http://www.newtheory.org ISSN: 2149-1402

Journal of New Theory

Received: 10.02.2015 *Year*: **2015,** *Number*: **5**, *Pages*: **80-91**
Accepted: 06.07.2015 *Original Article*[**]

Q-INTUITIONISTIC FUZZY SOFT SETS

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Abstract - In this paper, we first present the concept of Q- intuitionistic fuzzy soft sets which combine Q-intuitionistic fuzzy sets and soft sets. Basic properties are introduced along with illustrative examples. We also define its basic operations, namely equality, subset, complement, union, intersection, AND and OR, and study some related properties with supporting proofs. This concept is a generalization of Q-fuzzy soft sets.

***Keywords* -** *Intuitionistic fuzzy sets, Q-intuitionistic fuzzy sets, Q-intuitionistic fuzzy soft sets.*

1. Introduction

The concept of fuzzy sets was introduced by Zadeh [10] whose basic component is only a degree of membership. Atanassov [7] generalized this idea to intuitionistic fuzzy set (IFS in short) using a degree of membership and a degree of non-membership, under the constraint that the sum of the two degrees does not exceed one. The conception of IFS can be viewed as an appropriate /alternative approach in case where available information is not sufficient to define the impreciseness by the conventional fuzzy set. The idea of "intuitionistic Q-fuzzy set" was first published by Atanassov [8], as a generalization of the notion of fuzzy set.

In many fields, such as economics, engineering, environment, involve data that contain uncertainties. To understand and manipulate the uncertainties, there are many approaches such as probability theory, fuzzy set theory [10], intuitionistic fuzzy sets [7], rough set theory [20], etc. Each of these theories has its own difficulties as pointed out in [1]. To address these difficulties, Molodtsov[1] introduced the concept of soft set as a new mathematical tool for dealing with uncertainties that is free from difficulties. The main advantage of soft set theory in data analysis is that it does not need any grade of membership as in the fuzzy set theory. In soft set theory there is no limited condition to the description of the objects; so researchers can choose the form of parameter they need which greatly simplifies the decision making process and make the process more efficient in the

***Edited by* Oktay Muhtaroğlu *(Area Editor) and* Naim Çağman *(Editor-in-Chief).*
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absence of partial information. After Molodtsov's work, Maji et al.[14] introduced the concept of fuzzy soft set, a more generalized concept, which is a combination of fuzzy set and soft set and studied its properties and also discussed their properties. Also, Maji et al.[15] devoted the concept of intuitionistic fuzzy soft sets by combining intuitionistic fuzzy sets with soft sets.By using this definition of intuitionistic fuzzy soft sets many interesting applications of soft set theory have been expanded by some researchers [9, 11, 12 , 13, 16, 17,18]. Recently Adam et al. [5] defined a new concept called Q-fuzzy soft set which combine Q-fuzzy set and soft set. The same authors introduced the concept of multi Q-fuzzy set and a multi Q-fuzzyparameterized soft set [2], studied their operations and gave an application in decision making. Based on [5] and [8], we presented the concept of Q-intuitionistic fuzzy soft sets as ageneralization of Q-fuzzy soft sets.

The rest structure of this paper is as follows: part 2 presents some definitions which will be used in the sequel. Part 3 discusses the concept of Q-intuitionistic fuzzy soft set. Part 4 introduced the union, intersection, AND and OR operations on a Q-intuitionistic fuzzy soft set. Part 5 gives the conclusion.

2. Preliminaries

In this section we present the basic definitions of soft set theory, Q-fuzzy set, multi Q-fuzzy set, Q-fuzzy soft set, intuitionistic fuzzy set,Q- intuitionistic fuzzy set, intuitionistic fuzzy soft set and multi-Q intuitionistic fuzzy set required in this paper.

2.1. Soft Sets

Definition 2.1[1] A pair (F, E) is called a soft set over U, if and only if F is a mapping of E into the set of all subsets of the set U. In other words, the soft set is parameterized family of subsets of the set U.

As an illustration, let us consider the following example.

Example 2.2. Suppose that U is the set of houses under consideration, say $U = \{h_1, h_2,...,h_5\}$. Let E be the set of some attributes of such houses, say $E = \{e_1, e_2, . . ., e_8\}$, where e_1, e_2, . . ., e_8 stand for the attributes "beautiful", "costly", "in the green surroundings", "moderate", respectively.

