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SOFT π-OPEN SETS IN SOFT GENERALIZED TOPOLOGICAL SPACES

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Abstract – The main purpose of this paper is to study some interesting properties of the soft mapping $\pi : S(U)_E \rightarrow S(U)_E$ which satisfy the condition $\pi F_B \subset \pi F_D$ whenever $F_B \subset F_D \subset F_{\tilde{E}}$. A new class of generalized soft open sets, called soft π-open sets is introduced and studied their basic properties. A soft set $F_G \subset F_{\tilde{E}}$ is said to be a soft π-open set iff $F_G \subset \pi F_G$. The notions of soft interior and soft closure are generalized using these sets. We then introduce the concepts of soft π-interior $i_\pi F_G$, soft π-closure $c_\pi F_G$, soft $\pi^* F_G$ of a soft set $F_G \subset F_{\tilde{E}}$. Under suitable conditions on π, the soft π-interior $i_\pi F_G$ and the soft π-closure $c_\pi F_G$ of a soft set $F_G \subset F_{\tilde{E}}$ are easily obtained by explicit formulas. The soft μ-semi-open sets, soft μ-pre-open sets, soft μ-α-open sets and soft μ-β-open sets for a given Soft Generalized Topological Space $(F_{\tilde{E}}, \mu)$ can be obtained from soft π-open sets which are important for further research on soft generalized topology.

Keywords ***–*** *Soft sets, soft generalized topology, soft mapping, soft π-open sets, soft π-interior, soft π-closure.*

1 Introduction

The concept of soft set theory was introduced by Molodtsov [19] in 1999 as a mathematical tool for modeling uncertainties. Molodtsov successfully applied the soft set theory in several directions such as game theory, probability, Perron and Riemann Integration, theory of measurements [20]. Maji et al [17] and Naim Cagman et al. [5] have further modified the theory of soft sets which is similar to that of Molodtsov. After the introduction of the notion of soft sets, several researchers improved this concept. Cagman [6] presented the soft matrix theory and set up the maximum decision making method. D. Pei and D Miao [21] showed that soft sets are a class of special information systems. Babitha and Sunil [4] studied the soft set relation and discussed some related concepts. Kharal et al. [16] introduced soft functions over classes of soft sets. The notion of soft ideal is initiated for

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the first time by Kandil et al. [13]. Feng et al. [9] worked on soft semi rings, soft ideals and idealistic soft semi rings.

It is known that topology is an important area of mathematics, with many applications in the domain of computer science and physical sciences. Topological structure of soft sets was also studied by many researchers. Shabir and Naz [22] and Cagman [7] initiated the study of soft topology and soft topological spaces independently. Shabir and Naz defined soft topology on the collection of soft sets over an initial universe with a fixed set of parameters. On the other hand, Cagman et al. [7] introduced soft topology on a soft set and defined soft topological space. The notion of soft topology by Cagman is more general than that by Shabir and Naz. B Ahmad and S Hussain [1] explored the structures of soft topology using soft points. Weak forms of soft open sets were first studied by Chen [8]. He investigated soft semi-open sets in soft topological spaces and studied some properties of it. Arockiarani and Lancy [3] are defined soft β-open sets and continued to study weak forms of soft open sets in soft topological space. Akdag and Ozkan [2], defined soft α-open and soft α-closed sets in soft topological spaces and studied many important results and some properties of it. Soft pre-open sets were introduced by [3]. Kandil et al. [14] introduced a unification of some types of different kinds of subsets of soft topological spaces using the notion of γ-operations. Kandil et al. [15] generalize this unification of types of different kinds of subsets of soft topological spaces using the notion of γ-oprations to supra topological spaces. Soft generalized topology is relatively new and promising domain which can lead to the development of new mathematical models and innovative approaches that will significantly contribute to the solution of complex problems in engineering and environment. Jyothis and Sunil [10] introduced the notion of soft generalized topology (SGT) on a soft set and studied basic concepts of soft generalized topological spaces (SGTS). It is showed that a soft generalized topological space gives a parameterized family of generalized topological space. They also define and discuss the properties of soft generalized separation axioms which are important for further research on soft topology [12]. Jyothis and Sunil [11] introduced the concept of soft μ-compactness in soft generalized topological spaces as a generalization of compact spaces.

This paper is organized as follows. In section 2, we begin with the basic definitions and important results related to soft set theory which are useful for subsequent sections. In section 3, the definitions and basic theorems of soft generalized topology on an initial soft set are given. Finally in section 4, we study some interesting properties of the soft mapping $\pi : S(U)_E \rightarrow S(U)_E$ which satisfy the condition $\pi F_B \subset \pi F_D$ whenever $F_B \subset F_D \subset F_{\tilde{E}}$. We introduce the concept of soft π-open sets and study their basic properties. The most important special cases are obtained if μ is a SGT, i_μ and c_μ denote the soft μ-interior and soft μ-closure respectively, and $\pi = c_\mu i_\mu$, $\pi = i_\mu c_\mu$, $\pi = i_\mu c_\mu i_\mu$ and $\pi = c_\mu i_\mu c_\mu$. The corresponding soft π-open sets are called the soft μ-semi-open sets, soft μ-pre-open sets, soft μ-α-open sets and soft μ-β-open sets. Under suitable conditions on π, the soft π-interior $i_\pi F_G$ and the soft π-closure $c_\pi F_G$ of a soft set $F_G \subset F_{\tilde{E}}$ are easily obtained by explicit formulas.

2 Preliminaries

In this section we recall some definitions and results defined and discussed in [5, 10, 11, 16]. Throughout this paper U denotes the initial universe, E denotes the set of all possible parameters, $\mathcal{P}(U)$ is the power set of U and A is a nonempty subset of E.

Definition 2.1. A soft set F_A on the universe U is defined by the set of ordered pairs $F_A = \{(e, f_A(e)) \,/\, e \in E, f_A(e) \in \mathcal{P}(U)\}$, where $f_A : E \to \mathcal{P}(U)$ such that $f_A(e) = \emptyset$ if $e \notin A$. Here f_A is called an approximate function of the soft set F_A. The value of $f_A(e)$ may be arbitrary. Some of them may be empty, some may have nonempty intersection. The set of all soft sets over U with E as the parameter set will be denoted by $S(U)_E$ or simply S(U).

Definition 2.2. Let $F_A \in S(U)$. If $f_A(e) = \emptyset$ for all $e \in E$, then F_A is called an empty soft set, denoted by $F_\emptyset$. $f_A(e) = \emptyset$ means that there is no element in U related to the parameter e in E. Therefore we do not display such elements in the soft sets as it is meaningless to consider such parameters.

Definition 2.3. Let $F_A \in S(U)$. If $f_A(e) = U$ for all $e \in A$, then F_A is called an A-universal soft set, denoted by $F_{\tilde{A}}$. If A = E, then the A-universal soft set is called an universal soft set, denoted by $F_{\tilde{E}}$.

Definition 2.4. Let $F_A, F_B \in S(U)$. Then F_B is a soft subset of F_A (or F_A is a soft superset of F_B), denoted by $F_B \subseteq F_A$, if $f_B(e) \subseteq f_A(e)$, for all $e \in E$.

Definition 2.5. Let $F_A, F_B \in S(U)$. Then F_B and F_A are soft equal, denoted by $F_B = F_A$, if $f_B(e) = f_A(e)$, for all $e \in E$.

Definition 2.6. Let $F_A, F_B \in S(U)$. Then, the soft union of F_A and F_B, denoted by $F_A \cup F_B$, is defined by the approximate function $f_{A\cup B}(e) = f_A(e) \cup f_B(e)$.

Definition 2.7. Let $F_A, F_B \in S(U)$. Then, the soft intersection of F_A and F_B, denoted by $F_A \cap F_B$, is defined by the approximate function $f_{A\cap B}(e) = f_A(e) \cap f_B(e)$.

