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A FEW REMARKS ON FUZZY SOFT CONTINUOUS FUNCTIONS IN FUZZY SOFT TOPOLOGICAL SPACES

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Abstract – The aim of this paper is to appraise a few properties of fuzzy soft continuous functions and to define fuzzy soft compact set. Fuzzy soft continuous image of a fuzzy soft compact set is taken into account to be dealt with in this paper.

Keywords – *Soft sets, soft topology, soft bases, fuzzy soft functions, fuzzy soft compact sets.*

1 Introduction

Sometimes it is very complicated to come across precise solution of real life problems by using the classical mathematics. So in the present era such type problems are being solved approximately by using the concept of fuzzy set theory, soft set theory etc.

Fuzzy set theory, formulated by Zadeh [16] in 1965, is accepted as a potential mechanism for giving standardized technique to deal with uncertainties where classical theories fail to act upto. Thereafter a lot of works [5, 6, 15] have been done in this area during the last four decade. At that time a few uncertain problems were come out from engineering and computer sciences, which were not being solved by using the concepts of fuzzy set theory. To solve such problems, D. Molodtsov [11] formulated a new concept, viz. soft set theory, in 1999. Then many works [2, 3, 4, 8, 14] have been done in this field.

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Right now, different hybrid concepts are coming out, as a result the above said two concepts are combined together [9, 10] and it is termed as fuzzy soft set. Thereafter different notions have been generalized on fuzzy soft set and topology is one of them which has been generalized on it [1, 12, 13]. In fact, many researchers have engaged themselves to deal with the fuzzy soft function and to find its application on fuzzy soft topological spaces.

Fuzzy soft function was first originated by Kharal and Ahmad [7] in 2009. Then this definition has been amended by Atmaca and Zorlutuna [1] in 2013. For the simplicity, we have taken a small modification of this definition. Then we have established some theorems related to neighbourhood properties. In section 3, necessary and sufficient conditions have been established for a fuzzy soft function to be continuous in fuzzy soft topological spaces. In this section, also the notions of open mappings and closed mappings are being generalized for a fuzzy soft mapping and then a few properties of such mappings were established. In section 4, we have defined the concept of cover for a fuzzy soft set and compact fuzzy soft set, which are substantiated by considering three examples. The first example shows the existence of a cover of a fuzzy soft set. The second one is considered for the existence of an open cover of fuzzy soft set and the third one shows that every open cover may or may not have a finite subcover. Lastly, a theorem related to the continuous function and fuzzy soft compact set is taken into account to be dealt with in this paper.

2 Preliminary

This section contains some basic definitions and theorem which will be needed in the sequel. Throughout this paper, E is considered as the set of parameters and U being the course of universe.

Definition 2.1. [12] Let $A \subseteq E$. A fuzzy soft set over (U, E) is a mapping $F_A : E \to I^U$. For an element $e \in E$, we denote the image of e by $\mu^e_{F_A}$, where $\mu^e_{F_A} = \overline{0}$ if $e \in E \setminus A$ and $\mu^e_{F_A} \neq \overline{0}$ if $e \in A$.
The set of all fuzzy soft set over (U, E) is denoted by $FS(U, E)$.

Note 2.2. [12] If A is a null set, then F_A is said to be a null fuzzy soft set and it is denoted by Φ.

Definition 2.3. [12] A fuzzy soft set F_E is called absolute fuzzy soft set if $F_E(e) = \overline{1}$ for all $e \in E$. This fuzzy soft set is denoted by $\widetilde{E}$.

Definition 2.4. [12] Let F_A, $G_B \in FS(U, E)$. Then F_A is said to be fuzzy soft subset of G_B, denoted by $F_A \sqsubseteq G_B$ if $F_A(e) \subseteq G_B(e)$ for all $e \in A$.

Definition 2.5. [12] Let F_A, $G_B \in FS(U, E)$. Then the union of F_A and G_B is also a fuzzy soft set $F_A \sqcup G_B$, defined by $(F_A \sqcup G_B)(e) = F_A(e) \cup G_B(e)$ for all $e \in A \cup B$.
Following the arbitrary union of fuzzy subsets and the union of two fuzzy soft sets, the definition of arbitrary union of fuzzy soft sets can be described in the similarly fashion.

