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RELATIONS ON FP-SOFT SETS APPLIED TO DECISION MAKING PROBLEMS

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Abstract – In this work, we first define relations on the fuzzy parametrized soft sets and study their properties. We also give a decision making method based on these relations. In approximate reasoning, relations on the fuzzy parametrized soft sets have shown to be of a primordial importance. Finally, the method is successfully applied to a problems that contain uncertainties.

Keywords – *Soft sets, fuzzy sets, FP-soft sets, relations on FP-soft sets, decision making.*

1 Introduction

In 1999, the concept of soft sets was introduced by Molodtsov [25] to deal with problems that contain uncertainties. After Molodtsov, the operations of soft sets are given in [4, 23, 28] and studied their properties. Since then, based on these operations, soft set theory has developed in many directions and applied to wide variety of fields. For instance; on the theory of soft sets [2, 4, 5, 9, 20, 23, 24, 28], on the soft decision making [16, 17, 18, 21, 22, 27], on the fuzzy soft sets [7, 10, 11] and soft rough sets [16] are some of the selected works. Some authors have also studied the algebraic properties of soft sets, such as [1, 3, 6, 19, 26, 29, 30].

The fuzzy parametrized soft sets (FP-soft sets), firstly studied by Çağman *et al.* [8], is a fuzzy parameterized soft sets. Then, FP-soft sets theory and its applications studied in detail, for example [12, 13, 14]. In this paper, after given most of the fundamental definitions of the operations of fuzzy sets, soft sets and FP-soft sets in next section, we define relations on FP-soft sets and we also give their properties in Section 3. In Section 4, we define symmetric, transitive and reflexive relations on the FP-soft sets. In Section 5, we construct a decision making method based on the FP-soft sets. We also give an application which shows that this methods successfully works. In the final section, some concluding comments are presented.

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2 Preliminary

In this section, we give the basic definitions and results of soft set theory [25] and fuzzy set theory [31] that are useful for subsequent discussions.

Definition 2.1. *[31] Let U be the universe. A fuzzy set X over U is a set defined by a membership function μ_X representing a mapping*

$$\mu_X : U \to [0,1].$$

The value $\mu_X(x)$ for the fuzzy set X is called the membership value or the grade of membership of $x \in U$. The membership value represents the degree of x belonging to the fuzzy set X. Then a fuzzy set X on U can be represented as follows,

$$X = \{(\mu_X(x)/x) : x \in U, \mu_X(x) \in [0,1]\}.$$

Note that the set of all fuzzy sets on U will be denoted by $F(U)$.

Definition 2.2. *[15] t-norms are associative, monotonic and commutative two valued functions t that map from $[0,1] \times [0,1]$ into $[0,1]$. These properties are formulated with the following conditions:*

1. $t(0,0) = 0$ *and* $t(\mu_{X_1}(x), 1) = t(1, \mu_{X_1}(x)) = \mu_{X_1}(x),\ x \in E$
2. *If* $\mu_{X_1}(x) \le \mu_{X_3}(x)$ *and* $\mu_{X_2}(x) \le \mu_{X_4}(x)$, *then* $t(\mu_{X_1}(x), \mu_{X_2}(x)) \le t(\mu_{X_3}x), \mu_{X_4}(x))$
3. $t(\mu_{X_1}(x), \mu_{X_2}(x)) = t(\mu_{X_2}(x), \mu_{X_1}(x))$
4. $t(\mu_{X_1}(x), t(\mu_{X_2}(x), \mu_{X_3}(x))) = t(t(\mu_{X_1}(x), \mu_{X_2})(x), \mu_{X_3}(x))$

Definition 2.3. *[15] t-conorms or s-norm are associative, monotonic and commutative two placed functions s which map from $[0,1] \times [0,1]$ into $[0,1]$. These properties are formulated with the following conditions:*

1. $s(1,1) = 1$ *and* $s(\mu_{X_1}(x), 0) = s(0, \mu_{X_1}(x)) = \mu_{X_1}(x), x \in E$
2. *if* $\mu_{X_1}(x) \le \mu_{X_3}(x)$ *and* $\mu_{X_2}(x) \le \mu_{X_4}(x)$, *then* $s(\mu_{X_1}(x), \mu_{X_2}(x)) \le s(\mu_{X_3}(x), \mu_{X_4}(x))$
3. $s(\mu_{X_1}(x), \mu_{X_2}(x)) = s(\mu_{X_2}(x), \mu_{X_1}(x))$
4. $s(\mu_{X_1}(x), s(\mu_{X_2}(x), \mu_{X_3}(x))) = s(s(\mu_{X_1}(x), \mu_{X_2})(x), \mu_{X_3}(x))$

t-norm and t-conorm are related in a sense of lojical duality. Typical dual pairs of non parametrized t-norm and t-conorm are complied below:

