http://www.newtheory.org **ISSN: 2149-1402**

Journal of New Theory

Received: 21.04.2015 *Year*: **2015,** *Number*: **4**, *Pages*: **39-52**
Accepted: 30.04.2015 *Original Article*[**]

ON SOME DECOMPOSITIONS OF FUZZY SOFT CONTINUITY

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Abstract – In this article, some open-like fuzzy soft sets such as fuzzy soft semi-open set, fuzzy soft pre-open set, fuzzy soft α-open set and corresponding variants of fuzzy soft continuous functions are introduced and discussed. Some other variants of fuzzy soft sets such as fuzzy soft semi-preclosed set, fuzzy soft t-set, fuzzy soft α*-set, fuzzy soft regular open set, fuzzy soft *B*-set, fuzzy soft *C*-set and fuzzy soft *D*(α)-set are defined and some properties of these sets are studied and investigated. Some continuous-like functions are introduced and we obtained some decomposition of fuzzy soft continuity.

Keywords ***–*** *Soft sets, fuzzy sets, fuzzy soft sets, fuzzy soft B-sets, fuzzy soft B-continuous function, fuzzy soft C-continuous function, fuzzy soft D(α)-continuous function.*

1 Introduction

The notion of continuity is always considered as an important concept in topological study and investigations. It is seen from existing literatures that several weak forms of continuity were introduced both for general and fuzzy topology to investigate and find deep properties of continuity. Each of the weak forms of continuity is strictly weaker than continuity. Theoretically, for each weak form of continuity, there is another weak form of continuity such that both of them imply continuity. This gives rise to different decompositions of continuous function. A classical example towards decomposition of continuity is the paper of N. Levine [8]. Inception of concept of soft set of Molodtsov [10] opened different directions for subsequent rapid developments, encompassing various basic concepts and results of topology for their generalizations to soft settings.

[**]*Edited by* Irfan Deli *(Area Editor) and* Naim Çağman *(Editor-in-Chief).*
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In 2011, Shabir and Naz [14] initiated the study of soft topological spaces. In 2001, Maji et al. [9], introduced the concept of fuzzy soft set. Analytical part of fuzzy soft set theory practically began with the work of B. Tanay et al.[15]. Recently, some researchers have worked to find some decompositions of continuity in soft topological spaces. In this paper, we proposed to define some open-like fuzzy soft sets and investigate for some decompositions of fuzzy soft continuity.

In section 2, some open-like fuzzy soft sets such as fuzzy soft semi-open set, fuzzy soft pre-open set, fuzzy soft α-open set and corresponding variants of fuzzy soft continuous functions are introduced and discussed.

In section 3, we defined fuzzy soft semi-preclosed set, fuzzy soft t-set, fuzzy soft α*-set, fuzzy soft regular open set, fuzzy soft *B*-set, fuzzy soft *C*-set and fuzzy soft *D*(α)-set. We studied these sets and investigate some properties of these sets.

In section 4, we defined some continuous-like functions and we obtained some decompositions of fuzzy soft continuity.

2 Preliminaries

Definition 2.1. [10**]** Let $A \subseteq E$. A pair (F, A) is called a soft set over U if and only if F is a mapping given by $F : A \longrightarrow P(U)$ such that $F(e) = \varphi$ if $e \notin A$ and $F(e) \neq \varphi$ if $e \in A$, where φ stands for the empty set, U is an initial universe set, E is the set of parameters and $P(U)$ is the set of all subsets of U. Here F is called approximate function of the soft set (F, A) and the value $F(e)$ is a set called *e*–element of the soft set. In other words, the soft set is a parameterized family of subsets of the set U.

Definition 2.2. [9] Let U be an initial universe set, let E be a set of parameters, let $A \subseteq E$. A pair (F, A) is called a fuzzy soft set over U if and only if F is a mapping given by $F : A \longrightarrow I^U$ such that $F(e) = 0_U$ if $e \notin A$ and $F(e) \neq 0_U$ if $e \in A$, where $0_U(u) = 0$ for all $u \in U$. Here F is called approximate function of the fuzzy soft set (F, A) and the value $F(e)$ is a fuzzy set called *e*–element of the fuzzy soft set (F, A). Thus a fuzzy soft set (F, A) over U can be represented by the set of ordered pairs $(F, A) = \{ (e, F(e)) : e \in A,\ F(e) \in I^U \}$. In other words, the fuzzy soft set is a parameterized family of fuzzy subsets of the set U.

Definition 2.3. [3,4] A fuzzy soft set (F, A) over U is called a *null* fuzzy soft set, denoted by $\tilde{0}_E$, if $F(e) = 0_U$ for all $e \in A \subseteq E$.

Remark 2.4. According to the definition of fuzzy soft set, i.e., $F(e) \neq 0_U$ if $e \in A \subseteq E$, 0_U does not belong to the co-domain of F. Therefore, the concept of null fuzzy soft set can be defined as follows.

Definition 2.5. A fuzzy soft set (F, A) over U is called a *null* fuzzy soft set or an *empty* fuzzy soft set, whenever $A = \varphi$.

Definition 2.6. A fuzzy soft set (F, A) over U is said to be an *A–universal* fuzzy soft set if $F(e) = 1_U$ if $e \in A$, where $1_U(u) = 1$ for all $u \in U$.

An *A–universal* fuzzy soft set is denoted by $\tilde{1}_A$.

Definition 2.7. [13] A fuzzy soft set (F, A) over U is said to be an *absolute* fuzzy soft set or a *universal* fuzzy soft set if $A = E$ and $F(e) = 1_U$ for all $e \in E$.

An *absolute* fuzzy soft set is denoted by $\tilde{1}_E$.

Definition 2.8. [9] A fuzzy soft set (F, A) is said to be a fuzzy soft subset of a fuzzy soft set (G, B) over a common universe U if $A \subseteq B$ and $F(e) \leq G(e)$ for all $e \in A$.

We redefine fuzzy soft subset as follows.

