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SEMI-COMPACT SOFT MULTI SPACES

Mahmoud Raafat[1,*] <dr_mahmoudraafat2011@yahoo.com>
Sobhy El-Sheikh[1] <elsheikh33@hotmail.com>
Rajab Omar[1] <ben_omar1955@yahoo.com>

[1]*Department of Mathematics, Faculty of Education, Ain Shams University, Cairo, Egypt*

Abstract – In this paper, first we introduce the concepts of semi-open soft mset and semi-closed soft mset. Then, we discuss some relationships about those concepts. Finally, we introduce the notion of soft multi semi-compactness as a generalization to soft multi compactness and study their properties and theorems.

Keywords – *Soft multisets, Soft mset topology, Semi-open soft msets, Semi-closed soft msets, Semi-compact soft multi space.*

1 Introduction

The notion of a multiset is well established both in mathematics and computer science [1, 2, 4, 5, 12, 16, 19, 20]. In mathematics, a multiset is considered to be the generalization of a set. In classical set theory, a set is a well-defined collection of distinct objects. If repeated occurrences of any object is allowed in a set, then a mathematical structure, that is known as multiset (mset, for short), is obtained [3, 13, 16, 17, 18]. For the sake of convenience a mset is written as $\{k_1/x_1, k_2/x_2, ..., k_n/x_n\}$ in which the element x_i occurs k_i times. We observe that each multiplicity k_i is a positive integer. The number of occurrences of an object x in an mset A, which is finite in most of the studies that involve msets, is called its multiplicity or characteristic value, usually denoted by $m_A(x)$ or $C_A(x)$ or simply by A(x). One of the most natural and simplest examples is the mset of prime factors of a positive integer n. The number 504 has the factorization $504 = 2^3 3^2 7^1$ which gives the mset $M = \{3/x, 2/y, 1/z\}$ where $C_M(x) = 3$, $C_M(y) = 2$, $C_M(z) = 1$.

Classical set theory states that a given element can appear only once in a set, it assumes that all mathematical objects occur without repetition. Thus there is only one number four, one field of complex numbers, etc. So, the only possible relation

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[*]*Corresponding Author.*

between two mathematical objects is either they are equal or they are different. The situation in science and in ordinary life is not like this. In the physical world it is observed that there is enormous repetition. For instance, there are many hydrogen atoms, many water molecules, many strands of DNA, etc. Coins of the same denomination and year, electrons or grains of sand appear similar, despite being obviously separate. This leads to three possible relations between any two physical objects; they are different, they are the same but separate or they coincide and are identical. For the sake of definiteness we say that two physical objects are the same or equal, if they are indistinguishable, but possibly separate, and identical if they physically coincide.

The concept of soft msets which is combining soft sets and msets can be used to solve some real life problems. Also, this concept can be used in many areas, such as data storage, computer science, information science, medicine, engineering, etc. The concept of soft msets was introduced in [7]. Also, [6] soft multi connectedness was given. D. Tokat [14] was introduced compact soft multi spaces.

In this paper, we introduce the concept of semi-open and semi-closed sets in soft mset theory. In classical set theory, semi-open and semi-closed sets were first studied by N. Levine [15]. Since its introduction, semi-closed and semi-open sets have been studied by different authors [8, 9, 11, 21].

This paper begins with the initiation of semi-open soft msets and semi-closed soft msets in soft mset topology. Then, we focus on the study of various set theoretic properties of semi-open and semi-closed soft msets. Further we introduce the concept of semi-compactness in soft mset topological space along with certain characterizations.

2 Preliminary

Definition 2.1. [6] Let U be an universal mset, E be a set of parameters and $A \subseteq E$. Then, an order pair (F,A) is called a soft mset where F is a mapping given by $F : A \to P^*(U)$. For all $e \in A$, $F(e)$ mset represent by count function $C_{F(e)} : U^* \to N$ where N represents the set of non-negative integers and U^* represents the support set of U.

Let $U = \{2/x, 3/y, 1/z\}$ be a mset. Then, the support set of U is $U^* = \{x, y, z\}$.

Definition 2.2. [6] For two soft msets (F, A) and (G, B) over U, we say that (F, A) is a sub soft mset of (G, B) if:

1. $A \subseteq B$.
2. $C_{F(e)}(x) \leqslant C_{G(e)}(x)$, $\forall x \in U^*$, $e \in A \cap B$.

We write $(F, A)\widetilde{\subseteq}(G, B)$.

Definition 2.3. [6] Two soft msets (F, A) and (G, B) over U are said to be soft multi equal if (F, A) is a sub soft mset of (G, B) and (G, B) is a sub soft mset of (F, A).

Definition 2.4. [6] The union of two soft msets of (F, A) and (G, B) over U is the soft mset (H, C), where $C = A \cup B$ and $C_{H(e)}(x) = max\{C_{F(e)}(x), C_{G(e)}(x)\}$, $\forall e \in A \cup B$, $\forall x \in U^*$. We write $(F, A)\widetilde{\cup}(G, B)$.