Definition 2.6. [12] Let F_A, $G_B \in FS(U, E)$. Then the intersection of F_A and G_B is also a fuzzy soft set $F_A \sqcap G_B$, defined by $(F_A \sqcap G_B)(e) = F_A(e) \cap G_B(e)$ for all $e \in A \cap B$.

Definition 2.7. [12] Let τ be a collection of some fuzzy soft sets over (U, E). Then τ is said to be a fuzzy soft topology on (U, E) if τ satisfies the following properties
1. $\Phi, \widetilde{E} \in \tau$.
2. If F_A, $G_B \in \tau$ then $F_A \sqcap G_B \in \tau$.
3. If $F_{A_\alpha} \in \tau$ for all $\alpha \in \Lambda$, an index set, then $\sqcup_{\alpha \in \Lambda} F_{A_\alpha} \in \tau$.

Definition 2.8. [12] If τ is a fuzzy soft topology on (U, E), the triple $(U,\ E,\ \tau)$ is said to be a fuzzy soft topological space. Also each member of τ is called a fuzzy soft open set in $(U,\ E,\ \tau)$.

Definition 2.9. [12] A subfamily β of τ is called a fuzzy soft open base or simply a base of fuzzy soft topological space $(U,\ E,\ \tau)$ if the following conditions hold:
1. $\Phi \in \beta$.
2. $\sqcup\beta = \widetilde{E}$ i.e. for each $e \in E$ and $x \in U$, there exists $F_A \in \beta$ such that $\mu^e_{F_A}(x) = 1$.
3. If $F_A,\ G_B \in \beta$ then for each $e \in E$ and $x \in U$, there exists $H_C \in \beta$ such that $H_C \sqsubseteq F_A \sqcap G_B$ and $\mu^e_{H_C}(x) = \min\{\mu^e_{F_A}(x),\ \mu^e_{G_B}(x)\}$, where $C \subseteq A \cap B$.

Theorem 2.10. [12] Let β be a fuzzy soft base for a fuzzy soft topology τ_β on (U, E). Then $F_A \in \tau_\beta$ if and only if $F_A = \sqcup_{\alpha\in\Lambda} B_{A_\alpha}$ where $B_{A_\alpha} \in \beta$ for each $\alpha \in \Lambda$, Λ an index set.

Definition 2.11. [13] A fuzzy soft point F_e over (U, E) is a special fuzzy soft set, defined by

$$F_e(a) = \begin{cases} \mu_{F_e} & if \quad a = e, \quad where \quad \mu_{F_e} \neq \overline{0} \\ \overline{0} & if \quad a \neq e \end{cases}$$

Definition 2.12. [13] Let F_A be a fuzzy soft set over (U, E) and G_e be a fuzzy soft point over (U, E). Then we say that $G_e \in F_A$ if and only if $\mu_{G_e} \subseteq \mu^e_{F_A} = F_A(e)$ i.e., $\mu_{G_e}(x) \leq \mu^e_{F_A}(x)$ for all $x \in U$.

Definition 2.13. [13] A fuzzy soft set F_A is said to be a neighborhood of a fuzzy soft point G_e if there exists $H_B \in \tau$ such that $G_e \in H_B \sqsubseteq F_A$.
Then clearly, every open fuzzy soft set is a neighborhood of each of its points.

Definition 2.14. [13] The union of all fuzzy soft open subsets of F_A over (U, E) is called the interior of F_A and is denoted by $intF_A$.

2.1 Some Applications of Fuzzy Soft Functions

In this section we introduce some basic definitions and theorems of fuzzy soft functions.

In 2009, A Kharal and B. Ahmad first defined the fuzzy soft functions in their paper [7]. Then in 2013, S. Atmaca and I. Zorlutuna [1] have modified this definition. Here we also present this definition with a small modification as follows.