Definition 2.8. Let $F_A, F_B \in S(U)$. Then, the soft difference of F_A and F_B, denoted by $F_A \setminus F_B$, is defined by the approximate function $f_{A\setminus B}(e) = f_A(e) \setminus f_B(e)$.

Definition 2.9. Let $F_A \in S(U)$. Then, the soft complement of F_A, denoted by $(F_A)^c$, is defined by the approximate function $f_{A^c}(e) = (f_A(e))^c$, where $(f_A(e))^c$ is the complement of the set $f_A(e)$, that is, $(f_A(e))^c = U \setminus f_A(e)$ for all $e \in E$.

Cleary $((F_A)^c)^c = F_A$, $(F_\emptyset)^c = F_{\tilde{E}}$, and $(F_{\tilde{E}})^c = F_\emptyset$.

Definition 2.10. Let $F_A \in S(U)$. The soft power set of F_A, denoted by $\mathcal{P}(F_A)$, is defined by $\mathcal{P}(F_A) = \{F_{A_i} \,/\, F_{A_i} \subseteq F_A, i \in J \subseteq N\}$.

Theorem 2.11. Let $F_A, F_B, F_C \in S(U)$. Then,

(1) $F_A \cup F_A = F_A$.
(2) $F_A \cap F_A = F_A$.
(3) $F_A \cup F_\emptyset = F_A$.
(4) $F_A \cap F_\emptyset = F_\emptyset$.
(5) $F_A \cup F_{\tilde{E}} = F_{\tilde{E}}$.
(6) $F_A \cap F_{\tilde{E}} = F_A$.
(7) $F_A \cup (F_A)^c = F_{\tilde{E}}$.
(8) $F_A \cap (F_A)^c = F_\emptyset$.
(9) $F_A \cup F_B = F_B \cup F_A$.

(10) $F_A \cap F_B = F_B \cap F_A$.
(11) $(F_A \cup F_B)^c = (F_A)^c \cap (F_B)^c$.
(12) $(F_A \cap F_B)^c = (F_A)^c \cup (F_B)^c$.
(13) $(F_A \cup F_B) \cup F_C = F_A \cup (F_B \cup F_C)$.
(14) $(F_A \cap F_B) \cap F_C = F_A \cap (F_B \cap F_C)$.
(15) $F_A \cup (F_B \cap F_C) = (F_A \cup F_B) \cap (F_A \cup F_C)$.
(16) $F_A \cap (F_B \cup F_C) = (F_A \cap F_B) \cup (F_A \cap F_C)$.

Definition 2.12. [16] Let $S(U)_E$ and $S(V)_K$ be the families of all soft sets over U and V, respectively. Let $\varphi : U \to V$ and $\chi : E \to K$ be two mappings. The soft mapping

$$\varphi_\chi : S(U)_E \to S(V)_K$$

is defined as:

(1) Let F_A be a soft set in $S(U)_E$. The image of F_A under the soft mapping φ_χ is the soft set over V, denoted by $\varphi_\chi(F_A)$ and is defined by

$$\varphi_\chi(f_A)(k) = \begin{cases} \bigcup_{e \in \chi^{-1}(k) \cap A} \varphi\big(f_A(e)\big), & \text{if } \chi^{-1}(k) \cap A \neq \emptyset; \\ \emptyset, & \text{otherwise} \end{cases}$$

for all $k \in K$.

(2) Let G_B be a soft set in $S(V)_K$. The inverse image of G_B under the soft mapping φ_χ is the soft set over U, denoted by $\varphi_\chi{}^{-1}(G_B)$ and is defined by

$$\varphi_\chi{}^{-1}(g_B)(e) = \begin{cases} \varphi^{-1}(g_B(\chi(e))), & \text{if } \chi(e) \in B; \\ \emptyset, & \text{otherwise} \end{cases}$$

for all $e \in E$.

The soft mapping φ_χ is called injective, if φ and χ are injective. The soft mapping φ_χ is called surjective, if φ and χ are surjective.

The soft mapping from $S(U)_E$ to itself is denoted by $\varphi: S(U)_E \to S(U)_E$

Definition 2.13. Let $\varphi_\chi: S(U)_E \to S(V)_K$ and $\tau_\sigma: S(V)_K \to S(W)_L$, then the soft composition of the soft mappings φ_χ and τ_σ, denoted by $\tau_\sigma \circ \varphi_\chi$, is defined by $\tau_\sigma \circ \varphi_\chi = (\tau \circ \varphi)_{(\sigma \circ \chi)}$.

3 Soft Generalized Topological Spaces

Definition 3.1. [10] Let $F_A \in S(U)$. A Soft Generalized Topology (SGT) on F_A, denoted by μ or μ_{F_A} is a collection of soft subsets of F_A having the following properties:

(1) $F_\emptyset \in \mu$
(2) Any soft union of members of μ belongs to μ.

The pair (F_A, μ) is called a Soft Generalized Topological Space (SGTS)
Observe that $F_A \in \mu$ must not hold.

Definition 3.2. [10] A soft generalized topology μ on F_A is said to be strong if $F_A \in \mu$.

Definition 3.3. [10] Let (F_A, μ) be a SGTS. Then, every element of μ is called a soft μ–open set.

Definition 3.4. [10] Let (F_A, μ) be a SGTS and $F_B \subseteq F_A$. Then the collection $\mu_{F_B} = \{F_D \cap F_B \,/\, F_D \in \mu\}$ is called a Subspace Soft Generalized Topology (SSGT) on F_B. The pair (F_B, μ_{F_B}) is called a Soft Generalized Topological Subspace (SGTSS) of F_A.

Definition 3.5. [10] Let (F_A, μ) be a SGTS and $F_B \subseteq F_A$. Then the soft μ-interior of F_B denoted by $i_\mu(F_B)$ is defined as the soft union of all soft μ-open subsets of F_B.
Note that $i_\mu(F_B)$ is the largest soft μ-open set that is contained in F_B.

Theorem 3.6. [10] Let (F_A, μ) be a SGTS and $F_B \subseteq F_A$. Then F_B is a soft μ-open set if and only if $F_B = i_\mu(F_B)$.

Theorem 3.7. [10] Let (F_A, μ) be a SGTS and $F_G, F_H \subseteq F_A$. Then

(1) $i_\mu(i_\mu(F_G)) = i_\mu(F_G)$
(2) $F_G \subseteq F_H \Rightarrow i_\mu(F_G) \subseteq i_\mu(F_H)$
(3) $i_\mu(F_G) \cap i_\mu(F_H) \supseteq i_\mu(F_G \cap F_H)$
(4) $i_\mu(F_G) \cup i_\mu(F_H) \subseteq i_\mu(F_G \cup F_H)$
(5) $i_\mu(F_G) \subseteq F_G$.

Definition 3.8. [10] Let (F_A, μ) be a SGTS and $F_B \subseteq F_A$. Then F_B is said to be a soft μ-closed set if its soft complement $(F_B)^c$ is a soft μ-open set.

Theorem 3.9. [10] Let (F_A, μ) be a SGTS and $F_B \subseteq F_A$. Then the following conditions hold:

(1) The universal soft set $F_{\tilde{E}}$ is soft μ–closed.
(2) Arbitrary soft intersections of the soft μ–closed sets are soft μ–closed.

Definition 3.10. [10] Let (F_A, μ) be a SGTS and $F_B \subseteq F_A$. Then the soft μ-closure of F_B, denoted by $c_\mu(F_B)$ is defined as the soft intersection of all soft μ-closed super sets of F_B.
Note that $c_\mu(F_B)$ is the smallest soft μ-closed superset of F_B.

Theorem 3.11. [10] Let (F_A, μ) be a SGTS and $F_B \subseteq F_A$. F_B is a soft μ-closed set if and only if $F_B = c_\mu(F_B)$.