Definition 2.9. A fuzzy soft set (F, A) is said to be a fuzzy soft subset of a fuzzy soft set (G, B) over a common universe U if either $F(e) = 0_U$ for all $e \in A$ or $A \subseteq B$ and $F(e) \leq G(e)$ for all $e \in A$.

If a fuzzy soft set (F, A) is a fuzzy soft subset of a fuzzy soft set (G, B) we write $(F, A) \tilde{\subseteq} (G, B)$.

(F, A) is said to be a fuzzy soft superset of a fuzzy soft set (G, B) if (G, B) is a fuzzy soft subset of (F, A) and we write $(F, A) \tilde{\supseteq} (G, B)$.

Definition 2.10. [13] Two fuzzy soft sets (F, A) and (G, B) over a common universe are said to be equal, denoted by $(F, A) = (G, B)$, if $(F, A) \tilde{\subseteq} (G, B)$ and $(G, B) \tilde{\subseteq} (F, A)$. That is, if $F(e) \leq G(e)$ and $G(e) \leq F(e)$ for all $e \in E$.

Definition 2.11. [1,13] The intersection of two fuzzy soft sets (F, A) and (G, B) over a common universe U is the fuzzy soft set (H, C) where $C = A \cap B$ and $H(e) = F(e) \wedge G(e)$ for all $e \in C$ and we write $(H, C) = (F, A) \tilde{\cap} (G, B)$.

In particular, if $A \cap B = \varphi$ or $F(e) \wedge G(e) = 0_U$ for every $e \in A \cap B$, then $H(e) = 0_U$.

Definition 2.12. [9] The union of two fuzzy soft sets (F, A) and (G, B) over a common universe U is the fuzzy soft set (H, C) where $C = A \cup B$ and for all $e \in C$, $H(e) = F(e)$ if $e \in A - B$, $H(e) = G(e)$ if $e \in B - A$, $H(e) = F(e) \vee G(e)$ if $e \in A \cap B$. In this case we write $(H, C) = (F, A) \tilde{\cup} (G, B)$.

Definition 2.13. [9] The complement of a fuzzy soft set (F, A), denoted by $(F, A)^C$, is defined as $(F, A)^C = (F^C, \neg A)$, where $F^C : \neg A \to I^U$ is a mapping given by $F^C(e) = (F(\neg e))^C$ for all $e \in \neg A$.

Alternatively, the complement of a fuzzy soft set can be defined as follows.

Definition 2.14. [15] The fuzzy soft complement of a fuzzy soft set (F, A), denoted by $(F, A)^C$, is defined as $(F, A)^C = (F^C, A)$, where $F^C(e) = 1 - F(e)$ for every $e \in A$. Clearly, $((F, A)^C)^C = (F, A)$ and $(\tilde{1}_E)^C = \tilde{0}_E$ and $(\tilde{0}_E)^C = \tilde{1}_E$.

Proposition 2.15. Let (F, A) be a fuzzy soft set over (U, E). Then

1. $(F, A) \tilde{\cup} (F, A) = (F, A)$, $(F, A) \tilde{\cap} (F, A) = (F, A)$

2. $(F, A)\tilde{\cup}\,\tilde{0}_E = (F, A)$, $(F, A)\tilde{\cap}\,\tilde{0}_E = \tilde{0}_E$
3. $(F, A)\tilde{\cup}\,\tilde{1}_E = \tilde{1}_E$, $(F, A)\tilde{\cap}\,\tilde{1}_E = (F, A)$
4. $(F, A)\tilde{\cup}(F, A)^C = \tilde{1}_E$, $(F, A)\tilde{\cap}(F, A)^C = \tilde{0}_E$

Proposition 2.16. Let (F, A), $(G\ B)$, (H, C) be fuzzy soft sets over (U, E). Then

1. $(F, A)\tilde{\cup}(G\ B) = (G, B)\tilde{\cup}(F, A)$, $(F, A)\tilde{\cap}(G, B) = (G, B)\tilde{\cap}(F, A)$
2. $((F, A)\tilde{\cup}(G, B))^C = (G, B)^C\tilde{\cap}(F, A)^C$, $((F, A)\tilde{\cap}(G, B))^C = (G, B)^C\tilde{\cup}\ (F, A)^C$
3. $((F, A)\tilde{\cup}(G, B))\tilde{\cup}(H, C) = (F, A)\tilde{\cup}((G, B)\tilde{\cup}(H, C))$, $((F, A)\tilde{\cap}\ (G, B))\tilde{\cap}(H, C) = (F, A)\ \tilde{\cap}((G, B)\tilde{\cap}(H, C))$
4. $(F, A)\tilde{\cup}((G, B)\tilde{\cap}(H, C)) = ((F, A)\tilde{\cup}(G\ B))\tilde{\cap}((F, A)\tilde{\cup}(H, C))$, $(F, A)\tilde{\cap}((G, B))\tilde{\cup}(H, C)) = ((F, A)\ \tilde{\cap}(G, B))\tilde{\cup}((F, A)\tilde{\cap}(H, C))$

3 Fuzzy Soft Pre-open Set, Fuzzy soft α-open Set, Fuzzy Soft semi-open Set

In this section, we defined fuzzy soft pre-open set, fuzzy soft α-open set and we mentioned fuzzy soft semi-open set [5]. Then we defined the corresponding weaker forms of fuzzy soft continuous functions, namely, fuzzy soft pre-continuous, fuzzy soft α-continuous and fuzzy soft semi-continuous functions.

Let us recall the following definitions, propositions and theorems.

Definition 3.1. [13,15] A fuzzy soft topology τ on (U, E) is a family of fuzzy soft sets over (U, E), satisfying the following properties:

1. $\tilde{0}_E, \tilde{1}_E \in \tau$
2. If $(F, A), (G, B) \in \tau$ then $(F, A)\tilde{\cap}(G, B) \in \tau$.
3. If $(F, A)_\alpha \in \tau$, $\forall \alpha \in \Lambda$ then $\tilde{\bigcup}_{\alpha\in\Lambda} (F, A)_\alpha \in \tau$.