In this case, to define a soft set means to point out expensive houses, beautiful houses, and so on. For example, the soft set (K, A) that describes the "attractiveness of the houses" in the opinion of a buyer, say Thomas, may be defined like this:

$$A=\{e_1,e_2,e_3,e_4,e_5\};$$

$$K(e_1) = \{h_2, h_3, h_5\}, K(e_2) = \{h_2, h_4\}, K(e_3) = \{h_1\}, K(e_4) = U, K(e_5) = \{h_3, h_5\}.$$

2.2 Q-fuzzy Sets

Definition 2.3 Let X be a non-empty set and Q be a non-empty set. A Q-fuzzy subset A of X is a function

$$A : XxQ \to [\,0, 1\,].$$

Definition 2.4: The union of two Q-fuzzy subsets A and B of a set X is defined by

$$(A\cup B)(x,q) = \max \{\, A(x, q), B(x, q) \,\}$$

for all x in X and q in Q.

Definition 2.5: The intersection of two Q-fuzzy subsets A and B of a set X is defined by

$$(A \cap B)(x, q) = \min \{\, A(x, q), B(x, q) \,\}$$

for all x in X and q in Q.

2.3 Multi Q-fuzzy Sets

Definition 2.5 [2] Let I be a unit interval [0, 1], k be a positive integer. U be a universal set and Q be a non-empty set. A multi Q-fuzzy set $\tilde{A}_Q$ in U and q is a set of ordered sequences

$$\tilde{A}_Q=\{(u,q), \mu_i(u, q): u\in U, q \in Q\}$$

where μ_i: U ×Q → I^k.The function μ_1(u, q),μ_2(u, q),…,μ_k(u, q) is called membership function of multi Q-fuzzy set $\tilde{A}_Q$; and μ_1(u, q)+μ_2(u, q)+…+μ_k(u, q) ≤ 1, k is called the dimension of $\tilde{A}_Q$. The set of all multi Q-fuzzy sets of dimension k in U and Q is denoted by M^kQF(U).

2.4 Q-fuzzy Soft Sets

Definition 2.6 [5] Let U be a universal set, E be a set of parameters, and Q be a non-empty set. LetM^kQF(U) denote the power set of all multi Q-fuzzy subset of U with dimension k= 1. Let A ⊆ E. A pair (F_Q,A) is called a Q-fuzzy soft set (in short QF-soft set) over U where F_Q is a mapping given by

$$F_Q: A \to M^k QF(U) \text{ such that } F_Q(x) = \emptyset \text{ if } x \notin A.$$

Here a Q-fuzzy soft set can be represented by the set of ordered pairs

$$(F_Q,A)= \{(x,F_Q(x)): x \in U,\ F_Q(x) \in M^k QF(U)\,\}$$

Note that the set of all Q-fuzzy soft set over U will be denoted by QFS(U).

Definition 2.7 [5] Let (F_Q, A) and (G_Q, B) ∈QFS(U). The union of two Q-fuzzy soft sets (F_Q, A) and (G_Q, B), is the Q-fuzzy soft set (H_Q, C), written as (F_Q, A) ∪(G_Q, B) = (H_Q, C), where C = A∪ B for all e ∈ C and

$$H_Q(e)=\begin{cases} F_Q(e) if \ e \in A - B \ ; \\ G_Q(e) if \ e \in B - A; \\ F_Q(e) \cup G_Q(e) \ if \ e \in A \cap B. \end{cases}$$

Definition 2.8 [5] Let (F_Q, A) and (G_Q, B) ∈QFS(U). The intersection of two Q-fuzzy soft sets (F_Q, A) and (G_Q, B), is the Q-fuzzy soft set (H_Q, C), written as (F_Q, A) ∩(G_Q, B) = (H_Q, C), where C = A∩ B for all e ∈ C,

$$(H_Q, C)= \{e, \min \{\mu_{i_{F_Q}}(x, q), \mu_{i_{G_Q}}(y, q)\} : u \in U, q \in Q\} \text{and } i=1, 2,\dots,k.$$

2.5. Intuitionistic Fuzzy Sets

Definition 2.9 [7] Let U be an universe of discourse then the intuitionistic fuzzy set A is an object having the form A = {< x, μ_A(x), ω_A(x)>,x ∈ U},where the functions μ_A(x),

$$\omega_A(x) : U\rightarrow[0,1]$$

define respectively the degree of membership, and the degree of non-membership of the element x ∈ X to the set A with the condition.