Theorem 3.12. [10] Let (F_A, μ) be a SGTS and $F_B \subseteq F_A$. Then $i_\mu(F_B) \subseteq F_B \subseteq c_\mu(F_B)$

Theorem 3.13. [10] Let (F_A, μ) be a SGTS and $F_G, F_H \subseteq F_A$. Then

(1) $c_\mu(c_\mu(F_G)) = c_\mu(F_G)$
(2) $F_G \subseteq F_H \Rightarrow c_\mu(F_G) \subseteq c_\mu(F_H)$
(3) $c_\mu(F_G) \cap c_\mu(F_H) \supseteq c_\mu(F_G \cap F_H)$

(4) $c_\mu(F_G) \cup c_\mu(F_H) \subseteq c_\mu(F_G \cup F_H)$

4 Soft π-Open Sets

Consider the soft mapping $\pi : S(U)_E \to S(U)_E$ possessing the property of monotony, i.e, $F_B \subset F_D$ imply $\pi F_B \subset \pi F_D$. We denote the collection of all soft mapping having this property by Π. Consider the following conditions for a soft mapping $\pi \in \Pi$, $F_B \subset F_{\tilde{E}}$

(Π0) $\pi F_\emptyset = F_\emptyset$

(Π1) $\pi F_{\tilde{E}} = F_{\tilde{E}}$

(Π2) $\pi^2 F_B = \pi\pi F_B = \pi F_B$

(Π3) $F_B \subset \pi F_B$,

(Π4) $\pi F_B \subset F_B$,

(Π5) $\pi^2 F_B \subset \pi F_B$,

Example 4.1. The soft identity mapping *id*: $S(U)_E \to S(U)_E \in$ (Π0), (Π1), (Π2), (Π3), (Π4).

Let $(F_{\tilde{E}}, \mu)$ be a SGTS and $i_\mu : S(U)_E \to S(U)_E$ and $c_\mu : S(U)_E \to S(U)_E$ be the soft μ-interior and soft μ-closure operators respectively. If $\pi = i_\mu$, then $\pi \in$ (Π0), (Π2), (Π4). If $\pi = c_\mu$, then $\pi \in$ (Π1), (Π2), (Π3).

Definition 4.2. A soft set $F_G \subset F_{\tilde{E}}$ is said to be a soft π-open set iff $F_G \subset \pi F_G$.

Example 4.3. The following are some examples of soft π-open sets:

1. $F_\emptyset$ is always soft π-open for any $\pi \in \Pi$
2. $F_{\tilde{E}}$ is soft π-open iff $\pi \in$ (Π1)
3. Every soft set of the form πF_G is soft π-open if $\pi \in$ (Π2)
4. Every soft subset of $F_{\tilde{E}}$ is soft π-open if $\pi \in$ (Π3)
5. If $\pi \in$ (Π4), then F_G is soft π-open iff $F_G = \pi F_G$

Note: Let $(F_{\tilde{E}}, \mu)$ be a SGTS. Then F_G is soft i_μ-open (i.e, if $\pi = i_\mu$) iff $F_G \subset i_\mu F_G$. But $i_\mu F_G \subset F_G$. Thus F_G is soft i_μ-open iff $F_G = i_\mu F_G$ iff F_G is soft μ-open by theorem 3.6. Hence soft i_μ-open set coincides with the soft μ-open sets.

Theorem 4.4. Any soft union of soft π–open sets is soft π–open.

Proof. Let $\{F_{Bj}\}_{j\in J}$ be a collection of soft π-open sets. i.e, $F_{Bj} \subset \pi F_{Bj}$ $\forall$ $j \in J$. Let $F_B = \bigcup_{j\in J} F_{Bj}$. Now $F_{Bj} \subset F_B$ imply $\pi F_{Bj} \subset \pi F_B$ $\forall$ $j \in J$. Therefore $F_B = \bigcup_{j\in J} F_{Bj} \subset \bigcup_{j\in J} \pi F_{Bj} \subset \pi F_B$. i.e, $F_B \subset \pi F_B$. Hence F_B is soft π-open.■

Theorem 4.5. The collection of all soft π-open sets is a SGT.

Theorem 4.6. If μ is a SGT on $F_{\tilde{E}}$, then there is a soft mapping $\pi \in$ (Π0), (Π2), (Π4) such that μ is the collection of all soft π-open sets.

Proof. Define πF_G to be the soft union of all $F_H \in \mu$ satisfying $F_H \subset F_G$. Then clearly $\pi F_G \in \mu$ and $\pi F_G \subset F_G$, $\pi F_\emptyset = F_\emptyset$. Now $F_H \in \mu \Rightarrow \pi F_H = F_H \supset F_H$ so that the elements of μ are soft π-open, while $F_G \subset \pi F_G \Rightarrow \pi F_G = F_G$ and $F_G \in \mu$. Finally $\pi F_G \in \mu \Rightarrow \pi\pi F_G = \pi F_G$.■

Definition 4.7. Let $F_B \subset F_{\tilde{E}}$. The soft union of all soft π-open subsets of the soft set F_B is called the soft π-interior of F_B, and is denoted by $i_\pi F_B$.

Theorem 4.8. The soft set $i_\pi F_B$ is the largest soft π-open subset of F_B.

Note: Let $(F_{\tilde{E}}, \mu)$ be a SGTS and suppose $\pi = i_\mu$, then the soft set $i_{i\mu} F_B$ is the largest soft i_μ-open subset of F_B. Since soft i_μ-open sets are soft μ-open sets, $i_{i\mu} F_B$ is the largest soft μ-open subset of F_B. Hence $i_{i\mu} = i_\mu$.

Theorem 4.9. For any $\pi \in \Pi$ and $F_B \subset F_{\tilde{E}}$,

i) $i_\pi F_\emptyset = F_\emptyset$
ii) $i_\pi F_B = i_\pi i_\pi F_B$
iii) $i_\pi F_B \subset F_B$, and
iv) $i_\pi F_{\tilde{E}} = F_{\tilde{E}}$ iff $\pi F_{\tilde{E}} = F_{\tilde{E}}$

i.e, $i_\pi \in (\Pi 0)$, $(\Pi 2)$ and $(\Pi 4)$ for any $\pi \in \Pi$; $i_\pi \in (\Pi 1)$ iff $\pi \in (\Pi 1)$
Conversely if $\pi \in (\Pi 0)$, $(\Pi 2)$and $(\Pi 4)$, then $\pi = i_\pi$.

Proof. First show that i_π possess the property of monotony. Suppose $F_G \subset F_H$. By definition of i_π and by theorem 4.8, $i_\pi F_G \subset F_G$ and $i_\pi F_H \subset F_H$. $i_\pi F_H$ is the largest soft π-open subset of F_H. Hence $i_\pi F_G \subset i_\pi F_H$. Clearly $i_\pi F_\emptyset = F_\emptyset$. i.e, $i_\pi \in (\Pi 0)$. By definition 4.7, $i_\pi F_G \subset F_G$ for any $F_G \subset F_{\tilde{E}}$. i.e, $i_\pi \in (\Pi 4)$. By theorem 4.8, $i_\pi F_G$ is soft π –open, so $i_\pi(i_\pi F_G)$ = largest soft π-open subset of $i_\pi F_G = i_\pi F_G$. i.e, $i_\pi \in (\Pi 2)$. Again $i_\pi F_{\tilde{E}}$ = largest soft π-open subset of $F_{\tilde{E}} = F_{\tilde{E}}$ $\Leftrightarrow$ $F_{\tilde{E}}$ is a soft π-open set $\Leftrightarrow$ $\pi \in (\Pi 1)$.

