MALTEPE JOURNAL OF MATHEMATICS
ISSN:2667-7660, URL:HTTP://DERGIPARK.ORG.TR/TR/PUB/MJM
VOLUME VI ISSUE 2 (2024), PAGES 90-102, HTTPS://DOI.ORG/10.47087/MJM.1515500

NEUTROSOPHICATION β-COMPACTNESS

AHU ACIKGOZ[1], FERHAT ESENBEL[2] AND OSMAN MUCUK[3]
[1,2] DEPARTMENT OF MATHEMATICS, BALIKESIR UNIVERSITY,10145 BALIKESIR, TURKEY
[3] DEPARTMENT OF MATHEMATICS, ERCIYES UNIVERSITY, 38280 KAYSERI, TURKEY
[1]ORCID ID: 0000-0003-1468-8240
[2]ORCID ID: 0000-0001-5581-3621
[3]ORCID ID: 0000-0001-7411-2871

ABSTRACT. In this study, we first define the concept of neutrosophic β-open set. Then, using this new set definition, we present neutrosophic β-compact and neutrosophic β-closed spaces and examine their properties. Also, we classify these spaces using the concept of neutrosophic filterbase, which is introduced for the first time in this study. And, relationships between these different types and forms of compactness are investigated.

1. INTRODUCTION

The concept of compactness is one of the indispensable characters of general topology and other topology forms. In general topology, these concepts of β-open sets and β-continuous functions were first introduced by Abd El-Monsef [1]. Later, these concepts were adapted to different topology forms and some interesting properties of them were investigated in different forms of topology as in [4, 6, 8]. By using these notions, the basic classical results that have been going on in the general topology from the very beginning are generalized. The aim of this paper is to define and investigate the concepts of neutrosophic β-compactness and neutrosophic β-closed spaces. Also, the concept of neutrosophic filterbases is presented and by using this, we characterize these new types of compactness and space. Additionally, the relationship between these new types and some different forms of compactness in neutrosophic topology is clarified and a comparison between them is established.

2. PRELIMINARIES

In this section, we present the basic definitions related to neutrosophic set theory.

2020 *Mathematics Subject Classification.* Primary: 54A05; Secondaries: 54C10 ; 54D30 ; 54D101.
Key words and phrases. Neutrosophic β-open set, Neutrosophic β-compactness Neutrosophic β-closed space, Neutrosophic filterbases.

Submitted on July 12th, 2024. Accepted on September 20th, 2024.
Communicated by Huseyin CAKALLI and Nazlım Deniz ARAL.

Definition 2.1. ([10]) A neutrosophic set A on the universe set X is defined as:

$$A = \{\langle x, T_A(x), I_A(x), F_A(x)\rangle : x \in X\},$$

where $T,\ I,\ F : X \to\ ]^-0, 1^+[$ and $^-0 \leq T_A(x) + I_A(x) + F_A(x) \leq 3^+$.

Scientifically, membership functions, indeterminacy functions and non-membership functions of a neutrosophic set take value from real standart or nonstandart subsets of $]^-0, 1^+[$. However, these subsets are sometimes inconvenient to be used in real life applications such as economical and engineering problems. On account of this fact, we consider the neutrosophic sets, whose membership function, indeterminacy functions and non-membership functions take values from subsets of $[0, 1]$.

Definition 2.2. ([7]) Let X be a nonempty set. If r, t, s are real standard or non standard subsets of $]^-0, 1^+[$ then the neutrosophic set $x_{r,t,s}$ is called a neutrosophic point in X given by

$$x_{r,t,s}(x_p) = \begin{cases} (r, t, s), & \text{if } x = x_p \\ (0, 0, 1), & \text{if } x \neq x_p \end{cases}$$

For $x_p \in$X, it is called the support of $x_{r,t,s}$, where r denotes the degree of membership value, t denotes the degree of indeterminacy and s is the degree of non-membership value of $x_{r,t,s}$.

Definition 2.3. ([9]) Let A be a neutrosophic set over the universe set X. The complement of A is denoted by A^c and is defined by: $A^c = \left\{\left\langle x, F_{\tilde{F}(e)}(x), 1 - I_{\tilde{F}(e)}(x), T_{\tilde{F}(e)}(x)\right\rangle : x \in X\right\}$. It is obvious that $[A^c]^c = A$.

Definition 2.4. ([9]) Let A and B be two neutrosophic sets over the universe set X. A is said to be a neutrosophic subset of B if $T_A(x) \leq T_B(x), I_A(x) \leq I_B(x), F_A(x) \geq F_B(x)$, every $xinX$. It is denoted by $A \subseteq B$. A is said to be neutrosophic soft equal to B if $A \subseteq B$ and $B \subseteq A$. It is denoted by $A = B$.

Definition 2.5. ([9]) Let F_1 and F_2 be two neutrosophic soft sets over the universe set X. Then their union is denoted by $F_1 \cup F_2 = F_3$ is defined by:

$$F_3 = \{\langle x, T_{F_3}(x), I_{F_3}(x), F_{F_3}(x) : x \in X\rangle\},$$

where

$$T_{F_3(x)} = \max\{T_{F_1(x)}, T_{F_2}(x)\},$$

$$I_{F_3(x)} = \max\{I_{F_1(x)}, I_{F_2}(x)\},$$

$$F_{F_3(x)} = \min\{F_{F_1(x)}, F_{F_2}(x)\}.$$

Definition 2.6. ([9]) Let F_1 and F_2 be two neutrosophic soft sets over the universe set X. Then their intersection is denoted by $F_1 \cap F_2 = F_4$ is defined by:

$$F_4 = \{\langle x, T_{F_4}(x), I_{F_4}(x), F_{F_4}(x) : x \in X\rangle\},$$

where

$$T_{F_4(x)} = \min\{T_{F_1(x)}, T_{F_2}(x)\},$$

$$I_{F_4(x)} = \min\{I_{F_1(x)}, I_{F_2}(x)\},$$

$$F_{F_4(x)} = \max\{F_{F_1(x)}, F_{F_2}(x)\}.$$

Definition 2.7. ([9]) A neutrosophic set F over the universe set X is said to be a null neutrosophic set if $T_F(x) = 0$, $I_F(x) = 0$, $F_F(x) = 1$, every $x \in X$. It is denoted by 0_X.

Definition 2.8. ([9]) A neutrosophic set F over the universe set X is said to be an absolute neutrosophic set if $T_F(x) = 1$, $I_F(x) = 1$, $F_F(x) = 0$, every $x \in X$. It is denoted by 1_X.

