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Λ_g-CLOSED SETS WITH RESPECT TO AN IDEAL

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Abstract − In this paper, the notions of $\mathcal{I}_{\Lambda_g}$-closed sets and $\mathcal{I}_{\Lambda_g}$-open sets are introduced. Characterizations and properties of such notions are obtained. Suitable examples are given to substantiate each established notions.

Keywords − *Topological space, open set, λ-closed set, Λ_g-closed set, $\mathcal{I}_g$-closed set, $\mathcal{I}_{\pi g}$-closed set, ideal.*

1 Introduction and Preliminaries

In 1986, Maki [12] introduced the notion of Λ-sets in topological spaces. A Λ-set is a set A which is equal to its kernel (= saturated set) i.e to the intersection of all open supersets of A. Arenas et al [1] introduced and investigated the notion of λ-closed sets by involving Λ-sets and closed sets.

The notion of closed set is fundamental in the study of topological spaces. In 1970, Levine [11] introduced the concept of generalized closed sets in a topological space by comparing the closure of a subset with its open supersets. He defined a subset A of a topological space X to be generalized closed (briefly, g-closed) if cl(A) $\subseteq$ U whenever A $\subseteq$ U and U is open. This notion has been studied extensively in recent years by many topologists. After advent of g-closed sets, many generalizations of g-closed sets are being introduced and investigated by modern topologists.

An ideal on a set X is a non empty collection of subsets of X with heredity property which is also closed under finite unions. Quite Recently, Jafari and Rajesh [8] have introduced and studied the notion of generalized closed (g-closed) sets with respect to an ideal. Many generalizations of g-closed sets are being introduced and investigated by modern researchers. One among them is Λ_g-closed sets which were introduced by Caldas et al [2]. In this paper, we introduce and investigate the concept of Λ_g-closed sets with respect to an ideal.

Indeed ideals are very important tools in General Topology. It was the works of Newcomb [13], Rancin [14], Samuels [16] and Hamlett and Jankovic (see [4, 5, 6, 7, 9]) which motivated the research in applying topological ideals to generalize the most basic properties in General Topology. A nonempty

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collection $\mathcal{I}$ of subsets on a topological space (X, τ) is called a topological ideal [10] if it satisfies the following two conditions:

1. If A $\in \mathcal{I}$ and B $\subseteq$ A implies B $\in \mathcal{I}$ (heredity)
2. If A $\in \mathcal{I}$ and B $\in \mathcal{I}$, then A $\cup$ B $\in \mathcal{I}$ (finite additivity)

If A is a subset of a topological space (X, τ), cl(A) and int(A) denote the closure of A and the interior of A, respectively. Let A $\subseteq$ B $\subseteq$ X. Then cl_B(A) (resp. int_B(A)) denotes closure of A (resp. interior of A) with respect to B.

In this paper, we introduce and study the concept of Λ_g-closed sets with respect to an ideal, which is the extension of the concept of Λ_g-closed sets.

The following Definitions, Result, Lemma and Remarks are useful in the sequel.

Definition 1.1. *A subset A of a topological space (X, τ) is called regular open [17] if A = int(cl(A)).*

Definition 1.2. *The finite union of regular open sets is called π-open [18]. The complement of π-open set is called π-closed [18].*

Definition 1.3. *A subset A of a topological space (X, τ) is called*

1. *λ-closed [1] if A = L $\cap$ D, where L is a Λ-set and D is a closed set.*
2. *λ-open [1] if its complement is λ-closed.*
3. *Λ_g-closed [2] if cl(A)$\subseteq$U whenever A$\subseteq$U and U is λ-open.*
4. *π-generalized closed (briefly, πg-closed) [3] if cl(A) $\subseteq$ U whenever A $\subseteq$ U and U is π-open.*

Definition 1.4. *Let (X, τ) be a topological space and $\mathcal{I}$ be an ideal on X. A subset A of X is said to be generalized closed with respect to an ideal (briefly $\mathcal{I}_g$-closed) [8] if and only if cl(A)$-$B $\in \mathcal{I}$, whenever A $\subseteq$ B and B is open.*

Result 1.5. *For a subset of a topological space, the following properties hold:*

1. *Every closed set is Λ_g-closed but not conversely [2].*
2. *Every Λ_g-closed set is g-closed but not conversely [2].*
3. *Every closed set is λ-closed but not conversely [1, 2].*

Remark 1.6. *[8] Every g-closed set is $\mathcal{I}_g$-closed but not conversely.*