Definition 3.2. [13,15] If τ is a fuzzy soft topology on (U, E), the triple (U, E, τ) is said to be a fuzzy soft topological space. Each member of τ is called a fuzzy soft open set in (U, E, τ). The family of all Fuzzy soft open sets is denoted by $FSOS(U, E)$.

Definition 3.3. [12] Let (U, E, τ) be a fuzzy soft topological space. A fuzzy soft set is called fuzzy soft closed if its complement is a member of τ.

Proposition 3.4. [12] Let (U, E, τ) be a fuzzy soft topological space and let τ' be the collection of all fuzzy soft closed sets. Then

1. $\tilde{0}_E, \tilde{1}_E \in \tau'$
2. If $(F, A), (G\ B) \in \tau'$ then $(F, A)\tilde{\cup}(G, B) \in \tau'$.

3. If $(F, A)_\alpha \in \tau'$, $\forall \alpha \in \Lambda$ then $\tilde{\bigcap}_{\alpha \in \Lambda} (F, A)_\alpha \in \tau'$.

Definition 3.5.[12,15] Let (U, E, τ) be a fuzzy soft topological space. Let (F, A) be a fuzzy soft set over (U, E). Then the fuzzy soft closure of (F, A), denoted by $\overline{(F,A)}$, is defined as the intersection of all fuzzy soft closed sets which contain (F, A). That is, $\overline{(F,A)} = \tilde{\bigcap}\{(G, B) : (G, B)$ is fuzzy soft closed and $(F, A) \tilde{\subseteq} (G, B)\}$. Clearly, $\overline{(F,A)}$ is the smallest fuzzy soft closed set over (U, E) which contain (F, A). It is also clear that $\overline{(F,A)}$ is fuzzy soft closed and $(F, A) \tilde{\subseteq} \overline{(F,A)}$.

Theorem 3.6.[6] Let (U, E, τ) be a fuzzy soft topological space. Let (F, A) and (G, B) are fuzzy soft sets over (U, E). Then

1. $\overline{\tilde{0}_E} = \tilde{0}_E$, $\overline{\tilde{1}_E} = \tilde{1}_E$.
2. $(F, A) \tilde{\subseteq} \overline{(F,A)}$.
3. (F, A) is fuzzy soft closed if and only if $(F, A) = \overline{(F,A)}$.
4. $\overline{(\overline{(F,A)})} = \overline{(F,A)}$.
5. $(F, A) \tilde{\subseteq} (G, B)$ implies $\overline{(F,A)} \tilde{\subseteq} \overline{(G,B)}$.
6. $\overline{(F,A) \tilde{\cup} (G,B)} = \overline{(F,A)} \tilde{\cup} \overline{(G,B)}$.
7. $\overline{(F,A) \tilde{\cap} (G,B)} \tilde{\subseteq} \overline{(F,A)} \tilde{\cap} \overline{(G,B)}$

Definition 3.7. [12,15] Let (U, E, τ) be a fuzzy soft topological space. Let (F, A) be a fuzzy soft set over (U, E). Then the fuzzy soft interior of (F, A), denoted by $(F, A)^o$, is defined as the union of all fuzzy soft open sets contained in (F, A). That is, $(F, A)^o = \tilde{\bigcup}\{ (G, B) : (G, B)$ is fuzzy soft open and $(G, B) \tilde{\subseteq} (F, A) \}$. Clearly, $(F, A)^o$ is the largest fuzzy soft open set over (U, E) contained in (F, A). It is also clear that $(F, A)^o$ is fuzzy soft open and $(F, A)^o \tilde{\subseteq} (F, A)$.

Theorem 3.8. [6] Let (U, E, τ) be a fuzzy soft topological space. Let (F, A) and (G, B) are fuzzy soft sets over (U, E). Then

1. $(\tilde{0}_E)^o = \tilde{0}_E$ and $(\tilde{1}_E)^o = \tilde{1}_E$.
2. $(F, A)^o \tilde{\subseteq} (F, A)$.
3. $((F, A)^o)^o = (F, A)^o$.
4. (F, A) is a fuzzy soft open set if and only if $(F, A)^o = (F, A)$.
5. $(F, A) \tilde{\subseteq} (G, B)$ implies $(F, A)^o \tilde{\subseteq} (G, B)^o$.
6. $(F, A)^o \tilde{\cap} (G, B)^o = ((F, A) \tilde{\cap} (G, B))^o$.
7. $(F, A)^o \tilde{\cup} (G, B)^o \tilde{\subseteq} ((F, A) \tilde{\cup} (G, B))^o$.

We now define some open-like fuzzy soft sets.

Let us denote a family of fuzzy soft sets over (*U*, *E*) by *FSS*(*U*, *E*).

Definition 3.9. [5] Let (*U*, *E*, τ) be a fuzzy soft topological space. Let (*F*, *A*) $\in$ *FSS*(*U*, *E*). Then (*F*, *A*) is said to be fuzzy soft semi-open if (*F*, *A*) $\tilde{\subseteq}$ $\overline{(F,A)^o}$. The family of all fuzzy soft semi-open sets is denoted by *FSSOS*(*U*, *E*).

Example 3.10. Let $U = \{p, q, r\}$, $E = \{e_1, e_2, e_3, e_4\}$. $A = \{e_1\} \subseteq E$, $B = \{e_2\} \subseteq E$. Let us consider the following fuzzy soft sets over *(U, E).*

(*F*, *A*) = {*F*(e_1) = { *p/0.2, q/0.7, r/0.6*}, *F*(e_2) = { *p/0, q/0, r/0*}, *F*(e_3) = { *p/0, q/0, r/0*}, *F*(e_4) = { *p/0, q/0, r/0*}}
(*G*, *B*) = {*G*(e_1) = { *p/0, q/0, r/0*}, *G*(e_2) = { *p/0.1, q/0.3, r/0.2*}, *G*(e_3) = { *p/0, q/0, r/0*}, *G*(e_4) = { *p/0, q/0, r/0*}}
Let us consider the fuzzy soft topology $\tau = \{\tilde{0}_E, \tilde{1}_E, (G, B)\}$ over *(U, E).* Then (*F*, *A*) is fuzzy soft semi-open set.