$$0 \leq\mu_A(x)+ \omega_A(x)\leq 1.$$

For two IFS,

$$A_{IFS}= \{<x, \mu_A(x), \omega_A(x)> \mid x \in X \}$$

And

$$B_{IFS}= \{<x, \mu_B(x), \omega_B(x)> \mid x \in X \}$$

Then,

1. $A_{IFS} \subseteq B_{IFS}$ if and only if

$$\mu_A(x) \leq \mu_B(x), \omega_A(x) \geq \omega_B(x)$$

2. $A_{IFS} = B_{IFS}$ if and only if ,

$$\mu_A(x) =\mu_B(x) ,\omega_A(x) =\omega_B(x) \text{ for any } x \in X.$$

3. The complement of A_{IFS} is denoted by A^o_{IFS} and is defined by

$$A^{o}_{IFS} = \{<x, \omega_A(x), \mu_A(x) | x \in X \}$$

4. $A \cap B = \{<x, \min\{\mu_A(x), \mu_B(x)\}, \max\{\omega_A(x), \omega_B(x)\}>: x \in X \}$

5. $A \cup B = \{<x, \max\{\mu_A(x), \mu_B(x)\}, \min\{\omega_A(x), \omega_B(x)\}>: x \in X \}$

6. $0_{IFS} = (0, 1)$ and $1_{IFS} = (1, 0)$

As an illustration, let us consider the following example.

Example 2.10. Assume that the universe of discourse $U=\{x_1, x_2, x_3, x_4\}$. It may be further assumed that the values of x_1, x_2, x_3 and x_4 are in [0, 1] Then, A is an intuitionistic fuzzy set (IFS) of U, such that,

$$A = \{< x_1, 0.4, 0.6>, < x_2, 0.3, 0.7>, < x_3, 0.2, 0.8>, < x_4, 0.2, 0.8>\}$$

2.6. Q-intuitionistic Fuzzy Sets

Definition 2.11 [8] A Q-intuitionistic fuzzy subset A in X is defined as an object of the form

$$A = \{< (x, q), \mu_A(x, q), \nu_A(x, q) > / \ x \in X \text{ and } q \text{ in } Q\}$$

where $\mu_A: X \times Q \to [0, 1]$ and $\nu_A: X \times Q \to [0, 1]$ define the degree of membership and the degree of non-membership of the element x in X and q in A respectively and for every x in X and q in Q satisfying

$$0 \leq \mu_A(x, q) + \nu_A(x, q) \leq 1.$$

Definition 2.12 [8] If A is a Q-intuitionistic fuzzy subset A of X, then the complement of A, denoted A^C is the Q-intuitionistic fuzzy set of X, given by

$$A^C(x, q) = \{< (x, q), \nu_A(x, q), \mu_A(x, q) > / \ x \in X \text{ and } q \text{ in } Q\}.$$

Definition 2.13 [8] Let A and B be Q-intuitionistic fuzzy subsets of sets G and H respectively. The product of A and denoted by $A \times B$ is defined as

$$A \times B = \{< ((x, y), q), \mu_{A \times B}((x, y), q), \nu_{A \times B}((x, y), q) > / \ x \text{ in } G \text{ and } y \text{ in } H \text{ and } q \text{ in } Q\},$$

where

$$\mu_{A \times B}((x, y), q) = \min\{\mu_A(x, q), \mu_B(y, q)\} \text{ and } \nu_{A \times B}((x, y), q) = \max\{\nu_A(x, q), \nu_B(y, q)\}.$$

Definition 2.14 [8] Let A be a Q-intuitionistic fuzzy subset in a set S, the strongest Q-intuitionistic fuzzy relation on S, that is a Q-intuitionistic fuzzy relation on A is V given by

$$\mu_V((x, y), q) = \min\{\mu_A(x, q), \mu_B(y, q)\} \text{ and } \nu_V((x, y), q) = \max\{\nu_A(x, q), \nu_B(y, q)\},$$

for all x and y in S and q in Q.