Conversely, assume that $\pi \in (\Pi 0)$, $(\Pi 2)$ and $(\Pi 4)$. $\pi \in (\Pi 2) \Rightarrow \pi(\pi F_G) = \pi F_G \Rightarrow \pi F_G$ is soft π-open. $\pi \in (\Pi 4) \Rightarrow \pi F_G \subset F_G$ for any $F_G \subset F_{\tilde{E}}$. Therefore πF_G is a soft π-open subset of F_G. Next if $F_H \subset F_G$ is soft π-open, then $F_H \subset \pi F_H \subset \pi F_G$. So πF_G = largest soft π-open subset of F_G. Hence $i_\pi = \pi$.■

Theorem 4.10. A soft set F_G is soft i_π-open iff $F_G = i_\pi F_G$ iff F_G is soft π-open.

Proof. i_π possess the property of monotony. i.e, if $F_G \subset F_H$, then $i_\pi F_G \subset i_\pi F_H$. Also $i_\pi F_G \subset F_G$ for any $F_G \subset F_{\tilde{E}}$. Now F_G is soft i_π-open iff $F_G \subset i_\pi F_G$ iff $F_G = i_\pi F_G$ iff F_G is soft π-open by theorem 4.8.■

Definition 4.11. A soft set $F_G \subset F_{\tilde{E}}$ is soft π-closed iff its soft complement $(F_G)^c$ is soft π-open.

Note: 1) Since $F_\emptyset$ is always soft π-open, $F_{\tilde{E}}$ is always soft π-closed, for any $\pi \in \Pi$
2) $F_\emptyset$ is soft π-closed iff $F_{\tilde{E}}$ is soft π-open iff $\pi \in (\Pi 1)$
3) If $\pi \in (\Pi 3)$, every soft subset of $F_{\tilde{E}}$ is soft π-closed.

Theorem 4.12. Any soft intersection of soft π-closed sets is soft π-closed.

Proof. Suppose $\{F_{Gj}\}_{j\in J}$ be a collection of soft π-closed sets. Then $\{(F_{Gj})^c\}_{j\in J}$ is a collection of soft π-open sets. By theorem 4.4, $\bigcup_{j\in J}(F_{Gj})^c$ is soft π-open $\Rightarrow$ $(\bigcap_{j\in J}F_{Gj})^c$ is soft π-open $\Rightarrow$ $\bigcap_{j\in J}F_{Gj}$ is soft π-closed.■

Theorem 4.13. Let ξ be the collection of all soft π-closed sets. Then the following conditions hold.

1. The universal soft set $F_{\tilde{E}} \in \xi$.
2. Arbitrary soft intersection of members of ξ belongs to ξ.

Definition 4.14. The soft intersection of all soft π-closed supersets of F_G is called the soft π-closure of F_G and is denoted by $c_\pi F_G$.

Theorem 4.15. The soft set $c_\pi F_G$ is the smallest soft π-closed super set of F_G.

Note: Let $(F_{\tilde{E}}, \mu)$ be a SGTS and if $\pi = i_\mu$, then F_G is soft i_μ-closed set $\Leftrightarrow$ $(F_G)^c$ is soft i_μ-open $\Leftrightarrow$ $(F_G)^c$ is soft μ-open $\Leftrightarrow$ F_G is soft μ-closed. Hence soft i_μ-closed sets coincides with the soft μ-closed ones and $c_{i\mu} = c_\mu$

Definition 4.16. For any $\pi \in \Pi$ and $F_G \subset F_{\tilde{E}}$, $\pi^*F_G = [\pi(F_G)^c]^c$.

Theorem 4.17. For any $\pi \in \Pi$, the following conditions hold:

$\pi^* \in \Pi$, $(\pi^*)^* = \pi$, $\pi \in (\Pi0) \Leftrightarrow \pi^* \in (\Pi1)$, $\pi \in (\Pi1) \Leftrightarrow \pi^* \in (\Pi0)$, $\pi \in (\Pi2) \Leftrightarrow \pi^* \in (\Pi2)$, $\pi \in (\Pi3) \Leftrightarrow \pi^* \in (\Pi4)$, $(i_\pi)^* = c_\pi$.

Proof . Assume that $\pi \in \Pi$, i.e, if $F_G \subset F_H$, then $\pi F_G \subset \pi F_H$. Now $F_G \subset F_H \Rightarrow (F_G)^c \supset (F_H)^c \Rightarrow \pi(F_G)^c \supset \pi(F_H)^c \Rightarrow (\pi(F_G)^c)^c \subset (\pi(F_H)^c)^c \Rightarrow \pi^*F_G \subset \pi^*F_H$. Hence $\pi^* \in \Pi$. $\pi^*F_G = (\pi(F_G)^c)^c$. $\therefore$ $(\pi^*)^* F_G = [\pi^*(F_G)^c]^c = [(\pi F_G)^c]^c = \pi F_G$. Hence $(\pi^*)^* = \pi$. $\pi \in (\Pi0) \Leftrightarrow \pi F_\emptyset = F_\emptyset \Leftrightarrow (\pi F_\emptyset)^c = F_{\tilde{E}} \Leftrightarrow (\pi(F_{\tilde{E}})^c)^c = F_{\tilde{E}} \Leftrightarrow \pi^*F_{\tilde{E}} = F_{\tilde{E}} \Leftrightarrow \pi^* \in (\Pi1)$. $\pi \in (\Pi1) \Leftrightarrow \pi F_{\tilde{E}} = F_{\tilde{E}} \Leftrightarrow (\pi F_{\tilde{E}})^c = F_\emptyset \Leftrightarrow (\pi(F_\emptyset)^c)^c = F_\emptyset \Leftrightarrow \pi^*F_\emptyset = F_\emptyset \Leftrightarrow \pi^* \in (\Pi0)$. $\pi \in (\Pi2) \Leftrightarrow \pi(\pi(F_G)^c) = \pi(F_G)^c \Leftrightarrow [\pi(\pi(F_G)^c)]^c = [\pi(F_G)^c]^c \Leftrightarrow [\pi(\pi^*F_G)^c]^c = \pi^*F_G \Leftrightarrow \pi^*(\pi^*F_G) = \pi^*F_G \Leftrightarrow \pi^* \in (\Pi2)$. $\pi \in (\Pi3) \Leftrightarrow (F_G)^c \subset \pi(F_G)^c \Leftrightarrow F_G \supset (\pi(F_G)^c)^c \Leftrightarrow F_G \supset \pi^*F_G \Leftrightarrow \pi^* \in (\Pi4)$. $(i_\pi)^*F_G = (i_\pi(F_G)^c)^c$. By theorem 4.8, $i_\pi(F_G)^c$ is the largest soft π-open subset of $(F_G)^c$. Hence its soft complement coincides with the smallest soft π-closed super set of F_G. i.e, $(i_\pi)^*F_G = c_\pi F_G$ for any $F_G \subset F_{\tilde{E}}$. Hence $(i_\pi)^* = c_\pi$.■

Theorem 4.18. Let $(F_{\tilde{E}}, \mu)$ be a SGTS. Then $(i_\mu)^* = c_\mu$.

Proof. Take $\pi = i_\mu$ and since $i_{i\mu} = i_\mu$, the proof follows from theorem 4.17.■

Theorem 4.19. A soft set $F_G \subset F_{\tilde{E}}$ is soft π^*-closed $\Leftrightarrow$ $\pi F_G \subset F_G$.

Proof. F_G is soft π^*-closed $\Leftrightarrow$ $(F_G)^c$ is soft π^*-open $\Leftrightarrow$ $(F_G)^c \subset \pi^*(F_G)^c \Leftrightarrow (F_G)^c \subset (\pi F_G)^c \Leftrightarrow \pi F_G \subset F_G$.■

Theorem 4.20. For any $\pi \in \Pi$, $c_\pi \in (\Pi1), (\Pi2), (\Pi3)$; $c_\pi \in (\Pi0)$ iff $\pi \in (\Pi1)$. Conversely, if $\pi \in (\Pi1), (\Pi2), (\Pi3)$, then $\pi = c_{\pi^*}$.