Definition 3.11. Let (*U*, *E*, τ) be a fuzzy soft topological space. Let (*F*, *A*) $\in$ *FSS*(*U*, *E*). Then (*F*, *A*) is said to be

1. Fuzzy soft pre-open if (*F*, *A*) $\tilde{\subseteq}$ $(\overline{F,A})^o$,
2. Fuzzy soft α-open if (*F*, *A*) $\tilde{\subseteq}$ $(\overline{(F,A)^o})^o$.

The family of all Fuzzy soft pre-open (Fuzzy soft α-open) is denoted by *FSPOS*(*U*, *E*) (*FSαOS*(*U*, *E*)).

Remark 3.12 $\tilde{0}_E$ and $\tilde{1}_E$ are always fuzzy soft pre-open.

Remark 3.13 $\tilde{0}_E$ and $\tilde{1}_E$ are always fuzzy soft α-open.

Remark 3.14 Every fuzzy soft open set is a fuzzy soft pre-open set but not conversely.

Example 3.15 Let $U = \{p, q, r\}$, $E = \{e_1, e_2, e_3, e_4\}$. $A = \{e_1\} \subseteq E$, $B = \{e_3\} \subseteq E$. Let us consider the following fuzzy soft sets over *(U, E).*

(*F*, *A*) = {*F*(e_1) = { *p/0.1, q/0.7, r/0.9*}, *F*(e_2) = { *p/0, q/0, r/0*}, *F*(e_3) = { *p/0, q/0, r/0*}, *F*(e_4) = { *p/0, q/0, r/0*}}
(*G*, *B*) = {*G*(e_1) = { *p/0, q/0, r/0*}, *G*(e_2) = { *p/0, q/0, r/0*}, *G*(e_3) = { *p/0.4, q/0.2, r/0.7*}, *G*(e_4) = { *p/0, q/0, r/0*}}
Let us consider the fuzzy soft topology $\tau = \{\tilde{0}_E, \tilde{1}_E, (G, B)\}$ over *(U, E).* Then (*F*, *A*) is fuzzy soft pre-open set but (*F*, *A*) is not a fuzzy soft open.

Remark 3.16 Every fuzzy soft open set is a fuzzy soft α-open set but not conversely.

Example 3.17 Let $U = \{p, q, r\}$, $E = \{e_1, e_2, e_3\}$. $A = \{e_2\} \subseteq E$, $B = \{e_3\} \subseteq E$. Let us consider the following fuzzy soft sets over *(U, E).*

(*F*, *A*) = {*F*(e_1) = { *p/0, q/0, r/0*}, *F*(e_2) = { *p/0.7, q/0.6, r/0.5*}, *F*(e_3) = { *p/0, q/0, r/0*}}

$(G, B) = \{G(e_1) = \{ p/0, q/0, r/0\}, G(e_2) = \{ p/0, q/0, r/0\}, G(e_3) = \{ p/0.1, q/0.3, r/0.2\}\}$

Let us consider the fuzzy soft topology $\tau = \{\tilde{0}_E, \tilde{1}_E, (G, B)\}$ over (U, E). Then (F, A) is fuzzy soft α-open set but not a fuzzy soft open set.

Theorem 3.18. Let (U, E, τ) be a fuzzy soft topological space. Let (F, A) and (G, B) are fuzzy soft sets over (U, E). If either (F, A) is a fuzzy soft semi-open set or (G, B) is a fuzzy soft semi-open set $(\overline{(F,A)\tilde{\cap}(G,B)})^o = (\overline{(F,A)})^o \tilde{\cap} (\overline{(G,B)})^o$

Definition 3.19. [7] Let $FSS(U, E_1)$ and $FSS(V, E_2)$ be the families of all fuzzy soft sets over (U, E_1) and (V, E_2) respectively. Let $u : U \to V$ and $p : E_1 \to E_2$ be two functions. Then f_{pu} is called a fuzzy soft mapping from $FSS(U, E_1)$ to $FSS(V, E_2)$, denoted by $f_{pu} : FSS(U, E_1) \to FSS(V, E_2)$ and defined as follows:
(1) Let (F, A) be a fuzzy soft set in $FSS(U, E_1)$. Then the image of (F, A) under the fuzzy soft mapping f_{pu} is the fuzzy soft set over (V, E_2) defined by $f_{pu}((F, A))$, where

$$f_{pu}((F,A))(e_2)(y) = \begin{cases} \bigvee_{x \in u^{-1}(y)} \Big(\bigvee_{e_1 \in p^{-1}(e_2) \cap A} F(e_1) \Big)(x) & \text{if } u^{-1}(y) \neq \varphi, \text{ and } p^{-1}(e_2) \cap A \neq \varphi. \\ 0_V & \text{otherwise.} \end{cases}$$

(2) Let (G, B) be a fuzzy soft set in $FSS(V, E_2)$. Then the pre-image (inverse image) of (G, B) under the fuzzy soft mapping f_{pu} is the fuzzy soft set over (U, E_1) defined by $f^{-1}_{pu}((G, B))$, where

$$f^{-1}_{pu}((G,B))(e_1)(x) = \begin{cases} G(p(e_1))(u(x)) & \text{for } p(e_1) \in B \\ 0_U & \text{otherwise.} \end{cases}$$

Definition 3.20. If p and u are injective in definition 3.19, then the fuzzy soft mapping f_{pu} is said to be injective. If p and u are surjective then the fuzzy soft mapping f_{pu} is said to be surjective. If p and u are constant then f_{pu} is called constant.