2.7 Intuitionistic Fuzzy Soft Sets

Definition 2.15 [15] Let U be an initial universe set and $A \subset E$ be a set of parameters. Let IFS(U) denotes the set of all intuitionistic fuzzy subsets of U. The collection (F, A) is termed to be the intuitionistic fuzzy soft set over U, where F is a mapping given by

$$\boldsymbol{F}: \boldsymbol{A} \rightarrow \boldsymbol{IFS}(\boldsymbol{U}).$$

Example 2.16 Let U be the set of houses under consideration and E is the set of parameters. Each parameter is a word or sentence involving intuitionistic fuzzy words. Consider E ={beautiful, wooden, costly, very costly, moderate, green surroundings, in good repair, in bad repair, cheap, expensive}. In this case, to define an intuitionistic fuzzy soft set means to point out beautiful houses, wooden houses, houses in the green surroundings and so on. Suppose that, there are five houses in the universe U given by $U = \{h_1, h_2, \ldots, h_5\}$ and the set of parameters $A = \{e_1, e_2, e_3, e_4\}$,where e_1 stands for the parameter `beautiful', $e_2$ stands for the parameter `wooden', e_3 stands for the parameter `costly' and the parameter $e_4$ stands for `moderate'. Then the intuitionistic fuzzy set (F, A) is defined as follows:

$$(F,A) = \left\{ \begin{array}{l} \left(e_1 \left\{\frac{h_1}{(0.1,0.6)}, \frac{h_2}{(0.2,0.7)}, \frac{h_3}{(0.6,0.2)}, \frac{h_4}{(0.7,0.3)}, \frac{h_5}{(0.2,0.3)}\right\}\right) \\ \left(e_2 \left\{\frac{h_1}{(0.3,0.5)}, \frac{h_2}{(0.2,0.4)}, \frac{h_3}{(0.1,0.2)}, \frac{h_4}{(0.1,0.3)}, \frac{h_5}{(0.3,0.6)}\right\}\right) \\ \left(e_3 \left\{\frac{h_1}{(0.4,0.3)}, \frac{h_2}{(0.6,0.3)}, \frac{h_3}{(0.2,0.5)}, \frac{h_4}{(0.2,0.6)}, \frac{h_5}{(0.7,0.3)}\right\}\right) \\ \left(e_4 \left\{\frac{h_1}{(0.1,0.6)}, \frac{h_2}{(0.3, 0.6)}, \frac{h_3}{(0.6,0.4)}, \frac{h_4}{(0.4,0.2)}, \frac{h_5}{(0.5,0.3)}\right\}\right) \end{array} \right\}$$

2.8. Multi Q-intuitionistic Fuzzy Sets

Definition 2.17 [19] Let I be a unit interval [0, 1], k be a positive integer. U be a universal set and Q be a non-empty set. A multi Q-intuitionistic fuzzy set $\tilde{A}_Q$ in U and q is a set of ordered sequences

$$\tilde{A}_Q=\{(u,q), \mu_i(u, q),\nu_i(u, q): u\in U, q \in Q\}$$

where μ_i:U ×Q → I^k and ν_i:U ×Q → $I^k$and . The functions μ_1(u, q),μ_2(u, q),…,μ_k(u, q) is called membership function of multi Q-fuzzy set $\tilde{A}_Q$ and the functions ν_1(u, q),ν_2(u, q),…,ν_k(u, q) is called non-membership function of multi Q-intuitionistic fuzzy set $\tilde{A}_Q$; and $0 \leq \mu_i$(x, q) +ν_i(x, q)≤ 1, for i=1,2,….k. k is called the dimension of $\tilde{A}_Q$. The set of all multi Q- intuitionistic fuzzy sets of dimension k in U and Q is denoted by M^kQIF(U).

Example 2.18 [19] Let U= $\{u_1, u_2, u_3, u_4\}$ be a universal set, Q ={p, q } be a non- empty set , and k = 3 be a positive integer. If $\tilde{A}_Q$ is a function from U ×Q to I^3 then the set $\tilde{A}_Q$={((u_1, q),(0.2, 0.3),(0.4, 0.5),(0.4, 0.6)), ((u_1, p),(0.4, 0.5), (0.1, 0.2),(0.2, 0.4)), ((u_2,q),(0.3, 0.5),(0.2, 0.5), (0.3, 0.4))} is a multi Q- intuitionistic fuzzy sets in U and Q.

Remark 2.19: If ν_i(u, q) = 0, then the multi Q intuitionistic fuzzy set $\tilde{A}_Q$={(u,q), μ_i(u, q),ν_i(u, q): u ∈ U, q ∈ Q} degenerate to the multi Q fuzzy set $\tilde{A}_Q$={(u,q), μ_i(u, q) : u∈ U, q ∈ Q}

3. Q-Intuitionistic Fuzzy Soft Sets

In this section we introduce the concept Q- intuitionistic fuzzy soft set and define some properties of a Q- intuitionistic fuzzy soft set namely, null, absolute, subset, equality and complement, and give an example of Q- intuitionistic fuzzy soft set.