Definition 2.5. [6] The intersection of two soft msets of (F, A) and (G, B) over U is the soft mset (H, C), where $C = A \cap B$ and $C_{H(e)}(x) = min\{C_{F(e)}(x), C_{G(e)}(x)\}$, $\forall e \in A \cap B$, $\forall x \in U^*$. We write $(F, A)\widetilde{\cap}(G, B)$.

Definition 2.6. [6] A soft mset (F, A) over U is said to be a null soft mset denoted $\widetilde{\phi}$ if for all $e \in A$, $F(e) = \phi$.

Definition 2.7. [6] A soft mset (F, A) over U is said to be an absolute soft mset denoted $\widetilde{A}$ if for all $e \in A$, $F(e) = U$.

Definition 2.8. [6] Let V be a non-empty submset of U, then $\widetilde{V}$ denotes the soft mset (H, E) over U for which $H(e) = V$, for all $e \in E$.

In particular, (U, E) will be denoted by $\widetilde{U}$.

Definition 2.9. [6] The difference (H, E) between two soft msets (F, E) and (G, E) over U, denoted by $(F, E)\backslash(G, E)$, is defined as $H(e) = F(e)\backslash G(e)$ for all e∈E where $C_{H(e)}(x) = max\{C_{F(e)}(x) - C_{G(e)}(x), 0\}$, $\forall x \in U^*$.

Remark 2.1. [6] Let (F, E) be soft mset over U. If for all $e \in E$ and $a \in U^*$, $C_{F(e)}(a) = n$ $(n \geqslant 1)$ then we will write $a \in F(e)$ instead of $a \in^n F(e)$.

Definition 2.10. [6] Let (F, E) be a soft mset over U and a$\in U^*$. We say that $a \in (F, E)$ read as a belongs to the soft mset (F, E) whenever $a \in F(e)$ for all $e \in E$.

Note that for any $a \in U^*$, $a \notin (F, E)$, if $a \notin F(e)$ for some $e \in E$.

Definition 2.11. [6] Let $a \in U^*$, then (a, E) denotes the soft mset over U for which $a(e) = \{a\}$, for all $e \in E$.

Definition 2.12. [6] Let (F, E) be a soft mset over U and V be a non-empty submset of U. Then, the sub soft mset of (F, E) over V denoted by $({}^V F, E)$, is defined as follows:
${}^V F(e) = V \cap F(e)$, for all $e \in E$ where $C_{{}^V F(e)}(x) = min\{C_V(x), C_{F(e)}(x)\}$, $\forall x \in U^*$.

In other words $({}^V F, E) = \widetilde{V}\widetilde{\cap}(F, E)$.

Definition 2.13. [6] The complement of a soft mset (F, A) is denoted by $(F, A)^c$ and is defined by $(F, A)^c = (F^c, A)$ where $F^c : A \to P^*(U)$ is a mapping given by $F^c(e) = U\backslash F(e)$ for all $e \in A$ where $C_{F^c(e)}(x) = C_U(x) - C_{F(e)}(x)$, $\forall x \in U^*$.

Definition 2.14. [6] Let X be an universal mset and E be a set of parameters. Then, the collection of all soft msets over X with parameters from E is called a soft multi class and is denoted as $SMS(X_E)$.

Definition 2.15. [6] Let $\tau \subseteq SMS(X_E)$, then τ is said to be a soft multi topology on X if the following conditions hold:

1. $\widetilde{\phi}, \widetilde{X}$ belong to τ.
2. The union of any number of soft msets in τ belongs to τ.
3. The intersection of any two soft msets in τ belongs to τ.

τ is called a soft multi topology over X and the triple (X, τ, E) is called a soft multi topological space over X. Also, The members of τ are said to be open soft msets in X.

A soft mset (F, E) in $SMS(X_E)$ is said to be a closed soft mset in X, if its complement $(F, E)^c$ belongs to τ.

Definition 2.16. [6] Let X be universal mset, E be the set of parameters. Then:

- $\tau = \{\widetilde{\phi}, \widetilde{X}\}$ is called the indiscrete soft multi topology on X and (X, τ, E) is said to be an indiscrete soft multi space over X.
- Let τ be the collection of all soft msets over X. Then, τ is called the discrete soft multi topology on X and (X, τ, E) is said to be a discrete soft multi space over X.

Definition 2.17. [6] Let (X, τ, E) be a soft multi topological space over X and Y be a non-empty submset of X. Then,

$$\tau_Y = \{(^Y F, E) : (F, E) \in \tau\}$$

is said to be the soft multi topology on Y and (Y, τ_Y, E) is called a soft multi subspace of (X, τ, E).

Definition 2.18. [7] Let (X, τ, E) be a soft multi topological space over X and (F, E) be a soft mset over X. Then, the soft multi closure of (F, E), denoted by $cl(F, E)$ [or $\overline{(F, E)}$] is the intersection of all closed soft mset containing (F, E).