Definition 3.21. [2] Let (U, E_1, τ_1) and (V, E_2, τ_2) be two fuzzy soft topological spaces. A fuzzy soft mapping $f_{pu} : (U, E_1, \tau_1) \to (V, E_2, \tau_2)$ is called fuzzy soft continuous if $f^{-1}_{pu}((G, B)) \in \tau_1$ for all $(G, B) \in \tau_2$.

Definition 3.22. Let (U, E_1, τ_1) *and* (V, E_2, τ_2) be two fuzzy soft topological spaces. A fuzzy soft mapping $f_{pu} : (U, E_1, \tau_1) \to (V, E_2, \tau_2)$ is called

1. fuzzy soft pre-continuous if $f^{-1}_{pu}((G, B)) \in FSPOS(U, E_1)$ for all $(G, B) \in FSOS(V, E_2)$,
2. fuzzy soft α-continuous if $f^{-1}_{pu}((G, B)) \in FS\alpha OS(U, E_1)$ for all $(G, B) \in FSOS(V, E_2)$,
3. fuzzy soft semi-continuous if $f^{-1}_{pu}((G, B)) \in FSSOS(U, E_1)$ for all $(G, B) \in FSOS(V, E_2)$.

Remark 3.23. A fuzzy soft continuous mapping is fuzzy soft pre-continuous but not conversely.

Example 3.24. Let $U = \{p, q, r\}$, $E = \{e_1, e_2, e_3, e_4\}$. $A = \{e_1\} \subseteq E$, $B = \{e_3\} \subseteq E$. Let us consider the following fuzzy soft sets over *(U, E).*

$(F, A) = \{F(e_1) = \{p/0.1, q/0.7, r/0.9\}, F(e_2) = \{p/0, q/0, r/0\}, F(e_3) = \{p/0, q/0, r/0\}, F(e_4) = \{p/0, q/0, r/0\}\}$
$(G, B) = \{G(e_1) = \{p/0, q/0, r/0\}, G(e_2) = \{p/0, q/0, r/0\}, G(e_3) = \{p/0.4, q/0.2, r/0.7\}, G(e_4) = \{p/0, q/0, r/0\}\}$

Let us consider the fuzzy soft topology $\tau_1 = \{\tilde{0}_E, \tilde{1}_E, (G, B)\}$, and $\tau_2 = \{\tilde{0}_E, \tilde{1}_E, (F, A)\}$ over (U, E). We define the fuzzy soft mapping $f_{pu} : (U, E, \tau_1) \rightarrow (U, E, \tau_2)$ where $u : U \rightarrow U$ and $p : E \rightarrow E$ be a mapping defined as $u(p) = p$, $u(q) = q$, $u(r) = r$ and $p(e_1) = e_1$, $p(e_2) = e_2$, $p(e_3) = e_3$, $p(e_4) = e_4$. Now, $f^{-1}{}_{pu}((F, A)) = (F, A) \not\in (U, E, \tau_1)$ but (F, A) is fuzzy soft pre-open set. Thus $f_{pu} : (U, E, \tau_1) \rightarrow (U, E, \tau_2)$ is fuzzy soft pre-continuous; but not fuzzy soft continuous.

Remark 3.25. A fuzzy soft continuous mapping is fuzzy soft α-continuous but not conversely.

Example 3.26. Let U = $\{p, q, r\}$, E = $\{e_1, e_2, e_3\}$. $A = \{e_2\} \subseteq E$, $B = \{e_3\} \subseteq E$. Let us consider the following fuzzy soft sets over *(U, E).*

$(F, A) = \{F(e_1) = \{p/0, q/0, r/0\}, F(e_2) = \{p/0.7, q/0.6, r/0.5\}, F(e_3) = \{p/0, q/0, r/0\}\}$

$(G, B) = \{G(e_1) = \{p/0, q/0, r/0\}, G(e_2) = \{p/0, q/0, r/0\}, G(e_3) = \{p/0.1, q/0.3, r/0.2\}\}$

Let us consider the fuzzy soft topology $\tau_1 = \{\tilde{0}_E, \tilde{1}_E, (G, B)\}$, and $\tau_2 = \{\tilde{0}_E, \tilde{1}_E, (F, A)\}$ over *(U, E).* We define the fuzzy soft mapping $f_{up} : (U, E, \tau_1) \rightarrow (U, E, \tau_2)$ where $u : U \rightarrow U$ and $p : E \rightarrow E$ be a mapping defined as $u(p) = p, u(q) = q, u(r) = r, p(e_1) = e_1, p(e_2) = e_2, p(e_3) = e_3$
Now, $f_{up}^{-1}(F, A) = (F, A) \tilde{\not\in} (U, E, \tau_1)$ but (F, A) is fuzzy soft α-open set.

Thus $f_{up} : (U, E, \tau_1) \rightarrow (U, E, \tau_2)$ is fuzzy soft α-continuous; but not fuzzy soft continuous.

4 Fuzzy Soft *B*-Set, Fuzzy Soft *C*-Set, Fuzzy Soft *D*(α)-Set

In this section, we defined fuzzy soft semi-preclosed set, fuzzy soft t-set, fuzzy soft α*-set, fuzzy soft regular open set, fuzzy soft *B*-set, fuzzy soft *C*-set and fuzzy soft *D*(α)-set. We studied these sets and investigate some properties of these sets.

Definition 4.1. Let (U, E, τ) be a fuzzy soft topological space. Let $(F, A) \in FSS(U, E)$. Then (F, A) is said to be

1. fuzzy soft semi-preclosed set if $(\overline{(\overline{(F,A)})^o})^o \tilde{\subseteq}$ (F, A),
2. fuzzy soft *t*-set if $(F, A)^o = (\overline{(F,A)})^o$,
3. fuzzy soft α*-set if $(\overline{(\overline{(F,A)})^o})^o = (F, A)^o$,
4. fuzzy soft regular open [11] if $(F, A) = (\overline{(F,A)})^o$.

Example 4.2. $\tilde{0}_E$ and $\tilde{1}_E$ are always fuzzy soft semi pre-closed set, fuzzy soft *t*-set, fuzzy soft α*-set, fuzzy soft regular open set.

