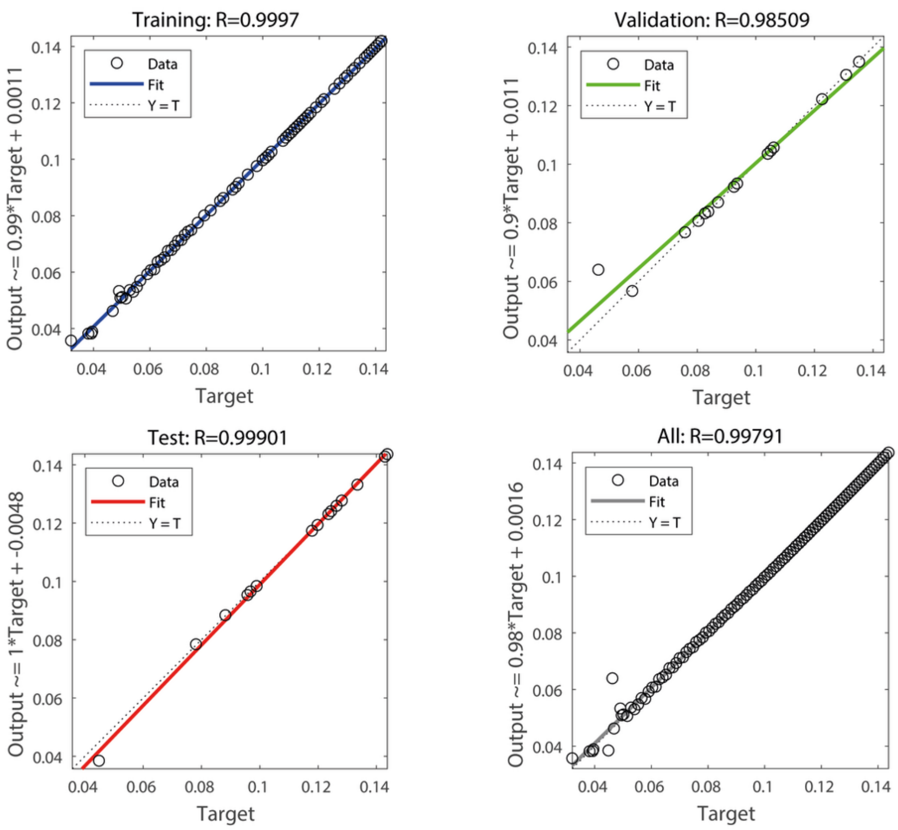

Figure 4. Graphical Representation of Analysis Results for ANN Prediction_1

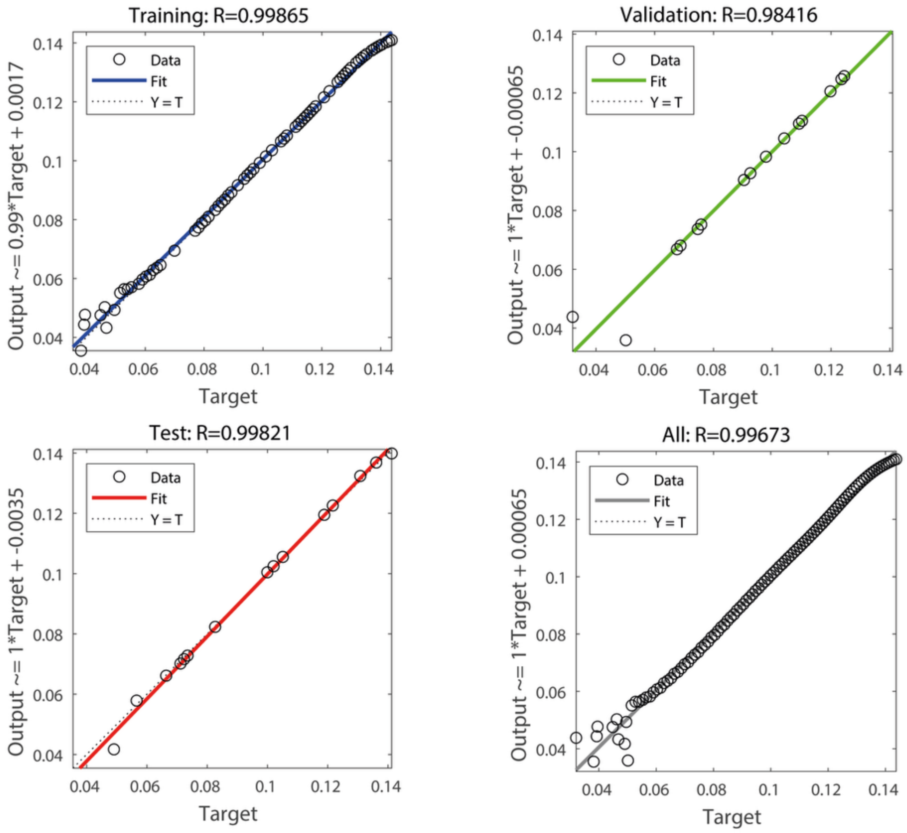

Figure 5. Graphical Representation of Analysis Results for ANN Prediction_2

Determining the ODE Model and Its Parameters

Let t denote the time parameter and the independent variable. The proposed general linear ODE mathematical model is as shown below.

$$
\begin{aligned}
\frac{dx}{dt} &= \theta_1 x + \theta_2 k + \theta_3 z + \theta_4 h + \theta_5 y + \theta_6 \\
\frac{dk}{dt} &= \theta_7 x + \theta_8 k + \theta_9 z + \theta_{10} h + \theta_{11} y + \theta_{12} \\
\frac{dz}{dt} &= \theta_{13} x + \theta_{14} k + \theta_{15} z + \theta_{16} h + \theta_{17} y + \theta_{18} \qquad (6) \\
\frac{dh}{dt} &= \theta_{19} x + \theta_{20} k + \theta_{21} z + \theta_{22} h + \theta_{23} y + \theta_{24} \\
\frac{dy}{dt} &= \theta_{25} x + \theta_{26} k + \theta_{27} z + \theta_{28} h + \theta_{29} y + \theta_{30}
\end{aligned}
$$

where $x \equiv x(t)$, $k \equiv k(t)$, $z \equiv z(t)$, $h \equiv h(t)$ and $y \equiv y(t)$ and the system (6) has to be finished with positive initial conditions $x(t_0) = x_0$, $k(t_0) = k_0$, $z(t_0) = z_0$, $h(t_0) = h_0$ and $y(t_0) = y_0$ for $t \geq t_0$. The dependent variables and their definitions used in the system in (6) are shown in Table 7 below.

The ODE model in (6) was solved with the Matlab R2023a program rungekutta45, and then the parameter values closest to the values in Table 1 (giving the minimum error) were found by using the lsqcurvefit function. The function lsqcurvefit, which is used as a tool in Matlab, is used to find the coefficients that will best fit a nonlinear function (with the logic of least squares). Find coefficients x that solve the problem minx $||F(x, x_{data}) - y_{data}||^2 2$ = minx $\sum i(F(x, x_{datai}) - y_{datai})^2$, given input data x_{data}, and the