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HOMOMORPHISM IN ROUGH LATTICE

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Abstract – In this paper, we introduce the concept of set-valued homomorphism of a lattice which is the generalization of ordinary lattice homomorphism. We construct generalized rough lower and upper approximations operators, by means of a set-valued mapping, which are also the generalized form of lower and upper approximations of a lattice, and the corresponding properties are investigated and finally we cite an example to show usefulness of the paper.

Keywords – *Rough Set, Lower and Upper approximations, Lattice, Equivalence Relation, Homomorphism.*

1 Introduction

In this section, we give some basic notions and results about generalized rough sets and lattices. The concept of rough sets was introduced by Pawlak [13], a mathematical tool for dealing with uncertainty or vagueness ([14],[20]). In rough set theory, rough set can be described by a pair of ordinary sets called lower and upper approximations. The theory of rough set is an extension of set theory. The study of the algebraic structure of the mathematical theory proves itself effective in making the applications more efficient. Such researches may not only provide more insight into rough set theory, but also hopefully developed methods for applications. Rough set has been studied from algebraic view points by many researchers. Pomykala [15] showed that the set of rough set forms a stone algebra. Iwinski [4] suggested a lattice theoretic approach to the rough set. Liu and Zhu [8] presented the structures of the approximations based on arbitrary binary relation. The generalized rough sets over fuzzy lattices have been explored by Liu [7]. Algebraic structure of T-rough set and corresponding lattice theory are explored in ([5], [2]) respectively. In mathematics, a lattice is a partially ordered set in which any two elements have a unique supremum and infimum. Lattice can also be characterized as algebraic structures satisfying certain axiomatic identities. Since the two definitions are equivalent, lattice theory draws on both order theory and universal algebra. In this paper, we consider a lattice as a universal set and study the rough sets in a lattice.

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2 Preliminaries

Definition 2.1. Let U and V be two non-empty universes. Let f be a set-valued mapping given by $f : U \longrightarrow P(V)$. Then the triple (U, V, f) is referred to as a generalized approximation space or generalized rough set. Any set-valued function from U to $P(V)$ defines a binary relation from U to V by setting $R_f = \{(x, y) : y \in f(x)\}$. Obviously, if R is an arbitrary relation from U to V, then it can be defined a set-valued mapping $f_R : U \longrightarrow P(V)$ by $f_R(x) = \{y \in V : (x, y) \in R\}$ where $x \in U$. For any set $A \subseteq V$, the lower and upper rough approximations $A \downarrow f$ and $A \uparrow f$, are defined by

$$A \downarrow_f \;=\; \{x \in U : f(x) \subseteq X\} \tag{1}$$
$$and \quad A \uparrow_f \;=\; \{x \in U : f(x) \cap A \neq \emptyset\}. \tag{2}$$

The pair $(A \downarrow_f, A \uparrow_f)$ is referred to as a generalized rough set. If a subset $A \subseteq V$satisfies that $A \downarrow_f = A \uparrow_f$, then A is called a definable set of (U, V, f). We denote all the definable sets of (U, V, f) by $Def(f)$.

Definition 2.2. Let L be a lattice and $\emptyset \neq A \subseteq L$. Then A is called a sublattice if $a, b \in A$ implies $a \vee b \in A$ and $a \wedge b \in A$.

Definition 2.3. Let L and K be two complete lattices. A mapping $f : L \longrightarrow P(K)$ is called a complete set-valued homomorphism if
$\bigvee_{i \in I} f(a_i) \subseteq f(\bigvee_{i \in I} a_i)$ $\bigwedge_{i \in I} f(a_i) \subseteq f(\bigwedge_{i \in I} a_i)$.
A set-valued mapping f is called a strong complete set-valued homomorphism
if $\bigvee_{i \in I} f(a_i) = f(\bigvee_{i \in I} a_i)$ and $\bigwedge_{i \in I} f(a_i) = f(\bigwedge_{i \in I} a_i)$.