1. Drastic product:

$$t_w(\mu_{X_1}(x), \mu_{X_2}(x)) = \begin{cases} min\{\mu_{X_1}(x), \mu_{X_2}(x)\}, & max\{\mu_{X_1}(x)\mu_{X_2}(x)\} = 1 \\ 0, & otherwise \end{cases}$$

2. Drastic sum:

$$s_w(\mu_{X_1}(x), \mu_{X_2}(x)) = \begin{cases} max\{\mu_{X_1}(x), \mu_{X_2}(x)\}, & min\{\mu_{X_1}(x)\mu_{X_2}(x)\} = 0 \\ 1, & otherwise \end{cases}$$

3. Bounded product:

$$t_1(\mu_{X_1}(x), \mu_{X_2}(x)) = max\{0, \mu_{X_1}(x) + \mu_{X_2}(x) - 1\}$$

4. Bounded sum:

$$s_1(\mu_{X_1}(x), \mu_{X_2}(x)) = min\{1, \mu_{X_1}(x) + \mu_{X_2}(x)\}$$

5. Einstein product:

$$t_{1.5}(\mu_{X_1}(x), \mu_{X_2}(x)) = \frac{\mu_{X_1}(x).\mu_{X_2}(x)}{2 - [\mu_{X_1}(x) + \mu_{X_2}(x) - \mu_{X_1}(x).\mu_{X_2}(x)]}$$

6. Einstein sum:

$$s_{1.5}(\mu_{X_1}(x), \mu_{X_2}(x)) = \frac{\mu_{X_1}(x) + \mu_{X_2}(x)}{1 + \mu_{X_1}(x).\mu_{X_2}(x)}$$

7. Algebraic product:

$$t_2(\mu_{X_1}(x), \mu_{X_2}(x)) = \mu_{X_1}(x).\mu_{X_2}(x)$$

8. Algebraic sum:

$$s_2(\mu_{X_1}(x), \mu_{X_2}(x)) = \mu_{X_1}(x) + \mu_{X_2}(x) - \mu_{X_1}(x).\mu_{X_2}(x)$$

9. Hamacher product:

$$t_{2.5}(\mu_{X_1}(x), \mu_{X_2}(x)) = \frac{\mu_{X_1}(x).\mu_{X_2}(x)}{\mu_{X_1}(x) + \mu_{X_2}(x) - \mu_{X_1}(x).\mu_{X_2}(x)}$$

10. Hamacher sum:

$$s_{2.5}(\mu_{X_1}(x), \mu_{X_2}(x)) = \frac{\mu_{X_1}(x) + \mu_{X_2}(x) - 2.\mu_{X_1}(x).\mu_{X_2}(x)}{1 - \mu_{X_1}(x).\mu_{X_2}(x)}$$

11. Minumum:

$$t_3(\mu_{X_1}(x), \mu_{X_2}(x)) = min\{\mu_{X_1}(x), \mu_{X_2}(x)\}$$

12. Maximum:

$$s_3(\mu_{X_1}(x), \mu_{X_2}(x)) = max\{\mu_{X_1}(x), \mu_{X_2}(x)\}$$

Definition 2.4. *[25]. Let U be an initial universe set and let E be a set of parameters. Then, a pair (F, E) is called a soft set over U if and only if F is a mapping or E into the set of aft subsets of the set U.*

In other words, the soft set is a parametrized family of subsets of the set U. Every set $F(\varepsilon)$, $\varepsilon \in E$, from this family may be considered as the set of ε-elements of the soft set (F, E), or as the set of ε-approximate elements of the soft set.

It is worth noting that the sets $F(\varepsilon)$ may be arbitrary. Some of them may be empty, some may have nonempty intersection.

In this definition, E is a set of parameters that are describe the elements of the universe U. To apply the soft set in decision making subset $A, B, C, ...$ of the parameters set E are needed. Therefore, Çağman and Enginoğlu [4] modified the definition of soft set as follows.