Definition 3.1 Let U be a universal set, E be a set of parameters, and Q be a non-empty set. LetM^kQIF(U) denote the power set of all multi Q- intuitionistic fuzzy subset of U with dimension k= 1. Let A ⊆ E. A pair (F_Q,A) is called a Q- intuitionistic fuzzy soft set (in short QIF-soft set) over U where F_Q is a mapping given by

$$F_Q: \text{A} \to M^k\text{QIF(U) such that } F_Q(x) = \emptyset \text{ if x} \notin \text{A.}$$

Here a Q- intuitionistic fuzzy soft set can be represented by the set of ordered pairs

$$(F_Q,\text{A})= \{(\text{x},F_Q(x)): \text{x} \in \text{U}, \ F_Q(x) \in M^k\text{QIF(U)}\}$$

Note that the set of all Q- intuitionistic fuzzy soft set over U will be denoted by QIFS(U).

Example 3.2 Let U= $\{u_1, u_2, u_3, u_4, u_5\}$ be a universal set, Q ={p, q, r} be a non- empty set , and E= $\{e_1, e_2, e_3, e_4, e_5\}$ be a set of parameters. If

A=$\{e_1, e_2, e_3\}$ ⊂ E,
$F_Q(e_1)$={((u_1,p),(0.2, 0.3)), ((u_1,q),(0.4, 0.5)), ((u_1,r),(0.3, 0.5))}
$F_Q(e_2)$={((u_1,p),(0.1, 0.4)), ((u_1,q),(0.2, 0.3)), ((u_1,r),(0.3, 0.5)), ((u_4,p),(0.2, 0.6)), ((u_4,q),(0.3, 0.4)), ((u_4,r),(0.2, 0.3))}
$F_Q(e_3)$={((u_1,p),(0.6, 0.3)), ((u_1,q),(0.4, 0.3)), ((u_1,r),(0.3, 0.2))},

then

(F_Q,A)={(e_1, {((u_1,p),(0.2, 0.3)), ((u_1,q),(0.4, 0.5)), ((u_1,r),(0.3, 0.5))}), (e_2, {((u_1,p),(0.1, 0.4)), ((u_1,q),(0.2, 0.3)), ((u_1,r),(0.3, 0.5)), ((u_4,p),(0.2, 0.6)), ((u_4,q),(0.3, 0.4)), ((u_4,r),(0.2, 0.3))}), (e_3, {((u_1,p),(0.6, 0.3)), ((u_1,q),(0.4, 0.3)), ((u_1,r),(0.3, 0.2))})}

is Q- intuitionistic fuzzy soft set.

Definition 3.3 Let (F_Q, A) ∈ QIFS(U). If F_Q(x) = Φ for all x ∈ E then (F_Q, A) is called a null QIF-S-set denoted by (Φ, A).

Example 3.4 (Φ,A)={(e_1, {((u_1,p),(0, 1)), ((u_1,q),(0, 1)), ((u_1,r),(0, 1))}), (e_2, {((u_1,p),(0, 1)), ((u_1,q),(0, 1)), ((u_1,r),(0, 1)), ((u_4,p),(0, 1)), ((u_4,q),(0, 1)), ((u_4,r),(0, 1))}), (e_3, {((u_1,p),(0, 1)), ((u_1,q),(0, 1)), ((u_1,r),(0, 1))})}.

Definition3.5 Let (F_Q, A) ∈ QIFS(U). If F_Q(x) = U for all x ∈ E then (F_Q, A) is called an absolute QIF-soft set denoted by (U, A).

Example 3.6 (U, A)={(e_1, {((u_1,p),(1, 0)), ((u_1,q),(1, 0)), ((u_1,r),(1, 0))}), (e_2, {((u_1,p),(1, 0)), ((u_1,q),(1, 0)), ((u_1,r),(1, 0)), ((u_4,p),(1, 0)), ((u_4,q),(0, 1)), ((u_4,r),(1, 0))}), (e_3, {((u_1,p),(1, 0)), ((u_1,q),(1, 0)), ((u_1,r),(1, 0))})}.

Definition 3.7 Let (F_Q, A), (H_Q, B) ∈ QIFS(U). then we say that (F_Q, A) is a QIF-soft subset of (H_Q,B), denoted by (F_Q, A) ⊆ (H_Q, B), if A ⊆ B and F_Q(x) ⊆ H_Q(x) for all x ∈ U.