observed output y_{data}, where x_{data} and y_data are matrices or vectors, and $F(x,x_{data})$ is a matrix-valued or vector-valued function of the same size as y_{data}. lsqcurvefit simply provides a convenient interface for data-fitting problems. Rather than compute the sum of squares, lsqcurvefit requires the user-defined function to compute the vector-valued function. Additionally, x = lsqcurvefit(fun, x_0, x_{data}, y_{data}) starts at x_0 and finds coefficients x to best fit the nonlinear function fun(x,x_{data}) to the data y_{data} (in the least-squares sense). y_{data} must be the same size as the vector (or matrix) F returned by fun [63] [64]. In differential equation systems, the parameter values used in the system are calculated with this function. In this context, the parameters used in the system can be found with the coefficients of the nonlinear curve that produces the closest results to the real values of the solution of the system based on the initial values of the variables [65] [66].

In this context, the approach in [67] was used. Thus, the parameters θ_i for i=1,2,...,30 obtained are given in Table 8.

Comparison of ANN and ODE Models and Discussion

In this section, the estimation results of two (2) ANN and one (1) ODE models for normalized original data and the effects of the input parameters on the result are analysed and discussed. In Table 9, ANN and ODE model prediction results for quantity of wastewater (y(t)) are presented for 10 original normalized data. When the results were examined, it was understood that both ANN models and ODE model could predict the original data with low MSE values. According to the comparative results, the lowest MSE value among the three models was with the ODE model.

If the parameter values obtained by the least squares method is rewritten in the suggested ODE model in (6), then we have the Eqs (7) as shown below.