Proof. Assume that $F_G \subset F_H \subset F_{\tilde{E}}$. By theorem 4.15, $c_\pi F_H$ is the smallest soft π-closed super set of F_H. But $F_H \supset F_G$. $\therefore$ $c_\pi F_H$ is a soft π-closed super set of F_G. Again by theorem

4.15, $c_\pi F_G$ is the smallest soft π-closed super set of F_G. Hence $c_\pi F_H \supset c_\pi F_G \Rightarrow c_\pi \in \Pi$. Since $F_{\tilde{E}}$ is a soft π-closed set, $c_\pi F_{\tilde{E}} = F_{\tilde{E}} \Rightarrow c_\pi \in (\Pi 1)$. By theorem 4.15, $c_\pi F_G$ is soft π-closed for any $F_G \subset F_{\tilde{E}}$. Therefore $c_\pi(c_\pi F_G) = c_\pi F_G \Rightarrow c_\pi \in (\Pi 2)$. By theorem 4.15, $c_\pi F_G$ is the smallest soft π-closed super set of F_G, $c_\pi F_G \supset F_G \Rightarrow c_\pi \in (\Pi 3)$. $c_\pi F_\emptyset = F_\emptyset \Leftrightarrow F_\emptyset$ is soft π-closed set $\Leftrightarrow F_{\tilde{E}}$ is soft π-open set $\Leftrightarrow F_{\tilde{E}} = \pi F_{\tilde{E}}$. Hence $c_\pi \in (\Pi 0)$ iff $\pi \in (\Pi 1)$.

Conversely, assume that $\pi \in (\Pi 1), (\Pi 2), (\Pi 3)$. Since $\pi \in (\Pi 2)$, $\pi(\pi F_G) = \pi F_G \Rightarrow \pi F_G$ is soft π^*-closed by theorem 4.19. Since $\pi \in (\Pi 3)$, πF_G is soft π^*-closed super set of F_G, for any $F_G \subset F_{\tilde{E}}$. If $F_H \supset F_G$ is a soft π^*-closed set, then by theorem 4.19, $\pi F_H \subset F_H$, so $F_H \supset \pi F_H \supset \pi F_G \supset F_G$. i.e, πF_G is the smallest soft π^*-closed super set of F_G. Hence $\pi = c_{\pi^*}$. ■

Theorem 4.21. Any soft set F_G is soft i_π-closed iff $F_G = c_\pi F_G$ iff F_G is soft π-closed.

Proof. By theorem 4.9, $i_\pi \in \Pi$. By theorem 4.17 and 4.19, F_G is soft i_π-closed $\Leftrightarrow F_G$ is soft $((i_\pi)^*)^*$-closed $\Leftrightarrow F_G$ is soft $(c_\pi)^*$-closed $\Leftrightarrow c_\pi F_G \subset F_G \Leftrightarrow c_\pi F_G = F_G \Leftrightarrow F_G$ is soft π-closed by theorem 4.15. ■

Theorem 4.22. If $\pi_1, \pi_2 \in \Pi$, $\pi_2\pi_1 \in \Pi$. If π_1 and $\pi_2 \in (\Pi 0), (\Pi 1), (\Pi 3), (\Pi 4)$, then $\pi_2\pi_1 \in (\Pi 0), (\Pi 1), (\Pi 3), (\Pi 4)$ and $(\pi_2\pi_1)^* = \pi_2^*\pi_1^*$.

Suppose the soft mappings $\theta, \sigma \in (\Pi 2)$. We will consider the soft mappings π that are the products of factors θ or σ. Only the products of alternating factors θ, σ need be taken into consideration.

Theorem 4.23. If $\theta, \sigma \in (\Pi 2)$, $\theta\sigma F_G \subset \sigma F_G$, $\theta\sigma F_G \subset \sigma\theta\sigma F_G$, and $\theta F_G \subset \sigma\theta F_G$ for any $F_G \subset F_{\tilde{E}}$. Then $\pi \in (\Pi 2)$ if π is a product of alternating factors θ and σ.

Proof. Clearly $\pi \in \Pi$ by theorem 4.22. Since $\theta\sigma F_G \subset \sigma F_G$, $\theta\sigma(\theta F_G) \subset \sigma(\theta F_G)$ and hence $\sigma\theta\sigma\theta(F_G) \subset \sigma\theta F_G$. Again since $\theta\sigma F_G \subset \sigma F_G$ and $\theta F_G \subset \sigma\theta F_G$, $\theta F_G \subset \sigma\theta F_G \Rightarrow \theta\theta F_G \subset \theta\sigma\theta F_G \Rightarrow \theta F_G \subset \theta\sigma\theta F_G \Rightarrow \sigma\theta F_G \subset \sigma\theta\sigma\theta F_G$. Hence $\sigma\theta\sigma\theta = \sigma\theta \Rightarrow \sigma\theta \in (\Pi 2)$. Since $\theta\sigma F_G \subset \sigma F_G \Rightarrow \sigma\theta\sigma F_G \subset \sigma\sigma F_G \Rightarrow \sigma\theta\sigma F_G \subset \sigma F_G \Rightarrow \theta\sigma\theta\sigma F_G \subset \theta\sigma F_G$. Again $\theta\sigma F_G \subset \sigma\theta\sigma F_G \Rightarrow \theta\theta\sigma F_G \subset \theta\sigma\theta\sigma F_G \Rightarrow \theta\sigma F_G \subset \theta\sigma\theta\sigma F_G$. Hence $\theta\sigma\theta\sigma = \theta\sigma \Rightarrow \theta\sigma \in (\Pi 2)$. Further, since $\theta\sigma\theta\sigma = \theta\sigma$, $(\theta\sigma\theta)(\theta\sigma\theta) = (\theta\sigma\theta\sigma)\theta = \theta\sigma\theta \Rightarrow \theta\sigma\theta \in (\Pi 2)$. $(\sigma\theta\sigma)(\sigma\theta\sigma) = \sigma(\theta\sigma\theta\sigma) = \sigma\theta\sigma \Rightarrow \sigma\theta\sigma \in (\Pi 2)$. $(\theta\sigma\theta\sigma)(\theta\sigma\theta\sigma) = (\theta\sigma)(\theta\sigma) \Rightarrow \theta\sigma\theta\sigma \in (\Pi 2)$. $(\sigma\theta\sigma\theta)(\sigma\theta\sigma\theta) = \sigma(\theta\sigma\theta\sigma)(\theta\sigma\theta) = \sigma(\theta\sigma)(\theta\sigma\theta) = \sigma(\theta\sigma\theta\sigma)\theta = \sigma(\theta\sigma)\theta \Rightarrow \sigma\theta\sigma\theta \in (\Pi 2)$. Again any alternating products of $k \geq 5$ factors is equal to another such product of $(k-2)$ factor and the statement holds for it. ■

Theorem 4.24. If $\theta \in (\Pi 2), (\Pi 4)$ and $\sigma \in (\Pi 2), (\Pi 3)$, then $\pi \in (\Pi 2)$ if π is any product of the factors θ and σ.

Proof. If $\theta \in (\Pi 2), (\Pi 4)$ and $\sigma \in (\Pi 2), (\Pi 3)$, then $\theta\sigma F_G \subset \sigma F_G$, $\theta\sigma F_G \subset \sigma\theta\sigma F_G$, and $\theta F_G \subset \sigma\theta F_G$ for any $F_G \subset F_{\tilde{E}}$. The proof follows from theorem 4.23.

Note: Let $(F_{\tilde{E}}, \mu)$ be a SGTS. Clearly the soft mappings $\theta = i_\mu \in (\Pi 2), (\Pi 4)$ and $\sigma = c_\mu \in (\Pi 2), (\Pi 3)$, so by theorem 4.24 any product of factor i_μ and c_μ is idempotent. In particular $i_\mu c_\mu i_\mu c_\mu = i_\mu c_\mu$ and $c_\mu i_\mu c_\mu i_\mu = c_\mu i_\mu$ so that any product of this kind is equal to one of the mappings i_μ, c_μ, $i_\mu c_\mu$, $c_\mu i_\mu$, $i_\mu c_\mu i_\mu$, $c_\mu i_\mu c_\mu$.