Proposition 3.8 Let (F_Q, A), (H_Q, B) ∈ QIFS(U).Then
1. (F_Q, A) ⊆ (U, E)
2. (Φ, A) ⊆ (F_Q, A)
3. If (F_Q, A) ⊆ (H_Q, B) and (H_Q, B) ⊆ (G_Q, C), then (F_Q, A) ⊆ (G_Q, C)

Proof. The proof can be easily obtained from Definition 3.7

Proposition 3.9 Let (F_Q, A), (H_Q, B) ∈ QIFS(U). If (F_Q, A) = (H_Q, B) and (H_Q, B) =(G_Q, C) Then (F_Q, A) = (G_Q, C).

Proof. The proof can be easily obtained from Definition 3.7

Definition 3.10 Let (F_Q, A) ∈ QIFS(U). Then, the complement of QIF-soft set denoted by $(F_Q, \text{A})^{\text{C}}$ is defined by $(F_Q, \text{A})^{\text{C}}$ = (F_Q^c, ¬A) where

$$F_Q^c : \neg\text{A} \rightarrow \text{QIF(U)}$$

is the mapping given by $F_Q^c(e)$ = Q - intuitionistic fuzzy complement for every e ∈ A.

Example 3.11 Consider example 3.2

(F_Q,A)={(e_1, {((u_1,p),(0.2, 0.3)), ((u_1,q),(0.4, 0.5)), ((u_1,r),(0.3, 0.5))}), (e_2, {((u_1,p),(0.1, 0.4)), ((u_1,q),(0.2, 0.3)), ((u_1,r),(0.3, 0.5)), ((u_4,p),(0.2, 0.6)), ((u_4,q),(0.3, 0.4)), ((u_4,r),(0.2, 0.3))}), (e_3, {((u_1,p),(0.6, 0.3)), ((u_1,q),(0.4, 0.3)), ((u_1,r),(0.3, 0.2))})}

Then

$(F_Q, \text{A})^{\text{C}}$={($e_1$, {(($u_1$,p),(0.3, 0.2)), (($u_1$,q),(0.5, 0.4)), (($u_1$,r),(0.5, 0.3))}), ($e_2$, {(($u_1$,p),(0.4, 0.1)), (($u_1$,q),(0.3, 0.2)), (($u_1$,r),(0.5, 0.3)), (($u_4$,p),(0.6, 0.2)), (($u_4$,q),(0.4, 0.3)), (($u_4$,r),(0.3, 0.2))}), ($e_3$, {(($u_1$,p),(0.3, 0.6)), (($u_1$,q),(0.3, 0.4)), (($u_1$,r),(0.2, 0.3))})}

Proposition 3.12 Let (F_Q, A) ∈ QIFS(U).Then
1. $\left((F_Q, \text{A})^c\right)^c$ = (F_Q, A)
2. $(\Phi, \text{A})^c$ = (U, E)
3. $(U, \text{E})^c$ = (Φ, E)

Proof. The proof can be easily obtained from Definition 3.7

4. Union and Intersection of Q-intuitionistic Fuzzy Soft Set.

In this section we introduce the union, intersection, AND and OR operations on a Q-intuitionistic fuzzy soft set.

Definition 4.1 Let (F_Q, A) and (G_Q, B) ∈ QIFS(U). The union of two Q- intuitionistic fuzzy soft sets (F_Q, A) and (G_Q, B), is the Q- intuitionistic fuzzy soft set (H_Q, C), written as (F_Q, A) ∪(G_Q, B) = (H_Q, C), where C = A∪ B for all e ∈ C and

$$H_Q(e)=\begin{cases} F_Q(e) if \ \ e \in A - B\ ; \\ G_Q(e) if \ \ e \in B - A; \\ F_Q(e) \cup G_Q(e)\ \ if \ \ e \in A \cap B. \end{cases}$$

Example 4.2 : Let U= $\{u_1, u_2, u_3, u_4, u_5\}$ be a universal set, Q ={p, q,r} be a non- empty set , and E= $\{e_1, e_2, e_3, e_4, e_5\}$ be a set of parameters. If A=$\{e_1, e_2, e_3\}$ ⊂ E, and B=$\{e_1, e_2, e_4\}$ ⊂ E

