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FURTHER DECOMPOSITIONS OF $*$-CONTINUITY[I]

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Abstract – In this paper, we introduce the notions of $^*g$-$\mathcal{I}$-LC*-sets, $\mathcal{I}^*_{*g}$-closed sets and $\mathcal{I}$-$^*g_t$-sets. Also we define the notions of $^*g$-$\mathcal{I}$-LC*-continuous maps, $\mathcal{I}^*_{*g}$-continuous maps, $\mathcal{I}$-$^*g_t$-continuous maps and obtain decompositions of $*$-continuity.

Keywords – *G-$\mathcal{I}$-LC*-set, $^*g$-$\mathcal{I}$-LC*-set, $\mathcal{I}^*_g$-closed set, $\mathcal{I}^*_{*g}$-closed set, $\mathcal{I}$-$^*g_t$-set.*

1 Introduction and Preliminaries

The concept of ideals in topological spaces is treated in the classic text by Kuratowski [13] and Vaidyanathaswamy [23]. The notion of $\mathcal{I}$-open sets in topological spaces was introduced by Jankovic and Hamlett [11]. Dontchev et al. [3] introduced and studied the notion of $\mathcal{I}_g$-closed sets. An ideal $\mathcal{I}$ on a topological space (X, τ) is a non-empty collection of subsets of X satisfying the following properties:

1. A $\in \mathcal{I}$ and B $\subseteq$ A imply B $\in \mathcal{I}$ (heredity);
2. A $\in \mathcal{I}$ and B $\in \mathcal{I}$ imply A $\cup$ B $\in \mathcal{I}$ (finite additivity).

A topological space (X, τ) with an ideal $\mathcal{I}$ on X is called an ideal topological space and is denoted by (X, τ, $\mathcal{I}$). For a subset A $\subseteq$ X, $A^*(\mathcal{I}) = \{x \in X : U \cap A \notin \mathcal{I}$ for every U $\in \tau$(x)$\}$, is called the local function [13] of A with respect to $\mathcal{I}$ and τ. We simply write A^* in case there is no chance for confusion. A Kuratowski closure operator cl*(.) for a topology $\tau^*(\mathcal{I})$ called the $*$-topology finer than τ is defined by cl*(A) = A $\cup$ A* [23]. Let (X, τ) denote a topological space on which no separation axioms are assumed unless explicity stated. In a topological space (X, τ), the closure and the interior of any subset A of X will be denoted by cl(A) and int(A), respectively. A subset A of a topological space (X, τ) is said to be semi-open [15] if A $\subseteq$ cl(int(A)). A subset A of a topological space (X, τ) is said to be g-closed [14] (resp. ω-closed [21]) if cl(A) $\subseteq$ U whenever A $\subseteq$ U and U is open (resp. semi-open) in X. The complement of g-closed (resp. ω-closed) set is said to be g-open (resp. ω-open).

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A subset A of a topological space (X, τ) is said to be *g-closed [9] (resp. g*-closed [24]) if cl(A) $\subseteq$ U whenever A $\subseteq$ U and U is ω-open (resp. g-open) in X. The complement of *g-closed (resp. g*-closed) set is said to be *g-open (resp. g*-open). The intersection of all *g-closed sets of X containing a subset A of X is denoted by *gcl(A). Notice that the intersection of two *g-open sets is again a *g-open. A subset A of an ideal topological space (X, τ, $\mathcal{I}$) is called $*$-closed [11] (resp. $*$-perfect [6]) if $A^* \subseteq A$ (resp. $A = A^*$).

Definition 1.1. *A subset A of a topological space (X, τ) is called*

1. *locally closed set [4] (briefly LC-set) if $A = U \cap V$, where U is open and V is closed.*
2. *$^*g$-LC^*-set [16] if $A = U \cap V$, where U is $^*g$-open and V is closed.*
3. *t-set [22] if int(cl(A)) = int(A).*
4. *$^*g_t$-set [16] if $A = C \cap D$, where C is $^*g$-open and D is a t-set.*

Definition 1.2. *A subset A of an ideal topological space (X, τ, $\mathcal{I}$) is called*

1. *t-$\mathcal{I}$-set [5] if int(cl*(A)) = int(A).*
2. *α^*-$\mathcal{I}$-set [5] if int(cl*(int(A))) = int(A).*
3. *$\mathcal{I}$-LC set [2] if $A = C \cap D$, where $C \in \tau$ and D is $*$-perfect.*
4. *weakly-$\mathcal{I}$-LC set [12] if $A = C \cap D$, where $C \in \tau$ and D is $*$-closed.*
5. *$C_{\mathcal{I}}$-set [5] if $A = C \cap D$, where $C \in \tau$ and D is an α^*-$\mathcal{I}$-set.*
6. *G-$\mathcal{I}$-LC^*-set [8] if $A = C \cap D$, where C is g-open and D is $*$-closed.*

Notice that the intersection of two t-$\mathcal{I}$-sets is again a t-$\mathcal{I}$-set.