Definition 2.4. A non-empty sub-set A of L is called a sublattice of L if $a \vee b \in A, a \wedge b \in A$ for all $a, b \in A$

Definition 2.5. A non-empty sub-set A of L is called a convex sublattice of L if $[a \vee b, a \wedge b] \subseteq A$ for any $a, b \in A$.

Definition 2.6. An equivalence relation E on L is a reflexive, symmetric, and transitive binary relation on L. If E is an equivalence relation on L then the equivalence class of $x \in L$ is the set $\{y \in L : (x, y) \in E\}$. We write it as $[x]_E$.

Let us illustrate this definition using the following example.
Example (1) Let $L = \{a, b, c, d\}$ such that $c \vee b = b, c \vee d = d, b \vee d = a$ and $K = \{u, v, x, y, z\}$ such that $z \vee x = x, y \vee z = y, x \vee y = v, u \vee v = u$. Consider the set-valued mapping $f : L \longrightarrow P(K)$ defined by $f(a) = \{u, v\}, f(b) = \{v, x\}, f(c) = y, f(d) = \{y, z\}$. Then f is set valued homomorphism but not a strong set valued homomorphism for $f(b) \vee f(c) = v \neq f(b \vee c) = f(a) = \{u, v\}$.
(2) Let g be a lattice homomorphism from L to K. Then the set valued mapping $f : L \longrightarrow P(K)$ defined by $f(a) = \{g(a) : a \in L\}$ is a strong set valued homomorphism. If L and K are complete, then f is a strong complete set valued homomorphism.

3 Rough Lattice and Set-valued Homomorphism

Rough set was originally proposed by Pawlak [13] with the consideration of an equivalence relation. An equivalence relation is sometimes difficult to be obtained in real-world problems due to vagueness and incompleteness of human knowledge. From this point of view, in this section, we introduce the concept of set-valued isomorphism of lattices. Let L and K be two lattices. A mapping f is a set-valued mapping from L to $P(K)$. Where $P(K)$ represents the set of all non-empty sub set of K. For $a, b \in L$, we define $f(a) \vee f(b) = \{x \vee y : x \in f(a), y \in f(b)\}$.
$f(a) \wedge f(b) = \{x \wedge y : x \in f(a), y \in f(b)\}$.

Definition 3.1. Let L and K be two lattices. A mapping $f : L \longrightarrow P(K)$ is called a set-valued homomorphism if
$f(a) \bigvee f(b) \subseteq f(a \bigvee b)$
$f(a) \bigwedge f(b) \subseteq f(a \bigwedge b)$ for all $a, b \in L$.
A set-valued mapping f is called a strong set-valued homomorphism if
$f(a) \bigvee f(b) = f(a \bigvee b)$
$f(a) \bigwedge f(b) = f(a \bigwedge b)$ for all $a, b \in L$.

Theorem 3.2. Let L, K be lattices and $f : L \longrightarrow P(K)$ be a strong set valued homomorphism. If A, B are two non empty subset of K, then $(1) f \downarrow (A) \vee f \downarrow (B) \subseteq f \downarrow (A \vee B)$
$(2) f \downarrow (A) \wedge f \downarrow (B) \subseteq f \downarrow (A \wedge B)$

Proof: (1) Assume that $x \in f \downarrow (A) \vee f \downarrow (B)$, then there exist $y \in f \downarrow (A), z \in f \downarrow (B)$ such that $x = y \vee z$. Since f is a strong set-valued homomorphism, we have $f(x) = f(y \vee z) = f(y) \vee f(z) \subseteq A \vee B$. So $x \in f \downarrow (A \vee B)$.
(2) Again we assume that $x \in f \downarrow (A) \wedge f \downarrow (B)$, then there exist $y \in f \downarrow (A), z \in f \downarrow (B)$ such that $x = y \wedge z$. Since f is a strong set-valued homomorphism, we have $f(x) = f(y \wedge z) = f(y) \wedge f(z) \subseteq A \wedge B$. So $x \in f \downarrow (A \wedge B)$.