(F_Q,A)={(e_1, {((u_1,p),(0.2, 0.3)), ((u_1,q),(0.4, 0.5)), ((u_1,r),(0.3, 0.5))}), (e_2, {((u_1,p),(0.1, 0.4)), ((u_1,q),(0.2, 0.3)), ((u_1,r),(0.3, 0.5)), ((u_4,p),(0.2, 0.6)), ((u_4,q),(0.3, 0.4)), ((u_4,r),(0.2, 0.3))}), (e_3, {((u_1,p),(0.6, 0.3)), ((u_1,q),(0.4, 0.3)), ((u_1,r),(0.3, 0.2))})}

And

(G_Q,B)={(e_1, {((u_1,p),(0.4, 0.5)), ((u_1,q),(0.3, 0.2)), ((u_1,r),(0.2, 0.4))}), (e_2, {((u_1,p),(0.3, 0.5)), ((u_1,q),(0.3, 0.6)), ((u_1,r),(0.4, 0.5)), ((u_4,p),(0.3, 0.6)), ((u_4,q),(0.2, 0.3)), ((u_4,r),(0.3, 0.5))}), (e_4, {((u_1,p),(0.6, 0.3)), ((u_1,q),(0.4, 0.3)), ((u_1,r),(0.3, 0.2))})}

Then

(H_Q, C)= {(e_1, {((u_1,p),(0.4, 0.3)), ((u_1,q),(0.4, 0.2)), ((u_1,r),(0.3, 0.4))}), (e_2, {((u_1,p),(0.3, 0.4)), ((u_1,q),(0.3, 0.3)), ((u_1,r),(0.4, 0.5)), ((u_4,p),(0.3, 0.6)), ((u_4,q),(0.3, 0.3)), ((u_4,r),(0.3, 0.3))}), (e_3, {((u_1,p),(0.6, 0.3)), ((u_1,q),(0.4, 0.3)), ((u_1,r),(0.3, 0.2))}), (e_4, {((u_1,p),(0.6, 0.3)), ((u_1,q),(0.4, 0.3)), ((u_1,r),(0.3, 0.2))})}

Definition 4.3 Let (F_Q, A) and (G_Q, B) ∈ QIFS(U). The intersection of two Q-intuitionistic fuzzy soft sets (F_Q, A) and (G_Q, B), is the Q- intuitionistic fuzzy soft set (H_Q, C), written as (F_Q, A) ∩(G_Q, B) = (H_Q, C), where C = A∩ B for all e ∈ C,

(H_Q, C)= {e, (min {$\mu_{i_{F_Q}}$(x, q), $\mu_{i_{G_Q}}$(y, q), max({$\nu_{i_{F_Q}}$(x, q), $\nu_{i_{G_Q}}$(y, q)))} : u∈ U, q ∈ Q} and i=1, 2,…,k.

Example 4.4 : Let U= $\{u_1, u_2, u_3, u_4, u_5\}$ be a universal set, Q ={p, q,r} be a non- empty set , and E= $\{e_1, e_2, e_3, e_4, e_5\}$ be a set of parameters. If A=$\{e_1, e_2, e_3\}$ ⊂ E, and B=$\{e_1, e_2, e_4\}$ ⊂ E

(F_Q,A)={(e_1, {((u_1,p),(0.2, 0.3)), ((u_1,q),(0.4, 0.5)), ((u_1,r),(0.3, 0.5))}), (e_2, {((u_1,p),(0.1, 0.4)), ((u_1,q),(0.2, 0.3)), ((u_1,r),(0.3, 0.5)), ((u_4,p),(0.2, 0.6)), ((u_4,q),(0.3, 0.4)), ((u_4,r),(0.2, 0.3))}), (e_3, {((u_1,p),(0.6, 0.3)), ((u_1,q),(0.4, 0.3)), ((u_1,r),(0.3, 0.2))})}

And

(G_Q,B)={(e_1, {((u_1,p),(0.4, 0.5)), ((u_1,q),(0.3, 0.2)), ((u_1,r),(0.2, 0.4))}), (e_2, {((u_1,p),(0.3, 0.5)), ((u_1,q),(0.3, 0.6)), ((u_1,r),(0.4, 0.5)), ((u_4,p),(0.3, 0.6)), ((u_4,q),(0.2, 0.3)), ((u_4,r),(0.3, 0.5))}), (e_4, {((u_1,p),(0.6, 0.3)), ((u_1,q),(0.4, 0.3)), ((u_1,r),(0.3, 0.2))})}