Clearly $0_X^c = 1_X$ and $1_X^c = 0_X$.

Definition 2.9. ([9]) Let $NS(X)$ be the family of all neutrosophic sets over the universe the set X and $\tau \subset NS(X)$. Then τ is said to be a neutrosophic topology on X if:
1) 0_X and 1_X belong to τ;
2) the union of any number of neutrosophic soft sets in τ belongs to τ;
3) the intersection of a finite number of neutrosophic soft sets in τ belongs to τ.
Then (X, τ) is said to be a neutrosophic topological space over X. Each member of τ is said to be a neutrosophic open set [9].

Definition 2.10. ([9]) Let (X, τ) be a neutrosophic topological space over X and F be a neutrosophic set over X. Then F is said to be a neutrosophic closed set iff its complement is a neutrosophic open set.

Definition 2.11. ([2]) A neutrosophic point $x_{r,t,s}$ is said to be *neutrosophic quasi-coincident* (*neutrosophic q-coincident*, for short) with F, denoted by $x_{r,t,s}\ q\ F$ if and only if $x_{r,t,s} \not\subseteq F^c$. If $x_{r,t,s}$ is not neutrosophic quasi-coincident with F, we denote by $x_{r,t,s}\ \tilde{q}\ F$.

Definition 2.12. ([2]) A neutrosophic set F in a neutrosophic topological space (X, τ) is said to be a *neutrosophic q-neighborhood* of a neutrosophic point $x_{r,t,s}$ if and only if there exists a neutrosophic open set G such that $x_{r,t,s}\ q\ G \subset F$.

Definition 2.13. ([2]) A neutrosophic set G is said to be *neutrosophic quasi-coincident* (*neutrosophic q-coincident*, for short) with F, denoted by $G\ q\ F$ if and only if $G \not\subseteq F^c$. If G is not neutrosophic quasi-coincident with F, we denote by G $\tilde{q}\ F$.

Definition 2.14. ([3]) A neutrosophic point $x_{r,t,s}$ is said to be a neutrosophic interior point of a neutrosophic set F if and only if there exists a neutrosophic open q-neighborhood G of $x_{r,t,s}$ such that $G \subset F$. The union of all neutrosophic interior points of F is called the neutrosophic interior of F and denoted by F°.

Definition 2.15. ([2]) A neutrosophic point $x_{r,t,s}$ is said to be a *neurosophic cluster point* of a neutrosophic set F if and only if every neutrosophic open q-neighborhood G of $x_{r,t,s}$ is q-coincident with F. The union of all neutrosophic cluster points of F is called the *neutrosophic closure* of F and denoted by F^-.

Definition 2.16. ([2]) Let f be a function from X to Y. Let B be a neutrosophic set in Y with members hip function $T_B(y)$, indeterminacy function $I_B(y)$ and non-membership function $F_B(y)$. Then, the inverse image of B under f, written as $f^{-1}(B)$, is a neutrosophic subset of X whose membership function, indeterminacy function and non-membership function are defined as $T_{f^{-1}(B)}(x) = T_B(f(x))$,

$I_{f^{-1}(B)}(x) = I_B(f(x))$ and $F_{f^{-1}(B)}(x) = F_B(f(x))$ for all x in X, respectively. Conversely, let A be a neutrosophic set in X with membership function $T_A(x)$, indeterminacy function $I_A(x)$ and non-membership function $F_A(x)$. The image of A under f, written as $f(A)$, is a neutrosophic subset of Y whose membership function, indeterminacy function and non-membership function are defined as

$$T_{f(A)}(y) = \begin{cases} sup_{z\in f^{-1}(y)}\{T_A(z)\}, & if\ f^{-1}(y)\ is\ not\ empty, \\ 0, & if\ f^{-1}(y)\ is\ empty, \end{cases}$$

$$I_{f(A)}(y) = \begin{cases} sup_{z\in f^{-1}(y)}\{I_A(z)\}, & if\ f^{-1}(y)\ is\ not\ empty, \\ 0, & if\ f^{-1}(y)\ is\ empty, \end{cases}$$

$$F_{f(A)}(y) = \begin{cases} sup_{z\in f^{-1}(y)}\{F_A(z)\}, & if\ f^{-1}(y)\ is\ not\ empty, \\ 0, & if\ f^{-1}(y)\ is\ empty, \end{cases}$$

for all y in Y, where $f^{-1}(y) = \{x : f(x) = y\}$, respectively.

3. Some Definitions

This section provides some new definitions that form the cornerstones of the sections that follow.

Definition 3.1. *A neutrosophic set F in a neutrosophic topological space (X, τ) is said to be*

Neutrosophic semiopen, if $F \subseteq \overline{F^\circ}$,
Neutrosophic preopen, $F \subseteq (\overline{F})^\circ$,
Neutrosophic β-open, $F \subseteq \overline{(\overline{F})^\circ}$. Equivalently, if there exists a neutrosophic preopen set A such that $A \subseteq F \subseteq \overline{A}$.

It is obvious that each neutrosophic semiopen and neutrosophic preopen neutrosophic set implies neutrosophic β-open.

Definition 3.2. *If, F be a neutrosophic set in neutrosophic topological space (X, τ) then, $F_p^- = \bigcap\{F : F \subseteq A,\ A\ is\ neutrosophic\ preclosed\}$ (resp. $F_p^\circ = \bigcup\{F : F \subseteq A,\ A\ is\ neutrosophic\ preopen\}$) is called a neutrosophic preclosure of F (resp. neutrosophic preinterior of F).*

Definition 3.3. *If, F be a neutrosophic set in neutrosophic topological space (X, τ) then, $F_s^- = \bigcap\{F : F \subseteq A,\ A\ is\ neutrosophic\ semiclosed\}$ (resp. $F_s^\circ = \bigcup\{F : F \subseteq A,\ A\ is\ neutrosophic\ semiopen\}$) is called a neutrosophic semiclosure of F (resp. neutrosophic semi-interior of F).*

Definition 3.4. *If, F be a neutrosophic set in neutrosophic topological space (X, τ) then, $F_\beta^- = \bigcap\{F : F \subseteq A,\ A\ is\ neutrosophic\ \beta - closed\}$ (resp. $F_s^\circ = \bigcup\{F : F \subseteq A,\ A\ is\ neutrosophic\ \beta - open\}$) is called a neutrosophic β-closure of F (resp. neutrosophic β-interior of F).*

It is obvious that $(F^c)_\beta^- = (F_\beta^\circ)^c$ and $(F^c)_\beta^\circ = (F_\beta^-)^c$.