Definition 2.19. [7] Let (X, τ, E) be a soft multi topological space over X and (F, E) be a soft mset over X. Then, the soft multi interior of (F, E), denoted by $int(F, E)$ [or $(F, E)^o$] is the union of all open soft mset contained in (F, E).

Definition 2.20. [14] Let (X, τ_1, E) and (Y, τ_2, K) be two soft mset topological spaces. Let $\varphi : X^* \to Y^*$ and $\psi : E \to K$ be two functions. Then the pair (φ, ψ) is called a soft multi function and denoted by $f = (\varphi, \psi) : (X, E) \to (Y, K)$ is defined as follows:
Let $(F, E) \widetilde{\subseteq} \widetilde{X}$. Then the image of (F,E) under soft multi function f is soft mset in $\widetilde{Y}$ defined by f(F,E), where for $k \in \psi(E) \subseteq K$ and $y \in Y^*$,

$$C_{f(F,E)(k)}(y) = \begin{cases} sup_{e\in\psi^{-1}(k)\cap E, x\in\varphi^{-1}(y)} C_{F(e)}(x), & \text{if } \psi^{-1}(k) \neq \phi, \varphi^{-1}(y) \neq \phi; \\ 0, & \text{otherwise.} \end{cases}$$

Let (G, K) be a soft mset in $\widetilde{Y}$. Then the inverse image of (G, K) under soft multi function f is soft mset in $\widetilde{X}$ defined by $f^{-1}(G, K)$, where for $e \in \psi^{-1}(K) \subseteq E$ and $x \in X^*$,

$$C_{f^{-1}(G,K)(e)}(x) = C_{G(\psi(e))}(\varphi(x)).$$

Theorem 2.1. [14] Let $f : X_E \to Y_K$ be a soft multi function, (F_i, A) soft msets in X_E and (G_i, B) soft msets in Y_K. Then:

1. $f(\widetilde{\bigcup}_{i\in I}(F_i, A_i)) = \widetilde{\bigcup}_{i\in I} f(F_i, A_i)$.
2. $f^{-1}(\widetilde{\bigcup}_{i\in I}(G_i, B)) = \widetilde{\bigcup}_{i\in I} f^{-1}(G_i, B)$.

3 Semi-open soft msets and semi-closed soft msets

Definition 3.1. A soft mset (S, E) in a soft mset topology (X,τ,E) is said to be semi open soft mset iff there exists an open soft mset (F, E) such that:
$C_{F(e)}(x) \leqslant C_{S(e)}(x) \leqslant C_{cl(F)(e)}(x)$ for all $x \in X^*$, $e \in E$.

Definition 3.2. A soft mset (S, E) in a soft mset topology (X,τ,E) is said to be semi closed soft mset iff there exist a closed soft mset (F, E) such that:
$C_{int(F)(e)}(x) \leqslant C_{S(e)}(x) \leqslant C_{F(e)}(x)$ for all $x \in X^*$, $e \in E$.

Note that the complement of semi-open soft mset is semi-closed soft mset.

Example 3.1. Let $X = \{2/x, 3/y, 1/z\}$ be a mset, $E = \{e_1, e_2\}$ be a set of parameters and $\tau = \{\widetilde{\phi}, \widetilde{X}, (F_1, E), (F_2, E), (F_3, E), (F_4, E), (F_5, E), (F_6, E)\}$, where:
$F_1(e_1) = \{2/x\}$, $F_1(e_2) = \{1/y\}$;
$F_2(e_1) = \{2/x, 1/y\}$, $F_2(e_2) = \{2/x, 1/y\}$;
$F_3(e_1) = X$, $F_3(e_2) = \{2/x\}$;
$F_4(e_1) = X$, $F_4(e_2) = \{2/x, 1/y\}$;
$F_5(e_1) = \{2/x, 1/y\}$, $F_5(e_2) = \{2/x\}$;
$F_6(e_1) = \{2/x\}$, $F_6(e_2) = \phi$.
Let (G, E) be a sub soft mset of X such that $G(e_1) = \{2/x, 2/y\}$, $G(e_2) = \{2/x, 2/y\}$. Then, $C_{F_1(e)}(x) \leqslant C_{G(e)}(x) \leqslant C_{cl(F_1)(e)}(x)$ for all $x \in X^*$, $e \in E$. Hence, (G, E) is semi-open soft mset.