Definition 1.7. *[15] Let (X, τ) be a topological space and $\mathcal{I}$ be an ideal on X. A subset A of X is said to be π-generalized closed with respect to an ideal (briefly $\mathcal{I}_{\pi g}$-closed) if and only if cl(A)$-$B $\in \mathcal{I}$, whenever A $\subseteq$ B and B is π-open.*

Remark 1.8. *[15] For several subsets defined above, we have the following implications.*

$$\begin{array}{ccccc} & & \mathcal{I}_g\text{-closed set} & \longrightarrow & \mathcal{I}_{\pi g}\text{-closed set} \\ & & \uparrow & & \uparrow \\ \text{closed set} & \longrightarrow & g\text{-closed set} & \longrightarrow & \pi g\text{-closed set} \end{array}$$

The reverse implications are not true.

Lemma 1.9. *[1] Let $A_i (i \in \mathcal{I})$ be subsets of a topological space (X, τ). The following properties hold:*

1. *If A_i is λ-closed for each $i \in I$, then $\cap_{i\in I} A_i$ is λ-closed.*
2. *If A_i is λ-open for each $i \in I$, then $\cup_{i\in I} A_i$ is λ-open.*

Recall that the intersection of a λ-closed set and a closed set is λ-closed.

Definition 1.10. *[2] A function $f : (X, \tau) \to (Y, \sigma)$ is called λ-irresolute if the inverse image of λ-open set of Y is λ-open in X.*

2 Λ_g-Closed Sets with Respect to an Ideal

Definition 2.1. *Let (X, τ) be a topological space and $\mathcal{I}$ be an ideal on X. A subset A of X is said to be Λ_g-closed with respect to an ideal (briefly $\mathcal{I}_{\Lambda_g}$-closed) if and only if cl(A)$-$B $\in \mathcal{I}$, whenever A $\subseteq$ B and B is λ-open.*

Remark 2.2. *Every Λ_g-closed set is $\mathcal{I}_{\Lambda_g}$-closed, but the converse need not be true, as this may be seen from the following Example.*

Example 2.3. *Let $X = \{a, b, c\}$, $\tau = \{\phi, X, \{a, c\}\}$ and $\mathcal{I} = \{\phi, \{b\}\}$. Then $\{a\}$ is $\mathcal{I}_{\Lambda_g}$-closed but not Λ_g-closed.*

The following Theorem gives a characterization of $\mathcal{I}_{\Lambda_g}$-closed sets.

Theorem 2.4. *A set A is $\mathcal{I}_{\Lambda_g}$-closed in (X, τ) if and only if F $\subseteq$ cl(A)$-$A and F is λ-closed in X implies F $\in \mathcal{I}$.*

Proof. Assume that A is $\mathcal{I}_{\Lambda_g}$-closed. Let F $\subseteq$ cl(A)$-$A. Suppose F is λ-closed. Then A $\subseteq$ X$-$F. By our assumption, cl(A)$-$(X$-$F) $\in \mathcal{I}$. But F $\subseteq$ cl(A)$-$(X$-$F) and hence F $\in \mathcal{I}$.

Conversely, assume that F $\subseteq$ cl(A)$-$A and F is λ-closed in X implies that F $\in \mathcal{I}$. Suppose A $\subseteq$ U and U is λ-open. Then cl(A)$-$U = cl(A) $\cap$ (X$-$U) is a λ-closed set in X, that is contained in cl(A)$-$A. By assumption, cl(A)$-$U $\in \mathcal{I}$. This implies that A is $\mathcal{I}_{\Lambda_g}$-closed.

Theorem 2.5. *If A and B are $\mathcal{I}_{\Lambda_g}$-closed sets of (X, τ), then their union A $\cup$ B is also $\mathcal{I}_{\Lambda_g}$-closed.*

Proof. Suppose A and B are $\mathcal{I}_{\Lambda_g}$-closed sets in (X, τ). If A $\cup$ B $\subseteq$ U and U is λ-open, then A $\subseteq$ U and B $\subseteq$ U. By assumption, cl(A)$-$U $\in \mathcal{I}$ and cl(B)$-$U $\in \mathcal{I}$ and hence cl(A $\cup$ B)$-$U = (cl(A)$-$U) $\cup$ (cl(B)$-$U) $\in \mathcal{I}$. That is A $\cup$ B is $\mathcal{I}_{\Lambda_g}$-closed.