Definition 3.5. *Let $f : (X, \tau) \to (Y, \sigma)$ be a function from a neutrosophic topological space (X, τ) into a neutrosophic topological space (Y, σ). The function f is said to be neutrosophic β-continuous, if $f^{-1}(A)$ is a neutrosophic β-open set of X, for each $A \in \sigma$.*

Definition 3.6. *Let $f : (X, \tau) \to (Y, \sigma)$ be a function from a neutrosophic topological space (X, τ) into a neutrosophic topological space (Y, σ). The function f is said to be neutrosophic $M\beta$-continuous, if $f^{-1}(A)$ is a neutrosophic β-open set of X, for each neutrosophic β-open set A in (Y, σ).*

Lemma 3.1. *Let $f : (X, \tau) \to (Y, \sigma)$ be a function from a neutrosophic topological space (X, τ) into a neutrosophic topological space (Y, σ). Then the following are equivalent:*

a) f is neutrosophic $M\beta$-continuous.
b) $f(F^{-}_{\beta}) \subseteq (f(F))^{-}_{\beta}$, for every neutrosophic set F in (X, τ):

Proof. *a)* $\implies$ *b)*: Let F be a neutrosophic set in (X, τ), then $f(F))^{-}_{\beta}$ is neutrosophic β-closed. By (a), $f^{-1}(f(F))^{-}_{\beta}$ is neutrosophic β-closed and so $f^{-1}(f(F))^{-}_{\beta} = (f^{-1}(f(F))^{-}_{\beta})^{-}_{\beta}$. Since $F \subseteq f^{-1}(f(F))$, $F^{-}_{\beta} \subseteq (f^{-1}(f(F)))^{-}_{\beta} \subseteq (f^{-1}((f(F))^{-}_{\beta}))^{-}_{\beta} = f^{-1}(f(F))^{-}_{\beta})$. Hence $f(F^{-}_{\beta}) \subseteq (f(F))^{-}_{\beta}$.
b) $\implies$ *a)*: Let G be a neutrosophic β-closed in (Y, σ): By (b), if $F = f^{-1}(G)$; then $(f^{-1}(G))^{-}_{\beta} \subseteq f^{-1}((f(f^{-1}(G)))^{-}_{\beta}) \subseteq f^{-1}(G^{-}_{\beta}) = f^{-1}(G)$. Since $f^{-1}(G) \subseteq (f^{-1}(G))^{-}_{\beta}$, then $f^{-1}(G) = (f^{-1}(G))^{-}_{\beta}$. Hence $f^{-1}(G)$ is a neutrosophic β-closed in set in (X, τ). Hence f is neutrosophic $M\beta$-continuous. □

Lemma 3.2. *Let $f : (X, \tau) \to (Y, \sigma)$ be a function from a neutrosophic topological space (X, τ) into a neutrosophic topological space (Y, σ). Then the following are equivalent:*

a) f is neutrosophic β-continuous.
b) $f(F^{-}_{\beta}) \subseteq (f(F))^{-}$, for every neutrosophic set F in (X, τ):

Proof. Obvious. □

Theorem 3.3. *If $f : (X, \tau) \to (Y, \sigma)$ is neutrosophic open function*[5], *then $f^{-1}(G^{-}) \subseteq (f^{-1}(G))^{-}$ for every neutrosophic set G in (Y, σ).*

Definition 3.7. *A collection of neutrosophic subsets ω of a neutrosophic topological space (X, τ) is said to form a neutrosophic filterbases if and only if, for every finite collection $\{F_i : i = 1, ..., n\}$, $\bigcap_{i=1}^{n} F_i \neq 0_x$.*

Definition 3.8. *A collection φ of neutrosophic sets in a neutrosophic topological space (X, τ) is said to be cover of a neutrosophic set G of X if and only if, $T_{(\cup_{F \in \varphi} F)}(x) = 0$, $I_{(\cup_{F \in \varphi} F)}(x) = 0$ and $F_{(\cup_{F \in \varphi} F)}(x) = 0$, where x is any support in G. A neutrosophic cover φ of a neutrosophic set G in a neutrosophic topological space (X, τ) is said to have a finite subcover if and only if, there exists a finite subcollection $\rho = \{F_1, ..., F_n\}$ of φ such that $T_{(\bigcup_{i=1}^{n} F_i)}(x) \geq T_G(x)$, $I_{(\bigcup_{i=1}^{n} F_i)}(x) \geq I_G(x)$, and $F_{(\bigcup_{i=1}^{n} F_i)}(x) \geq F_G(x)$, where x is any support in G.*

Definition 3.9. *A neutrosophic topological space (X, τ) is said to be neutrosophic strongly compact if and only if, every neutrosophic preopen cover of X has a finite subcover.*

Definition 3.10. *A neutrosophic topological space (X, τ) is said to be neutrosophic semicompact if and only if, every neutrosophic semiopen cover of X has a finite subcover.*

Definition 3.11. *A neutrosophic topological space (X, τ) is said to be neutrosophic almost compact if and only if, every neutrosophic open cover of X has a finite subcollection whose closures cover X .*

Definition 3.12. *A neutrosophic topological space (X, τ) is said to be neutrosophic S-closed if and only if, every neutrosophic semiopen cover of X has a finite subcollection whose closures cover X.*

Definition 3.13. *A neutrosophic topological space (X, τ) is said to be neutrosophic s-closed if and only if, every neutrosophic semiopen cover of X has a finite subcollection whose semiclosures cover X.*

Definition 3.14. *A neutrosophic topological space (X, τ) is said to be neutrosophic P-closed if and only if, every neutrosophic preopen cover of X has a finite subcollection whose preclosures cover X.*

4. Neutrosophic β-Compact Space

In this subheading, we introduce the term neutrosophic β-compactness, which forms the basis of our study. Then, we examine the relationship of this new concept we introduced with other concepts introduced for the first time in this study and conduct an in-depth research on its properties.