Definition 3.3. Let (X,τ,E) be a soft mset topology. Then:

1. The semi closure of a soft mset (G, E) is denoted by $scl(G, E)$ and defined as $scl(G, E) = \widetilde{\cap}\{(F, E) : (G, E)\widetilde{\subseteq}(F, E),\ (F, E)\ is\ semi-closed\ soft\ mset\}$, where $C_{(scl(G))(e)}(x) = min\{C_{F(e)}(x) : C_{G(e)}(x) \leqslant C_{F(e)}(x),\ (F, E)\ is\ semi-closed\ soft\ mset\}$; for all $x \in X^*$, $e \in E$.
2. The semi interior of a soft mset (G, E) is denoted by $sint(G, E)$ and defined as $sint(G, E) = \widetilde{\cup}\{(F, E) : (F, E)\widetilde{\subseteq}(G, E), (F, E)\ is\ semi-open\ soft\ mset\}$, where
$C_{(sint(G))(e)}(x) = max\{C_{F(e)}(x) : C_{F(e)}(x) \leqslant C_{G(e)}(x),\ (F, E)\ is\ semi-open\ soft\ mset\}$; for all $x \in X^*$, $e \in E$.

Theorem 3.1. Let (X,τ,E) be a soft mset topology. Then, arbitrary union of semi-open soft msets is a semi-open soft mset.

Proof. Let $\{(T_\lambda, E) : \lambda \in \Lambda\}$ be a collection of semi-open soft msets. Since, (T_λ, E) is a semi-open soft mset, then there exists an open soft mset (O_λ, E) for each λ such that $C_{O_\lambda(e)}(x) \leqslant C_{T_\lambda(e)}(x) \leqslant C_{cl(O_\lambda)(e)}(x)$ for all $x \in X^*$, $e \in E$ and $\lambda \in \Lambda$. Now, taking arbitrary union over λ, $C_{\cup_\lambda O_\lambda(e)}(x) \leqslant C_{\cup_\lambda T_\lambda(e)}(x) \leqslant C_{\cup_\lambda cl(O_\lambda)(e)}(x) = C_{cl(\cup_\lambda O_\lambda)(e)}(x)$ for all $x \in X^*$, $e \in E$. This imply $(\cup_\lambda T_\lambda, E)$ is a semi-open soft mset, because $(\cup_\lambda O_\lambda, E)$ is an open soft mset being the arbitrary union of open soft msets.

Remark 3.1. Finite intersection of semi-open soft msets may not be semi-open soft mset. As shown in the following example.

Example 3.2. Let $X = \{3/x, , 3/y, 2/z, 1/d\}$ be a mset, $E = \{e_1, e_2\}$ be a set of parameters and $\tau = \{\widetilde{\phi}, \widetilde{X}, (F_1, E), (F_2, E), (F_3, E), (F_4, E)\}$, where:
$F_1(e_1) = \{2/x, 1/y\}$, $F_1(e_2) = \{2/x, 1/y\}$;
$F_2(e_1) = \{2/z, 1/d\}$, $F_2(e_2) = \{1/x, 3/y\}$;
$F_3(e_1) = \{2/x, 1/y, 2/z, 1/d\}$, $F_3(e_2) = \{2/x, 3/y\}$;
$F_4(e_1) = \phi$, $F_4(e_2) = \{1/x, 1/y\}$.
Let (G, E), (F, E) be two sub soft mset of X such that $G(e_1) = \{2/x, 2/y\}$, $G(e_2) = \{2/x, 2/y\}$ and $F(e_1) = \{2/z, 1/d\}$, $F(e_2) = \{3/x, 3/y\}$. Then, (G, E), (F, E) are semi-open soft msets. But, $(G \cap F)(e_1) = \phi$, $(G \cap F)(e_2) = \{2/x, 2/y\}$. Hence, $(G \cap F, E)$ is not semi-open soft mset.

Remark 3.2. The collection of all semi-open msets doesn't form a soft mset topology since intersection of two semi-open soft msets may not be semi-open soft mset.

Theorem 3.2. The union of a semi-open soft mset with an open soft mset is aslo a semi-open soft mset.

Proof. Let (O, E) be an open soft mset and (S, E) be a semi-open soft mset. So, there exist an open soft mset (F, E) such that $C_{F(e)}(x) \leqslant C_{S(e)}(x) \leqslant C_{cl(F)(e)}(x)$ for all $x \in X^*$, $e \in E$. Therefore, $C_{(F\cup O)(e)}(x) \leqslant C_{(S\cup O)(e)}(x) \leqslant C_{(cl(F)\cup O)(e)}(x)$. Now, we have $C_{(cl(F)\cup O)(e)}(x) \leqslant C_{(cl(F)\cup cl(O))(e)}(x) = C_{(cl(F\cup O))(e)}(x)$. Hence, $C_{(F\cup O)(e)}(x) \leqslant C_{(S\cup O)(e)}(x) \leqslant C_{(cl(F\cup O))(e)}(x)$. Then, $(S \cup O, E)$ is a semi-open soft mset.

Corollary 3.1. Let (X,τ,E) be a soft mset topological space, then arbitrary intersection of semi-closed soft msets is a semi-closed soft mset.

Proof. Immediate.

Remark 3.3. Union of two semi-closed soft msets may not be a semi-closed soft mset. As shown in the following example.