Theorem 4.25. Let $(F_{\tilde{E}}, \mu)$ be a SGTS. Then the soft mappings i_μ, c_μ, $i_\mu c_\mu$, $c_\mu i_\mu$, $i_\mu c_\mu i_\mu$, $c_\mu i_\mu c_\mu$ are all belong to $(\Pi 2)$.

Proof. Take $\pi_1 = i_\mu$ and $\pi_2 = c_\mu$, where $i_\mu F_G$ be the soft μ-interior of the soft set F_G and $c_\mu F_G$ be the soft μ-closure of the soft set F_G w.r. t. the SGT μ. Clearly the soft mappings $\pi_1 = i_\mu \in$ (Π2), (Π4) and $\pi_2 = c_\mu$ (Π2), (Π3). So by theorem 4.24, the soft mappings i_μ, c_μ, $i_\mu c_\mu$, $c_\mu i_\mu$, $i_\mu c_\mu i_\mu$, $c_\mu i_\mu c_\mu$ are all belong to (Π2)■

Definition 4.26. Let $(F_{\tilde{E}}, \mu)$ be a SGTS. Then a soft set $F_G \subset F_{\tilde{E}}$ is said to be a soft μ-semi-open set iff $F_G \subset c_\mu i_\mu F_G$ (i.e, the case when $\pi = c_\mu i_\mu$). The class of all soft μ-semi-open sets is denoted by $\delta_{(\mu)}$ or δ_μ.

Definition 4.27. Let $(F_{\tilde{E}}, \mu)$ be a SGTS. Then a soft set $F_G \subset F_{\tilde{E}}$ is said to be a soft μ-pre-open set iff $F_G \subset i_\mu c_\mu F_G$ (i.e, the case when $\pi = i_\mu c_\mu$). The class of all soft μ-pre-open sets is denoted by $\rho_{(\mu)}$ or ρ_μ.

Definition 4.28. Let $(F_{\tilde{E}}, \mu)$ be a SGTS. Then a soft set $F_G \subset F_{\tilde{E}}$ is said to be a soft μ-α-open set iff $F_G \subset i_\mu c_\mu i_\mu F_G$ (i.e, the case when $\pi = i_\mu c_\mu i_\mu$). The class of all soft μ-α-open set is denoted by $\alpha_{(\mu)}$ or α_μ.

Definition 4.29. Let $(F_{\tilde{E}}, \mu)$ be a SGTS. Then a soft set $F_G \subset F_{\tilde{E}}$ is said to be a soft μ-β-open sets iff $F_G \subset c_\mu i_\mu c_\mu F_G$ (i.e, the case when $\pi = c_\mu i_\mu c_\mu$). The class of all soft μ-β-open set is denoted by $\beta_{(\mu)}$ or β_μ.

Example 4.30. Let $U = \{h_1, h_2, h_3\}$, $E = \{e_1, e_2\}$ and $\mu = \{F_\emptyset, F_A, F_{\tilde{E}}\}$ where $F_A = \{(e_1, \{h_3\}), (e_2, \{h_1\})\}$. Then $(F_{\tilde{E}}, \mu)$ is a SGTS. The Soft set $F_G = \{(e_1, \{h_1, h_3\}), (e_2, \{h_1\})\}$ is a soft μ-semi-open sets

Example 4.31. Let $U = \{h_1, h_2, h_3\}$, $E = \{e_1, e_2\}$ and $\mu = \{F_\emptyset, F_B, F_{\tilde{E}}\}$ where $F_B = \{(e_1, \{h_1, h_2\}), (e_2, \{h_1, h_3\})\}$. Then $(F_{\tilde{E}}, \mu)$ is a SGTS. The Soft sets $F_G = \{(e_1, \{h_2, h_3\}), (e_2, \{h_2\})\}$, $F_H = \{(e_1, \{h_1, h_3\}), (e_2, \{h_2\})\}$ are soft μ-pre-open sets

Example 4.32. Let $U = \{h_1, h_2, h_3\}$, $E = \{e_1, e_2\}$ and $\mu = \{F_\emptyset, F_D, F_{\tilde{E}}\}$ where $F_D = \{(e_1, \{h_1\}), (e_2, \{h_2\})\}$. Then $(F_{\tilde{E}}, \mu)$ is a SGTS. The Soft set $F_G = \{(e_1, \{h_1, h_2\}), (e_2, \{h_2\})\}$ is a soft μ-α-open sets

Example 4.33. Let $U = \{h_1, h_2, h_3, h_4\}$, $E = \{e_1\}$ and $\mu = \{F_\emptyset, F_P, F_Q, F_R, F_S, F_{\tilde{E}}\}$ where $F_P = \{(e_1, \{h_4\})\}$, $F_Q = \{(e_1, \{h_1\})\}$, $F_R = \{(e_1, \{h_1, h_4\})\}$, $F_S = \{(e_1, \{h_1, h_3, h_4\})\}$. Then $(F_{\tilde{E}}, \mu)$ is a SGTS. The Soft set $F_G = \{(e_1, \{h_3, h_4\})\}$ is a soft μ-β-open sets

Theorem 4.34. Let $(F_{\tilde{E}}, \mu)$ be a SGTS. Then δ_μ, ρ_μ, α_μ and β_μ are SGT's.

Proof. Follows from theorem 4.5.■

Now consider the soft mappings $\pi \in$ (Π5)

Theorem 4.35. If $\pi \in \Pi$, then every soft π-open set is soft π^n–open and $\pi F_G \subset \pi^n F_G$ for $n \in N$.

Proof. Suppose F_G is soft π-open. Then $F_G \subset \pi F_G$. Now $F_G \subset \pi F_G \Rightarrow \pi^m F_G \subset \pi^{m+1} F_G \Rightarrow F_G \subset \pi F_G \subset \pi^n F_G \Rightarrow F_G$ is soft π^n–open.■

Theorem 4.36. If $\pi \in$ (Π5), then $\pi^n F_G \subset \pi F_G$ and soft π-open sets and soft π^n–open sets coincide, $n \in N$.

Proof. Suppose $\pi \in (\Pi 5)$, then $\pi^2 F_G \subset \pi F_G \Rightarrow \pi^{m+1} F_G \subset \pi^m F_G$ and $\pi^n F_G \subset \pi F_G$. Hence by theorem 4.35, $\pi F_G = \pi^n F_G$. Threfore soft π-open sets and soft π^n–open sets coincide.■

Theorem 4.37. If $\theta, \sigma \in (\Pi 5)$ satisfies $\theta\sigma F_G \subset \sigma F_G$, then any product of factors θ and σ belong to $(\Pi 5)$. If both θ and σ occur among the factors of a product of this kind, then

(1) soft $\theta\pi'\sigma$-open $\Rightarrow$ soft $\theta\sigma$-open
(2) soft $\theta\pi'\theta$-open $\Rightarrow$ soft $\theta\sigma\theta$-open
(3) soft $\sigma\pi'\theta$-open $\Rightarrow$ soft $\sigma\theta$-open
(4) soft $\sigma\pi'\sigma$-open $\Rightarrow$ soft $\sigma\theta\sigma$-open

The converse implication is true if no factor θ is immediately followed by another such factor.