Definition 4.1. *A neutrosophic topological space (X, τ) is said to be neutrosophic β-compact if and only if, for every family φ of neutrosophic β-open sets such that $\cup_{G\in\varphi} G = 1_X$, there is a finite subfamily $\omega \subseteq \varphi$ such that $\cup_{G\in\omega} G = 1_X$.*

Definition 4.2. *A neutrosophic set δ in a neutrosophic topological space (X, τ) is said to be neutrosophic β -compact relative to (X, τ) if and only if, for every family φ of neutrosophic β-open sets such that $T_{(\cup_{F\in\varphi} F)}(x) \geq T_\delta(x)$, $I_{(\cup_{F\in\varphi} F)}(x) \geq I_\delta(x)$ and $F_{(\cup_{F\in\varphi} F)}(x) \leq F_\delta(x)$, where x is any support in G, there is a finite subfamily $\rho \subseteq \varphi$ such that $T_{(\cup_{F\in\rho} F)}(x) \geq T_\delta(x)$, $I_{(\cup_{F\in\rho} F)}(x) \geq I_\delta(x)$ and $F_{(\cup_{F\in\rho} F)}(x) \leq F_\delta(x)$, where x is any support in G.*

Remark. *Since each of neutrosophic semiopenness and neutrosophic preopenness implies neutrosophic β-openness, it is clear that every neutrosophic β-compactness implies each of neutrosophic strongly compactness and neutrosophic semicompactness. But the converse need not to be true.*

Theorem 4.1. *A neutrosophic topological space (X, τ) is neutrosophic β-compact if and only if, for every collection $\{F_i : i \in I\}$ of neutrosophic β-closed neutrosophic sets in (X, τ) having the finite intersection property $\bigcap_{i\in I} F_i \neq 0_x$.*

Proof. Let $\{F_i : i \in I\}$ be a collection of neutrosophic β-closed sets with the finite intersection property. Suppose that $\bigcap_{i\in I} F_i = 0_X$. Then, $\bigcup_{i\in I} F_i^c = 1_X$. Since $\{F_i^c : i \in I\}$ is a collection of neutrosophic β-open sets cover of X, then from the neutrosophic β-compactness of X it follows that there exists a finite subset $J \subseteq I$ such that $\bigcup_{j\in J} F_j^c = 1_X$. Then, $\bigcap_{j\in J} F_j = 0_X$, which gives a contradiction and therefore $\bigcap_{i\in I} F_i \neq 0_X$.
Conversely, Let $\{F_i : i \in I\}$ be a collection of neutrosophic β-open sets cover of

X. Suppose that for every finite subset $J \subseteq I$, we have $\bigcup_{j\in J} F_j \neq 1_X$. Then, $\bigcap_{j\in J} F_J^c \neq 0_X$. Hence $\{F_i^c : i \in I\}$ satisfies the finite intersection property. Then, from the hypothesis we have $\bigcap_{i\in I} F_i^c \neq 0_X$, which implies $\bigcup_{j\in J} F_j \neq 1_X$ and this contradicting that $\{F_i : i \in I\}$ is a neutrosophic β-open cover of X. Thus, X is neutrosophic β-compact. □

Now, we give some results of neutrosophic β-compactness in terms of neutrosophic filterbases.

Theorem 4.2. *A neutrosophic topological space (X, τ) is neutrosophic β-compact if and only if, every filterbases μ in (X, τ), $\bigcap_{H\in\mu} H^{\bar{}}_\beta \neq 0_X$.*

Proof. Let φ be a neutrosophic β-open cover of X and φ have no a finite subcover. Then, for every finite subcollection $\{F_1, ..., F_n\}$ of φ, there exists $x \in X$ such that $T_{F_j}(x) < 1$ or $I_{F_j}(x) < 1$ or $F_{F_j}(x) > 0$ for every $j = 1, ..., n$. Then, $T_{F_j^c}(x) > 0$ or $I_{F_j^c}(x) > 0$ or $F_{F_j^c}(x) < 1$. So, $\bigcap_{j\in J} F_j^c \neq 0_X$. $\{F_j^c : F_j \in \varphi\}$ forms a filterbases in (X, τ). Since φ is neutrosophic β-open cover of X , then $T_{F_j}(x) = 1$, $I_{F_j}(x) = 1$ and $F_{F_j}(x) = 0$ for every $x \in X$ and hence $\bigcap_{F_j\in\varphi}(F_j^c)^{\bar{}}_\beta = \bigcap_{F_j\in\varphi} F_j^c = 0_X$ which is a contradiction. Then, every neutrosophic β-open cover of X has a finite subcover and hence X is neutrosophic β-compact.
Conversely, suppose there exists a filterbases μ such that $\bigcap_{H\in\mu} H^{\bar{}}_\beta = 0_X$. So, $\bigcup_{H\in\mu}(H^{\bar{}}_\beta)^c = 1_X$. Hence, $\{(H^{\bar{}}_\beta)^c : H \in \mu\}$ is a neutrosophic β-open cover in (X, τ). Since X is neutrosophic β-compact, then there exists a finite subcover $\{((H_1)^{\bar{}}_\beta)^c, \ldots\ldots\ldots\ldots\ldots, ((H_n)^{\bar{}}_\beta)^c\}$. Then, $\bigcup_{i=1}^n((H_1)^{\bar{}}_\beta)^c = 1_X$ and hence $\bigcup_{i=1}^n(H_i)^c = 1_X$. So, $\bigcap_{i=1}^n H_i = 0_X$, which is a contradiction, since the H_i are members of filterbases μ. Therefore, $\bigcap_{H\in\mu} H^{\bar{}}_\beta \neq 0_X$ every filterbases μ. □

Theorem 4.3. *A neutrosophic set α in a neutrosophic topological space (X, τ) is neutrosophic β-compact relative to X if and only if, for every filterbase μ such that every finite number of members of μ is neutrosophic quasi-coincident with α,*