Definition 2.5. *[4] Let U be a universe, E be a set of parameters that are describe the elements of U, and $A \subseteq E$. Then, a soft set F_A over U is a set defined by a set valued function f_A representing a mapping*

$$f_A : E \to P(U) \text{ such that } f_A(x) = \emptyset \text{ if } x \in E - A \quad (1)$$

where f_A is called approximate function of the soft set F_A. In other words, the soft set is a parametrized family of subsets of the set U, and therefore it can be written a set of ordered pairs

$$F_A = \{(x, f_A(x)) : x \in E, f_A(x) = \emptyset \text{ if } x \in E - A\}$$

The subscript A in the f_A indicates that f_A is the approximate function of F_A. The value $f_A(x)$ is a set called *x-element* of the soft set for every $x \in E$.

Definition 2.6. *[8] Let F_X be a soft set over U with its approximate function f_X and X be a fuzzy set over E with its membership function μ_X. Then, a $FP-$soft sets Γ_X, is a fuzzy parameterized soft set over U, is defined by the set of ordered pairs*

$$\Gamma_X = \{(\mu_X(x)/x, f_X(x)) : x \in E\}$$

where $f_X : E \to P(U)$ such that $f_X(x) = \emptyset$ if $\mu_X(x) = 0$ is called approximate function and $\mu_X : E \to [0, 1]$ is called membership function of $FP-$soft set Γ_X. The value $\mu_X(x)$ is the degree of importance of the parameter x and depends on the decision-maker's requirements.

Note that the sets of all $FP-$soft sets over U will be denoted by $FPS(U)$.

3 Relations on the FP-Soft Sets

In this section, after given the cartesian products of two FP-soft sets, we define a relations on FP-soft sets and study their desired properties.

Definition 3.1. *Let $\Gamma_X, \Gamma_Y \in FPS(U)$. Then, a cartesian product of Γ_X and Γ_Y, denoted by $\Gamma_X \widehat{\times} \Gamma_Y$, is defined as*

$$\Gamma_X \widehat{\times} \Gamma_Y = \Big\{ \big(\mu_{X \widehat{\times} Y}(x,y)/(x,y), f_{X \widehat{\times} Y}(x,y)\big) : (x,y) \in E \times E \big) \Big\}$$

where

$$f_{X \widehat{\times} Y}(x,y) = f_X(x) \cap f_Y(y)$$

and

$$\mu_{X \widehat{\times} Y}(x,y) = min\{\mu_X(x), \mu_Y(y)\}$$

Here $\mu_{X \widehat{\times} Y}(x,y)$ is a t-norm.

Example 3.2. *Let $U = \{u_1, u_2, u_3, u_4, u_5, u_6, u_7, u_8, u_9, u_{10}, u_{11}, u_{12}, u_{13}, u_{14}, u_{15}\}$, $E = \{x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8\}$, and $X = \{0.5/x_1, 0.7/x_2, 0.3/x_3, 0.9/x_4, 0.6/x_5\}$ and $Y = \{0.9/x_3, 0.1/x_6, 0.7/x_7, 0.3/x_8\}$ be two fuzzy subsets of E. Suppose that*

$$\Gamma_X = \Big\{(0.5/x_1, \{u_1, u_3, u_4, u_6, u_7, u_8, u_{11}, u_{12}, u_{13}, u_{15}\}), (0.7/x_2, \{u_3, u_7, u_8, u_{14}, u_{15}\}), (0.3/x_3, \{u_1, u_2, u_4, u_5, u_6, u_9, u_{10}, u_{12}, u_{13}\}), (0.9/x_4, \{u_2, u_4, u_6, u_8, u_{12}, u_{13}\}), (0.6/x_5, \{u_3, u_4, u_6, u_7, u_9, u_{13}, u_{15}\})\Big\}$$

$$\Gamma_Y = \Big\{(0.9/x_3, \{u_1, u_5, u_6, u_9, u_{10}, u_{13}\}), (0.1/x_6, \{u_3, u_5, u_7, u_8, u_9, u_{11}, u_{15}\}), (0.7/x_7, \{u_2, u_5, u_9, u_{10}, u_{11}, u_{14}\}), (0.3/x_8, \{u_2, u_5, u_8, u_{10}, u_{12}, u_{14}\})\Big\}$$