Proof. Since $\theta, \sigma \in (\Pi 5)$, by theorem 4.36, we have $\theta^n F_G \subset \theta F_G$ and $\sigma^n F_G \subset \sigma F_G$ for $n \in N$ and since $\theta\sigma F_G \subset \sigma F_G$, $(\theta\sigma)^n F_G \subset \sigma^n F_G \subset \sigma F_G$. $\therefore$ $\theta^n\theta^n F_G \subset \theta\theta^n F_G \subset \theta^2\theta^{n-1} F_G \subset \theta\theta^{n-1} F_G \subset \theta^n F_G$. Hence $\pi \in (\Pi 5)$ if $\pi = \theta^n$. Suppose π is a product of factors θ and σ, containing atleast one factor σ. Then π can be written in the form $\pi_1\sigma\pi_2$, where π_1 and π_2 (may be empty) are products of factors θ and σ. Then $\pi\pi F_G = \pi_1\sigma\pi_2\pi_1\sigma\pi_2 F_G$. Since $\theta^n F_G \subset \theta F_G$, $\sigma^n F_G \subset \sigma F_G$ and $(\theta\sigma)^n F_G \subset \sigma F_G$, in the product $\sigma\pi_2\pi_1\sigma$, each group of factors θ^p can be replaced by θ, each group of factors σ^q can be replaced by σ, and then $(\theta\sigma)^r$ can be replaced by σ. Therefore $\pi\pi F_G = \pi_1\sigma\pi_2\pi_1\sigma\pi_2 F_G \subset \pi_1\sigma\sigma\pi_2 F_G \subset \pi_1\sigma\pi_2 F_G = \pi F_G$. Hence $\pi \in (\Pi 5)$.

Consider (1). Suppose F_G is soft $\theta\pi'\sigma$-open, where π' is any product of both the factors θ and σ. Then $F_G \subset \theta\pi'\sigma F_G$, Now consider the product $\theta\pi'\sigma$, by theorem 4.36, each group of factors θ^p can be replaced by θ and each group of factors σ^q can be replaced by σ, so we can write $\theta\pi'\sigma F_G \subset (\theta\sigma)^n F_G$. $\therefore$ $F_G \subset \theta\pi'\sigma F_G \subset (\theta\sigma)^n F_G = \theta\sigma(\theta\sigma)^{n-1} F_G \subset \theta\sigma\sigma^{n-1} F_G \subset \theta\sigma^n F_G \subset \theta\sigma F_G$ for a suitable $n \in N$. Hence F_G is soft $\theta\sigma$-open. Conversely suppose that F_G is soft $\theta\sigma$-open and no factor θ is followed by another one in $\pi = \theta\pi'\sigma$. Then $F_G \subset \theta\sigma F_G \Rightarrow F_G \subset (\theta\sigma)^m F_G$ by theorem 4.35, where m is the number of the factors σ in the product π. Apply the condition $\theta\sigma F_G \subset \sigma F_G$ repeatedly, then it is easy to show that $(\theta\sigma)^m F_G \subset \theta\pi'\sigma F_G$. Hence F_G is soft $\theta\pi'\sigma$-open.

Consider (2). Suppose F_G is soft $\theta\pi'\theta$-open, where π' is any product of both the factors θ and σ. Then $F_G \subset \theta\pi'\theta F_G$. By theorem 4.36, we can write $\theta\pi'\theta F_G \subset (\theta\sigma)^k\theta F_G$. Since $(\theta\sigma)^n F_G \subset \theta\sigma F_G$, $(\theta\sigma)^k\theta F_G \subset \theta\sigma\theta F_G$. Hence $F_G \subset \theta\sigma\theta F_G \Rightarrow F_G$ is soft $\theta\sigma\theta$-open. Conversely assume that F_G is soft $\theta\sigma\theta$-open and no factor θ is followed by another one in $\pi = \theta\pi'\theta$. Then $F_G \subset \theta\sigma\theta F_G \Rightarrow (\theta\sigma)^j\theta F_G \subset (\theta\sigma)^j\theta\theta\sigma\theta F_G \subset (\theta\sigma)^j\theta\sigma\theta F_G = (\theta\sigma)^{j+1}\theta F_G$ for $j \in N$. i.e, $F_G \subset \theta\sigma\theta F_G \subset (\theta\sigma)^{j+1}\theta F_G$ and as above $\theta\sigma\theta F_G \subset (\theta\sigma)^m\theta F_G \subset \theta\pi'\theta F_G$ by the repeated application of the condition $\theta\sigma F_G \subset \sigma F_G$. Hence F_G is soft $\theta\pi'\theta$-open.

Consider (3). Suppose F_G is soft $\sigma\pi'\theta$-open, where π' is any product of both the factors θ and σ. Then $F_G \subset \sigma\pi'\theta F_G$. Since $(\theta\sigma)^n F_G \subset \theta\sigma F_G$ and $\sigma\theta \in (\Pi 5)$, we can write $\sigma\pi'\theta F_G \subset (\sigma\theta)^k F_G = \sigma(\theta\sigma)^{k-1}\theta F_G \subset \sigma(\theta\sigma)\theta F_G \subset \sigma\theta F_G$ for some $k \in N$. Hence $F_G \subset \sigma\theta F_G \Rightarrow F_G$ is soft $\sigma\theta$-open. Conversely assume that F_G is soft $\sigma\theta$-open and no factor θ is followed by another one in $\pi = \sigma\pi'\theta$. Then $F_G \subset \sigma\theta F_G \Rightarrow F_G \subset (\sigma\theta)^m F_G$, by theorem 4.35. Since $\theta\sigma F_G \subset \sigma F_G$, it is easy to show that $(\sigma\theta)^m F_G \subset \sigma\pi'\theta F_G$. Hence $F_G \subset \sigma\pi'\theta F_G \Rightarrow F_G$ is soft $\sigma\pi'\theta$-open.

Consider (4). Suppose F_G is soft $\sigma\pi'\sigma$-open, then $F_G \subset \sigma\pi'\sigma F_G$. Since $(\sigma\theta)^k F_G \subset \sigma\theta F_G$, we can write $\sigma\pi'\sigma F_G \subset (\sigma\theta)^k\sigma F_G \subset \sigma\theta\sigma F_G$. Thus $F_G \subset \sigma\pi'\sigma F_G \Rightarrow F_G \subset \sigma\theta\sigma F_G \Rightarrow F_G$ is soft $\sigma\theta\sigma$-open. Conversely assume that F_G is soft $\sigma\theta\sigma$-open and no factor θ is followed by another one in $\pi = \sigma\pi'\sigma$. Then $F_G \subset \sigma\theta\sigma F_G \Rightarrow (\sigma\theta)^j\sigma F_G \subset (\sigma\theta)^j\sigma\sigma\theta\sigma F_G \subset (\sigma\theta)^j\sigma\theta\sigma F_G = (\sigma\theta)^{j+1}\sigma F_G$. Hence $\sigma\theta\sigma F_G \subset (\sigma\theta)^{m-1}\sigma F_G$. Since $\theta\sigma F_G \subset \sigma F_G$, it is easy to show that $(\sigma\theta)^{m-1}\sigma F_G \subset \sigma\pi'\sigma F_G$. Hence $F_G \subset \sigma\pi'\sigma F_G \Rightarrow F_G$ is soft $\sigma\pi'\sigma$-open.■

Theorem 4.38. If $\theta \in (\Pi4)$ and $\sigma \in (\Pi5)$ then the statements of theorem 4.37 are valid; moreover, soft $\theta\sigma\theta$–open $\Leftrightarrow$ (soft $\theta\sigma$-open and soft $\sigma\theta$–open) $\Rightarrow$ (soft $\theta\sigma$-open or soft $\sigma\theta$-open) $\Rightarrow$ soft $\sigma\theta\sigma$-open $\Rightarrow$ soft σ-open.