Definition 1.3. *[8] A subset A of an ideal topological space (X, τ, $\mathcal{I}$) is said to be $\mathcal{I}_g^*$-closed if $A^* \subseteq U$ whenever $A \subseteq U$ and U is g-open in X.*

Definition 1.4. *[7] A function f : (X, τ, $\mathcal{I}$) $\rightarrow$ (Y, σ) is said to be $*$-continuous if $f^{-1}(V)$ is $*$-closed in (X, τ, $\mathcal{I}$) for every closed set A in (Y, σ).*

Definition 1.5. *A function f : (X, τ) $\rightarrow$ (Y, σ) is said to be $^*g$-LC^*-continuous [16] (resp. g^*-continuous [24], $^*g_t$-continuous [16]) if $f^{-1}(A)$ is $^*g$-LC^*-set (resp. g^*-closed, $^*g_t$-set) in (X, τ) for every closed set A of (Y, σ).*

Definition 1.6. *A function f : (X, τ, $\mathcal{I}$) $\rightarrow$ (Y, σ) is said to be $\mathcal{I}_g^*$-continuous [8] (resp. G-$\mathcal{I}$-LC^*-continuous [8], weakly-$\mathcal{I}$-LC continuous [7]) if $f^{-1}(V)$ is $\mathcal{I}_g^*$-closed (resp. G-$\mathcal{I}$-LC^*-set, weakly-$\mathcal{I}$-LC set) in (X, τ, $\mathcal{I}$) for every closed set V in (Y, σ).*

For a subset A of an ideal topological space (X, τ, $\mathcal{I}$), if A is $*$-closed, then by [17] A is weakly-$\mathcal{I}$-LC. Also by Definition 1.3 it follows that if A is $*$-closed, then A is I_g^*-closed.

Lemma 1.7. *[11] Let (X, τ, $\mathcal{I}$) be an ideal topological space and A, B subsets of X. Then the following properties hold:*

1. *If $A \subseteq B$ then $A^* \subseteq B^*$;*
2. *$A^* = cl(A^*) \subseteq cl(A)$;*
3. *$(A^*)^* \subseteq A^*$;*
4. *$(A \cup B)^* = A^* \cup B^*$.*

Proposition 1.8. *[5] Let (X, τ, $\mathcal{I}$) be an ideal topological space and A a subset of X. Then the following hold:*

1. *If A is a t-$\mathcal{I}$-set, then A is an α^*-$\mathcal{I}$-set.*
2. *If A is an α^*-$\mathcal{I}$-set, then A is a $C_{\mathcal{I}}$-set.*

Remark 1.9. *[1] The following hold in an ideal topological space (X, τ, $\mathcal{I}$).*

$\ast$-*perfect* $\longrightarrow$ $\ast$-*closed* $\longrightarrow$ *t-$\mathcal{I}$-set* $\longrightarrow$ *α^*-$\mathcal{I}$-set*

Remark 1.10. *[19] The following hold in a topological space (X, τ).*

$$\begin{array}{ccc} \textit{closed} & \longrightarrow & \omega\textit{-closed} \\ \downarrow & & \downarrow \\ {}^*g\textit{-closed} & \longrightarrow & g\textit{-closed} \end{array}$$

Notice that ω-closed sets and *g-closed sets are independent of each other.

2 *g-$\mathcal{I}$-LC*-sets

Definition 2.1. *A subset A of an ideal topological space (X, τ, $\mathcal{I}$) is said to be an *g-$\mathcal{I}$-LC*-set if A = C $\cap$ D, where C is *g-open and D is $\ast$-closed.*

Proposition 2.2. *Let (X, τ, $\mathcal{I}$) be an ideal topological space and A $\subseteq$ X. Then the following hold:*

1. *If A is *g-open, then A is an *g-$\mathcal{I}$-LC*-set;*
2. *If A is $\ast$-closed, then A is an *g-$\mathcal{I}$-LC*-set;*
3. *If A is weakly-$\mathcal{I}$-LC set, then A is an *g-$\mathcal{I}$-LC*-set;*
4. *If A is an *g-$\mathcal{I}$-LC*-set, then A is an G-$\mathcal{I}$-LC*-set.*

The converses of Proposition 2.2 need not be true as seen from the following Examples.

Example 2.3. *Let X = $\{a, b, c\}$, τ = $\{\emptyset, X, \{c\}\}$ and $\mathcal{I}$ = $\{\emptyset, \{a\}\}$. Then *g-$\mathcal{I}$-LC*-sets are P(X) and *g-open sets are $\emptyset$, X, $\{a\}$, $\{b\}$, $\{c\}$, $\{a, c\}$, $\{b, c\}$. It is clear that $\{a, b\}$ is *g-$\mathcal{I}$-LC*-set but it is not *g-open.*

Example 2.4. *In Example 2.3, *g-$\mathcal{I}$-LC*-sets are P(X) and $\ast$-closed sets are $\emptyset$, X, $\{a\}$, $\{a, b\}$. It is clear that $\{a, c\}$ is *g-$\mathcal{I}$-LC*-set but it is not $\ast$-closed.*

Example 2.5. *In Example 2.3, *g-$\mathcal{I}$-LC*-sets are P(X) and weakly-$\mathcal{I}$-LC sets are $\emptyset$, X, $\{a\}$, $\{c\}$, $\{a, b\}$. It is clear that $\{b, c\}$ is *g-$\mathcal{I}$-LC*-set but it is not weakly-$\mathcal{I}$-LC set.*

Example 2.6. *Let X = $\{a, b, c\}$, τ = $\{\emptyset, X, \{a\}, \{b, c\}\}$ and $\mathcal{I}$ = $\{\emptyset, \{c\}\}$. Then G-$\mathcal{I}$-LC*-sets are P(X) and *g-$\mathcal{I}$-LC*-sets are $\emptyset$, X, $\{a\}$, $\{c\}$, $\{a, c\}$, $\{b, c\}$. It is clear that $\{a, b\}$ is G-$\mathcal{I}$-LC*-set but it is not *g-$\mathcal{I}$-LC*-set.*