Remark 2.6. *The intersection of two $\mathcal{I}_{\Lambda_g}$-closed sets need not be an $\mathcal{I}_{\Lambda_g}$-closed as shown by the following Example.*

Example 2.7. *Let $X = \{a, b, c, d\}$, $\tau = \{\phi, X, \{a\}, \{d\}, \{a, d\}\}$ and $\mathcal{I} = \{\phi, \{c\}\}$. Then $A = \{a, b\}$ and $B = \{a, c\}$ are $\mathcal{I}_{\Lambda_g}$-closed but their intersection $A \cap B = \{a\}$ is not $\mathcal{I}_{\Lambda_g}$-closed.*

Remark 2.8. *Every $\mathcal{I}_{\Lambda_g}$-closed set is $\mathcal{I}_g$-closed but not conversely.*

Example 2.9. *Let $X = \{a, b, c\}$, $\tau = \{\phi, X, \{b\}, \{b, c\}\}$ and $\mathcal{I} = \{\phi\}$. Then $\{a, b\}$ is $\mathcal{I}_g$-closed but not $\mathcal{I}_{\Lambda_g}$-closed.*

Remark 2.10. *For several subsets defined above, we have the following implications.*

$$\begin{array}{ccccccc} & & \mathcal{I}_{\Lambda_g}\text{-closed set} & \longrightarrow & \mathcal{I}_g\text{-closed set} & \longrightarrow & \mathcal{I}_{\pi g}\text{-closed set} \\ & & \uparrow & & \uparrow & & \uparrow \\ \text{closed set} & \longrightarrow & \Lambda_g\text{-closed set} & \longrightarrow & g\text{-closed set} & \longrightarrow & \pi g\text{-closed set} \end{array}$$

The reverse implications are not true.

Theorem 2.11. *If A is $\mathcal{I}_{\Lambda_g}$-closed and A $\subseteq$ B $\subseteq$ cl(A) in (X, τ), then B is $\mathcal{I}_{\Lambda_g}$-closed in (X, τ).*

Proof. Suppose A is $\mathcal{I}_{\Lambda_g}$-closed and A $\subseteq$ B $\subseteq$ cl(A) in (X, τ). Suppose B $\subseteq$ U and U is λ-open. Then A $\subseteq$ U. Since A is $\mathcal{I}_{\Lambda_g}$-closed, we have cl(A)$-$U $\in \mathcal{I}$. Now B $\subseteq$ cl(A). This implies that cl(B)$-$U $\subseteq$ cl(A)$-$U $\in \mathcal{I}$. Hence B is $\mathcal{I}_{\Lambda_g}$-closed in (X, τ).

Theorem 2.12. *Let $A \subseteq Y \subseteq X$ and suppose that A is $\mathcal{I}_{\Lambda_g}$-closed in (X, τ). Then A is $\mathcal{I}_{\Lambda_g}$-closed relative to the subspace Y of X, with respect to the ideal $\mathcal{I}_Y = \{F \subseteq Y : F \in \mathcal{I}\}$.*

Proof. Suppose A $\subseteq$ U $\cap$ Y and U is λ-open in (X, τ), then A $\subseteq$ U. Since A is $\mathcal{I}_{\Lambda_g}$-closed in (X, τ), we have cl(A)$-$U $\in \mathcal{I}$. Now (cl(A) $\cap$ Y)$-$(U$\cap$Y) = (cl(A)$-$U) $\cap$ Y $\in \mathcal{I}$, whenever A $\subseteq$ U $\cap$ Y and U is λ-open. Hence A is $\mathcal{I}_{\Lambda_g}$-closed relative to the subspace Y.

Theorem 2.13. *Let A be an $\mathcal{I}_{\Lambda_g}$-closed set and F be a closed set in (X, τ), then A $\cap$ F is an $\mathcal{I}_{\Lambda_g}$-closed set in (X, τ).*

Proof. Let A ∩ F ⊆ U and U is λ-open. Then A ⊆ U ∪ (X−F). Since A is $\mathcal{I}_{\Lambda_g}$-closed, we have cl(A)−(U ∪ (X−F)) ∈ $\mathcal{I}$. Now, cl(A ∩ F) ⊆ cl(A) ∩ F = (cl(A) ∩ F)−(X−F). Therefore, cl(A ∩ F)−U ⊆ (cl(A) ∩ F)−(U ∩ (X−F)) ⊆ cl(A)−(U ∪ (X−F)) ∈ $\mathcal{I}$. Hence A ∩ F is $\mathcal{I}_{\Lambda_g}$-closed in (X, τ).