Definition 2.15. Let $FS(U, E)$ and $FS(V, E')$ be two collections of fuzzy soft sets over (U, E) and (V, E') respectively. Let $f : U \to V$ and $g : E \to E'$. Then a function $S : FS(U, E) \to FS(V, E')$ is defined by
$S(F_A)(\alpha) = \mu^\alpha_{S(F_A)}$ for all $F_A \in FS(U, E)$ and $\alpha \in E'$
where

$$\mu^\alpha_{S(F_A)}(y) = \begin{cases} \max\limits_{x\in f^{-1}(y),\, e\in g^{-1}(\alpha)} \mu^e_{F_A}(x) & \text{if } f^{-1}(y) \neq \phi \quad \text{and} \quad g^{-1}(\alpha) \neq \phi \\ 0 & \text{otherwise,} \end{cases}$$

for $y \in V$.
and $S^{-1}(H_B)(e) = \mu^e_{S^{-1}(H_B)}$ for all $H_B \in FS(V, E')$ and $e \in E$
where $\mu^e_{S^{-1}(H_B)}(x) = \mu^{g(e)}_{H_B}(f(x))$, for $x \in U$.

Definition 2.16. Let (U, E, τ) and (V, E', τ') be two fuzzy soft topological spaces and $S : (U, E, \tau) \rightarrow (V, E', \tau')$ be a fuzzy soft function. Then S is said to be
1. an open mapping if and only if $S(F_A) \in \tau'$ for all $F_A \in \tau$.
2. a closed mapping if and only if $S(F_A)$ is closed in (V, E', τ') for every fuzzy soft closed set F_A in (U, E, τ).
3. a continuous mapping [1] if and only if $S^{-1}(H_B) \in \tau$ for all $H_B \in \tau'$.

Theorem 2.17. Let $S : (U, E, \tau) \rightarrow (V, E', \tau')$ be a closed map. Then for any fuzzy soft set $H_B \in FS(V, E')$ and any fuzzy soft open set F_A containing $S^{-1}(H_B)$, there exists an open set I_C containing H_B such that $S^{-1}(I_C) \sqsubseteq F_A$.

Proof. Let $I_C = \widetilde{E}' \setminus S(\widetilde{E} \setminus F_A) = \widetilde{E}' \setminus S(F_A^c)$.
At first we show that $H_B \sqsubseteq I_C$.
Let $\alpha \in E'$ and $y \in V$. Now $I_C(\alpha) = \overline{1} - \mu^{\alpha}_{S(F_A^c)}$.
If $f^{-1}(y) = \phi$ or $g^{-1}(\alpha) = \phi$, then $\mu^{\alpha}_{S(F_A^c)}(y) = 0$. That is, $\mu^{\alpha}_{I_C}(y) = 1$. So $\mu^{\alpha}_{H_B}(y) \subseteq \mu^{\alpha}_{I_C}(y)$.
If not, then let $f(x) = y$ and $g(e) = \alpha$. Since $S^{-1}(H_B) \sqsubseteq F_A$, $\mu^e_{S^{-1}(H_B)}(x) = \mu^{g(e)}_{H_B}(f(x)) = \mu^{\alpha}_{H_B}(y) \leq \mu^e_{F_A}(x)$.

Now
$$\begin{aligned}\mu^{\alpha}_{I_C}(y) &= 1 - \mu^{\alpha}_{S(F_A^c)}(y)\\ &= 1 - \max\{\mu^e_{F_A^c}(x) : x \in f^{-1}(y) \text{ and } e \in g^{-1}(\alpha)\}\\ &= \min\{1 - \mu^e_{F_A^c}(x) : x \in f^{-1}(y) \text{ and } e \in g^{-1}(\alpha)\}\\ &= \min\{\mu^e_{F_A}(x) : x \in f^{-1}(y) \text{ and } e \in g^{-1}(\alpha)\}\\ &\geq \mu^{\alpha}_{H_B}(y).\end{aligned}$$
Therefore $\mu^{\alpha}_{I_C}(y) \geq \mu^{\alpha}_{H_B}(y)$ for all $\alpha \in E'$ and $y \in V$. So, $H_B \sqsubseteq I_C$.
Again since S is closed and F_A is a fuzzy soft open set in (U, E, τ), I_C is open set in (V, E', τ') containing H_B.
Now $S^{-1}(I_C) = S^{-1}(\widetilde{E'} \setminus S(F_A^c))$. Let $e \in E$ and $x \in U$.