Theorem 3.3. Let L, K be two lattices. Then (1) Let $f : L \longrightarrow P(K)$ be a set-valued homomorphism. If A is a sublattice of K and $f \uparrow (A)$ is non-empty subset of L, then $f \uparrow (A)$ is a sublattice of L
(2) Let $f : L \longrightarrow P(K)$ be a strong set-valued homomorphism. If A is a sublattice of K and $f \downarrow (A)$ is non-empty subset of L, then $f \downarrow (A)$ is a sublattice of L.

Proof: (1) Assume that $x, y \in f \uparrow (A)$ there exist $a, b \in A$ such that $a \in f(x), b \in f(y)$. Since f is a set-valued homomorphism and A is a sublattice, we have $a \vee b \in f(x) \vee f(y) \subseteq f(x \vee y)$ and $a \vee b \in A$. So $a \vee b \in f(x \vee y) \cap A$ which implies that $x \vee y \in f \uparrow (A)$. Similarly, we have $x \wedge y \in f \uparrow (A)$
(2) Assume that $x, y \in f \downarrow (A)$, we have $f(x) \subseteq A, f(y) \subseteq A$. Since f is a strong set-valued homomorphism and A is a sublattice, we have $f(x \wedge y) = f(x) \wedge f(y)$ which implies that $x \vee y \in f \downarrow (A)$. Similarly, we have $x \wedge y \in f \downarrow (A)$.

Theorem 3.4. Let $f : L \longrightarrow P(K)$ be a set-valued homomorphism of lattices. If A, B are down-sets of K, then $f(A \cap B) = fA) \cap f(B)$.

Proof: Assume that $x \in f \uparrow (A) \cap f \uparrow (B)$, there exist $y \in A, z \in B$ such that $y.z \in f(x)$. Since A, B are down-sets, we have $y \wedge z \in A \cap B$. f is a set-valued homomorphism, we have $y \wedge z \in f(x) \cap f(x) \subseteq f(x \wedge x) = f(x)$. So $y \wedge z \in f(x) \cap (A \cap B)$ which implies $x \in f(A \cap B)$. We also know that $f \uparrow (A \cap B) \subseteq f \uparrow (A) \cap f \uparrow (B)$. Thus, we get the conclusion easily.

Definition 3.5. [5] Let E be an equivalence relation on L, then E is called a full congruence relation if $(a, b) \in E$ and $(c, d) \in E \Leftrightarrow (a \vee c, b \vee d) \in E$ and $(a \wedge c, b \wedge d) \in E$ for all $a, b, c, d \in L$.

Definition 3.6. Let E be an equivalence relation on L, then $(a, b) \in E \Leftrightarrow (a \vee x, b \vee x), (a \wedge x, b \wedge x) \in E$ for all $x \in L$. If A, B are non empty subset of L, for any $a \in L$ we define $A \vee B = \{x \vee y : x \in A, y \in B\}$
$a \wedge A = \{x \wedge y : x \in A\}$
$A \wedge B = \{x \wedge y : x \in A, y \in B\}$.

Theorem 3.7. Let E be an equivalence relation on $L$and if $a, b \in L$, then
(1) $[a]_E \vee [b]_E \subseteq [a \vee b]_E$
(2) $[a]_E \wedge [b]_E \subseteq [a \wedge b]_E$

Proof: Suppose $z \in [a]_E \vee [b]_E$ then there exist $x \in [a]_E$ and $y \in [b]_E$ such that $z = x \vee y$. Since $(a, x) \in E$ and $(b, y) \in E$, we have $(a \vee b, x \vee y) \in E$, namely $(a \vee b, z) \in E$, so $z \in [a \vee b]_E$
Again, suppose $z \in [a]_E \wedge [b]_E$ then there exist $x \in [a]_E$ and $y \in [b]_E$ such that $z = x \wedge y$. Since $(a, x) \in E$ and $(b, y) \in E$, we have $(a \wedge b, x \wedge y) \in E$, namely $(a \wedge b, z) \in E$, so $z \in [a \wedge b]_E$.