When the normalized values of Table 1 are compared with Eqs. (7), the results obtained is shown in Figure 6 below.

$$\begin{aligned}
\frac{dx}{dt} &= 0.26850x + 1.94674k + 0.14510z + 2.63931h + 0.62586y + 15.33129 \\
\frac{dk}{dt} &= 0.12103x + 0.57664k + 0.08480z + 0.81543h + 0.19972y + 4.08364 \\
\frac{dz}{dt} &= 0.30291x + 2.26687k + 0.15939z + 3.06476h + 0.72651y + 17.96241 \\
\frac{dh}{dt} &= -0.29066x - 4.12683k - 0.00916z - 5.69884h - 1.11164y - 34.75096 \\
\frac{dy}{dt} &= 0.37755x + 3.15511k + 0.17317z + 4.30412h + 0.96812y + 25.30082.
\end{aligned} \tag{7}$$

Table 7. State Variables

Variable	Definition
x(t)	**Population of the Municipality Served by the WWTP at time t**
k(t)	**WWTPs Capacity at time t**
z(t)	**Number of WWTPs at time t**
h(t)	**Amount of Daily Wastewater Discharged per Person in Municipalities at time t**
y(t)	**Amount of Wastewater Treated in WWTPs at time t**

Table 8. Rate Constants of Equations in (6)

θ_1=0.26850	**θ_7=0.12103**	**θ_13=0.30291**	**θ_19=-0.29066**	**θ_25=0.37755**
θ_2=1.94674	**θ_8=0.57664**	**θ_14=2.26687**	**θ_20=-4.12683**	**θ_26=3.15511**
θ_3=0.14510	**θ_9=0.08480**	**θ_15=0.15939**	**θ_21=-0.00916**	**θ_27=0.17317**
θ_4=2.63931	**θ_10=0.81543**	**θ_16=3.06476**	**θ_22=-5.69884**	**θ_28=4.30412**
θ_5=0.62586	**θ_11=0.19972**	**θ_17=0.72651**	**θ_23=-1.11164**	**θ_29=0.96812**
θ_6=15.33129	**θ_12=4.08364**	**θ_18=17.96241**	**θ_24=-34.75096**	**θ_30=25.30082**

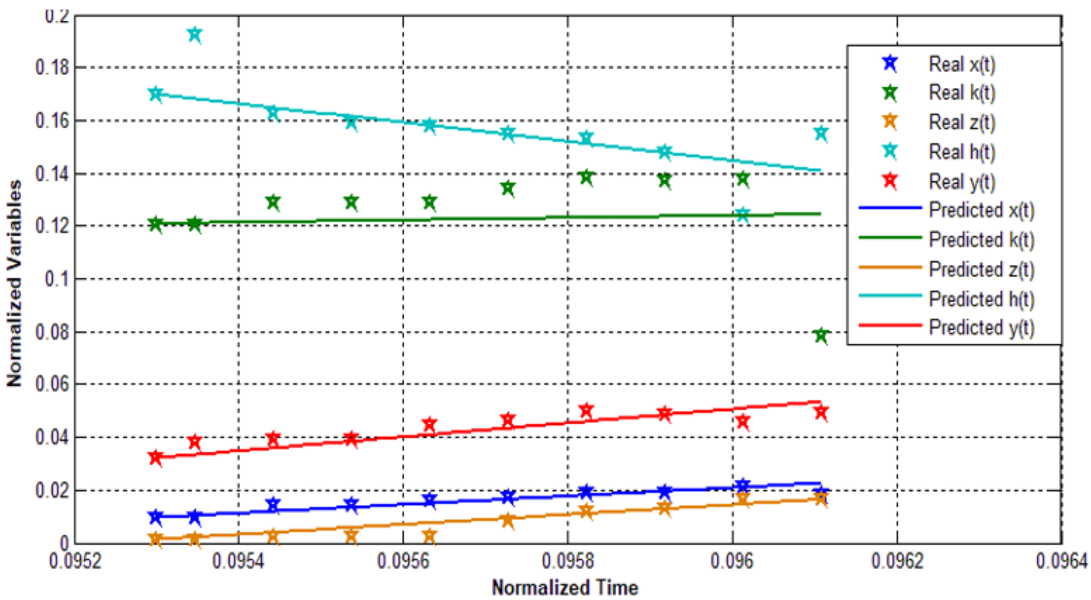

Figure 6. Graphical representation of the proposed ODE system

Table 9. The prediction performances of the proposed models according to the normalized values of Table 1