$$(\bigcap_{H\in\mu} H^{\bar{}}_\beta) \cap \alpha \neq 0_X.$$

Proof. Suppose that α is not be neutrosophic β-compact relative to X, then there exists a neutrosophic β-open cover φ of α, which has a finite subcover. Then, for every finite subcover δ of φ, $\alpha \nsubseteq \bigcup_{H_i\in\delta} H_i$. So, $\bigcap_{H_i\in\delta} H_i^c \nsubseteq \alpha^c \neq 0_X$. Then, $\mu = \{H_i)^c : H_i \in \varphi\}$ forms a filterbases and $(\bigcap_{H_i\in\delta} H_i^c)q\alpha$. From our assumption, $(\bigcap_{H_i\in\delta}(H_i^c)^{\bar{}}_\beta) \cap \alpha \neq 0_X$. Clearly, $(\bigcap_{H_i\in\delta} H_i^c \cap \alpha \neq 0_X$. Then, $(\bigcap_{H_i\in\delta} H_i^c \neq 0_X$. This implies that $(\bigcup_{H_i\in\delta} H_i \neq 1_X$. From this contradiction, α is neutrosophic β-compact relative to X. Conversely, suppose that there exists a filterbases μ such that every finite number of members of μ is neutrosophic quasi-coincident with α and $(\bigcap_{H\in\mu} H^{\bar{}}_\beta) \cap \alpha = 0_X$.
This implies $\alpha \subseteq (\bigcap_H \in \mu H^{\bar{}}_\beta)^c$. So, $\alpha \subseteq \bigcup_H \in \mu(H^{\bar{}}_\beta)^c$. Then, $\varphi = \{(H^{\bar{}}_\beta)^c : H \in \mu\}$ is neutrosophic β-open cover of α. Since α is neutrosophic β-compact relative to X, then there exists a finite subcover δ of φ, say $\delta = \{((H_i)^{\bar{}}_\beta)^c, \ldots\ldots\ldots\ldots\ldots, ((H_n)^{\bar{}}_\beta)^c\}$, such that $\alpha \subseteq \bigcup_{i=1}^n((H_i)^{\bar{}}_\beta)^c$. Then, $\bigcap_{i=1}^n(H_i)^{\bar{}}_\beta \subseteq \alpha^c$. This means that $(\bigcap_{i=1}^n(H_i)^{\bar{}}_\beta)\tilde{q}\alpha$. From thi contradiction, it is clear that, for every filterbase μ such that every finite number of members of μ is neutrosophic quasi-coincident with α, $(\bigcap_{H\in\mu} H^{\bar{}}_\beta) \cap \alpha \neq 0_X$. □

Theorem 4.4. *Every neutrosophic β-closed subset of a neutrosophic β-compact space is neutrosophic β-compact relative to X.*

Proof. Let φ be a neutrosophic filterbases in X such that $\alpha q(\bigcap\{H : H \in \delta\})$ holds for every finite subcollection δ of φ and a neutrosophic β-closed set α. Consider $\varphi^* = \{\alpha\} \cup \varphi$. For any finite subcollection δ^* of φ^*, if $\alpha \notin \delta^*$ then $\bigcap \delta^* \neq 0_X$. If $\alpha \in \delta^*$ and since $\alpha q(\bigcap\{H : H \in \delta^* - \{\alpha\}\})$, then $\bigcap \delta^* \neq 0_X$. Hence δ^* is a neutrosophic filterbases in X. Since X is neutrosophic β-compact, then $(\bigcap_H \in \varphi^*(\bar{H_\beta}) \neq 0_X$. So, $(\bigcap_H \in \varphi(\bar{H_\beta}) \cap \alpha = (\bigcap_H \in \varphi(\bar{H_\beta}) \cap \bar{\alpha_\beta} \neq 0_X$. Hence by Theorem 4.3, we have α is neutrosophic β-compact relative to X. □

Theorem 4.5. *If a function $f : (X, \tau) \to (Y, \sigma)$ is neutrosophic $M\beta$-continuous and α is neutrosophic β-compact relative to X, then so is $f(\alpha)$.*

Proof. Let $\{F_i : i \in I\}$ be a neutrosophic β-open cover of $f(\alpha)$. Since f is neutrosophic $M\beta$-continuous, $\{f^{-1}(F_i) : i \in I\}$ is neutrosophic β-open cover of α. Since α is neutrosophic β-compact relative to X, there is a finite subfamily $\{f^{-1}(F_i) : i = 1, \ldots\ldots, n\}$ such that $\alpha \subseteq \bigcup_{i=1}^{n} f^{-1}(F_i)$. Then, $f(\alpha) \subseteq f(\bigcup_{i=1}^{n} f^{-1}(F_i)) \subseteq \bigcup_{i=1}^{n} F_i$. Therefore, $f(\alpha)$ is neutrosophic β-compact relative to Y. □

Lemma 4.6. *If $f : (X, \tau) \to (Y, \sigma)$ is neutrosophic open and neutrosophic continuous function, then f is neutrosophic $M\beta$-continuous.*

Proof. Let ω be a neutrosophic β-open set in Y; then $\omega \subseteq \overline{(\overline{\omega})^\circ}$. So, $f^{-1}(\omega) \subseteq f^{-1}(\overline{(\overline{\omega})^\circ}) \subseteq \overline{f^{-1}((\overline{\omega})^\circ)}$. Since f is neutrosophic continuous[5], then $f^{-1}((\overline{\omega})^\circ) = (f^{-1}((\overline{\omega})^\circ))$. Also by Theorem 3.3, $f^{-1}\underline{((\overline{\omega})^\circ) = (f^{-1}((\overline{\omega})^\circ))^\circ} \subseteq (f^{-1}(\overline{\omega}))^\circ \subseteq \overline{(f^{-1}(\omega))^\circ}$. Thus, $f^{-1}(\omega) \subseteq \overline{f^{-1}((\overline{\omega})^\circ)} \subseteq \overline{((\overline{f^{-1}(\omega)}))^\circ})$. This implies that $f^{-1}(\omega)$ is neutrosophic β-open. So, f is neutrosophic $M\beta$-continuous. □

Corollary 4.7. *Let $f : (X, \tau) \to (Y, \sigma)$ be neutrosophic open and neutrosophic continuous function. Consider that X is neutrosophic β-compact. Then, $f(X)$ is neutrosophic β-compact .*

Proof. It is follows directly from Lemma 4.6 and Theorem 4.5. □

Definition 4.3. *A function $f : (X, \tau) \to (Y, \sigma)$ is said to be neutrosophic $M\beta$-open if and only if, the image of every neutrosophic β-open set in X is neutrosophic β-open in Y.*

Theorem 4.8. *Let $f : (X, \tau) \to (Y, \sigma)$ be a neutrosophic $M\beta$-open bijective function and Y is neutrosophic β-compact, then X neutrosophic β-compact.*

Proof. Let $\{F_i : i \in I\}$ be a neutrosophic β-open cover of X. Then, $\{f(F_i) : i \in I\}$ is a neutrosophic β-open cover of Y. Since Y is neutrosophic β-compact, there is a finite subset $J \subseteq I$ such that $\{F_j : j \in J\}$ is a neutrosophic β-open cover of Y. So, $1_Y = \bigcup_{j \in J} f(F_i)$. Since f is a neutrosophic $M\beta$-open bijective function, $1_X = f^{-1}(1_Y) = f^{-1}(f(\bigcup_{j \in J} F_j)) = \bigcup_{j \in J} F_j$. Therefore, X is neutrosophic β-compact. □