Theorem 2.7. *Let (X, τ, $\mathcal{I}$) be an ideal topological space and A be an *g-$\mathcal{I}$-LC*-subset of X. Then the following hold:*

1. *If B is a $\ast$-closed set, then A $\cap$ B is an *g-$\mathcal{I}$-LC*-set;*
2. *If B is an *g-open set, then A $\cap$ B is an *g-$\mathcal{I}$-LC*-set;*
3. *If B is an *g-$\mathcal{I}$-LC*-set, then A $\cap$ B is an *g-$\mathcal{I}$-LC*-set.*

Proof. (1) Let B be $\ast$-closed and A is *g-$\mathcal{I}$-LC*-set, then A $\cap$ B = (C $\cap$ D) $\cap$ B = C $\cap$ (D $\cap$ B), where D $\cap$ B is $\ast$-closed. Hence A $\cap$ B is an *g-$\mathcal{I}$-LC*-set.

(2) Let B be *g-open and A is *g-$\mathcal{I}$-LC*-set, then A $\cap$ B = (C $\cap$ D) $\cap$ B = (C $\cap$ B) $\cap$ D, where C $\cap$ B is *g-open. Hence A $\cap$ B is an *g-$\mathcal{I}$-LC*-set.

(3) Let A and B be *g-$\mathcal{I}$-LC*-sets, then A $\cap$ B = (C $\cap$ D) $\cap$ (U $\cap$ V) = (C $\cap$ U) $\cap$ (D $\cap$ V), where C $\cap$ U is *g-open and D $\cap$ V is $\ast$-closed. Hence A $\cap$ B is an *g-$\mathcal{I}$-LC*-set.

Remark 2.8. *The union of any two *g-$\mathcal{I}$-LC*-sets need not be an *g-$\mathcal{I}$-LC*-set.*

Example 2.9. *Let X = $\{a, b, c\}$, τ = $\{\emptyset, X, \{a, b\}\}$ and $\mathcal{I}$ = $\{\emptyset, \{a\}\}$. Then *g-$\mathcal{I}$-LC*-sets are $\emptyset$, X, $\{a\}$, $\{b\}$, $\{c\}$, $\{a, b\}$, $\{a, c\}$. It is clear that A = $\{b\}$ and B = $\{c\}$ are *g-$\mathcal{I}$-LC*-sets, but their union A$\cup$B = $\{b, c\}$ is not *g-$\mathcal{I}$-LC*-set.*

Definition 2.10. *[20] A subset A of an ideal topological space (X, τ, $\mathcal{I}$) is said to be $\mathcal{I}^*_{*g}$-closed if A^* $\subseteq$ U whenever A $\subseteq$ U and U is $^*g$-open in X. The complement of $\mathcal{I}^*_{*g}$-closed set is called $\mathcal{I}^*_{*g}$-open.*

Theorem 2.11. *[20] If (X, τ, $\mathcal{I}$) is any ideal topological space and A $\subseteq$ X, then the following are equivalent.*

1. *A is $\mathcal{I}^*_{*g}$-closed,*
2. *cl*(A) $\subseteq$ U whenever A $\subseteq$ U and U is *g-open in X,*
3. *For all $x \in$ cl*(A), *gcl($\{x\}$)$\cap$A $\neq \emptyset$,*
4. *cl*(A) $-$ A contains no nonempty *g-closed set,*
5. *A* $-$ A contains no nonempty *g-closed set.*

Proposition 2.12. *Let (X, τ, $\mathcal{I}$) be an ideal topological space and A be a subset of X. If A is $\mathcal{I}^*_g$-closed, then A is $\mathcal{I}^*_{*g}$-closed.*

The converse of Proposition 2.12 need not be true as seen from the following Example.

Example 2.13. *In Example 2.6, $\mathcal{I}^*_{*g}$-closed sets are P(X) and $\mathcal{I}^*_g$-closed sets are $\emptyset$, X, $\{a\}$, $\{c\}$, $\{a, c\}$, $\{b, c\}$. It is clear that $\{a, b\}$ is $\mathcal{I}^*_{*g}$-closed set but it is not $\mathcal{I}^*_g$-closed.*

Definition 2.14. *A subset A of an ideal topological space (X, τ, $\mathcal{I}$) is said to be an $\mathcal{I}$-$^*g_t$-set if A = C $\cap$ D, where C is $^*g$-open and D is a t-$\mathcal{I}$-set.*

Proposition 2.15. *Let (X, τ, $\mathcal{I}$) be an ideal topological space and A be a subset of X. If A is an $^*g$-$\mathcal{I}$-LC*-set, then A is an $\mathcal{I}$-$^*g_t$-set.*

The converse of Proposition 2.15 need not be true as seen from the following Example.

Example 2.16. *Let X = $\{a, b, c, d\}$, τ = $\{\emptyset, X, \{a\}, \{a, b, c\}\}$ and $\mathcal{I}$ = $\{\emptyset, \{a\}\}$. Then $\mathcal{I}$-$^*g_t$-sets are $\emptyset$, X, $\{a\}$, $\{b\}$, $\{c\}$, $\{d\}$, $\{a, b\}$, $\{a, c\}$, $\{a, d\}$, $\{b, c\}$, $\{b, d\}$, $\{c, d\}$, $\{a, b, c\}$, $\{b, c, d\}$ and *g-$\mathcal{I}$-LC*-sets are $\emptyset$, X, $\{a\}$, $\{b\}$, $\{c\}$, $\{d\}$, $\{a, b\}$, $\{a, c\}$, $\{a, d\}$, $\{b, c\}$, $\{a, b, c\}$, $\{b, c, d\}$. It is clear that $\{c, d\}$ is $\mathcal{I}$-$^*g_t$-set but it is not *g-$\mathcal{I}$-LC*-set.*

Proposition 2.17. *Let (X, τ, $\mathcal{I}$) be an ideal topological space and A be a subset of X. If A is an $\mathcal{I}$-$^*g_t$-set, then A is a $C_{\mathcal{I}}$-set.*

The converse of Proposition 2.17 need not be true as seen from the following Example.