Then, the cartesian product of Γ_X and Γ_Y is obtained as follows

$$\Gamma_X \widehat{\times} \Gamma_Y = \Big\{(0.5/(x_1, x_3), \{u_1, u_6, u_{13}\}), (0.1/(x_1, x_6), \{u_3, u_7, u_8, u_{11}, u_{15}\}), (0.5/(x_1, x_7), \{u_{11}\}), (0.3/(x_1, x_8), \{u_8, u_{12}\}), (0.7/(x_2, x_3), \emptyset), (0.1/(x_2, x_6), \{u_3, u_7, u_8\}), (0.7(x_2, x_7), \{u_{14}\}), (0.3/(x_2, x_8), \{u_8, u_{14}\}), (0.3/(x_3, x_3), \{u_1, u_5, u_6, u_9, u_{10}, u_{13}\}), (0.1/(x_3, x_6), \{u_5, u_9\}), (0.3/(x_3, x_7), \{u_2, u_5, u_9, u_{10}\}), (0.3/(x_3, x_8), \{u_2, u_5, u_{10}, u_{12}\}), (0.9/(x_4, x_3), \{u_6\}), (0.1/(x_4, x_6), \emptyset), (0.7/(x_4, x_7), \{u_2, u_6\}), (0.3/(x_4, x_8), \{u_2, u_8, u_{12}\}), (0.6/(x_5, x_3), \{u_6, u_9, u_{13}\}), (0.1/(x_5, x_6), \{u_3, u_7, u_9, u_{11}, u_{15}\}), (0.6/(x_5, x_7), \{u_2\}), (0.3/(x_5, x_8), \emptyset)\Big\}$$

Definition 3.3. *Let $\Gamma_X, \Gamma_Y \in FPS(U)$. Then, an FP-soft relation from Γ_X to Γ_Y, denoted by R_F, is an FP-soft subset of $\Gamma_X \widehat{\times} \Gamma_Y$. Any FP-soft subset of $\Gamma_X \times \Gamma_Y$ is called a FP-relation on Γ_X.*

Note that if $\alpha = (\mu_X(x), f_X(x)) \in \Gamma_X$ and $\beta = (\mu_Y(y), f_Y(y)) \in \Gamma_Y$, then

$$\alpha R_F \beta \Leftrightarrow \big(\mu_{X \widehat{\times} Y}(x,y)/(x,y), f_{X \widehat{\times} Y}(x,y)\big) \in R_F$$

Example 3.4. *Let us consider the Example 3.2. Then, we define an FP-soft relation R_F, from Γ_Y to Γ_X, as follows*

$$\alpha R_F \beta \Leftrightarrow \mu_{X \widehat{\times} Y}(x_i, x_j)/(x_i, x_j)) \geq 0.3 \quad (1 \leq i, j \leq 3)$$

Then

$$R_F = \Big\{(0.5/(x_1, x_3), \{u_1, u_6, u_{13}\}), ((0.5/(x_1, x_7), \{u_{11}\}), (0.3/(x_1, x_8), \{u_8, u_{12}\}), (0.7(x_2, x_7), \{u_{14}\}), (0.3/(x_2, x_8), \{u_8, u_{14}\}), (0.3/(x_3, x_3), \{u_1, u_5, u_6, u_9, u_{10}, u_{13}\}), (0.3/(x_3, x_7), \{u_2, u_5, u_9, u_{10}\}), (0.3/(x_3, x_8), \{u_2, u_5, u_{10}, u_{12}\}), (0.9/(x_4, x_3), \{u_6\}), (0.7/(x_4, x_7), \{u_2, u_6\}), (0.3/(x_4, x_8), \{u_2, u_8, u_{12}\}), (0.6/(x_5, x_3), \{u_6, u_9, u_{13}\}), (0.6/(x_5, x_7), \{u_2\})\Big\}$$

Definition 3.5. *Let $\Gamma_X, \Gamma_Y \in FPS(U)$ and R_F be an FP-soft relation from Γ_X to Γ_Y. Then domain and range of R_F respectively is defined as*

$$\begin{array}{rcl} D(R_F) & = & \{\alpha \in F_A : \alpha R_F \beta\} \\ R(R_F) & = & \{\beta \in F_B : \alpha R_F \beta\}. \end{array}$$