Definition 2.14. *Let (X, τ) be a topological space and $\mathcal{I}$ be an ideal on X. A subset A ⊆ X is said to be Λ_g-open with respect to an ideal (briefly $\mathcal{I}_{\Lambda_g}$-open) if and only if X−A is $\mathcal{I}_{\Lambda_g}$-closed.*

Theorem 2.15. *A set A is $\mathcal{I}_{\Lambda_g}$-open in (X, τ) if and only if F− U ⊆ int(A), for some U ∈ $\mathcal{I}$, whenever F ⊆ A and F is λ-closed.*

Proof. Suppose A is $\mathcal{I}_{\Lambda_g}$-open. Suppose F ⊆ A and F is λ-closed. We have X−A ⊆ X−F. By assumption, cl(X−A) ⊆ (X−F) ∪ U, for some U ∈ $\mathcal{I}$. This implies X−((X−F) ∪ U) ⊆ X− (cl(X−A)) and hence F−U ⊆ int(A).

Conversely, assume that F ⊆ A and F is λ-closed. Then F−U ⊆ int(A), for some U ∈ $\mathcal{I}$. Consider an λ-open set G such that X−A ⊆ G. Then X−G ⊆ A. By assumption, (X−G)−U ⊆ int(A) = X−cl(X−A). This gives that X−(G ∪ U) ⊆ X−cl(X−A). Then, cl(X−A) ⊆ G ∪ U, for some U ∈ $\mathcal{I}$. This shows that cl(X−A)−G ∈ $\mathcal{I}$. Hence X−A is $\mathcal{I}_{\Lambda_g}$-closed.

Recall that the sets A and B are said to be separated if cl(A) ∩ B = ϕ and A ∩ cl(B) = ϕ.

Theorem 2.16. *If A and B are separated $\mathcal{I}_{\Lambda_g}$-open sets in (X, τ), then A ∪ B is $\mathcal{I}_{\Lambda_g}$-open.*

Proof. Suppose A and B are separated $\mathcal{I}_{\Lambda_g}$-open sets in (X, τ) and F be a λ-closed subset of A ∪ B. Then F ∩ cl(A) ⊆ $(A \cup B)$ ∩ cl(A) = (A ∩ cl(A)) ∪ (B ∩ cl(A)) = $A \cup \phi = A$ and F ∩ cl(B) ⊆ (A ∪ B) ∩ cl(B) = (A ∩ cl(B)) ∪ (B ∩ cl(B)) = $\phi \cup B = B$. By assumption and by Theorem 2.15, (F ∩ cl(A))−U_1 ⊆ int(A) and (F ∩ cl(B))−U_2 ⊆ int(B), for some U_1, U_2 ∈ $\mathcal{I}$. It means that ((F ∩ cl(A))−int(A)) ∈ $\mathcal{I}$ and ((F ∩ cl(B))−int(B)) ∈ $\mathcal{I}$. Then ((F ∩ cl(A))−int(A)) ∪ ((F ∩ cl(B))−int(B)) ∈ $\mathcal{I}$. Hence (F ∩ (cl(A) ∪ cl(B))−(int(A) ∪ int(B))) ∈ $\mathcal{I}$. But F = F ∩ (A ∪ B) ⊆ F ∩ cl(A ∪ B), and we have F−int(A ∪ B) ⊆ (F ∩ cl(A ∪ B))−int(A ∪ B) ⊆ (F ∩ cl(A ∪ B))−(int(A) ∪ int(B)) ∈ $\mathcal{I}$. Hence, F−U ⊆ int(A ∪ B), for some U ∈ $\mathcal{I}$. This proves that A ∪ B is $\mathcal{I}_{\Lambda_g}$-open.

Corollary 2.17. *Let A and B be $\mathcal{I}_{\Lambda_g}$-closed sets and suppose X−A and X−B are separated in (X, τ). Then A ∩ B is $\mathcal{I}_{\Lambda_g}$-closed.*

Corollary 2.18. *If A and B are $\mathcal{I}_{\Lambda_g}$-open sets in (X, τ), then A ∩ B is $\mathcal{I}_{\Lambda_g}$-open.*

Proof. If A and B are $\mathcal{I}_{\Lambda_g}$-open, then X−A and X−B are $\mathcal{I}_{\Lambda_g}$-closed. By Theorem 2.5, X−(A ∩ B) is $\mathcal{I}_{\Lambda_g}$-closed, which implies A ∩ B is $\mathcal{I}_{\Lambda_g}$-open.