5. Neutrosophic β-Closed Spaces

Definition 5.1. *A neutrosophic set α in a neutrosophic topological space (X,τ) is said to be a neutrosophic βq-neighborhood of a neutrosophic point $x_{r,t,s}$ in X, if there exists a neutrosophic β-open set $F \subseteq \alpha$ such that $x_{r,t,s}qF$.*

Proof. Let $x_{r,t,s} \in \bar{\alpha_\beta}$ and there exists a neutrosophic βq-neighborhood G of $x_{r,t,s}$, $G\tilde{q}\alpha$. Then there exists a neutrosophic open set $F \subseteq G$ in X such that $x_{r,t,s}qF$, which implies $F\tilde{q}\alpha$ and hence $\alpha \subseteq F^c$. Since F^c neutrosophic β-closed, $\bar{\alpha_\beta} \subseteq F^c$. From $x_{r,t,s} \notin F^c$, we see that $x_{r,t,s} \notin \bar{\alpha_\beta}$. This is a contradiction. Conversely, let $x_{r,t,s} \notin \bar{\alpha_\beta} = \bigcap\{F : F is \beta - closed in X, \alpha \subseteq F\}$. Then, there exists a neutrosophic β-closed set F such that $x_{r,t,s} \notin F$ and $\alpha \subseteq F$. Hence, $x_{r,t,s}qF^c$ and $\alpha\tilde{q}F^c$, where F^c is neutrosophic β-open in X. Then, F^c is a neutrosophic βq-neighborhood of $x_{r,t,s}$ and $\alpha\tilde{q}F^c$. □

Definition 5.2. *A neutrosophic topological space (X,τ) is said to be neutrosophic β-closed if and only if, for every family φ of neutrosophic β-open sets such that $\bigcup_{H\in\varphi} H = 1_X$, there is a finite subfamily $\delta \subseteq \varphi$ such that $\bigcup_{H\in\delta} \bar{H_\beta} = 1_X$.*

Remark. *From the above definition and other types of neutrosophic compactness, one can draw the following diagram:*

$$\begin{array}{lll} Neutrosophic\ semi-compact & \rightarrow Neutrosophic\ s-closed & \rightarrow Neutrosophic\ S-closed \\ \downarrow & \downarrow & \\ Neutrosophic\ \beta-compact & \rightarrow Neutrosophic\ \beta-closed & \\ \downarrow & \downarrow & \\ Neutrosophic\ strongly\ compact & \rightarrow Neutrosophic\ P-closed & \end{array}$$

Example 5.1. *Let (X,τ) be a neutrosophic topological and neutrosphic sets α_n be defined as $\alpha_n = \{\langle x, 1-\frac{1}{n}, 1-\frac{1}{n}, \frac{1}{n}\rangle : x \in X\}$ for each $n \in N^+$. Consider a neutrosophic topological space (X,τ) that $\{\alpha_n : n \in N^+\}$ is a neutrosophic base for τ. Then, $\{\alpha_n : n \in N^+\}$ is obviously a neutrosophic β-open cover of X. In (X,τ), $\bar{(\alpha_n)_\beta} = 1_X$ for each $n \geq 3$. Then, X is neutrosophic β-closed but not neutrosophic β-compact.*

Remark. *Example 5.1 also shows that:*

(i) Each of the concepts neutrosphic s-closed, neutrosphic S-closed and neutrosphic P-closed spaces does not imply neutrosphic β-compact.
(ii) Since $\{\alpha_n : n \in N^+\}$ is also neutrosophic semiopen cover of X, then X is also neutrosophic β-closed space but not neutrosophic semi-compact space.
(iii) Since $\{\alpha_n : n \in N^+\}$ is also neutrosophic preopen cover of X, then X is also neutrosophic β-closed space but not neutrosophic strongly compact space.

Example 5.2. *Let $X = [0,1]$ and consider the following neutrosophic sets $\alpha_1 = \{\langle x, \frac{1,7}{\sqrt{3}}, \frac{1,7}{\sqrt{3}}, 1-\frac{1,7}{\sqrt{3}}\rangle : x \in X\}$, $\alpha_2 = \{\langle x, \frac{1,73}{\sqrt{3}}, \frac{1,73}{\sqrt{3}}, 1-\frac{1,73}{\sqrt{3}}\rangle : x \in X\}$, $\alpha_3 = \{\langle x, \frac{1,732}{\sqrt{3}}, \frac{1,732}{\sqrt{3}}, 1-\frac{1,732}{\sqrt{3}}\rangle : x \in X\}$, $\alpha_4 = \{\langle x, \frac{1,7320}{\sqrt{3}}, \frac{1,7320}{\sqrt{3}}, 1-\frac{1,7320}{\sqrt{3}}\rangle : x \in X\}$, $\alpha_5 = \{\langle x, \frac{1,73205}{\sqrt{3}}, \frac{1,73205}{\sqrt{3}}, 1-\frac{1,73205}{\sqrt{3}}\rangle : x \in X\}$, where $\sqrt{3} = 1,7320508075688\ldots\ldots$ Let $\varphi = \{\alpha_i : i \in N^+\} \cup \{0_X, 1_X\}$. It is clear that φ is a neutrosophic topology on*

X. Now, the collection $\{\alpha_i : i \in N^+\}$ is a neutrosophic semiopen (resp. neutrosophic preopen) cover of X but not has a finite subcover. So X is not neutrosophic semicompact space (resp. neutrosophic strongly compact space). Since the neutrosophic semi-closure (resp. neutrosophic pre-closure) of every neutrosophic semiopen (resp. neutrosophic preopen) set of X is 1_X, then X is neutrosophic s-closed (resp. neutrosophic P-closed).

Remark. *Example 5.2 is also shows that each of the concepts neutrosophic S-closed and neutrosophic P-closed spaces does not imply each of neutrosophic semicompact and neutrosophic strongly compact spaces.*

Remark. *From the remark just after, Definition 4.2, Example 5.1, the remark just after Example 5.1, Example 5.2 and the remark just after Example 5.2, it is clear that:*
(i) Neutrosophic S-closed and neutrosophic P-closed spaces are independent notions.
(ii) Neutrosophic S-closed and neutrosophic strongly compact spaces are independent notions.
(iii) Neutrosophic P-closed and neutrosophic semicompact spaces are independent notions.
(iv) Neutrosophic β-compact, neutrosophic semicompact and neutrosophic strongly compact spaces are independent notions.