Then

(H_Q, C)= {(e_1, {((u_1,p),(0.2, 0.5)), ((u_1,q),(0.3, 0.5)), ((u_1,r),(0.2, 0.5))}), (e_2, {((u_1,p),(0.1, 0.5)), ((u_1,q),(0.2, 0.6)), ((u_1,r),(0.3, 0.5)), ((u_4,p),(0.2, 0.6)), ((u_4,q),(0.2, 0.4)), ((u_4,r),(0.2, 0.5))})}

Proposition 4.5 Let (F_Q, A), (G_Q, B) and (H_Q, C) ∈ QIFS(U).Then
1. (F_Q, A) ∪(Φ, A) = (F_Q, A)
2. (F_Q, A) ∪(U, A) = (U, A)
3. (F_Q, A) ∪(F_Q, A) = (F_Q, A)
4. (F_Q, A) ∪(G_Q, B) = (G_Q, B) ∪(F_Q, A)
5. (F_Q, A) ∪((G_Q, B) ∪ (H_Q, C)) = ((F_Q, A) ∪(G_Q, B)) ∪ (H_Q, C)

Proof. The proof can be easily obtained from Definition 4.1

Proposition 4.6 Let (F_Q, A), (G_Q, B) and (H_Q, C) ∈ QIFS(U).Then
1. (F_Q, A) ∩(Φ, A) = (Φ, A)
2. (F_Q, A) ∩(U, A) = (F_Q, A)
3. (F_Q, A) ∩(F_Q, A) = (F_Q, A)
4. (F_Q, A) ∩(G_Q, B) = (G_Q, B) ∩(F_Q, A)
5. (F_Q, A) ∩((G_Q, B) ∩(H_Q, C)) = ((F_Q, A) ∩(G_Q, B)) ∩ (H_Q, C)

Proof. The proof are straightforward.

Proposition 4.7 Let (F_Q, A), (G_Q, B) and (H_Q, C) ∈ QIFS(U).Then
1. $((F_Q,\text{A}) \cap (G_Q,\text{B}))^c = (F_Q,\text{A})^c \cup (G_Q,\text{B})^c$
2. $((F_Q,\text{A}) \cup (G_Q,\text{B}))^c = (F_Q,\text{A})^c \cap (G_Q,\text{B})^c$

The proof are straightforward by using the properties of a multi Q–intuitionistic fuzzy sets.

Proposition 4.8 Let (F_Q, A), (G_Q, B) and (H_Q, C) $\in$ QIFS(U) .Then

1. (F_Q, A) $\cap$((G_Q, B) $\cup$(H_Q, C)) = ((F_Q, A) $\cap$(G_Q, B)) $\cup$ ((F_Q, A) $\cap$(H_Q, C))
2. (F_Q, A) $\cup$((G_Q, B) $\cap$(H_Q, C)) = ((F_Q, A) $\cup$(G_Q, B)) $\cap$ ((F_Q, A) $\cup$(H_Q, C))

Definition 4.9 Let (F_Q, A) and (G_Q, B) $\in$ QIFS(U). Then (F_Q, A) AND (G_Q, B) is the Q-intuitionistic fuzzy soft set denoted by (F_Q, A) $\wedge$ (G_Q, B) and defined by

$$(F_Q, A) \wedge (G_Q, B) = (H_Q, A\times B)$$

where $H_Q(\alpha, \beta) = F_Q(\alpha) \cap G_Q(\beta)$ for all $\alpha \in$ A and $\beta \in$ B, is the operation of intersection of two Q-intuitionistic fuzzy sets.

Definition 4.10 Let (F_Q, A) and (G_Q, B) $\in$ QIFS(U). Then (F_Q, A) OR (G_Q, B) is the Q-intuitionistic fuzzy soft set denoted by (F_Q, A) $\vee$ (G_Q, B) and defined by

$$(F_Q, A) \vee (G_Q, B) = (H_Q, A\times B)$$

where $H_Q(\alpha, \beta) = F_Q(\alpha) \cup G_Q(\beta)$ for all $\alpha \in$ A and $\beta \in$ B, is the operation of union of two Q-intuitionistic fuzzy sets.

Conclusion

In this paper we have introduced the concept of Q-intuitionistic fuzzy soft sets and studied some related properties with supporting proofs. The equality, subset, complement, union, intersection, AND or OR operations have been defined on the Q-intuitionistic fuzzy soft sets. This new extension will provide a significant addition to existing theories for handling uncertainties, and lead to potential areas of further research and pertinent applications.

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