Remark 4.3. It is clear from definition that in a fuzzy soft topological space (U, E, τ), every fuzzy soft regular open set is fuzzy soft open set, but the converse is not true, which follows from the following example.

Example 4.4. Let U = $\{a, b, c\}$, E = $\{e_1, e_2, e_3, e_4\}$. $A = \{e_1, e_2\} \subseteq E$, $B = \{e_1, e_2, e_3\} \subseteq E$. Let us consider the following fuzzy soft sets over (U, E).

$(F, A) = \{F(e_1) = \{a/0.5, b/0.2, c/0\}, F(e_2) = \{a/0.7, b/0.6, c/0.3\}, F(e_3) = \{a/0, b/0, c/0\}, F(e_4) = \{a/0, b/0, c/0\}\}$
$(G, B) = \{G(e_1) = \{a/0.5, b/0.3, c/0\}, G(e_2) = \{a/0.7, b/0.8, r/0.5\}, G(e_3) = \{a/0.4, b/0.9, c/0.8\}, G(e_4) = \{a/0, b/0, c/0\}\}$
Let us consider the fuzzy soft topology $\tau_1 = \{\tilde{0}_E, \tilde{1}_E, (F, A), (G, B)\}$, over (U, E). Now,
$(F, A)^C = (F^C, A) = \{F^C(e_1) = \{a/0.5, b/0.8, c/1\}, F^C(e_2) = \{a/0.3, b/0.4, c/0.7\}, F^C(e_3) = \{a/1, b/1, c/1\}, F^C(e_4) = \{a/1, b/1, c/1\}\}$
and
$(G, B)^C = (G^C, B) = \{G^C(e_1) = \{a/0.5, b/0.7, c/1\}, G^C(e_2) = \{a/0.3, b/0.2, c/0.5\}, G^C(e_3) = \{a/0.6, b/0.1, c/0.2\}, G^C(e_4) = \{a/1, b/1, c/1\}\}$
and clearly, $(F, A)^C$ and $(G, B)^C$ are fuzzy soft closed sets.
Then the fuzzy soft closure of (F, A), is the intersection of all fuzzy soft closed sets containing (F, A). That is $\overline{(F,A)} = \tilde{1}_E$
The fuzzy soft interior of $\overline{(F,A)}$, is the union of all fuzzy soft open sets contained in $\overline{(F,A)}$.
That is $(\overline{(F,A)})^o = (\tilde{1}_E)^o = \tilde{1}_E$
Hence, (F, A) is open but not a fuzzy soft regular open set.

Remark 4.5. A fuzzy soft *t*-set and fuzzy soft α*-set may not be fuzzy soft regular open set, which follows from the following example.

Example 4.6. Let $U = \{a, b\}$, $E = \{e_1, e_2\}$,
Let us consider the following fuzzy soft sets over (U, E).
$(F, E) = \{F(e_1) = \{a/0.1, b/0.1\}, F(e_2) = \{a/0.1, b/0.2\}\}$
$(G, E) = \{G(e_1) = \{a/0.2, b/0.2\}, G(e_2) = \{a/0.1, b/0.2\}\}$
$(H, E) = \{H(e_1) = \{a/0.2, b/0.7\}, H(e_2) = \{a/0.2, b/0.7\}\}$

$(I,E)=\{I(e_1)=\{a/0.9,b/0.9\},I(e_2)=\{a/0.7,b/0.7\}\}$
$(J,E)=\{J(e_1)=\{a/0.9,b/1\},J(e_2)=\{a/0.7,b/0.9\}\}$
Let us consider the fuzzy soft topology $\tau=\{\tilde{0}_E,\tilde{1}_E,(F,E),(G,E),(H,E),(I,E),(J,E)\}$ over *(U, E)*.
Now, $(E,F)^C=\{F^C(e_1)=\{a/0.9,b/0.9\},F^C(e_2)=\{a/0.9,b/0.8\}\}$
$(G,E)^C=\{G^C(e_1)=\{a/0.8,b/0.8\},G^C(e_2)=\{a/0.9,b/0.8\}\}$
$(H,E)^C=\{H^C(e_1)=\{a/0.8,b/0.3\},H^C(e_2)=\{a/0.8,b/0.3\}\}$
$(I,E)^C=\{I^C(e_1)=\{a/0.1,b/0.1\},I^C(e_2)=\{a/0.3,b/0.3\}\}$
$(J,E)^C=\{J^C(e_1)=\{a/0.1,b/0\},J^C(e_2)=\{a/0.3,b/0.1\}\}$
Clearly, $(F,E)^C,(G,E)^C,(H,E)^C,(I,E)^C$ and $(J,E)^C$ are fuzzy soft closed sets.
Obviously, $(F,E),(G,E),(H,E),(I,E)$ are fuzzy soft α*-sets and also fuzzy soft regular open sets.
Let us consider the fuzzy soft set (K,E) over (U,E) defined as
$(K,E)=\{K(e_1)=\{a/0.4,b/0.5\},K(e_2)=\{a/0.3,b/0.4\}\}$. Then (K,E) is a fuzzy soft t-set and also fuzzy soft α*-set but not a fuzzy soft regular open set.

Definition 4.7. Let (U, E, τ) be a fuzzy soft topological space. Let (F, A) $\in FSS(U, E)$. Then (F, A) is said to be

1. fuzzy soft B-set if (F, A) = (G, B) $\tilde{\cap}$ (H, C), where (G, B) $\in \tau$ and (H, C) is a fuzzy soft t-set,
2. fuzzy soft C-set if (F, A) = (G, B) $\tilde{\cap}$ (H, C), where (G, B) $\in \tau$ and (H, C) is a fuzzy soft α*-set,
3. fuzzy soft D(α)-set if $(F, A)^o = (F, A) \tilde{\cap} (\overline{(F,A)^o})^o$.

Remark 4.8. $\tilde{0}_E$ and $\tilde{1}_E$ are always fuzzy soft B-set, fuzzy soft C-set, fuzzy soft D(α)-set.