Definition 3.8. [5] Let E be a full congruence relation, then E is called a complete full congruence relation if $[a]_E \vee [b]_E = [a \vee b]_E$ and $[a]_E \wedge [b]_E = [a \wedge b]_E$ for all $a, b \in L$.

Theorem 3.9. Let E be a complete equivalence relation on L, if A, B are non- empty subsets of L, then $E \uparrow (A) \vee E \uparrow (B) \subseteq E \uparrow (A \vee B)$ and $E \uparrow (A) \wedge E \uparrow (B) \subseteq E \uparrow (A \wedge B)$

Proof: Let us suppose $z \in E \uparrow (A) \vee E \uparrow (B)$, then there exist $x \in E \uparrow (A)$ and $y \in E \uparrow (B)$ such that $z = x \vee y$. Then $[x]_E \cap A \neq \emptyset$ and $[y]_E \cap B \neq \emptyset$, so there exist $a \in [x]_E, a \in A$ and $b \in [y]_E, b \in B$. Then $a \vee b \in [x]_E \vee [y]_E \subseteq [x \vee y]_E = [z]_E$ and $a \vee b \in A \vee B$. Thus $[z]_E \cap (A \vee B) \neq \emptyset$ and hence $z \in E \uparrow (A \vee B)$.
Next suppose $z \in E \uparrow (A) \wedge E \uparrow (B)$, then there exist $x \in E \uparrow (A)$ and $y \in E \uparrow (B)$ such that $z = x \wedge y$. Then $[x]_E \cap A \neq \emptyset$ and $[y]_E \cap B \neq \emptyset$, so there exist $a \in [x]_E, a \in A$ and $b \in [y]_E, b \in b$. Then $a \wedge b \in [x]_E \wedge [y]_E \subseteq [x \wedge y]_E = [z]_E$ and $a \wedge b \in A \wedge B$. Thus $[z]_E \cap (A \wedge B) \neq \emptyset$ and hence $z \in E \uparrow (A \wedge B)$.

Theorem 3.10. Let E be a complete equivalence relation on L, if A, B are non empty subsets of L, then $E \downarrow (A) \vee E \downarrow (B) \subseteq E \uparrow (A \vee B)$ and
$E \downarrow (A) \wedge E \downarrow (B) \subseteq E \downarrow (A \wedge B)$

Proof: Suppose $z \in E \downarrow (A) \vee E \downarrow (B)$, then there exist $x \in E \downarrow (A)$ and $y \in E \downarrow (B)$ such that $z = x \vee y$. Then $[x]_E \subseteq$ and $[y]_E \subseteq B$ then $[x]_E \vee [y]_E \subseteq A \vee B$. Since E is a full complete equivalence relation on L, we have $[x]_E \vee [y]_E = [x \vee y]_E = [z]_E$, namely $[z]_E \subseteq A \vee B$. Hence $z \in f \downarrow (A \vee B)$.
Again suppose $z \in E \downarrow (A) \wedge E \downarrow (B)$, then there exist $x \in E \downarrow (A)$ and $y \in E \downarrow (B)$ such that $z = x \wedge y$. Then $[x]_E \subseteq$ and $[y]_E \subseteq B$ then $[x]_E \wedge [y]_E \subseteq A \wedge B$. Since E is a complete equivalence relation on L, we have $[x]_E \wedge [y]_E = [x] \wedge y]_E = [z]_E$, namely $[z]_E \subseteq A \wedge B$. Hence $z \in f \downarrow (A \wedge B)$.