Therefore
$$\begin{aligned}\mu^e_{S^{-1}(I_C)}(x) &= \mu^e_{S^{-1}(\widetilde{E'} \setminus S(F_A^c))}(x)\\ &= \mu^{g(e)}_{\widetilde{E'} \setminus S(F_A^c)}(f(x))\\ &= 1 - \mu^{g(e)}_{S(F_A^c)}(f(x))\\ &= 1 - \max\{\mu^{e_1}_{F_A^c}(x_1) : e_1 \in g^{-1}(g(e)) \text{ and } x_1 \in f^{-1}(f(x))\}\\ &= \min\{1 - \mu^{e_1}_{F_A^c}(x_1) : e_1 \in g^{-1}(g(e)) \text{ and } x_1 \in f^{-1}(f(x))\}\\ &= \min\{\mu^{e_1}_{F_A}(x_1) : e_1 \in g^{-1}(g(e)) \text{ and } x_1 \in f^{-1}(f(x))\}\\ &\leq \mu^e_{F_A}(x).\end{aligned}$$
Therefore $\mu^e_{S^{-1}(I_C)}(x) \leq \mu^e_{F_A}(x)$ for all $e \in E$ and $x \in U$.
Hence $S^{-1}(I_C) \sqsubseteq F_A$.

Theorem 2.18. Let $S : (U, E, \tau) \rightarrow (V, E', \tau')$ be an open map. Then for any fuzzy soft set $H_B \in FS(V, E')$ and any fuzzy soft closed set F_A containing $S^{-1}(H_B)$, there exists a closed set I_C containing H_B such that $S^{-1}(I_C) \sqsubseteq F_A$.

Proof. Same as previous theorem.

Theorem 2.19. The following four properties of a map $S : (U, E, \tau) \to (V, E', \tau')$ are equivalent
1. S is an open map.
2. $S(intF_A) \sqsubseteq int(S(F_A))$ for all $F_A \in FS(U, E)$.
3. S sends each member of a fuzzy soft open base for τ to a fuzzy soft open set in (V, E', τ').
4. For each fuzzy soft point F_e over (U, E) and each neighborhood G_A of F_e, there exists a neighborhood H_B of $S(F_e)$ in (V, E') such that $H_B \sqsubseteq S(G_A)$

Proof. (1) $\Rightarrow$ (2)
Since $intF_A \sqsubseteq F_A$, $S(intF_A) \sqsubseteq S(F_A)$.
By hypothesis, $S(intF_A)$ is open in (V, E', τ'). Again $int(S(F_A))$ is the union of all fuzzy soft open subsets of $S(F_A)$ over (V, E', τ'). Hence $S(intF_A) \sqsubseteq int(S(F_A))$.

(2) $\Rightarrow$ (3)
Let F_A be a member of fuzzy soft open base for (U, E, τ).
Then $S(F_A) = S(intF_A) \sqsubseteq int(S(F_A)) \sqsubseteq S(F_A)$. That is, $S(F_A) = int(S(F_A))$. Therefore $S(F_A)$ is an open set in (V, E', τ').

(3) $\Rightarrow$ (4)
Let F_e be a fuzzy soft point and G_A be a neighborhood of F_e. Then there exists a member I_C of fuzzy soft base for τ such that $F_e \in I_C \sqsubseteq G_A$. So, $S(F_e) \in S(I_C) \sqsubseteq S(G_A)$. By (3), $S(I_C)$ is a open fuzzy soft set in (V, E', τ'). So, there exists a neighborhood $S(I_C)$ of $S(F_e)$ such that $S(I_C) \sqsubseteq S(G_A)$.

(4) $\Rightarrow$ (1)
Let G_A be a fuzzy soft open set in (U, E, τ). Then by hypothesis, for each point $F_e \in G_A$, there exists a neighborhood H_B of $S(F_e)$ such that $S(F_e) \in H_B \sqsubseteq S(G_A)$. That is, there exists a fuzzy soft open set I_{F_e} containing $S(F_e)$ such that $I_{F_e} \sqsubseteq S(G_A)$. So, $S(G_A) = \sqcup\{I_{F_e} : F_e \in G_A\}$. Which shows that $S(G_A)$ is open fuzzy soft set. This completes the proof.

Theorem 2.20. A mapping $S : (U, E, \tau) \to (V, E', \tau')$ is closed if and only if $\overline{S(F_A)} \sqsubseteq S(\overline{F_A})$ for every $F_A \in FS(U, E)$.