Example 2.18. *Let X = $\{a, b, c, d\}$, τ = $\{\emptyset, X, \{d\}, \{a, c\}, \{a, c, d\}\}$ and $\mathcal{I}$ = $\{\emptyset, \{a\}, \{d\}, \{a, d\}\}$. Then $C_{\mathcal{I}}$-sets are P(X) and $\mathcal{I}$-$^*g_t$-sets are $\emptyset$, X, $\{a\}$, $\{b\}$, $\{c\}$, $\{d\}$, $\{a, b\}$, $\{a, c\}$, $\{a, d\}$, $\{b, d\}$, $\{c, d\}$, $\{a, b, c\}$, $\{a, b, d\}$, $\{a, c, d\}$. It is clear that $\{b, c, d\}$ is $C_{\mathcal{I}}$-set but it is not $\mathcal{I}$-$^*g_t$-set.*

Remark 2.19. *From the above discussion, we have the following implications:*

$$\begin{array}{ccccccc} & & G\text{-}\mathcal{I}\text{-}LC^*\text{-set} & & & & \\ & & \uparrow & & & & \\ *\text{-closed} & \longrightarrow & {}^*g\text{-}\mathcal{I}\text{-}LC^*\text{-set} & \longrightarrow & \mathcal{I}\text{-}{}^*g_t\text{-set} & \longrightarrow & C_{\mathcal{I}}\text{-set} \\ & & \uparrow & & & & \\ \mathcal{I}\text{-}LC\text{-set} & \longrightarrow & \text{weakly-}\mathcal{I}\text{-}LC\text{ set} & & & & \end{array}$$

Theorem 2.20. *Let (X, τ, $\mathcal{I}$) be an ideal topological space and A be an $\mathcal{I}$-$^*g_t$-subset of X. Then the following hold:*

1. *If B is a t-$\mathcal{I}$-set, then A $\cap$ B is an $\mathcal{I}$-$^*g_t$-set;*
2. *If B is an $^*g$-open set, then A $\cap$ B is an $\mathcal{I}$-$^*g_t$-set;*
3. *If B is an $\mathcal{I}$-$^*g_t$-set, then A $\cap$ B is an $\mathcal{I}$-$^*g_t$-set.*

Remark 2.21. *The union of any two $\mathcal{I}$-$^*g_t$-sets need not be an $\mathcal{I}$-$^*g_t$-set.*

Example 2.22. *Let $X = \{a, b, c\}$, $\tau = \{\emptyset, X, \{c\}, \{b, c\}\}$ and $\mathcal{I} = \{\emptyset, \{a\}\}$. Then $\mathcal{I}$-$^*g_t$-sets are $\emptyset$, X, $\{a\}$, $\{b\}$, $\{c\}$, $\{a, b\}$, $\{b, c\}$. It is clear that $A = \{a\}$ and $B = \{c\}$ are $\mathcal{I}$-$^*g_t$-sets but their union $A \cup B = \{a, c\}$ is not $\mathcal{I}$-$^*g_t$-set.*

Theorem 2.23. *The following are equivalent for a subset A of an ideal topological space $(X, \tau, \mathcal{I})$:*

1. *A is $*$-closed;*
2. *A is a weakly-$\mathcal{I}$-LC set and an $\mathcal{I}_g^*$-closed set [10];*
3. *A is an $^*g$-$\mathcal{I}$-LC^*-set and an $\mathcal{I}_g^*$-closed set;*
4. *A is an $^*g$-$\mathcal{I}$-LC^*-set and an $\mathcal{I}_{*g}^*$-closed set;*
5. *A is an $\mathcal{I}$-$^*g_t$-set and an $\mathcal{I}_{*g}^*$-closed set.*

Proof. (1) ⇒ (2): This is obvious.
(2) ⇒ (3): Follows from Proposition 2.2.
(3) ⇒ (4): Follows from Proposition 2.12.
(4) ⇒ (5): Follows from Proposition 2.15.
(5) ⇒ (1): Let A be an $\mathcal{I}$-$^*g_t$-set and $\mathcal{I}_{*g}^*$-closed set. Since A is an $\mathcal{I}$-$^*g_t$-set, A = C ∩ D, where C is *g-open and D is a t-$\mathcal{I}$-set. Now A ⊆ C and A is $\mathcal{I}_{*g}^*$-closed implies $A^* \subseteq C$. Also A ⊆ D and D is a t-$\mathcal{I}$-set implies $\mathrm{int}(D) = \mathrm{int}(\mathrm{cl}^*(D)) = \mathrm{int}(D \cup D^*) \supseteq \mathrm{int}(D) \cup \mathrm{int}(D^*)$. This shows that $\mathrm{int}(D^*) \subseteq \mathrm{int}(D)$. Thus $D^* \subseteq D$ and hence $A^* \subseteq D$. Therefore $A^* \subseteq C \cap D = A$. Hence A is $*$-closed.

Remark 2.24.