Theorem 2.19. *If int(A) ⊆ B ⊆ A and A is $\mathcal{I}_{\Lambda_g}$-open in (X, τ), then B is $\mathcal{I}_{\Lambda_g}$-open in X.*

Proof. Suppose int(A) ⊆ B ⊆ A and A is $\mathcal{I}_{\Lambda_g}$-open. Then X−A ⊆ X−B ⊆ cl(X−A) and X−A is $\mathcal{I}_{\Lambda_g}$-closed. By Theorem 2.11, X−B is $\mathcal{I}_{\Lambda_g}$-closed and hence B is $\mathcal{I}_{\Lambda_g}$-open.

Theorem 2.20. *Let (X, τ) be a topological space. Then a set A is $\mathcal{I}_{\Lambda_g}$-closed in X if and only if cl(A)−A is $\mathcal{I}_{\Lambda_g}$-open in X.*

Proof. Necessity: Suppose F ⊆ cl(A)−A and F be λ-closed. Then by Theorem 2.4, F ∈ $\mathcal{I}$. This implies that F−U = ϕ, for some U ∈ $\mathcal{I}$. Clearly, F−U ⊆ int(cl(A)−A). By Theorem 2.15, cl(A)−A is $\mathcal{I}_{\Lambda_g}$-open.

Sufficiency: Suppose A ⊆ G and G is λ-open in (X, τ). Then cl(A) ∩ (X−G) ⊆ cl(A) ∩ (X−A) = cl(A)−A. By hypothesis and by Theorem 2.15, (cl(A) ∩ (X−G))−U ⊆ int(cl(A)−A) = ϕ, for some U ∈ $\mathcal{I}$. This implies that cl(A) ∩ (X−G) ⊆ U ∈ $\mathcal{I}$ and hence cl(A)−G ∈ $\mathcal{I}$. Thus, A is $\mathcal{I}_{\Lambda_g}$-closed.

Theorem 2.21. *Let f : (X, τ) → (Y, σ) be λ-irresolute and closed. If A ⊆ X is $\mathcal{I}_{\Lambda_g}$-closed in X, then f(A) is f($\mathcal{I}$)$_{\Lambda_g}$-closed in (Y, σ), where f($\mathcal{I}$) = {f(U) : U ∈ $\mathcal{I}$}.*

Proof. Suppose A ⊆ X and A is $\mathcal{I}_{\Lambda_g}$-closed. Suppose f(A) ⊆ G and G is λ-open in Y. Then A ⊆ f^{-1}(G). By definition, cl(A)−f^{-1}(G) ∈ $\mathcal{I}$ and hence f(cl(A))−G ∈ f($\mathcal{I}$). Since f is closed, cl(f(A)) ⊆ cl(f(cl(A))) = f(cl(A)). Then cl(f(A))−G ⊆ f(cl(A))−G ∈ f($\mathcal{I}$) and hence f(A) is f($\mathcal{I}$)$_{\Lambda_g}$-closed in Y.

3 Conclusion

Topology is an area of Mathematics concerned with the properties of space that are preserved under continuous deformations including stretching and bending, but not tearing. By the middle of the 20th century, topology had become a major branch of Mathematics.

Topology as a branch of Mathematics can be formally defined as the study of qualitative properties of certain objects that are invariant under a certain kind of transformation especially those properties that are invariant under a certain kind of equivalence and it is the study of those properties of geometric configurations which remain invariant when these configurations are subjected to one-to-one bicontinuous transformations or homeomorphisms. Topology operates with more general concepts than analysis. Differential properties of a given transformation are nonessential for topology but bicontinuity is essential. As a consequence, topology is often suitable for the solution of problems to which analysis cannot give the answer.

Though the concept of topology has been identified as a difficult territory in Mathematics, we have taken it up as a challenge and cherishingly worked out this research study. It can also further up the understanding of basic structure of classical mathematics and offers new methods and results in obtaining significant results of classical mathematics. Moreover it also has applications in some important fields of Science and Technology.

In this paper, the notions of $\mathcal{I}_{\Lambda_g}$-closed sets and $\mathcal{I}_{\Lambda_g}$-open sets are introduced. Furthermore the relations with other notions connected with the notions of $\mathcal{I}_{\Lambda_g}$-closed sets and $\mathcal{I}_{\Lambda_g}$-open are investigated.

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