Theorem 4.9. Let (U, E, τ) be a fuzzy soft topological space. Then the following statements are equivalent:

1. (F, A) is fuzzy soft α*-set.
2. (F, A) is fuzzy soft semi-preclosed set.
3. (F, A) is fuzzy soft regular open set.

Proof: Straight forward.

Theorem 4.10. Let (U, E, τ) be a fuzzy soft topological space. Then we have the following results:

1. A fuzzy soft semi-open set (F, A) is fuzzy soft t-set if and only if (F, A) is fuzzy soft α*-set.

2. A fuzzy soft α-open set (*F*, *A*) is fuzzy soft α*-set if and only if (*F*, *A*) is fuzzy soft regular open set.

Proof: (1) Let (*F*, *A*) be fuzzy soft semi-open and fuzzy soft *t*-set. Since (*F*, *A*) is a fuzzy soft semi-open set, $\overline{(F,A)^o} = \overline{(F,A)}$. Then $(F, A)^o$ = $(\overline{F,A})^o = ((\overline{F,A)^o})^o$. Hence (*F*, *A*) is fuzzy soft α*-set.

Conversely, let (*F*, *A*) be fuzzy soft semi-open and fuzzy soft α*-set. Since (*F*, *A*) is a fuzzy soft semi-open set, $\overline{(F,A)^o} = \overline{(F,A)}$. Then $(\overline{F,A})^o = ((\overline{F,A)^o})^o = (F, A)^o$. Hence (*F*, *A*) is fuzzy soft *t*-set.

(2) Let (*F*, *A*) be fuzzy soft α-open and fuzzy soft α*-set. Then by theorem 3.1, (*F*, *A*) is fuzzy soft semi-preclosed. Since (*F*, *A*) is fuzzy soft α-open, we have $((\overline{F,A)^o})^o =$ (*F*, *A*) and so $(\overline{F,A})^o = ((\overline{F,A)^o})^o$ = (*F*, *A*). Hence (*F*, *A*) is fuzzy soft regular open set.

Conversely, proof is obvious.

Theorem 4.11. Let (*U*, *E*, τ) be a fuzzy soft topological space. If (*F*, *A*) is fuzzy soft t-set, then (*F*, *A*) is fuzzy soft α*-set.

Proof: (1) Let (*F*, *A*) is fuzzy soft *t*-set. Then $(F, A)^o = (\overline{F,A})^o$. We have $((\overline{F,A)^o})^o = (\overline{F,A})^o = (F, A)^o$. Hence is (*F*, *A*) is fuzzy soft α*-set.

Theorem 4.12. Let (*U*, *E*, τ) be a fuzzy soft topological space. Then

(1) Every fuzzy soft α*-set is fuzzy soft *C*-set.

(2) Every fuzzy soft open set is fuzzy soft *C*-set.

Proof: The proof of (1) and (2) are obvious since $\tilde{1}_E$ is both fuzzy soft open and fuzzy soft α*-set.

Theorem 4.13. Every fuzzy soft t-set in a fuzzy soft topological space (*U*, *E*, τ) is fuzzy soft *B*-set.

Proof: Let a fuzzy soft set (*F*, *A*) in a fuzzy soft topological space (*U*, *E*, τ) be fuzzy soft *t*-set Let (*G*, *B*) = $\tilde{1}_E \in \tau$. Then (*F*, *A*) = $(G, B) \tilde{\cap} (F, A)$ and hence (*F*, *A*) is fuzzy soft *B*-set.

Theorem 4.14. Every fuzzy soft t-set in a fuzzy soft topological space (*U*, *E*, τ) is fuzzy soft *C*-set.

Proof: Let a fuzzy soft set (*F*, *A*) in a fuzzy soft topological space (*U*, *E*, τ) be fuzzy soft *t*-set. Then by theorem 3.5, (*F*, *A*) is fuzzy soft *B*-set. As (*F*, *A*) is fuzzy soft *B*-set, (*F*, *A*) = $(G, B) \tilde{\cap} (H, C)$, where $(G, B) \in \tau$ and (*H*, *C*) is a fuzzy soft *t*-set. Then $(H, C)^o = (\overline{H,C})^o \tilde{\supseteq} ((\overline{H,C)^o})^o \tilde{\supseteq} (H, C)^o$. Hence $(H, C)^o = ((\overline{H,C)^o})^o$. Therefore, (*F*, *A*) is fuzzy soft *C*-set.

Remark 4.15. Converse of the theorem 3.6 is not always true.

Example 4.16. Let $U = \{p, q, r\}$, $E = \{e_1, e_2, e_3, e_4\}$. $A = \{e_1\} \subseteq E$, $B = \{e_3\} \subseteq E$ and $C = \{e_4\} \subseteq E$. Let us consider the following fuzzy soft sets over (U, E).

$(F, A) = \{F(e_1) = \{p/0.3, q/0.4, r/0.4\}, F(e_2) = \{p/0, q/0, r/0\}, F(e_3) = \{p/0, q/0, r/0\}, F(e_4) = \{p/0, q/0, r/0\}\}$
$(G, B) = \{G(e_1) = \{p/0, q/0, r/0\}, G(e_2) = \{p/0, q/0, r/0\}, G(e_3) = \{p/0.4, q/0.5, r/0.5\}, G(e_4) = \{p/0, q/0, r/0\}\}$
$(H, C) = \{H(e_1) = \{p/0, q/0, r/0\}, H(e_2) = \{p/0, q/0, r/0\}, H(e_3) = \{p/0, q/0, r/0\}, H(e_4) = \{p/0.7, q/0.6, r/0.6\}\}$
Let us consider the fuzzy soft topology $\tau = \{\tilde{0}_E, \tilde{1}_E, (F, A), (G, B)\}$ over (U, E). Then (H, C) is fuzzy soft C-set but not fuzzy soft t-set.