Proof. Obvious.

Proposition 2.21. [7] Let $S : (U, E) \to (V, E')$ be a fuzzy soft function. Then $S^{-1}(\sqcup_{\alpha\in\Lambda} H_{B_\alpha}) = \sqcup_{\alpha\in\Lambda} S^{-1}(H_{B_\alpha})$ where $H_{B_\alpha} \in FS(V, E')$ for all $\alpha \in \Lambda$.

Proof.
$$\begin{aligned} \sqcup_{\alpha\in\Lambda} S^{-1}(H_{B_\alpha})(e) &= \cup\{\mu^e_{S^{-1}(H_{B_\alpha})} : \alpha \in \Lambda\} \\ &= \cup\{\mu^{g(e)}_{H_{B_\alpha}} : \alpha \in \Lambda\} \\ &= \mu^{g(e)}_{\sqcup_{\alpha\in\Lambda} H_{B_\alpha}} \\ &= \mu^e_{S^{-1}(\sqcup_{\alpha\in\Lambda} H_{B_\alpha})} \\ &= S^{-1}(\sqcup_{\alpha\in\Lambda} H_{B_\alpha})(e) \text{ for all } e \in E. \end{aligned}$$
Therefore $\sqcup_{\alpha\in\Lambda} S^{-1}(H_{B_\alpha}) = S^{-1}(\sqcup_{\alpha\in\Lambda} H_{B_\alpha})$.

Theorem 2.22. Let (U, E, τ) and (V, E', τ') be two fuzzy soft topological spaces and β be a basis for the topology τ'. Then $S : (U, E, \tau) \to (V, E', \tau')$ is continuous if and only if $S^{-1}(F_B)$ is open in (U, E, τ) for every $F_B \in \beta$.

Proof. If S is continuous, then the theorem is obvious.
Suppose $S^{-1}(F_B)$ is open in (U,E,τ) for every $F_B \in \beta$. We now show that S is continuous. Let G_A be an open fuzzy soft set in (V,E',τ'). Then by theorem 2.10 G_A is a union of some member of β, say $G_A = \sqcup G_{A_\alpha}$ where $G_{A_\alpha} \in \beta$ and union is taken over $\alpha \in \Lambda$.
Thus $S^{-1}(G_A) = S^{-1}(\sqcup G_{A_\alpha}) = \sqcup S^{-1}(G_{A_\alpha})$ by proposition 2.21.
By assumption, each $S^{-1}(G_{A_\alpha})$ is open in (U,E,τ). Therefore $S^{-1}(G_A)$ is open fuzzy soft set. Hence S is continuous.

2.2 Fuzzy Soft Compact Sets

Definition 2.23. Let F_A be a fuzzy soft set. A collection $\mathcal{C}$ of fuzzy soft sets $\{F_{A_\alpha} : \alpha \in \Lambda\}$, Λ being the index set, is said to be a cover of F_A if $F_A \sqsubseteq \sqcup\{F_{A_\alpha} : \alpha \in \Lambda\}$.
If $\mathcal{C}$ be a collection of fuzzy soft open sets, then $\mathcal{C}$ is said to be an open cover of F_A.

Example 2.24. Let $A = \{e_1, e_2, e_3\}$ and $U = \{x, y\}$. Also let $A_1 = \{e_1, e_2\}$, $A_2 = \{e_2, e_3\}$, $A_3 = \{e_1, e_3\}$
$\mu^{e_1}_{F_{A_1}} = \{.2, .3\}$, $\mu^{e_2}_{F_{A_1}} = \{.1, .3\}$
$\mu^{e_2}_{F_{A_2}} = \{.2, .5\}$, $\mu^{e_3}_{F_{A_2}} = \{.5, .7\}$
$\mu^{e_1}_{F_{A_3}} = \{.1, .4\}$, $\mu^{e_3}_{F_{A_3}} = \{.7, .7\}$
Let F_A be a fuzzy soft set, where $\mu^{e_1}_{F_A} = \{.2, .35\}$, $\mu^{e_2}_{F_A} = \{.1, .4\}$, $\mu^{e_3}_{F_A} = \{.6, .65\}$.
Then obviously, $F_A \sqsubseteq F_{A_1} \sqcup F_{A_2} \sqcup F_{A_3}$. Therefore $\{F_{A_1}, F_{A_2}, F_{A_3}\}$ is a cover of F_A.