1. *The notions of weakly-$\mathcal{I}$-LC sets and $\mathcal{I}_g^*$-closed sets are independent [10].*
2. *The notions of $^*g$-$\mathcal{I}$-LC^*-sets and $\mathcal{I}_g^*$-closed sets are independent.*
3. *The notions of $^*g$-$\mathcal{I}$-LC^*-sets and $\mathcal{I}_{*g}^*$-closed sets are independent.*
4. *The notions of $\mathcal{I}$-$^*g_t$-sets and $\mathcal{I}_{*g}^*$-clo sed sets are independent.*

Example 2.25. *In Example 2.22, we have weakly-$\mathcal{I}$-LC sets are $\emptyset$, X, $\{a\}$, $\{b\}$, $\{c\}$, $\{a, b\}$, $\{b, c\}$ and $\mathcal{I}_g^*$-closed sets are $\emptyset$, X, $\{a\}$, $\{a, b\}$, $\{a, c\}$. It is clear that $\{b, c\}$ is weakly-$\mathcal{I}$-LC set but it is not $\mathcal{I}_g^*$-closed and $\{a, c\}$ is $\mathcal{I}_g^*$-closed set but it is not weakly-$\mathcal{I}$-LC set.*

Example 2.26. *In Example 2.22, we have *g-$\mathcal{I}$-LC^*-sets are $\emptyset$, X, $\{a\}$, $\{b\}$, $\{c\}$, $\{a, b\}$, $\{b, c\}$ and $\mathcal{I}_g^*$-closed sets are $\emptyset$, X, $\{a\}$, $\{a, b\}$, $\{a, c\}$. It is clear that $\{b, c\}$ is *g-$\mathcal{I}$-LC^*-set but it is not $\mathcal{I}_g^*$-closed and $\{a, c\}$ is $\mathcal{I}_g^*$-closed set but it is not *g-$\mathcal{I}$-LC^*-set.*

Example 2.27. *In Example 2.22, we have *g-$\mathcal{I}$-LC^*-sets are $\emptyset$, X, $\{a\}$, $\{b\}$, $\{c\}$, $\{a, b\}$, $\{b, c\}$ and $\mathcal{I}_{*g}^*$-closed sets are $\emptyset$, X, $\{a\}$, $\{a, b\}$, $\{a, c\}$. It is clear that $\{b, c\}$ is *g-$\mathcal{I}$-LC^*-set but it is not $\mathcal{I}_{*g}^*$-closed and $\{a, c\}$ is $\mathcal{I}_{*g}^*$-closed set but it is not *g-$\mathcal{I}$-LC^*-set.*

Example 2.28. *In Example 2.22, we have $\mathcal{I}$-$^*g_t$-sets are $\emptyset$, X, $\{a\}$, $\{b\}$, $\{c\}$, $\{a, b\}$, $\{b, c\}$ and $\mathcal{I}_{*g}^*$-closed sets are $\emptyset$, X, $\{a\}$, $\{a, b\}$, $\{a, c\}$. It is clear that $\{b, c\}$ is $\mathcal{I}$-$^*g_t$-set but it is not $\mathcal{I}_{*g}^*$-closed and $\{a, c\}$ is $\mathcal{I}_{*g}^*$-closed set but it is not $\mathcal{I}$-$^*g_t$-set.*

3 Decompositions of $*$-continuity

Definition 3.1. *A function $f : (X, \tau, \mathcal{I}) \to (Y, \sigma)$ is said to be $\mathcal{I}_{*g}^*$-continuous (resp. $^*g$-$\mathcal{I}$-LC^*-continuous, $\mathcal{I}$-$^*g_t$-continuous) if $f^{-1}(V)$ is $\mathcal{I}_{*g}^*$-closed (resp. $^*g$-$\mathcal{I}$-LC^*-set, $\mathcal{I}$-$^*g_t$-set) in $(X, \tau, \mathcal{I})$ for every closed set V in (Y, σ).*

Remark 3.2.

1. *Every $*$-continuous function is weakly $\mathcal{I}$-LC continuous [10].*
2. *Every weakly $\mathcal{I}$-LC continuous function is $^*g$-$\mathcal{I}$-LC^*-continuous.*
3. *Every $*$-continuous function is $\mathcal{I}_g^*$-continuous [10].*
4. *Every $\mathcal{I}_g^*$-continuous function is $\mathcal{I}_{*g}^*$-continuous.*

Example 3.3. *Let $X = Y = \{a, b, c\}$, $\tau = \{\emptyset, X, \{c\}, \{b, c\}\}$, $\sigma = \{\emptyset, Y, \{a\}, \{a, b\}\}$ and $\mathcal{I} = \{\emptyset, \{a\}\}$. Then weakly-$\mathcal{I}$-LC sets are $\emptyset$, X, $\{a\}$, $\{b\}$, $\{c\}$, $\{a, b\}$, $\{b, c\}$ and $*$-closed sets are $\emptyset$, X, $\{a\}$, $\{a, b\}$. Let $f : (X, \tau, \mathcal{I}) \to (Y, \sigma)$ be an identity function. It is clear that $f^{-1}(\{b, c\}) = \{b, c\}$ is not $*$-closed set. Hence f is weakly $\mathcal{I}$-LC continuous but not $*$-continuous function.*

Example 3.4. *Let $X = Y = \{a, b, c\}$, $\tau = \{\emptyset, X, \{c\}\}$, $\sigma = \{\emptyset, Y, \{a\}, \{a, b\}\}$ and $\mathcal{I} = \{\emptyset, \{a\}\}$. Then weakly-$\mathcal{I}$-LC sets are $\emptyset$, X, $\{a\}$, $\{c\}$, $\{a, b\}$ and $^*g$-$\mathcal{I}$-LC*-sets are $P(X)$. Let $f : (X, \tau, \mathcal{I}) \to (Y, \sigma)$ be an identity function. It is clear that $f^{-1}(\{b, c\}) = \{b, c\}$ is not weakly-$\mathcal{I}$-LC set. Hence f is $^*g$-$\mathcal{I}$-LC* continuous but not weakly $\mathcal{I}$-LC continuous function.*