Theorem 4.17. Let (U, E, τ) be a fuzzy soft topological space. Then (F, A) is fuzzy soft open set if and only if it is both fuzzy soft α-open and fuzzy soft C-set.

Proof: If (F, A) is fuzzy soft open set then clearly (F, A) is fuzzy soft α-open as well as fuzzy soft C-set.
Conversely, let (F, A) be both fuzzy soft α-open and fuzzy soft C-set. Since (F, A) is fuzzy soft C-set, there exist $(G, B) \in \tau$ and a fuzzy soft α*-set (H, C) such that $(F, A) = (G, B) \tilde{\cap} (H, C)$. Since (F, A) is fuzzy soft α-open, we get $(F, A) \tilde{\subseteq} (\overline{(F, A)^o})^o = (\overline{((G, B) \tilde{\cap} (H, C))^o})^o = (\overline{G, B})^o \tilde{\cap} (\overline{(H, C)^o})^o = (\overline{G, B})^o \tilde{\subseteq} (H, C)^o$. Therefore, $(F, A) = (G, B) \tilde{\cap} (H, C) \tilde{\subseteq} (G, B) \tilde{\cap} [(\overline{G, B})^o \tilde{\cap} (H, C)^o] = (G, B)) \tilde{\cap} (H, C)^o \tilde{\subseteq} (F, A)$. Consequently, $(F, A) = (G, B) \tilde{\cap} (H, C)^o$. Hence (F, A) is fuzzy soft open set.

Theorem 4.18. Let (U, E, τ) be a fuzzy soft topological space. Then (F, A) is fuzzy soft open set if and only if it is both fuzzy soft pre-open and fuzzy soft B-set.

Proof: If (F, A) is fuzzy soft open set then clearly (F, A) is fuzzy soft pre-open as well as fuzzy soft B-set.

Conversely, let (F, A) be both fuzzy soft pre-open and fuzzy soft B-set. Since (F, A) is fuzzy soft B-set, there exist $(G, B) \in \tau$ and a fuzzy soft t-set (H, C) such that $(F, A) = (G, B) \tilde{\cap} (H, C)$. Since (F, A) is fuzzy soft pre-open, we get $(F, A) \tilde{\subseteq} (\overline{F, A})^o = (\overline{(G, B) \tilde{\cap} (H, C)})^o = (\overline{G, B})^o \tilde{\cap} (\overline{H, C})^o = (\overline{G, B})^o \tilde{\subseteq} (H, C)^o$. Therefore, $(F, A) = (G, B) \tilde{\cap} (H, C) \tilde{\subseteq} (G, B) \tilde{\cap} [(\overline{G, B})^o \tilde{\cap} (H, C)^o] = (G, B)) \tilde{\cap} (H, C)^o \tilde{\subseteq} (F, A)$. As a consequence, $(F, A) \in \tau$.

Theorem 4.19. Let (U, E, τ) be a fuzzy soft topological space. Then (F, A) is fuzzy soft open set if and only if it is both fuzzy soft α-open and fuzzy soft D(α)-set.

Proof: If (F, A) is fuzzy soft open set then clearly (F, A) is fuzzy soft α-open as well as fuzzy soft D(α)-set. Conversely, let (F, A) be both fuzzy soft α-open and fuzzy soft D(α)-set. Since (F, A) is fuzzy soft D(α)-set, $(F, A)^o = (F, A) \tilde{\cap} (\overline{(F, A)^o})^o$. Since (F, *A*) is fuzzy soft *α*-open, we have $(F, A) \tilde{\subseteq} ((\overline{F, A})^o)^o$. Then $(F, A) \tilde{\cap} (F, A) = (F, A)$ $\tilde{\subseteq} ((\overline{F, A})^o)^o \tilde{\cap} (F, A)$. Hence $(F, A) \tilde{\subseteq} (F, A)^o$. As a consequence, $(F, A) \in \tau$.

5 Decomposition of Fuzzy Soft Continuity

In this section, we obtained some decomposition of fuzzy soft continuity.

Definition 5.1. Let (U, E_1, τ_1) and (V, E_2, τ_2) be two fuzzy soft topological spaces. A fuzzy soft mapping $f_{pu} : (U, E_1, \tau_1) \rightarrow (V, E_2, \tau_2)$ is called

1. fuzzy soft *C*-continuous if $f^{-1}_{pu}((G, B))$ is fuzzy soft *C*-set for all $(G, B) \in \tau_2$,
2. fuzzy soft *B*-continuous if $f^{-1}_{pu}((G, B))$ is fuzzy soft *B*-set for all $(G, B) \in \tau_2$,
3. fuzzy soft *D*(α)-continuous if $f^{-1}_{pu}((G, B))$ is fuzzy soft *D*(α)-set for all $(G, B) \in \tau_2$.

Theorem 5.2. Let (U, E_1, τ_1) and (V, E_2, τ_2) be two fuzzy soft topological spaces. A fuzzy soft mapping $fpu : (U, E_1, \tau_1) \rightarrow (V, E_2, \tau_2)$ is fuzzy soft continuous function if and only if it is both fuzzy soft α-continuous and fuzzy soft *C*-continuous.

Proof: The proof follows from theorem 4.17.

Theorem 5.3. Let (U, E_1, τ_1) and (V, E_2, τ_2) be two fuzzy soft topological spaces. A *fuzzy soft mapping* $f_{pu} : (U, E_1, \tau_1) \rightarrow (V, E_2, \tau_2)$ is fuzzy soft continuous function if and only if it is both fuzzy soft pre-continuous and fuzzy soft *B*-continuous.

Proof: The proof follows from theorem 4.18.

Theorem 5.4. Let (U, E_1, τ_1) and (V, E_2, τ_2) be two fuzzy soft topological spaces. A fuzzy soft mapping $f_{pu} : (U, E_1, \tau_1) \rightarrow (V, E_2, \tau_2)$ is fuzzy soft continuous function if and only if it is both fuzzy soft α-continuous and fuzzy soft *D*(α)-continuous.

Proof: The proof follows from theorem 4.19.

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