Example 3.6. *Let us consider the Example 3.4.*

$$D(R_F) = \Big\{(0.5/x_1, \{u_1, u_3, u_4, u_6, u_7, u_8, u_{11}, u_{12}, u_{13}, u_{15}\}), (0.7/x_2, \{u_3, u_7, u_8, u_{14}, u_{15}\}), (0.3/x_3, \{u_1, u_2, u_4, u_5, u_6, u_9, u_{10}, u_{12}, u_{13}\}), (0.9/x_4, \{u_2, u_4, u_6, u_8, u_{12}, u_{13}\}), (0.6/x_5, \{u_3, u_4, u_6, u_7, u_9, u_{13}, u_{15}\})\Big\}$$

$$R(R_F) = \Big\{(0.9/x_3, \{u_1, u_5, u_6, u_9, u_{10}, u_{13}\}), (0.7/x_7, \{u_2, u_5, u_9, u_{10}, u_{11}, u_{14}\}), (0.3/x_8, \{u_2, u_5, u_8, u_{10}, u_{12}, u_{14}\})\Big\}$$

Definition 3.7. *Let R_F be an FP-soft relation from Γ_X to Γ_Y. Then R_F^{-1} is from Γ_Y to Γ_X is defined as*

$$\alpha R_F^{-1} \beta = \beta R_F \alpha$$

Example 3.8. *Let us consider the Example 3.4. Then, ${R_F}^{-1}$ is from Γ_Y to Γ_X is obtained by*

$${R_F}^{-1} = \Big\{(0.5/(x_3, x_1), \{u_1, u_6, u_{13}\}), ((0.5/(x_7, x_1), \{u_{11}\}), (0.3/(x_8, x_1), \{u_8, u_{12}\}), (0.7(x_7, x_2), \{u_{14}\}), (0.3/(x_8, x_2), \{u_8, u_{14}\}), (0.3/(x_3, x_3), \{u_1, u_5, u_6, u_9, u_{10}, u_{13}\}), (0.3/(x_7, x_3), \{u_2, u_5, u_9, u_{10}\}), (0.3/(x_8, x_3), \{u_2, u_5, u_{10}, u_{12}\}), (0.9/(x_3, x_4), \{u_6\}), (0.7/(x_7, x_4), \{u_2, u_6\}), (0.3/(x_8, x_4), \{u_2, u_8, u_{12}\}), (0.6/(x_3, x_5), \{u_6, u_9, u_{13}\}), (0.6/(x_7, x_5), \{u_2\})\Big\}$$

Proposition 3.9. *Let R_{F_1} and R_{F_2} be two FP-soft relations. Then*

1. $(R_{F_1}^{-1})^{-1} = R_{F_1}$
2. $R_{F_1} \subseteq R_{F_2} \Rightarrow R_{F_1}^{-1} \subseteq R_{F_2}^{-1}$

Proof:

1. $\alpha (R_{F_1}^{-1})^{-1} \beta = \beta R_{F_1}^{-1} \alpha = \alpha R_{F_1} \beta$
2. $\alpha R_{F_1} \beta \subseteq \alpha R_{F_2} \beta \Rightarrow \beta R_{F_1}^{-1} \alpha \subseteq \beta R_{F_2}^{-1} \alpha \Rightarrow R_{F_1}^{-1} \subseteq R_{F_2}^{-1}$

Definition 3.10. *If R_{F_1} is a fuzzy parametrized soft relation from Γ_X to Γ_Y and R_{F_2} is a fuzzy parametrized soft relation from Γ_Y to Γ_Z, then a composition of two FP-soft relations R_{F_1} and R_{F_2} is defined by*

$$\alpha (R_{F_1} \circ R_{F_2}) \gamma = (\alpha R_{F_1} \beta) \wedge (\beta R_{F_2} \gamma)$$

Proposition 3.11. *Let R_{F_1} and R_{F_2} be two FP-soft relation from Γ_X to Γ_Y. Then, $(R_{F_1} \circ R_{F_2})^{-1} = R_{F_2}^{-1} \circ R_{F_1}^{-1}$*

Proof:

$$\begin{array}{rl} \alpha (R_{F_1} \circ R_{F_2})^{-1}) \gamma & = \gamma (R_{F_1} \circ R_{F_2}) \alpha \\ & = (\gamma R_{F_1} \beta) \wedge (\beta R_{F_2} \alpha) \\ & = (\beta R_{F_2} \alpha) \wedge (\gamma R_{F_1} \beta) \\ & = (\alpha R_{F_2}^{-1} \beta) \wedge (\beta R_{F_1}^{-1} \gamma) \\ & = \alpha (R_{F_2}^{-1} \circ R_{F_1}^{-1}) \gamma \end{array}$$

Therefore we obtain

$(R_{F_1} \circ R_{F_2})^{-1} = R_{F_2}^{-1} \circ R_{F_1}^{-1}$

Definition 3.12. *An FP-soft relation R_F on Γ_X is said to be an FP-soft symmetric relation if* $\alpha R_{F_1}\beta \Rightarrow \beta R_{F_1}\alpha, \forall\alpha,\beta \in \Gamma_X$.