Theorem 3.11. Suppose E_1, E_2 are two complete equivalence relations on L, A is a non empty subset of L, then $(E_1 \cap E_2) \uparrow (A) \subseteq E_1 \uparrow (A) \cap E_2 \uparrow (A)$

Proof: Let us suppose that $x \in (E_1 \cap E_2) \uparrow (A)$, then $[x]_{E_1 \cap E_2} \neq \emptyset$. So there exists $a \in [x]_{E_1 \cap E_2} \cap A$. Since $(a, x) \in E_1 \cap E_2$, then $(a, x) \in E_1$ and $(a, x) \in E_2$. Thus $a \in [x]_{E_1}$ and $a \in [x]_{E_2}$. Then $[x]_{E_1} \cap A \neq \emptyset$ and $[x]_{E_2} \cap A \neq \emptyset$. Therefore, $x \in E_1 \uparrow (A)$ and $x \in E_2 \uparrow (A)$. Hence $x \in E_1 \uparrow (A) \cap E_2 \uparrow (A)$. Thus $(E_1 \cap E_2) \uparrow (A) \subseteq E_1 \uparrow (A) \cap E_2 \uparrow (A)$.

Definition 3.12. Let L be a complete lattice and let $k \in L$. k is said to be compact if, for every subset S of $L, k \leq \bigvee S \Rightarrow k \leq \bigvee T$ for some finite subset T of S. The set of all compact elements of L is denoted $K(L)$. A complete lattice L is said to be algebraic if, for each $a \in L$; $a = \bigvee\{k \in k(L) : k \leq a\}$.

Definition 3.13. Let $(L, \leq)$ and $(K, \leq)$ be two lattices and $A \in P(K)$ where $P(K)$ denotes the set of all non-empty subsets of K. Let $f : L \to P(K)$ be a set-valued mapping. The lower and upper approximations of A under f are defined by $f \downarrow (A) = \{x \in L : f(x) \subseteq A\} f \uparrow (A) = \{x \in L : f(x) \cap A \neq \emptyset\}$.

Definition 3.14. The pair $(f \downarrow (A), f \uparrow (A))$ is referred to as the generalized rough set with respect to A, induced by f or f- rough set with respect to A.

Example Let (L, E) be an approximation space and $f : L \to P(L)$ be a set-valued mapping where $f(x) = [x]_E$ for all $x \in L$, then for any $A \subseteq L, f \downarrow (A)$ and $f \uparrow (A)$ are lower and upper approximations respectively.

Proposition 3.15. Let L and K be two lattices and $A, B \in P(K)$. Let $f : L \to P(K)$ be a set-valued mapping. Then the following assertions hold:
$(i) f \uparrow (A \cup B) = f \uparrow (A) \cup f \uparrow (B)$;
$(ii) f \downarrow (A \cap B) = f \downarrow (A) \cap f \downarrow (B)$;
$(iii) A = B$ implies $f \downarrow (A) = f \downarrow (B)$ and $f \uparrow (A) = f \uparrow (B)$;
$(iv) f \downarrow (A) \cup f \downarrow (B) = f \downarrow (A \cup B)$; $(v) f \uparrow (A \cap B) = f \uparrow (A) \cap f \uparrow (B)$.

Definition 3.16. A non-empty subset K of L is a sublattice of the lattice $(L, \bigwedge, \bigvee)$ if $a \vee b, a \wedge b \in K$ for all $a, b \in K$.

Definition 3.17. If A and B are non-empty subsets of L, we define $A \bigwedge B$ and $A \bigvee B$ as follows: $A \bigwedge B = \{a \wedge b : a \in A, b \in B\}; A \bigvee B = \{a \vee b : a \in A, b \in B\}$.

Definition 3.18. Let L and K be two lattices and $f : L \to P(K)$ be a set- valued mapping. f is called a set-valued homomorphism if $(i) f(x \wedge y) = f(x) \bigwedge f(y); (ii) f(x \vee y) = f(x) \bigvee f(y)$, for all $x, y \in L$.

Lemma 3.19. Let L and K be two lattices and $f : L \to P(K)$ be a set-valued homomorphism. If S is a sublattice of K and $f \downarrow (S) \neq \emptyset$, and $f \uparrow (S) \neq \emptyset$, then $f \downarrow (S)$ and $f \uparrow (S)$ are sublattices of L.