Theorem 5.1. *A neutrosophic topological space (X, τ) is neutrosophic β-closed if and only if, for every neutrosophic β-open filterbases φ in (X, τ), $\bigcap_{H\in\varphi} H^-_\beta \neq 0_X$.*

Proof. Let δ be a neutrosophic β-open cover of X and let for every finite subfamily ρ of δ, $\bigcup_{B\in\rho} B^-_\beta \neq 1_X$. Then, $0_X \subset \bigcap_{B\in\rho}(B^-_\beta)^c$. Thus, $\varphi = \{(B^-_\beta)^c : B \in \delta\}$ forms a neutrosophic β-open filterbases in X. Since δ is a neutrosophic β-open cover of X, $\bigcap_{B\in\delta} B^c = 0_X$. For $B \subseteq B^-_\beta$, $(B^-_\beta)^c \subseteq B^c$. As B^c is neutrosophic β-closed, $((B^-_\beta)^c)^-_\beta \subseteq B^c$. So, $\bigcap_{B\in\delta}((B^-_\beta)^c)^-_\beta = 0_X$. This contradicts with our assumption. This means that every neutrosophic β-open cover of X has a finite subfamily ρ such that $\bigcup_{B\in\rho} B^-_\beta = 1_X$. Hence, X is neutrosophic β-closed.
Conversely, suppose there exists a neutrosophic β-open filterbases φ in X such that $\bigcap_{H\in\varphi} H^-_\beta = 0_X$. So, $\bigcup_{H\in\varphi}(H^-_\beta)^c = 1_X$. Then, $\delta = \{(H^-_\beta)^c : B \in \varphi\}$ is a neutrosophic β-open cover of X. Since X is neutrosophic β-closed, then δ has a finite subfamily ρ such that $\bigcup_{H\in\rho}((H^-_\beta)^c)^-_\beta = 1_X$. So, $\bigcap_{H\in\rho}(((H^-_\beta)^c)^-_\beta)^c = 0_X$. Thus, $\bigcap_{H\in\rho} H = 0_X$. This contradicts with our assumption that φ is a neutrosophic filterbese in (X, τ). □

Definition 5.3. *A neutrosophic set α in a neutrosophic topological space (X, τ) is said to be neutrosophic β-closed relative to X if and only if, for every family δ of neutrosophic β-open sets such that $\bigcup_{B\in\delta} B = \alpha$, there is a finite subfamily $\rho \subseteq \delta$ such that $\alpha \subseteq \bigcup_{B\in\rho} B^-_\beta$.*

Theorem 5.2. *A neutrosophic set α in a neutrosophic topological space (X, τ) is neutrosophic β-closed relative to X if and only if, every neutrosophic β-open filterbases φ in (X, τ), $(\bigcap_{H\in\varphi} H^-_\beta) \cap \alpha = 0_X$, there exists a finite subfamily ω of φ such that $(\bigcap_{H\in\omega} H)\tilde{q}\alpha$.*

Proof. Let α be a neutrosophic β-closed relative to X; suppose φ is a neutrosophic β-open filterbases in (X, τ) such that for every finite subfamily ω of φ,

$(\bigcap_{H\in\omega} H)q\alpha$ but $(\bigcap_{H\in\varphi} H^{-}_{\beta})\cap\alpha = 0_X$. For every support x in α, $T_{(\bigcap_{H\in\varphi} H^{-}_{\beta})}(x) = 0$, $I_{(\bigcap_{H\in\varphi} H^{-}_{\beta})}(x) = 0$ and $F_{(\bigcap_{H\in\varphi} H^{-}_{\beta})}(x) = 1$. This implies that, for every support x in α, $T_{(\bigcap_{H\in\varphi}(H^{-}_{\beta})^c)}(x) = 1$, $I_{(\bigcap_{H\in\varphi}(H^{-}_{\beta})^c)}(x) = 1$ and $F_{(\bigcap_{H\in\varphi}(H^{-}_{\beta})^c)}(x) = 0$. Then, $\delta = \{(H^{-}_{\beta})^c) : B\in\varphi\}$ is neutrosophic β-open cover of α and hence there exists a finite subfamily $\omega\subseteq\varphi$ such that $\alpha\subseteq(\bigcup_{H\in\omega}((H^{-}_{\beta})^c)^{-}_{\beta}$. So, $\bigcap_{H\in\omega}(((H^{-}_{\beta})^c)^{-}_{\beta})^c\subseteq\alpha^c$. Then, $\bigcap_{H\in\omega}(((H^{-}_{\beta})^{\circ}_{\beta}\subseteq\alpha^c$. This means that $\bigcap_{H\in\omega} H\subseteq\alpha^c$. Therefore, $(\bigcap_{H\in\omega} H)\tilde{q}\alpha^c$. This is a contradiction.

Conversely, let α not be a neutrosophic β-closed set relative to X; then there exists a neutrosophic β-open cover δ of α such that every finite subfamily $\rho\subseteq\delta$, $\bigcup_{B\in\rho} B^{-}_{\beta}\nsubseteq\alpha$. Then, $\alpha^c\nsubseteq(B^{-}_{\beta})^c$. This implies that $\bigcap_{B\in\rho}(B^{-}_{\beta})^c\neq 0_X$. So, $\varphi = \{(B^{-}_{\beta})^c : B\in\delta\}$ forms a neutrosophic β-open filterbases in (X,τ). Let there exists a finite subfamily $\{(B^{-}_{\beta})^c : B\in\rho\}$ such that $(\bigcap_{B\in\rho}(B^{-}_{\beta})^c)\tilde{q}\alpha$. Then, $\alpha\subseteq\bigcup_{B\in\rho B^{-}_{\beta}}$. Hence, exists a finite subfamily $\rho\subseteq\delta$ such that $\alpha\subseteq\bigcup_{B\in\rho B^{-}_{\beta}}$, which is a contradiction. Then, for each finite subfamily $\omega = \{(B^{-}_{\beta})^c : B\in\rho\}$ of φ, we have $(\bigcap_{B\in\rho}(B^{-}_{\beta})^c)q\alpha$. From our assumption, $(\bigcap_{B\in\delta}((B^{-}_{\beta})^c)^{-}_{\beta}\cap\alpha\neq 0_X$. So, $\bigcap_{B\in\delta}(((B^{-}_{\beta})^c)^{-}_{\beta})^c\cup\alpha^c\neq 1_X$. Clearly, $\bigcap_{B\in\delta}(((B^{-}_{\beta})^c)^{-}_{\beta})^c\cup\alpha^c\neq 1_X$. Then, $\bigcup_{B\in\delta}(((B^{-}_{\beta})^{\circ}_{\beta}\neq 1_X$. So, $\bigcup_{B\in\delta} B\neq 1_X$. This contradicts with the fact that δ is a neutrosophic β-open cover of α. Therefore α is neutrosophic β-closed relative to X. □