Order	Original Data y(t)	y(t) according to ANN Prediction_1	y(t) according to ANN Prediction_2	y(t) according to ODE Model
1	**0.0320719**	**0.0357**	**0.0438**	**0.03207**
2	**0.03819688**	**0.0382**	**0.0355**	**0.03337**
3	**0.03926269**	**0.0382**	**0.0443**	**0.03574**
4	**0.03956154**	**0.0389**	**0.0477**	**0.03855**
5	**0.0448479**	**0.0385**	**0.0476**	**0.04071**
6	**0.04682247**	**0.0462**	**0.0433**	**0.04351**
7	**0.05016399**	**0.0512**	**0.0359**	**0.04589**
8	**0.04905854**	**0.0533**	**0.0417**	**0.04847**
9	**0.04625831**	**0.0640**	**0.0503**	**0.05084**
10	**0.04956095**	**0.0509**	**0.0493**	**0.05343**
ERROR VALUE (MSE)		**0.00004**	**0.00005**	**0.00001**
R2		**0.7475**	**0.6251**	**0.9363**
MAPE		**0.0724 (or 7.24%)**	**0.1105 (or 11.05%)**	**0.0231 (or 2.31%)**

As shown in Table 8, the results obtained show the success of the ODE model in predicting the original data. In real world, the selected variables may not always be in sufficient number to mathematically analyse the relationship between them and make predictions. Making predictions about variables, even with little data, can be vital. In this way, precautions can be taken by developing various control strategies. Especially in ANN analysis, it is important to increase the data sufficiently in such cases. One of the first methods that comes to mind for this is ARIMA. According to the working principle of ARIMA, in order to increase the data, it increases the data according to at least two previous values of the relevant prediction. In ANN analysis using data augmented in this way, the output tends to be a linear function of the inputs. However, in most real-world problems these relationships are non-linear. The activation function chosen for ANN in this study is non-linear tangent hyperbolic. In this sense, the ODE model in this study, whose solutions are nonlinear, was found to be quite successful in predicting the original data compared to the ANN activation function.

On the other hand, one of the aims of this study is to make predictions for the future and analyse the results of these predictions after determining the best model based on the original data. Therefore, for the period until 2100 (approximately 80 years later)

The projections are regarding the following as illustrated in Figure 7 below::

a) The change graph of Quantity of wastewater to be treated over the years, and

b) The change graph of the ratio of the Quantity of wastewater to be treated value to population size over the years

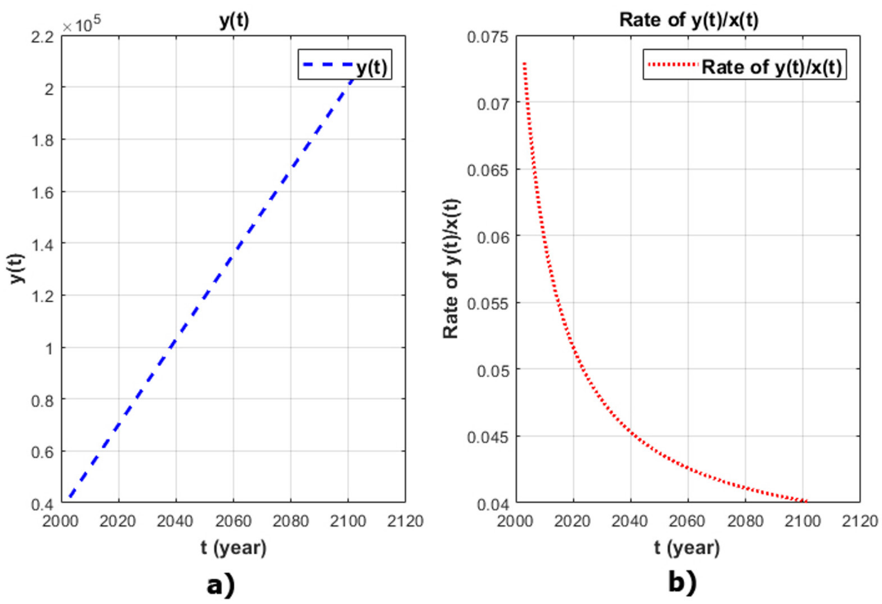

Figure 7. Analysis graph of wastewater amount according to various inputs

The results of the ODE model in Eqs. (7) are given in Table 10 and predictions have been made for the quantity of treated wastewater in WWTPs and population of the Municipality.