Example 3.3. From Example 3.2, Let (G, E), (F, E) be two sub soft mset of X such that $G(e_1) = \{1/x, 1/y, 2/z, 1/d\}$, $G(e_2) = \{1/x, 1/y, 2/z, 1/d\}$ and $F(e_1) = \{3/x, 3/y\}$, $F(e_2) = \{2/z, 1/d\}$. Then, (G, E), (F, E) are two semi-closed soft msets. Moreover, $(G \cup F)(e_1) = X$, $(G \cup F)(e_2) = \{1/x, 1/y, 2/z, 1/d\}$ but $(G \cup F, E)$ is not semi-closed soft mset.

Theorem 3.3. Every open soft mset is a semi-open soft mset.

Proof. Immediate.

Note that the converse of Theorem 3.3 is not true as shown in this example.

Example 3.4. From Example 3.1, (G, E) is semi-open soft mset but it is not open soft mset.

Theorem 3.4. If (S, E) is a semi-open soft mset such that $C_{S(e)}(x) \leqslant C_{N(e)}(x) \leqslant C_{cl(S)(e)}(x)$ for all $x \in X^*$, $e \in E$. Then, the soft mset (N, E) is also a semi-open soft mset.

Proof. As (S, E) is a semi-open soft mset, there exists an open soft mset (O, E) such that $C_{O(e)}(x) \leqslant C_{S(e)}(x) \leqslant C_{cl(O)(e)}(x)$ for all $x \in X^*$, $e \in E$. Then by hypothesis, $C_{O(e)}(x) \leqslant C_{N(e)}(x) \leqslant C_{cl(S)(e)}(x) \leqslant C_{cl(O)(e)}(x)$ for all $x \in X^*$, $e \in E$. Hence, (N, E) is a semi-open soft mset.

Corollary 3.2. If (F, E) is a semi-closed soft mset in a soft mset topology (X,τ,E) such that $C_{int(F)(e)}(x) \leqslant C_{S(e)}(x) \leqslant C_{F(e)}(x)$ for all $x \in X^*$, $e \in E$. Then the soft mset (S, E) is also a semi-closed soft mset.

Proof. Immediate.

Theorem 3.5. For a soft mset topology, the following conditions are equivalent:

1. (S,E) is a semi-open soft mset.
2. $C_{S(e)}(x) \leqslant C_{cl(int(S))(e)}(x)$.
3. $C_{int(cl(S^c))(e)}(x) \leqslant C_{S^c(e)}(x)$, where S^c is the complement of S.
4. (S^c, E) is a semi-closed soft mset.

Proof. (1⇒2) Let (S, E) be a semi-open soft mset. So, there exist an open soft mset (O, E) such that $C_{O(e)}(x) \leqslant C_{S(e)}(x) \leqslant C_{cl(O)(e)}(x)$ for all $x \in X^*$, $e \in E$. Since, (O, E) is an open soft mset, then $C_{S(e)}(x) \leqslant C_{cl(int(O))(e)}(x)$. Since $C_{O(e)}(x) \leqslant C_{S(e)}(x)$, then $C_{cl(int(O))(e)}(x) \leqslant C_{cl(int(S))(e)}(x)$. Thus, we have $C_{S(e)}(x) \leqslant C_{cl(int(S))(e)}(x)$.

(2⇒3) Taking complement of (2). Then, $C_{int(cl(S^c))(e)}(x) \leqslant C_{S^c(e)}(x)$.

(3⇒4) Since, $(cl(S^c), E)$ is a closed soft mset such that $C_{int(cl(S^c)(e)}(x) \leqslant C_{S^c(e)}(x) \leqslant C_{cl(S^c)(e)}(x)$ for all $x \in X^*$, $e \in E$. So, (S^c, E) is a semi-closed soft mset.

(4⇒1) Since, (S^c, E) is a semi-closed soft mset. Then, there exist a closed soft mset (F, E) such that $C_{int(F)(e)}(x) \leqslant C_{S^c(e)}(x) \leqslant C_{F(e)}(x)$. Therefore, $C_{F^c(e)}(x) \leqslant C_{S(e)}(x) \leqslant C_{cl(F^c)(e)}(x)$ for all $x \in X^*$, $e \in E$. Then, (S, E) is a semi-open soft mset.

4 Semi-Compactness

Definition 4.1. A collection $\{(T_\lambda, E) : \lambda \in \Lambda\}$ of soft msets is said to be a cover of a soft mset (F, E) if $C_{F(e)}(x) \leqslant C_{\widetilde{\bigcup}_\lambda T_\lambda(e)}(x)$ for all $x \in X^*$, $e \in E$. Then, we say (F, E) is covered by $\{(T_\lambda, E) : \lambda \in \Lambda\}$. Also, If each (T_λ, E) is a semi-open soft mset, then the cover is said to be a semi open cover.

If $C_{\widetilde{X}(e)}(x) \leqslant C_{\widetilde{\bigcup}_{\lambda} T_{\lambda}(e)}(x)$ for all $x \in X^*$, $e \in E$. Then, we say $\widetilde{X}$ is covered by $\{(T_\lambda, E) : \lambda \in \Lambda\}$.