Definition 3.13. *An FP-soft relation R_F on Γ_X is said to be an FP-soft transitive relation if $R_F \circ R_F \subseteq R_F$, that is, $\alpha R_F\beta$ and $\beta R_F\gamma \Rightarrow \alpha R_F\gamma, \forall\alpha,\beta,\gamma \in \Gamma_X$.*

Definition 3.14. *An FP-soft relation R_F on Γ_X is said to be an FP-soft reflexive relation if $\alpha R_F\alpha, \forall\alpha \in \Gamma_X$.*

Definition 3.15. *An FP-soft relation R_F on Γ_X is said to be an FP-soft equivalence relation if it is symmetric, transitive and reflexive.*

Example 3.16. *Let $U = \{u_1, u_2, u_3, u_4, u_5, u_6, u_7, u_8\}$, $E = \{x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8\}$ and $X = \{0.5/x_1, 0.7/x_2, 0.3/x_3\}$ be a fuzzy subsets over E. Suppose that*

$$\Gamma_X = \Big\{(0.5/x_1,\{u_1,u_3,u_4,u_6,u_7,u_8\}),(0.7/x_2,\{u_3,u_7,u_8\}), (0.3/x_3,\{u_1,u_2,u_4,u_5,u_6,u_9\})\Big\}$$

Then, a cartesian product on Γ_X is obtained as follows

$$\begin{aligned}\Gamma_X\hat{\times}\Gamma_X = \Big\{&(0.5/(x_1,x_1),\{u_1,u_3,u_4,u_6,u_7,u_8\}),\\ &(0.5/(x_1,x_2),\{u_3,u_7,u_8\}),(0.3/(x_1,x_3),\{u_1,u_4,u_6\}),\\ &(0.5/(x_2,x_1),\{u_3,u_7,u_8\}),(0.7/(x_2,x_2),\{u_3,u_7,u_8\}),\\ &(0.3/(x_3,x_1),\{u_1,u_4,u_6\}),(0.3/(x_3,x_3),\{u_1,u_2,u_4,u_5,u_6,u_9\})\Big\}\end{aligned}$$

Then, we get a fuzzy parametrized soft relation R_F on F_X as follows

$$\alpha R_F\beta \Leftrightarrow \mu_{X\hat{\times}Y}(x_i,x_j)/(x_i,x_j)) \geq 0.3 \quad (1 \leq i,j \leq 3)$$

Then

$$\begin{aligned}R_F = \Big\{&(0.5/(x_1,x_1),\{u_1,u_3,u_4,u_6,u_7,u_8\}),(0.5/(x_1,x_2),\{u_3,u_7,u_8\}),\\ &(0.3/(x_1,x_3),\{u_1,u_4,u_6\}),(0.5/(x_2,x_1),\{u_3,u_7,u_8\}),\\ &(0.7/(x_2,x_2),\{u_3,u_7,u_8\}),(0.3/(x_3,x_1),\{u_1,u_4,u_6\}),\\ &(0.3/(x_3,x_3),\{u_1,u_2,u_4,u_5,u_6,u_9\})\Big\}\end{aligned}$$

R_F on Γ_X is an FP-soft equivalence relation because it is symmetric, transitive and reflexive.

Proposition 3.17. *If R_F is symmetric if and only if R_F^{-1} is so.*

Proof: If R_F is symmetric, then $\alpha R_F^{-1}\beta = \beta R_F\alpha = \alpha R_F\beta = \beta R_F^{-1}\alpha$. So, R_F^{-1} is symmetric.
Conversely, if R_F^{-1} is symmetric, then $\alpha R_F\beta = \alpha(R_F^{-1})^{-1}\beta = \beta(R_F^{-1})\alpha = \alpha(R_F^{-1})\beta = \beta R_F\alpha$ So, R_F is symmetric.