Proof. If $\theta \in (\Pi4)$, then $\theta F_G \subset F_G \Rightarrow \theta\theta F_G \subset \theta F_G \Rightarrow \theta \in (\Pi5)$ and also $\theta\sigma F_G \subset \sigma F_G$. Now the hypotheses of theorem 4.37 are fulfilled. Further, $\theta\sigma(\theta F_G) \subset \theta\sigma F_G$ and $\theta(\sigma\theta F_G) \subset \sigma\theta F_G$; i.e F_G is soft $\theta\sigma\theta$–open $\Rightarrow F_G \subset \theta\sigma\theta F_G \subset \theta\sigma F_G$ and $F_G \subset \theta\sigma\theta F_G \subset \sigma\theta F_G \Rightarrow F_G$ is both soft $\theta\sigma$-open and soft $\sigma\theta$–open. Conversely assume that F_G is both soft $\theta\sigma$-open and soft $\sigma\theta$-open. Then $F_G \subset \theta\sigma F_G$ and $F_G \subset \sigma\theta F_G \Rightarrow F_G \subset \theta\sigma F_G \cap \sigma\theta F_G \Rightarrow F_G \subset \theta\sigma F_G \subset \theta\sigma(\sigma\theta F_G) \subset \theta\sigma\theta F_G \Rightarrow F_G$ is soft $\theta\sigma\theta$–open. Again, F_G is soft $\theta\sigma$-open or soft $\sigma\theta$-open $\Rightarrow F_G \subset \theta\sigma F_G$ or $F_G \subset \sigma\theta F_G \Rightarrow F_G \subset \theta\sigma\theta\sigma F_G$ or $F_G \subset \sigma\theta\sigma\theta F_G$ respectively. Hence $F_G \subset \sigma\theta\sigma F_G$ by $\theta \in (\Pi4)$ $\Rightarrow F_G$ is soft $\sigma\theta\sigma$-open. And F_G is soft $\sigma\theta\sigma$-open $\Rightarrow F_G \subset \sigma\theta\sigma F_G \subset \sigma F_G$ by $\theta \in (\Pi4)$ and $\sigma \in (\Pi5) \Rightarrow F_G$ is soft σ-open.■

Note: Let $(F_{\tilde{E}}, \mu)$ be a SGTS. Then we can say that a soft set F_G is soft $i_\mu c_\mu i_\mu$–open iff it is both soft $i_\mu c_\mu$–open and soft $c_\mu i_\mu$–open.

Theorem 4.39. If $\pi \in (\Pi5)$ and F_G is soft π-open then $c_{\pi*}F_G = \pi F_G$.

Proof. Since $\pi \in (\Pi5)$, $\pi\pi F_G \subset \pi F_G \Rightarrow \pi F_G$ is soft π^*-closed by theorem 4.19. If $F_H \supset F_G$ is soft π^*-closed, then $F_H \supset \pi F_H \supset \pi F_G$. Hence $\pi F_G \supset F_G$ is the smallest soft π^*-closed super set of F_G. Hence $c_{\pi*}F_G = \pi F_G$.■

Theorem 4.40. For any $\pi \in \Pi$ and $F_G \subset F_{\tilde{E}}$, we have $i_\pi F_G \subset F_G \cap \pi F_G$.

Proof. Suppose $F_H \subset F_G$ is soft π-open. Then $F_H \subset \pi F_H \subset \pi F_G$ so that $F_H \subset F_G \cap \pi F_G$. Hence $i_\pi F_G \subset F_G \cap \pi F_G$.■

Theorem 4.41. Let $(F_{\tilde{E}}, \mu)$ be a SGTS and if $\pi = c_\mu i_\mu$ or $\pi = i_\mu c_\mu i_\mu$, then $i_\pi F_G = F_G \cap \pi F_G$ for any $F_G \subset F_{\tilde{E}}$.

Proof. Clearly $i_\mu F_G \subset c_\mu i_\mu F_G$ for $F_G \subset F_{\tilde{E}}$ and $i_\mu F_G \subset c_\mu i_\mu F_G \Rightarrow i_\mu i_\mu F_G \subset i_\mu c_\mu i_\mu F_G \Rightarrow i_\mu F_G \subset i_\mu c_\mu i_\mu F_G$. Therefore $i_\mu F_G \subset F_G \cap c_\mu i_\mu F_G$ and $i_\mu F_G \subset F_G \cap i_\mu c_\mu i_\mu F_G$. Hence $i_\mu i_\mu F_G \subset i_\mu(F_G \cap c_\mu i_\mu F_G)$ and $i_\mu i_\mu F_G \subset i_\mu(F_G \cap i_\mu c_\mu i_\mu F_G)$. i.e, $i_\mu F_G \subset i_\mu(F_G \cap c_\mu i_\mu F_G)$ and $i_\mu F_G \subset i_\mu(F_G \cap i_\mu c_\mu i_\mu F_G)$. Therefore $c_\mu i_\mu F_G \subset c_\mu i_\mu(F_G \cap c_\mu i_\mu F_G)$ and $i_\mu c_\mu i_\mu F_G \subset i_\mu c_\mu i_\mu(F_G \cap i_\mu c_\mu i_\mu F_G)$. Hence $F_G \cap c_\mu i_\mu F_G \subset c_\mu i_\mu(F_G \cap c_\mu i_\mu F_G)$ and $F_G \cap i_\mu c_\mu i_\mu F_G \subset i_\mu c_\mu i_\mu(F_G \cap i_\mu c_\mu i_\mu F_G)$. ie, $F_G \cap \pi F_G \subset \pi(F_G \cap \pi F_G)$ for $\pi = c_\mu i_\mu$ or $\pi = i_\mu c_\mu i_\mu$. i.e, $F_G \cap \pi F_G$ is soft π-open for $\pi = c_\mu i_\mu$ or $\pi = i_\mu c_\mu i_\mu$. Thus $F_G \cap \pi F_G \subset i_\pi F_G$ in these two cases. But by theorem 4.40, $i_\pi F_G \subset F_G \cap \pi F_G$. Hence $i_\pi F_G = F_G \cap \pi F_G$ for $\pi = c_\mu i_\mu$ or $\pi = i_\mu c_\mu i_\mu$.■

Theorem 4.42. For $\pi \in \Pi$ and $F_G \subset F_{\tilde{E}}$, $i_\pi F_G = F_G \cap \pi F_G$ for $F_G \subset F_{\tilde{E}}$ is true iff $c_\pi F_G = F_G \cup \pi^* F_G$.

Proof. Suppose $i_\pi F_G = F_G \cap \pi F_G$ is true. Then by theorem 4.17, $c_\pi F_G = (i_\pi)^* F_G = [i_\pi(F_G)^c]^c = [(F_G)^c \cap \pi(F_G)^c]^c = F_G \cup [\pi(F_G)^c]^c = F_G \cup \pi^* F_G$. Conversely, suppose that $c_\pi F_G = F_G \cup \pi^* F_G$. Then $i_\pi F_G = (c_\pi)^* F_G = [c_\pi(F_G)^c]^c = [F_G{}^c \cup \pi^*(F_G)^c]^c = F_G \cap [\pi^*(F_G)^c]^c = F_G \cap \pi F_G$, by theorem 4.17.■

Theorem 4.43. Let $(F_{\tilde{E}}, \mu)$ be a SGTS. Then $c_\pi F_G = F_G \cup \pi^* F_G$ is true if $\pi = c_\mu i_\mu$ or $\pi = i_\mu c_\mu i_\mu$.

Proof. The proof follows from theorem 4.41 and 4.42.

Conclusion

In the present work, we mainly study some interesting properties of the soft mapping $\pi : S(U)_E \rightarrow S(U)_E$ which satisfy the condition $\pi F_B \subset \pi F_D$ whenever $F_B \subset F_D \subset F_{\tilde{E}}$. The concept of soft π-open set is introduced and established some of their properties. The notions of soft interior and soft closure are generalized using these sets and under suitable conditions on π, the soft π-interior $i_\pi F_G$ and the soft π-closure $c_\pi F_G$ of a soft set $F_G \subset F_{\tilde{E}}$ are easily obtained by explicit formulas. We expect that results in this paper will be a basis for applications of soft π-open sets in soft set theory and will promote the further study on soft generalized topology to carry out general frame work for the applications in practical life.

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