Proof: Let $x, y \in f \downarrow (S)$, by definition we have $f(x), f(y) \subseteq S$. Since S is a sublattice of K, we have $f(x \vee y) = f(x) \bigvee f(y) \subseteq S$ and $f(x \wedge y) = f(x) \bigwedge f(y) = S$. It shows that $x \vee y; x \wedge y \in f \downarrow (S)$. Moreover, let $x, y \in f \uparrow (S)$, by definition, $f(x) \bigcap S \neq \emptyset$ and $f(y) \bigcap S \neq \emptyset$. Suppose $a \in f(x) \cap S$ and $b \in f(y) \cap S$. Since S is a sublattice of K, we have $a \vee b \in S$ and $a \wedge b \in f(x) \bigvee f(y) = f(x \vee y)$. It implies that $a \vee b \in f(x \vee y) \bigcap S$. Hence $f(x \vee y) \bigcap S \neq \emptyset$. It means that $x \vee y \in \uparrow (S)$. Again, $a \wedge b \in S$ and $a \wedge b \in f(x) \bigvee f(y)$. So that $a \wedge b \in f(x \wedge y) f \cap S$. Therefore $f(x \wedge y) \bigcap S \neq \emptyset$. It means that $x \wedge y \in f \uparrow (S)$.

Lemma 3.20. Let L and K be two lattices and $f : L \to P(K)$ be a set- valued homomorphism. If S is a sublattice of K and $f \downarrow (S) \neq \emptyset \neq f \uparrow (S)$, then $(f \downarrow (S), f \uparrow (S))$ is a rough sublattice of L.

Proposition 3.21. Let L and K be two lattices and $f : L \to P(K)$ be a set-valued homomorphism. If A, B be non-empty subsets of K , then $(1) f \downarrow (A) \bigvee f \downarrow (B) \subseteq f \downarrow (A \bigvee B); (2) f \downarrow (A) \bigwedge f \downarrow (B) \subseteq f \downarrow (A \bigwedge B)$.

Proof: Suppose z be any element of $f \downarrow (A) \bigvee f \downarrow (B)$. Then $z = a \vee b$ for some $a \in f \downarrow (A)$ and $b \in f \downarrow (B)$. By definition, $f(a) \subseteq A$ and $f(b) \subseteq B$. Since $f(a \vee b) = f(a) \bigvee f(b) = \{x \vee y : x \in f(a), y \in f(b)\} \subseteq \{x \vee y : x \in A, y \in B\} = A \bigvee B$, we imply that $a \vee b \in f \downarrow (A \bigvee B)$ and so $z \in f \downarrow (A \bigvee B)$. The proof of (2) is similar to the proof of (1).
The following examples show that the converse of above proposition is not true.
Example (1): Let $L = \{x_0, x_1, x_2, \ldots, x_7\}$, where $x_0 < x_1 < x_2 < x_3 < x_4 < x_5 < x_6 < x_7$. Let $a \vee b = max\{a, b\}$ and $a \wedge b = min\{a, b\}$ for all $a, b \in L$. Then $(L, \vee, \wedge)$ is a lattice. If we consider equivalence classes $[x_0] = \{x_0, x_1, x_2\}, [x_3] = \{x_3, x_4\}, [x_5] = \{x_5, x_6, x_7\}$ and $f : L \to P(L)$ be a set-valued homomorphism with $f(x) = [x]$ for all $x \in L$. Let $A = \{x_3, x_4, x_5, x_7\}, B = \{x_0, x_1, x_2, x_3, x_6\}$. Then $A \bigvee B = \{x_3, x_4, x_5, x_6, x_7\}, f \downarrow (A \bigvee B) = \{x_3, x_4, x_5, x_6, x_7\}, f \downarrow (A) = \{x_3, x_4\}, f \downarrow (B) = \{x_0, x_1, x_2\}$ and $f \downarrow (A) \bigvee f \downarrow (B) = \{x_3, x_4\}$. And so $f \downarrow (A \bigvee B) \not\subseteq f \downarrow (A) \bigvee f \downarrow (B)$.
(2): Let $L = [0, 1]$ and $f : L \to P(L)$ be a set-valued homomorphism with $f(x) = [0, x]$ for all $x \in L$. And let $A = \{0, \frac{1}{2}\}, B = \{\frac{1}{3}, \frac{1}{2}\}$. Then $f \downarrow (A) = \{0\}, f \downarrow (B) = \emptyset, f \downarrow (A \bigwedge B) = \{0\}, f \downarrow (A) \bigwedge f \downarrow (B) = \emptyset$. Therefore $f \downarrow (A \bigwedge B) \not\subseteq f \downarrow (A) \bigwedge f \downarrow (B)$.