Table 10. The forecast results of the ODE model for the years 2020-2100

t	x(t)	y(t)	t	x(t)	y(t)	t	x(t)	y(t)
2003.015	**576561.1**	**42068.01**	**2009.756**	**886398.4**	**53186.52**	**2060.693**	**3215076**	**136740.4**
2003.083	**579692.7**	**42180.39**	**2011.458**	**964577.8**	**55991.93**	**2065.616**	**3438977**	**144773**
2003.152	**582824.3**	**42292.77**	**2016.381**	**1190571**	**64101.43**	**2070.54**	**3662669**	**152798**
2003.22	**585955.9**	**42405.14**	**2021.305**	**1416354**	**72203.21**	**2075.463**	**3886154**	**160815.4**
2003.288	**589087.4**	**42517.52**	**2026.228**	**1641927**	**80297.27**	**2080.387**	**4109431**	**168825.1**
2003.628	**604744.4**	**43079.38**	**2031.152**	**1867291**	**88383.63**	**2085.31**	**4332500**	**176827.2**
2003.969	**620400.4**	**43641.2**	**2036.075**	**2092444**	**96462.29**	**2090.233**	**4555363**	**184821.7**
2004.309	**636055.4**	**44202.99**	**2040.999**	**2317389**	**104533.3**	**2095.157**	**4778018**	**192808.6**
2004.649	**651709.4**	**44764.73**	**2045.922**	**2542124**	**112596.6**	**2100.08**	**5000466**	**200787.9**
2006.352	**729964.3**	**47572.92**	**2050.846**	**2766650**	**120652.2**			
2008.054	**808193.9**	**50380.18**	**2055.769**	**2990967**	**128700.1**			

When Figure 7 is considered, the decrease in the average amount of wastewater treatment per capita may result from a combination of factors. These factors may include technological advances, water management policies, community awareness and administrative support. This situation can be considered as a positive development in terms of environmental sustainability and water resources management.

The ANN and ODE based wastewater forecasting model developed in this study offers municipalities and wastewater treatment plants a practical tool with which to make informed decisions. The model facilitates the optimisation of wastewater treatment plant capacity, thereby preventing plants from becoming inadequate or over-utilising capacity in response to future demands. Furthermore, the model facilitates strategic planning by aiding infrastructure maintenance and expansion decisions based on wastewater production trends. The integration of the model into smart city infrastructure will enable real-time monitoring and long-term planning for sustainable wastewater management. With regard to policy recommendations, it would be prudent for governments to consider investing in AI-enabled smart infrastructure, introducing regulations mandating infrastructure upgrades in line with predictive models, and promoting water reuse and recycling practices. Such measures will contribute to the more efficient management of future wastewater demand and the protection of precious water resources.

CONCLUSION

In this study, two distinct ANN and one ODE model were employed to forecast the time series of annual total wastewater volume in Kayseri province, Turkey. The models identify the impact factors of wastewater discharge, predict the amount of urban wastewater in the coming years, and analyse the impact degree of all factors. Upon evaluation of the ODE model exhibiting the highest performance among the three models, the following results were obtained in the test phase: R^2 = 0.9363, MSE = 0.00001, and MAPE = 0.0231. These results demonstrate that the model exhibits an acceptable degree of accuracy in predicting the volume of wastewater in the Kayseri province. It is important to note that future predictions for the quantity of wastewater using the model proposed in this study should not be considered as a replacement for direct measurements. Nevertheless, the method may result in a discrepancy in grid data for modelling and urban water management, particularly for research purposes, in instances where measured data are unavailable (such as for the year 2100) or legally restricted. By evaluating discharged waters in conjunction with land use plans and modelling various future scenarios by analysing agricultural productivity factors, it is possible to simulate changes in land use habits.

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DATA AVAILABILITY STATEMENT

The author confirm that the data that supports the findings of this study are available within the article. Raw data that support the finding of this study are available from the corresponding author, upon reasonable request.

CONFLICT OF INTEREST

The author declared no potential conflicts of interest with respect to the research, authorship, and/or publication of this article.

USE OF AI FOR WRITING ASSISTANCE

Not declared.

ETHICS

There are no ethical issues with the publication of this manuscript.

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