Example 2.25. Here we recall the example 3.8 of our paper[13] as
Let $E = \{e_1,\ e_2,\ e_3\}$, $U = \{a,\ b,\ c\}$ and $A,\ B,\ C$ be the subsets of E, where $A = \{e_1,\ e_2\}$, $B = \{e_2,\ e_3\}$ and $C = \{e_1,\ e_3\}$ and also let $\tau = \{\phi,\ \widetilde{E},\ F_A,\ G_B,\ H_{e_2},\ I_E,\ J_B,\ K_{e_2}\}$ be a fuzzy soft topology over (U,E) where $F_A,\ G_B,\ H_{e_2},\ I_E,\ J_B,\ K_{e_2}$ are fuzzy soft set over (U,E), defined as follows
$\mu^{e_1}_{F_A} = \{.5,\ .75,\ .4\}$, $\mu^{e_2}_{F_A} = \{.3,\ .8,\ .7\}$,
$\mu^{e_2}_{G_B} = \{.4,\ .6,\ .3\}$, $\mu^{e_3}_{G_B} = \{.2,\ .4,\ .45\}$,
$\mu_{H_{e_2}} = \{.3,\ .6,\ .3\}$,
$\mu^{e_1}_{I_E} = \{.5,\ .75,\ .4\}$, $\mu^{e_2}_{I_E} = \{.4,\ .8,\ .7\}$, $\mu^{e_3}_{I_E} = \{.2,\ .4,\ .45\}$,
$\mu^{e_2}_{J_B} = \{.4,\ .8,\ .7\}$, $\mu^{e_3}_{J_B} = \{.2,\ .4,\ .45\}$,
$\mu_{K_{e_2}} = \{.3,\ .8,\ .7\}$.
Now consider a fuzzy soft set L_B as follows
$\mu^{e_2}_{L_B} = \{.37, .7, .3\}$, $\mu^{e_3}_{L_B} = \{.1, .4, .4\}$
Then $L_B \sqsubseteq F_A \sqcup G_B$. Therefore $\{F_A, G_B\}$ is an open cover of L_B.

Example 2.26. Let $E = \{e_1, e_2, e_3, \cdots\}$, $U = \{x_1, x_2, x_3, \cdots, x_p\}$ where p is any positive integer and for any $e_j \in E$, F_{e_j} be a fuzzy soft point over $(U,\ E)$.
For any subset A of E, we define a fuzzy soft set F_A as follows

$$\mu^{e_j}_{F_A}(x_i) = \begin{cases} \mu_{F_{e_j}}(x_i) & \text{if } e_j \in A \\ 0 & \text{otherwise.} \end{cases}$$

where $i = 1, 2, 3, \cdots, p$ and $j = 1, 2, 3, \cdots$.
Let $\tau = \{\phi,\ \widetilde{E},\ F_{e_1}, F_{e_2}, \cdots, F_{\{e_1,e_2\}}, F_{\{e_2,e_3\}}, \cdots, F_{\{e_1,e_2,e_3\}}, F_{\{e_2,e_3,e_4\}}, \cdots, F_{\{e_1,e_2,e_3,\cdots,e_m\}}, F_{\{e_2,e_3,e_4,\cdots,e_{m+1}\}}, F_{\{e_3,e_4,e_5,\cdots,e_{m+2}\}}, \cdots\}$.

Then clearly, τ is a fuzzy soft topology on (U, E). Now let us consider a fuzzy soft set G_A as follows

$$\mu^{e_j}_{G_A}(x_i) = \begin{cases} \frac{\mu_{F_{e_j}}(x_i)}{2} & \text{if } e_j \in A \\ 0 & \text{otherwise.} \end{cases}$$

where $i = 1, 2, \cdots, p$ and $j = 1, 2, \cdots$ and A is an infinite subset of E.
Then clearly, $G_A \sqsubseteq F_{e_1} \sqcup F_{e_2} \sqcup \cdots$
Therefore $\{F_{e_1}, F_{e_2}, \cdots\}$ is an open cover of G_A. But it has no finite subcover.
Again if we consider a fuzzy soft set H_B where B is a finite subset of E. Then obviously, every open cover of H_B has a finite subcover.