Definition 4.2. Any subcollection of a semi-open cover is said to be semi subcover if it covers $\widetilde{X}$.

Definition 4.3. Any subcollection of a semi open cover where each element is a whole sub soft mset is said to be semi whole subcover if it covers $\widetilde{X}$.

Definition 4.4. A soft mset topology (X,τ,E) is said to be a semi compact space if every semi open cover of $\widetilde{X}$ has a finite semi open subcover i.e., for any collection $\{(T_\lambda, E) : \lambda \in \Lambda\}$ of semi-open soft msets covering $\widetilde{X}$, there exist a finite subcollection $\{(T_{\lambda_i}, E) : i = 1, 2, 3, ..., n\}$ such that $C_{\widetilde{X}(e)}(x) \leqslant C_{\widetilde{\cup} T_{\lambda_i}(e)}(x)$ for all $x \in X^*$, $e \in E$, $i = 1, 2, 3, ..., n$.

Example 4.1.

1. Every finite soft mset topological space is a semi-compact soft multi space.
2. Any indiscrete soft mset topological space is a semi-compact soft multi space.

Remark 4.1. Every compact soft multi space is semi-compact soft multi space.

Definition 4.5. A soft mset topological space (X,τ,E) is said to be:

1. a semi-whole compact soft multi space if every semi open cover of $\widetilde{X}$ has a finite semi-whole subcover.
2. a semi-partial whole compact soft multi space if every semi open cover of $\widetilde{X}$ has a finite semi-partial whole subcover.
3. a semi-full compact soft multi space if every semi open cover of $\widetilde{X}$ has a finite semi-full subcover.

Definition 4.6. An arbitrary collection $S = \{(O_1, E), (O_2, E), ...\}$ of soft msets is said to have finite intersection property (FIP) if intersection of elements of every finite subcollection $\{(O_1, E), (O_2, E), ..., (O_n, E)\}$ of S is non-empty.
i.e., $C_{\widetilde{\cap} O_i(e)}(x) \neq C_{\widetilde{\phi}(e)}(x)$, $x \in X^*$, $e \in E$, $i = 1, 2, 3, ..., n$.

Theorem 4.1. Let (X,τ,E) be a soft mset topology. Therefore, (X,τ,E) is semi compact iff every collection $C = \{(T_\lambda, E) : \lambda \in \Lambda\}$ of semi closed soft msets in $\widetilde{X}$ having the FIP is such that $C_{\widetilde{\bigcap} T_\lambda(e)}(x) \neq C_{\widetilde{\phi}(e)}(x)$, $x \in X^*$, $e \in E$, $\lambda \in \Lambda$.

Proof. ($\Rightarrow$) Let (X,τ,E) be a semi compact space and $\{(T_\lambda, E) : \lambda \in \Lambda\}$ be a collection of semi-closed soft msets with FIP such that $C_{(\widetilde{\bigcap}_{\lambda \in \Lambda} T_\lambda)(e)}(x) = C_{\widetilde{\phi}(e)}(x)$. Then, $C_{(\widetilde{\bigcup}_{\lambda \in \Lambda} T^c_\lambda)(e)}(x) = C_{\widetilde{X}(e)}(x)$. Therefore, $\{(T^c_\lambda, E) : \lambda \in \Lambda\}$ forms semi open cover of $\widetilde{X}$. So, there exist $\{(T^c_{\lambda_i}, E) : i = 1, 2, 3, ..., n\}$ such that $C_{(\widetilde{\bigcup}_{i=1,..,n} T^c_{\lambda_i})(e)}(x) = C_{\widetilde{X}(e)}(x)$. Thus, $C_{(\widetilde{\bigcap}_{i=1,..,n} T_{\lambda_i})(e)}(x) = C_{\widetilde{\phi}(e)}(x)$ for all $x \in X^*$, $e \in E$, which is a contradiction.

($\Leftarrow$) Let every collection $C = \{(T_\lambda, E) : \lambda \in \Lambda\}$ of semi-closed soft msets having FIP be such that $C_{(\widetilde{\bigcap}_{\lambda\in\Lambda} T_\lambda)(e)}(x) \neq C_{\widetilde{\phi}(e)}(x)$. Assume that (X,$\tau$,E) is not semi compact space. Then, there exist a semi open cover $\{(S_\lambda, E) : \lambda \in \Lambda\}$ of $\widetilde{X}$ which has no finite subcover of $\widetilde{X}$. Hence, $C_{\widetilde{X}(e)}(x) > C_{(\widetilde{\bigcup}_{i=1,..,n} S_{\lambda_i})(e)}(x)$. Therefore, $C_{\widetilde{\phi}(e)}(x) \leqslant C_{(\widetilde{\bigcap}_{i=1,..,n} S^c_{\lambda_i})(e)}(x)$, which is a contradiction with hypothesis.