Proposition 3.18. *R_F is symmetric if and only if $R_F^{-1} = R_F$*

Proof: If R_F is symmetric, then $\alpha R_F^{-1}\beta = \beta R_F\alpha = \alpha R_F\beta$. So, $R_F^{-1} = R_F$.
Conversely, if $R_F^{-1} = R_F$, then $\alpha R_F\beta = \alpha R_F^{-1}\beta = \beta R_F\alpha$. So, R_F is symmetric.

Proposition 3.19. *If R_{F_1} and R_{F_2} are symmetric relations on Γ_X, then $R_{F_1} \circ R_{F_2}$ is symmetric on Γ_X if and only if $R_{F_1} \circ R_{F_2} = R_{F_2} \circ R_{F_1}$*

Proof: If R_{F_1} and R_{F_2} are symmetric, then it implies $R_{F_1}^{-1} = R_{F_1}$ and $R_{F_2}^{-1} = R_{F_2}$. We have $(R_{F_1} \circ R_{F_2})^{-1} = R_{F_2}^{-1} \circ R_{F_1}^{-1}$. then $R_{F_1} \circ R_{F_2}$ is symmetric. It implies $R_{F_1} \circ R_{F_2} = (R_{F_1} \circ R_{F_2})^{-1} = R_{F_2}^{-1} \circ R_{F_1}^{-1} = R_{F_2} \circ R_{F_1}$.
Conversely, $(R_{F_1} \circ R_{F_2})^{-1} = R_{F_2}^{-1} \circ R_{F_1}^{-1} = R_{F_2} \circ R_{F_1} = R_{F_1} \circ R_{F_2}$. So, $R_{F_1} \circ R_{F_2}$ is symmetric.

Corollary 3.20. *If R_F is symmetric, then R_F^n is symmetric for all positive integer n, where*

$$R_F^n = \underbrace{R_F \circ R_F \circ ... \circ R_F}_{n\ times}$$

Proposition 3.21. *If R_F is transitive, then R_F^{-1} is also transitive.*

Proof:

$$\begin{aligned} \alpha R_F^{-1}\beta &= \beta R_F \alpha \supseteq \beta(R_F \circ R_F)\alpha \\ &= (\beta R_F \gamma) \wedge (\gamma R_F \alpha) \\ &= (\gamma R_F \alpha) \wedge (\beta R_F \gamma) \\ &= (\alpha R_F^{-1}\gamma) \wedge (\gamma R_F^{-1}\beta) \\ &= \alpha(R_F^{-1} \circ R_F^{-1})\beta \end{aligned}$$

So, $R_F^{-1} \circ R_F^{-1} \subseteq R_F^{-1}$. The proof is completed.

Proposition 3.22. *If R_F is transitive then $R_F \circ R_F$ is so.*

Proof:

$$\begin{aligned} \alpha(R_F \circ R_F)\beta &= (\alpha R_F \gamma) \wedge (\gamma\text{-}R_F\beta) \\ &= \alpha(R_F \circ R_F)\gamma \wedge \gamma(R_F \circ R_F)\beta \\ &= \alpha(R_F \circ R_F \circ R_F \circ R_F)\beta \end{aligned}$$

So, $\alpha(R_F \circ R_F \circ R_F \circ R_F)\beta \subseteq \alpha(R_F \circ R_F)\beta$. The proof is completed.

Proposition 3.23. *If R_F is reflexive then R_F^{-1} is so.*

Proof: $\alpha R_F^{-1}\beta = \beta R_F \alpha \subseteq \alpha R_F \alpha = \alpha R_F^{-1}\alpha$ and $\beta R_F^{-1}\alpha = \alpha R_F \beta \subseteq \alpha R_F \alpha = \alpha R_F^{-1}\alpha$. The proof is completed.

Proposition 3.24. *If R_F is symmetric and transitive, then R_F is reflexive.*

Proof: Proof can be made easily by using Definition 4.1, Definition 4.2 and Definition 4.3.