Definition 2.27. A fuzzy soft set F_A of a fuzzy soft topological space (U, E, τ) is said to be a compact fuzzy soft set if every open cover of F_A has a finite subcover.

Result 2.28. If $F_A \sqsubseteq S^{-1}(H_B)$, then $S(F_A) \sqsubseteq H_B$.

Proof. Let $F_A \sqsubseteq S^{-1}(H_B)$. Then $\mu^e_{F_A} \subseteq \mu^e_{S^{-1}(H_B)}$ for all $e \in E$.
That is, $\mu^e_{F_A}(x) \leq \mu^e_{S^{-1}(H_B)}(x) = \mu^{g(e)}_{H_B}(f(x))$ for all $e \in E$ and $x \in U$.
Now let $\alpha \in E'$ and $y \in V$.
Then
$$\begin{aligned} \mu^{\alpha}_{S(F_A)}(y) &= \max\{\mu^e_{F_A}(x) : x \in f^{-1}(y) \text{ and } e \in g^{-1}(\alpha)\} \\ &\leq \max\{\mu^e_{S^{-1}(H_B)}(x) : x \in f^{-1}(y) \text{ and } e \in g^{-1}(\alpha)\} \\ &= \max\{\mu^{g(e)}_{H_B}(f(x)) : x \in f^{-1}(y) \text{ and } e \in g^{-1}(\alpha)\} \\ &= \mu^{\alpha}_{H_B}(y) \end{aligned}$$
Therefore $\mu^{\alpha}_{S(F_A)}(y) \leq \mu^{\alpha}_{H_B}(y)$ for all $\alpha \in E'$ and $y \in V$. Hence $S(F_A) \sqsubseteq H_B$.

Theorem 2.29. Let $S : (U, E, \tau) \to (V, E', \tau')$ be a continuous mapping. Suppose F_A is a compact subset of $\widetilde{E}$. Then $S(F_A)$ is also compact.

Proof. Let $\mathcal{V}$ be an open cover of $S(F_A)$ and $\mathcal{U} = \{S^{-1}(G_B) : G_B \in \mathcal{V}\}$. We now show that $\mathcal{U}$ is a cover of F_A. Let $e \in A$ and H_e be any point of F_A. Then $S(H_e) \in S(F_A)$. Then there exists a subset $\{G_{B_\alpha} : \alpha \in \Lambda\}$ of $\mathcal{V}$ such that $S(H_e) \in \sqcup\{G_{B_\alpha} : \alpha \in \Lambda\}$. Therefore $H_e \in S^{-1}(\sqcup_{\alpha\in\Lambda} G_{B_\alpha}) = \sqcup_{\alpha\in\Lambda} S^{-1}(G_{B_\alpha})$ which shows that $\mathcal{U}$ is a cover of F_A. Again since S is continuous, each member of $\mathcal{U}$ is open. So, $\mathcal{U}$ is an open cover of F_A. Since F_A is compact, there exist finitely many members of $\mathcal{U}$, say $S^{-1}(G_{B_1})$, $S^{-1}(G_{B_2})$, $\cdots$, $S^{-1}(G_{B_n})$ where $G_{B_1}, G_{B_2}, \cdots, G_{B_n} \in \mathcal{V}$ such that
$$\begin{aligned} F_A &\sqsubseteq S^{-1}(G_{B_1}) \sqcup S^{-1}(G_{B_2}) \sqcup \cdots \sqcup S^{-1}(G_{B_n}) \\ &= S^{-1}(G_{B_1} \sqcup G_{B_2} \sqcup \cdots \sqcup G_{B_n}) \text{ by proposition 2.21.} \end{aligned}$$
Therefore $S(F_A) \sqsubseteq G_{B_1} \sqcup G_{B_2} \sqcup \cdots \sqcup G_{B_n}$ by result 2.28.
So, $\{G_{B_1}, G_{B_2}, \cdots, G_{B_n}\}$ is a cover of $S(F_A)$. Hence $S(F_A)$ is compact.

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