Proposition 3.22. [3] Let L and K be two lattices and $f : L \to P(K)$ be a set-valued homomorphism. If A, B be non-empty subsets of K, then
(1) $f \uparrow (A) \bigvee f \uparrow (B) \subseteq f \uparrow (A \bigvee B)$; (2)$f \uparrow (A) \bigwedge f \uparrow (B) \subseteq f \uparrow (A \bigwedge B)$.

Proof: (1) Let us suppose that $z \in f \uparrow (A) \bigvee f \uparrow (B)$. Then $z = a \vee b$ for some $a \in f \uparrow (A)$ and $b \in f \uparrow (B)$. Hence, $f(a) \bigcap A \neq \emptyset$ and $f(b) \bigcap B \neq \emptyset$ and so there exist $x \in f(a) \bigcap A$ and $y \in f(b) \bigcap B$. Therefore, $x \vee y \in A \bigvee B$ and $x \vee y \in f(a) \bigvee f(b) = f(a \vee b)$. Thus $x \vee y \in f(a \vee b) \bigcap (A \bigvee B)$ which implies that $f(a \vee b) \bigcap f(A \bigvee B) \neq \emptyset$. So $z = a \vee b \in f \uparrow (A \bigvee B)$.
(2). The proof is obvious as that of (1).
The following examples show that the converse of above proposition is not true.
Example (i) Let $L = \{0, x_1, x_2, x_3, x_4, x_5, 1\}$ be the lattice and $f : L \to P(L)$ be a set-valued homomorphism with $f(x) = \{x_5\}$ for all $x \in L$. And let $A = \{x_2, x_3\}, B = \{x_1, x_2\}$. Then $f \uparrow (A) =$

$\emptyset, f\uparrow(B)=\emptyset, f\uparrow(A\bigvee B)=L, f\uparrow(A)\bigvee f\uparrow(B)=\emptyset$. Therefore $f\uparrow(A\bigvee B)\subseteq f\uparrow(A)\bigvee f\uparrow(B)$.

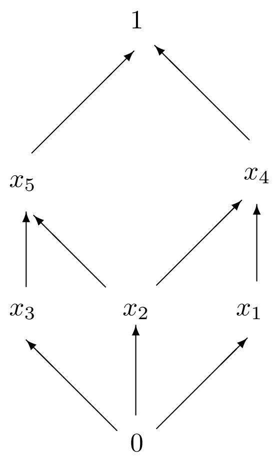

Figure-1: Lattice Structure of L

(ii) Let L be the above lattice and $f: L\rightarrow P(L)$ be a set-valued homomorphism with $f(x)=\{x_2\}$ for all $x\in L$. And let $A=\{x_4\}, B=\{x_5\}$. Then $f\uparrow(A)=\emptyset, f\uparrow(B)=\emptyset, f\uparrow(A\bigwedge B)=L, f\uparrow(A)Vf\uparrow(B)=\emptyset$. Therefore $f\uparrow(A\bigwedge B)\not\subseteq f\uparrow(A)\bigwedge f\uparrow(B)$.

4 Conclusion

The paper is devoted to the application of rough lattice determined by Pawlak's Information System in which the concept of upper and lower approximations of a subset in a lattice are considered and studied their algebraic properties. By some indiscernibility relation, we have shown that the entries of the indiscernibility relation of an information system forms a rough lattice. Some properties of set valued homomorphism are obtained which shall be very practical in the theory and application of rough lattice.

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