Example 3.5. *Let $X = Y = \{a, b, c\}$, $\tau = \{\emptyset, X, \{c\}, \{b, c\}\}$, $\sigma = \{\emptyset, Y, \{b\}\}$ and $\mathcal{I} = \{\emptyset, \{a\}\}$. Then $*$-closed sets are $\emptyset$, X, $\{a\}$, $\{a, b\}$ and $\mathcal{I}_g^*$-closed sets are $\emptyset$, X, $\{a\}$, $\{a, b\}$, $\{a, c\}$. Let $f : (X, \tau, \mathcal{I}) \to (Y, \sigma)$ be an identity function. It is clear that $f^{-1}(\{a, c\}) = \{a, c\}$ is not $*$-closed set. Hence f is $\mathcal{I}_g^*$-continuous but not $*$-continuous function.*

Example 3.6. *Let $X = Y = \{a, b, c\}$, $\tau = \{\emptyset, X, \{a\}, \{b, c\}\}$, $\sigma = \{\emptyset, Y, \{c\}, \{b, c\}\}$ and $\mathcal{I} = \{\emptyset, \{c\}\}$. Then $\mathcal{I}_{*g}^*$-closed sets are $P(X)$ and $\mathcal{I}_g^*$-closed sets are $\emptyset$, X, $\{a\}$, $\{c\}$, $\{a, c\}$, $\{b, c\}$. Let $f : (X, \tau, \mathcal{I}) \to (Y, \sigma)$ be an identity function. It is clear that $f^{-1}(\{a, b\}) = \{a, b\}$ is not $\mathcal{I}_g^*$-closed set. Hence f is $\mathcal{I}_{*g}^*$-continuous but not $\mathcal{I}_g^*$-continuous function.*

Definition 3.7. *[20] A subset A of a topological space (X, τ) is said to be $^*g^*$-closed if $cl(A) \subseteq U$ whenever $A \subseteq U$ and U is $^*g$-open in X.*

Definition 3.8. *A function $f : (X, \tau) \to (Y, \sigma)$ is said to be $^*g^*$-continuous if $f^{-1}(A)$ is $^*g^*$-closed set in (X, τ) for every closed set A of (Y, σ).*

Remark 3.9.

1. *Every g^*-continuous function is $^*g^*$-continuous.*
2. *Every $^*g$-LC*-continuous function is $^*g_t$-continuous.*

Example 3.10. *Let $X = Y = \{a, b, c\}$, $\tau = \{\emptyset, X, \{c\}, \{a, b\}\}$ and $\sigma = \{\emptyset, Y, \{a, c\}\}$. Then $^*g^*$-closed sets are $P(X)$ and g^*-closed sets are $\emptyset$, X, $\{c\}$, $\{a, b\}$. Let $f : (X, \tau) \to (Y, \sigma)$ be an identity function. It is clear that $f^{-1}(\{b\}) = \{b\}$ is not g^*-closed set. Hence f is $^*g^*$-continuous but not g^*-continuous function.*

Example 3.11. *Let $X = Y = \{a, b, c, d\}$, $\tau = \{\emptyset, X, \{b\}, \{b, c, d\}\}$ and $\sigma = \{\emptyset, Y, \{b\}, \{b, d\}\}$. Then $^*g_t$-sets are $\emptyset$, X, $\{a\}$, $\{b\}$, $\{c\}$, $\{d\}$, $\{a, c\}$, $\{a, d\}$, $\{b, c\}$, $\{b, d\}$, $\{c, d\}$, $\{a, c, d\}$, $\{b, c, d\}$ and $^*g$-LC*-sets are $\emptyset$, X, $\{a\}$, $\{b\}$, $\{c\}$, $\{d\}$, $\{b, c\}$, $\{b, d\}$, $\{c, d\}$, $\{a, c, d\}$, $\{b, c, d\}$. Let $f : (X, \tau) \to (Y, \sigma)$ be an identity function. It is clear that $f^{-1}(\{a, c\}) = \{a, c\}$ is not $^*g$-LC*-set. Hence f is $^*g_t$-continuous but not $^*g$-LC*-continuous function.*

Proposition 3.12. *Let $f : (X, \tau, \mathcal{I}) \to (Y, \sigma)$ be $\mathcal{I}_{*g}^*$-continuous and $g : (Y, \sigma) \longrightarrow (Z, \eta)$ be continuous. Then $g \circ f : (X, \tau, \mathcal{I}) \longrightarrow (Z, \eta)$ is $\mathcal{I}_{*g}^*$-continuous.*

Theorem 3.13. *For a function $f : (X, \tau, \mathcal{I}) \to (Y, \sigma)$, the following are equivalent:*

1. *f is $*$-continuous;*
2. *f is weakly $\mathcal{I}$-LC continuous and $\mathcal{I}_g^*$-continuous [10];*
3. *f is $^*g$-$\mathcal{I}$-LC*-continuous and $\mathcal{I}_g^*$-continuous;*
4. *f is $^*g$-$\mathcal{I}$-LC*-continuous and $\mathcal{I}_{*g}^*$-continuous;*
5. *f is $\mathcal{I}$-$^*g_t$-continuous and $\mathcal{I}_{*g}^*$-continuous.*

Proof. Immediately follows from Theorem 2.23.