Definition 5.4. *A neutrosophic set α in a neutrosophic topological space (X,τ) is said to be neutrosophic β-regular, if it is both neutrosophic β-open and neutrosophic β-closed set.*

Proposition 5.3. *If α is neutrosophic β-open set in (X,τ), then α^{-}_{β} is neutrosophic β-regular.*

Proof. Since α^{-}_{β} is neutrosophic β-closed, we must show that α^{-}_{β} is neutrosophic β-open. Since α is neutrosophic β-open in (X,τ), $\vartheta\subseteq\alpha\subseteq\vartheta^{-}$ holds for some neutrosophic preopen set ϑ in (X,τ). Therefore, we have $\vartheta\subseteq\vartheta^{-}_{\beta}\subseteq\alpha^{-}_{\beta}\subseteq\vartheta^{-}$ and hence α^{-}_{β} is neutrosophic β-open. □

Theorem 5.4. *For a neutrosophic topological space (X,τ), the following are equivalent:*

(a) (X,τ) is a neutrosophic β-space.
(b) Every neutrosophic β-regular cover of X has a finite subcover.
(c) For every collection $\{B_i : i\in I\}$ of neutrosophic β-regular sets such that $(\bigcap_{i\in I} B_i = 0_X$, there exists a finite subset $G\subseteq I$ such that $(\bigcap_{i\in G} B_i = 0_X$.

Proof. It is obvious from Proposition 5.3 and from the facts that, for every collection $\{B_i : i\in I\}$, $(\bigcup_{i\in I} B_i)^c = \bigcap_{i\in I}(B_i)^c$, $(\bigcap_{i\in I} B_i)^c = \bigcup_{i\in I}(B_i)^c$ and B is neutrosophic β-open set if and only if, B^c is neutrosophic β-closed set. □

Theorem 5.5. *Let $f : (X,\tau)\to(Y,\sigma)$ be a neutrosophic β-continuous surjection function. If (X,τ) is neutrosophic β-closed space, then (Y,σ) is neutrosophic almost compact.*

Proof. Let $\{B_i : i\in I\}$ be a neutrosophic open cover of Y. Then, $\{f^{-1}(B_i) : i\in I\}$ is a neutrosophic β-open cover of X. By hypothesis, there exists a finite subset $G\subseteq I$ such that $(\bigcup_{i\in G}(f^{-1}(B_i))^{-}_{\beta} = 1_X$. From the surjectivity of f and by Lemma 3.2,

$1_Y = f(1_X) = f((\bigcup_{i\in G}(f^{-1}(B_i))^{-}_{\beta}) \subseteq \bigcup_{i\in G}(f(f^{-1}(B_i)))^{-}_{\beta} = \bigcup_{i\in G}(B_i)^{-}$. Hence (Y,σ) is neutrosophic almost compact. □

Using Lemma 3.1, we have also the following theorem which can proved similarly to Theorem 5.5.

Theorem 5.6. *If $f : (X,\tau) \to (Y,\sigma)$ is neutrosophic $M\beta$-continuous surjection function and (X,τ) is neutrosophic β-closed space, then (Y,σ) is so.*

6. Conclusion

In our article, first of all, all the factors that make it necessary for us to do this study and the definitions that form the cornerstones of our study are given in the introduction and preliminaries section, respectively. In the third chapter, some types of open sets and continuities, which have already been introduced and whose properties have been examined in detail in general topology, but have never been introduced before in neutrosophic spaces, are given and some properties of these concepts are given. In the fourth and fifth sections, some types of space are given using the new concepts given in the third section, and the relationships between these spaces are shown through diagrams. In order to better demonstrate these relationships, reverse examples were also given.

Conflict of interests

The authors declare that there is no conflict of interests.

Acknowledgment

The authors admit that the statements of some results in this paper were presented at the Seventh International Conference of Mathematical Sciences (ICMS 2023), [4].

References

[1] M.E. Abd El-Monsef, S.N. El-Deeb, and R.A. Mahmoud, β-open sets and β-continuous Mappings, Bull. Fac. Sci. Assiut Univ. **12** (1983) 77-90.
[2] A. Acikgoz, H. Çakalli, F. Esenbel, and Lj. D.R. Kočinac, A quest of G-continuity in neutrosophic spaces, Math. Meth. Appl. Sci. **44** (9) (2021) 7834-7844.
[3] A. Acikgoz, and F. Esenbel, Neutrosophic Connected topological spaces, Fasc. Math. **66** (2023) 5-22.
[4] A. Acikgoz, F. Esenbel and O. Mucuk, Neutrosophication β-Compactness, Seventh International Conference of Mathematical Sciences (ICMS 2023) Abstract Book page 17 Maltepe University Istanbul Turkey.
[5] A. Acikgoz, and F. Esenbel F. Neutrosophic seperation axioms, Maltepe J Math. **5** (2), (2023) 32–40.
[6] B. Ganesan, On fuzzy β-compact spaces and fuzzy β-extremally disconnected spaces, Kybernetika [cybernetics]. **33**(3) (1997) 271-277.
[7] R. Dhavaseelan, S. Jafari, C. Ozel, and C.M. A Al-Shumrani, Generalized neutrosophic contra-continuity, In: F Smarandache, S Pramanik, eds. New Trends in Neutrosophic Theory and Applications Vol. II (2018) 365-380.
[8] I.M. Hanafy, Fuzzy β-Compactness and Fuzzy β-Closed Spaces, Turk J Math. **28** (2004) 281–293.
[9] A.A. Salama, and S.A. Alblowi, Neutrosophic set and neutrosophic topological spaces, IOSR J. Math. **3** (2012) 31-35.

[10] F. Smarandache, Neutrosophic set, a generalisation of the intuitionistic fuzzy sets, Int. J. Pure Appl. Math. **24** (2005) 287-297.

Ahu Acikgoz,
Department of Mathematics, Balikesir University,10145 Balikesir, Turkey
Email address: `ahuacikgoz@gmail.com`

Ferhat Esenbel,
Department of Mathematics, Balikesir University,10145 Balikesir, Turkey
Email address: `fesenbel@gmail.com`

Osman Mucuk,
Department of Mathematics, Erciyes University,32280 Kayseri, Turkey
Email address: `mucuk@erciyes.edu.tr`