Theorem 4.2. Let (X,τ,E) be a soft mset topology. Therefore, (X,τ,E) is semi compact iff every collection $C = \{(T_\lambda, E) : \lambda \in \Lambda\}$ of soft msets in $\widetilde{X}$ having the FIP is such that $C_{(\widetilde{\bigcap}_{\lambda\in\Lambda} scl(T_\lambda))(e)}(x) \neq C_{\widetilde{\phi}(e)}(x)$.

Proof. ($\Rightarrow$) Let (X,τ,E) be a semi-compact. Assume that $C = \{(T_\lambda, E) : \lambda \in \Lambda\}$ be a collection of soft msets in $\widetilde{X}$ having the FIP be such that $C_{(\widetilde{\bigcap}_{\lambda\in\Lambda} scl(T_\lambda))(e)}(x) = C_{\widetilde{\phi}(e)}(x)$. Then, $C_{(\widetilde{\bigcup}_{\lambda\in\Lambda} scl(T_\lambda)^c)(e)}(x) = C_{\widetilde{X}(e)}(x)$. Therefore, $\{(scl(T_\lambda, E))^c : \lambda \in \Lambda\}$ forms a semi open cover of $\widetilde{X}$. Since, (X,τ,E) is semi compact. Then, there exist a finite subcover $\{(scl(T_{\lambda_i}, E))^c : i = 1, 2, 3, ..., n\}$ such that $C_{[\widetilde{\bigcup}_{i=1,..,n} (scl(T_{\lambda_i}))^c](e)}(x) = C_{\widetilde{X}(e)}(x)$. Then, $C_{[\widetilde{\bigcap}_{i=1,..,n} scl(T_{\lambda_i})](e)}(x) = C_{\widetilde{\phi}(e)}(x)$. Therefore, $C_{(\widetilde{\bigcap}_{i=1,..,n} T_{\lambda_i})(e)}(x) \leqslant C_{\widetilde{\phi}(e)}(x)$, which is a contradiction with the FIP.

($\Leftarrow$) Sufficiency. Assume that (X,τ,E) is not a semi-compact. Then, there exist a semi open cover $\{(T_\lambda, E) : \lambda \in \Lambda\}$ which has no finite subcover. So, for all finite subcollection $\{(T_{\lambda_i}, E) : i = 1, 2, 3, ..., n\}$, we have $C_{(\widetilde{\bigcup}_{i=1,..,n} T_{\lambda_i})(e)}(x) < C_{\widetilde{X}(e)}(x)$. Thus, $C_{(\widetilde{\bigcap}_{i=1,..,n} {T_{\lambda_i}}^c)(e)}(x) \geqslant C_{\widetilde{\phi}(e)}(x)$. Hence, $\{({T_\lambda}^c, E) : \lambda \in \Lambda\}$ is a family of semi-closed soft msets with FIP. Now, $C_{(\widetilde{\bigcup}_{\lambda\in\Lambda} T_\lambda)(e)}(x) \geqslant C_{\widetilde{X}(e)}(x)$. Then, $C_{(\widetilde{\bigcap}_{\lambda\in\Lambda} {T_\lambda}^c)(e)}(x) = C_{\widetilde{\phi}(e)}(x)$. Therefore, $C_{[\widetilde{\bigcap}_{\lambda\in\Lambda} scl({T_\lambda}^c)](e)}(x) = C_{\widetilde{\phi}(e)}(x)$, which is a contradiction with hypothesis.

Remark 4.2. The Theorems 4.1 and 4.2 hold for semi whole (resp. partial whole and full) compact spaces .

Theorem 4.3. Let (X,τ,E) be a soft mset topology and (Y,τ_Y,E) be its subspace. Let (A, E) be a soft mset such that $C_{A(e)}(x) \leqslant C_{\widetilde{Y}(e)}(x) \leqslant C_{\widetilde{X}(e)}(x)$. Then, (A, E) is τ-semi compact iff (A, E) is τ_Y-semi compact.

Proof. ($\Rightarrow$) Let (A, E) be a τ-semi compact and $\{(K_\lambda, E) : \lambda \in \Lambda\}$ be τ_Y-semi open cover of (A, E). So, there exist τ-semi open soft msets $\{(S_\lambda, E) : \lambda \in \Lambda\}$ such that $C_{K_\lambda(e)}(x) = C_{(\widetilde{Y}\widetilde{\bigcap} S_\lambda)(e)}(x)$ for all $x \in X^*$, $e \in E$, $\lambda \in \Lambda$. Now, $C_{A(e)}(x) \leqslant C_{\widetilde{Y}(e)}(x) \leqslant C_{\widetilde{\bigcup}_{\lambda\in\Lambda} K_\lambda(e)}(x) \leqslant C_{\widetilde{\bigcup}_{\lambda\in\Lambda} S_\lambda(e)}(x)$. Therefore, $\{(S_\lambda, E) : \lambda \in \Lambda\}$ forms a τ-semi open cover of (A, E). So, there exist a finite subcover $\{(S_{\lambda_i}, E) : i = 1, 2, 3, ..., n\}$ such that $C_{A(e)}(x) \leqslant C_{(\widetilde{\bigcup}_{i=1,..,n} S_{\lambda_i})(e)}(x)$. Since, $C_{A(e)}(x) \leqslant C_{\widetilde{Y}(e)}(x)$, then $C_{A(e)}(x) \leqslant C_{[\widetilde{Y}\widetilde{\bigcap}(\widetilde{\bigcup}_{i=1,..,n} S_{\lambda_i})](e)}(x) = C_{[\widetilde{\bigcup}_{i=1,..,n} (\widetilde{Y}\widetilde{\bigcap} S_{\lambda_i})](e)}(x) = C_{(\widetilde{\bigcup}_{i=1,..,n} K_{\lambda_i})(e)}(x)$. Thus, (A, E) is τ_Y-semi compact.