Corollary 3.14. *Let $(X, \tau, \mathcal{I})$ be an ideal topological space and $\mathcal{I} = \{\phi\}$, for a function $f : (X, \tau, \mathcal{I}) \to (Y, \sigma)$, the following are equivalent:*

1. *f is continuous;*
2. *f is LC-continuous and g^*-continuous [10];*
3. *f is $^*g$-LC*-continuous and g^*-continuous;*
4. *f is $^*g$-LC*-continuous and $^*g^*$-continuous;*
5. *f is $^*g_t$-continuous and $^*g^*$-continuous.*

4 On $\mathcal{I}^*_{*g}$-normal Spaces

Definition 4.1. *An ideal topological space (X, τ, $\mathcal{I}$) is said to be $\mathcal{I}^*_{*g}$-normal, if for any two disjoint closed sets F and G in (X, τ, $\mathcal{I}$) there exist disjoint $\mathcal{I}^*_{*g}$-open sets U and V such that F $\subseteq$ U and G $\subseteq$ V.*

Theorem 4.2. *For an ideal topological space (X, τ, $\mathcal{I}$), the following are equivalent:*

1. *(X, τ, $\mathcal{I}$) is $\mathcal{I}^*_{*g}$-normal.*
2. *For each closed set F and for each open set V containing F, there exists an $\mathcal{I}^*_{*g}$-open set U such that F $\subseteq$ U $\subseteq$ cl*(U) $\subseteq$ V.*

Proof. (1) $\Rightarrow$ (2): Let F be a closed subset of X and D be an open set such that F $\subseteq$ D. Then F and X $-$ D are disjoint closed sets in X. Therefore, by hypothesis there exist disjoint $\mathcal{I}^*_{*g}$-open sets U and V such that F $\subseteq$ U and X $-$ D $\subseteq$ V. Hence F $\subseteq$ U $\subseteq$ X $-$ V $\subseteq$ D. Now with D being open it is also *g-open and since X $-$ V is $\mathcal{I}^*_{*g}$-closed, we have F $\subseteq$ U $\subseteq$ cl*(U) $\subseteq$ cl*(X $-$ V) $\subseteq$ D.

(2) $\Rightarrow$ (1): Let F and G be two disjoint closed subsets of X. Then by hypothesis, there exists an $\mathcal{I}^*_{*g}$-open set U such that F $\subseteq$ U $\subseteq$ cl*(U) $\subseteq$ X $-$ G. If we take W = X $-$ cl*(U), then U and W are the required disjoint $\mathcal{I}^*_{*g}$-open sets containing F and G respectively. Hence (X, τ, $\mathcal{I}$) is $\mathcal{I}^*_{*g}$-normal.

Theorem 4.3. *Let (X, τ, $\mathcal{I}$) be $\mathcal{I}^*_{*g}$-normal. Then the following statements are true.*

1. *If F is closed and A is an *g-closed set such that A $\cap$ F = ϕ, then there exist disjoint $\mathcal{I}^*_{*g}$-open sets U and V such that A $\subseteq$ U and F $\subseteq$ V.*
2. *If A is closed and B is an *g-open set containing A, then there exists $\mathcal{I}^*_{*g}$-open set U such that A $\subseteq$ int*(U) $\subseteq$ U $\subseteq$ B.*
3. *If A is *g-closed and B is an open set containing A, then there exists $\mathcal{I}^*_{*g}$-open set U such that A $\subseteq$ U $\subseteq$ cl*(U) $\subseteq$ B.*

Proof. (1) Since A $\cap$ F = ϕ, A $\subseteq$ X $-$ F, where X $-$ F is open and hence ω-open. Hence by hypothesis, cl(A) $\subseteq$ X $-$ F. Since cl(A) $\cap$ F = ϕ and X is $\mathcal{I}^*_{*g}$-normal, there exist disjoint $\mathcal{I}^*_{*g}$-open sets U and V such that cl(A) $\subseteq$ U and F $\subseteq$ V. The proofs of (2) and (3) are similar.

Definition 4.4. *A function f : (X, τ, $\mathcal{I}$) $\rightarrow$ (Y, σ, $\mathcal{J}$) is said to be $\mathcal{I}^*_{*g}$-irresolute if f^{-1}(V) is $\mathcal{I}^*_{*g}$-open in (X, τ, $\mathcal{I}$) for every $\mathcal{J}^*_{*g}$-open set V in (Y, σ, $\mathcal{J}$).*

Theorem 4.5. *If f : (X, τ, $\mathcal{I}$) $\rightarrow$ (Y, σ, $\mathcal{J}$) is an $\mathcal{I}^*_{*g}$-irresolute closed injection and Y is an $\mathcal{J}^*_{*g}$-normal space, then X is $\mathcal{I}^*_{*g}$-normal.*

Proof. Let F and G be disjoint closed sets of X. Since f is a closed injection, f(F) and f(G) are disjoint closed sets of Y. Now from the $\mathcal{J}^*_{*g}$-normality of Y, there exist disjoint $\mathcal{J}^*_{*g}$-open sets U and V such that f(F) $\subseteq$ U and f(G) $\subseteq$ V. Also since, f is $\mathcal{I}^*_{*g}$-irresolute, f^{-1}(U) and f^{-1}(V) are disjoint $\mathcal{I}^*_{*g}$-open sets containing F and G respectively. Hence by Definition 4.1, it follows that X is $\mathcal{I}^*_{*g}$-normal.