Definition 3.25. *Let $\Gamma_X \in FPS(U)$, R_F be an FP-soft equivalence relation on Γ_X and $\alpha \in R_F$. Then, an equivalence class of α, denoted by $[\alpha]_{R_F}$, is defined as*

$$[\alpha]_{R_F} = \{\beta : \alpha R_F \beta\}.$$

Example 3.26. *Let us consider the Example 3.16. Then an equivalence class of $(x_1, \{u_1, u_3, u_4, u_6, u_7, u_8\})$ will be as follows.*

$$[(0.5/x_1, \{u_1, u_3, u_4, u_6, u_7, u_8\})]_{R_F} = \Big\{(0.5/x_1, \{u_1, u_3, u_4, u_6, u_7, u_8\}), (0.7/x_2, \{u_3, u_7, u_8\}), (0.3/x_3, \{u_1, u_2, u_4, u_5, u_6, u_9\})\Big\}$$

4 Decision Making Method

In this section, we construct a soft fuzzification operator and a decision making method on FP-soft relations.

Definition 4.1. *Let $\Gamma_X \in FPS(U)$ and R_F be a FP-soft relation on Γ_X. Then fuzzification operator, denoted by s_{R_F}, is defined by*

$$s_{R_F} : R_F \to F(U), \quad s_{R_F}(X \times X, U) = \{\mu_{R_F}(u)/u : u \in U\}$$

where

$$\mu_{R_F}(u) = \frac{1}{|X \times X|} \sum_j \sum_i \mu_{R_F}(x_i, x_j)\chi(u)$$

and where

$$\chi(u) = \begin{cases} 1, & u \in f_{R_F}(x_i, x_j) \\ 0, & u \notin f_{R_F}(x_i, x_j) \end{cases}$$

Note that $|X \times X|$ is the cardinality of $X \times X$.

Now; we can construct a decision making method on FP-soft relation by the following algorithm;

1. construct a feasible fuzzy subset X over E,
2. construct a FP-soft set Γ_X over U,
3. construct a FP-soft relation R_F over Γ_X according to the requests,
4. calculate the fuzzification operator s_{R_F} over R_F,
5. select the objects, from s_{R_F}, which have the largest membership value.

Example 4.2. *A customer, Mr. X, comes to the auto gallery agent to buy a car which is over middle class. Assume that an auto gallery agent has a set of different types of car* $U = \{u_1, u_2, u_3, u_4, u_5, u_6, u_7, u_8\}$, *which may be characterized by a set of parameters* $E = \{x_1, x_2, x_3, x_4\}$. *For* $i = 1, 2, 3, 4$ *the parameters* x_i *stand for "safety", "cheap", "modern" and "large", respectively. If Mr. X has to consider own set of parameters, then we select a car on the basis of the set of customer parameters by using the algorithm as follows.*

1. *Mr X constructs a fuzzy sets* X *over* E,
   $X = \{0.5/x_1, 0.7/x_2, 0.3/x_3\}$
2. *Mr X constructs a FP-soft set* Γ_X *over* U,

$$\Gamma_X = \{(0.5/x_1, \{u_1, u_3, u_4, u_6, u_7, u_8\}), (0.7/x_2, \{u_3, u_7, u_8\}), (0.3/x_3, \{u_1, u_2, u_4, u_5, u_6, u_9\}$$

3. *the fuzzy parametrized soft relation* R_F *over* Γ_X *is calculated according to the Mr X's requests (The car must be a over middle class, it means the membership degrees are over 0.5),*

$$R_F = \Big\{(0.5/(x_1, x_1), \{u_1, u_3, u_4, u_6, u_7, u_8\}), (0.5/(x_1, x_2), \{u_3, u_7, u_8\}), (0.5/(x_2, x_1), \{u_3, u_7, u_8\}), (0.7/(x_2, x_2), \{u_3, u_7, u_8\})\Big\}$$

4. *the soft fuzzification operator* s_{R_F} *over* R_F *is calculated as follows*

$$s_{R_F} = \Big\{(0.055/u_1, 0.0/u_2, 0.244/u_3, 0.055/u_4, 0.0/u_5, 0.055/u_6, 0.244/u_7, 0.244/u_8\}\Big\}$$

5. *now, select the optimum alternative objects* u_3, u_7 *and* u_8 *which have the biggest membership degree 0.244 among the others.*

5 Conclusion

We first gave most of the fundamental definitions of the operations of fuzzy sets, soft sets and FP-soft sets are presented. We then defined relations on FP-soft sets and studied some of their properties. We also defined symmetric, transitive and reflexive relations on the FP-soft sets. Finally, we construct a decision making method and gave an application which shows that this method successfully works. We have used a t-norm, which is minimum operator, the above relation. However, application areas the relations can be expanded using the above other norms in the future.

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