5 Conclusion

Topology is an area of Mathematics concerned with the properties of space that are preserved under continuous deformations including stretching and bending, but not tearing. By the middle of the 20th century, topology had become a major branch of Mathematics.

Topology as a branch of Mathematics can be formally defined as the study of qualitative properties of certain objects that are invariant under a certain kind of transformation especially those properties that are invariant under a certain kind of equivalence and it is the study of those properties of geometric configurations which remain invariant when these configurations are subjected to one-to-one bicontinuous transformations or homeomorphisms. Topology operates with more general concepts than analysis. Differential properties of a given transformation are nonessential for topology but bicontinuity is essential. As a consequence, topology is often suitable for the solution of problems to which analysis cannot give the answer.

Though the concept of topology has been identified as a difficult territory in Mathematics, we have taken it up as a challenge and cherishingly worked out this research study. Ideal Topology is a generalization of topology in classical mathematics, but it also has its own unique characteristics. It can also further up the understanding of basic structure of classical mathematics and offers new methods and results in obtaining significant results of classical mathematics. Moreover it also has applications in some important fields of Science and Technology.

We introduce the notions of $^*g$-$\mathcal{I}$-LC*-sets, $\mathcal{I}^*_{*g}$-closed sets and $\mathcal{I}$-$^*g_t$-sets. Also we define the notions of $^*g$-$\mathcal{I}$-LC*-continuous maps, $\mathcal{I}^*_{*g}$-continuous maps, $\mathcal{I}$-$^*g_t$-continuous maps and obtain decompositions of $*$-continuity.

References

[1] A. Acikgoz, T. Noiri and S. Yuksel, On $*$-operfect sets and α-$*$-closed sets, Acta Math. Hungar., 127(1-2)(2010), 146-153.

[2] J. Dontchev, On pre-$\mathcal{I}$-open sets and a decomposition of $\mathcal{I}$-continuity, Banyan Math. J., 2(1996).

[3] J. Dontchev, M. Ganster and T. Noiri, Unified operation approach of generalized closed sets via topological ideals, Math. Japan., 49(1999), 395-401.

[4] M. Ganster and I. L. Reilly, Locally closed sets and LC-continuous functions, Internat. J. Math. Math. Sci., 12(3)(1989), 417-424.

[5] E. Hatir and T. Noiri, On decompositions of continuity via idealization, Acta Math. Hungar., 96(4)(2002), 341-349.

[6] E. Hayashi, Topologies defined by local properties, Math. Ann., 156(1964), 205-215.

[7] V. Inthumathi, S. Krishnaprakash and M. Rajamani, Strongly $\mathcal{I}$ locally closed sets and decompositions of $*$-continuity, Acta Math. Hungar., 130(4)(2011), 358-362.

[8] S. Jafari, J. Jayasudha and K. Viswanathan, G-$\mathcal{I}$-LC*-sets and decompositions of $*$-continuity, IOSR-Jour. of Math., 2(2)(2012), 43-46.

[9] S. Jafari, T. Noiri, N. Rajesh and M. L. Thivagar, Another generalization of closed sets, Kochi J. Math., 3(2008), 25-38.

[10] S. Jafari, K. Viswanathan and J. Jayasudha, Another decomposition of $*$-continuity via ideal topological spaces, Jordan Journal of Mathematics and Statistics, 6(4)(2013), 285-295.

[11] D. Jankovic and T. R. Hamlett, New topologies from old via ideals, Amer. Math. Monthly, 97(4)(1990), 295-310.

[12] A. Keskin, T. Noiri and S. Yuksel, Decompositions of $\mathcal{I}$-continuity and continuity, Commun. Fac. Sci. Univ. Ankara Series A1, 53(2004), 67-75.

[13] K. Kuratowski, Topology, Vol. I, Academic press, New York, 1966.

[14] N. Levine, Generalized closed sets in topology, Rend. Circ. Mat. Palermo., (2)19(1970), 89-96.

[15] N. Levine, Semi-open sets and semi-continuity in topological spaces, Amer. Math. Monthly, 70(1963), 36-41.

[16] A. Muthulakshmi, M. Kamaraj, O. Ravi and R. Chitra, Decompositions of *g-coninuity, Journal of Advanced Research in Scientific Computing, 6(1)(2014), 16-24.

[17] M. Navaneethakrishnan, A study on ideal topological spaces, Ph.D. Thesis, Manonmaniam Sundaranar University, Tirunelveli, (2009).

[18] M. Rajamani, V. Inthumathi and S. Krishnaprakash, On $\mathcal{I}^{*\alpha}_g$-closed sets and $\mathcal{I}^{*\alpha}_g$-continuity, Jordan J. Math. Stat., 5(3)(2012), 201-208.

[19] O. Ravi, S. Tharmar, M. Sangeetha and J. Antony Rex Rodrigo, *g-closed sets in ideal topological spaces, Jordan Journal of Mathematics and Statistics, 6(1)(2013), 1-13.

[20] O. Ravi, V. Rajendran, K. Indirani and R. Senthil Kumar, $\mathcal{I}^*_{*g}$-closed sets, submitted.

[21] M. Sheik John, A study on generalized closed sets and continuous maps in topological and bitopological spaces, Ph.D. Thesis, Bharathiar University, Coimbatore, (2002).

[22] J. Tong, A decomposition of continuity in topological spaces, Acta Math. Hungar., 54(1989), 51-55.

[23] R. Vaidyanathaswamy, Set topology, Chelsea Publishing Company, New York, (1946).

[24] M. K. R. S. Veera Kumar, Between closed sets and g-closed sets, Mem. Fac. Kochi Univ. Math., 21(2000), 1-19.