($\Leftarrow$) Let $\{(S_\lambda, E) : \lambda \in \Lambda\}$ be τ-semi open cover of (A, E). Putting $C_{G_\lambda(e)}(x) = C_{(\tilde{Y}\tilde{\bigcap}S_\lambda)(e)}(x)$ for all $x \in X^*$, $e \in E$, $\lambda \in \Lambda$. Since, $C_{A(e)}(x) \leqslant C_{\tilde{Y}(e)}(x)$ and $C_{A(e)}(x) \leqslant C_{\tilde{\bigcup}_{\lambda\in\Lambda}S_\lambda(e)}(x)$, then $C_{A(e)}(x) \leqslant C_{(\tilde{Y}\tilde{\bigcap}(\tilde{\bigcup}_{\lambda\in\Lambda}S_\lambda))(e)}(x) = C_{\tilde{\bigcup}_{\lambda\in\Lambda}(\tilde{Y}\tilde{\bigcap}S_\lambda)(e)}(x) = C_{\tilde{\bigcup}_{\lambda\in\Lambda}G_\lambda(e)}(x)$. So, $\{(G_\lambda, E) : \lambda \in \Lambda\}$ is τ_Y-semi open cover of (A, E). By hypothesis, there exist a finite subcollection $\{(G_{\lambda_i}, E) : i = 1, 2, 3, ..., n\}$ such that $C_{A(e)}(x) \leqslant C_{(\tilde{\bigcup}_{i=1,..,n}G_{\lambda_i})(e)}(x) = C_{[\tilde{\bigcup}_{i=1,..,n}(\tilde{Y}\tilde{\bigcap}S_{\lambda_i})](e)}(x) \leqslant C_{(\tilde{\bigcup}_{i=1,..,n}S_{\lambda_i})(e)}(x)$. This implies that (A, E) is τ-semi compact .

Definition 4.7. A soft multi function $f : (X, \tau_1, E) \to (Y, \tau_2, K)$ is said to be irresolute soft multi function if $f^{-1}(G, K)$ is τ_1-semi open (resp. closed) for every (G, K) is τ_2-semi open (resp. closed).

Theorem 4.4. Let $f : (X, \tau_1, E) \to (Y, \tau_2, K)$ be a bijection irresolute soft multi function. If (H, E) is a τ_1-semi compact, then $f(H, E)$ is a τ_2-semi compact.

Proof. Assume that $(H, E)\tilde{\subseteq}\tilde{X}$ is a τ_1-semi compact. Let $\{(G_\lambda, K) : \lambda \in \Lambda\}$ be a τ_2-semi open cover of $f(H, E)$ i.e., $C_{f(H)(k)}(y) \leqslant C_{(\tilde{\bigcup}_{\lambda\in\Lambda}G_\lambda)(k)}(y)$ for all $y \in Y^*$, $k \in K$. Therefore, $C_{H(e)}(x) = C_{(f^{-1}(f(H)))(e)}(x) \leqslant C_{(f^{-1}(\tilde{\bigcup}_{\lambda\in\Lambda}G_\lambda))(e)}(x) = C_{(\tilde{\bigcup}_{\lambda\in\Lambda}f^{-1}(G_\lambda))(e)}(x)$ for all $x \in X^*$, $e \in E$. Since, (G_λ, K) is τ_2-semi open cover of $f(H, E)$ and f is irresolute soft multi function, then $f^{-1}(G_\lambda, K)$ is τ_1-semi open cover of (H, E). Thus, $C_{H(e)}(x) \leqslant C_{(\tilde{\bigcup}_{i=1,2,...,n}f^{-1}(G_{\lambda_i}))(e)}(x)$ for all $x \in X^*$, $e \in E$. Then, $C_{f(H)(k)}(y) \leqslant C_{f(\tilde{\bigcup}_{i=1,2,..,n}f^{-1}(G_{\lambda_i}))(k)}(y) = C_{(\tilde{\bigcup}_{i=1,2,..,n}G_{\lambda_i})(k)}(y)$ for all $y \in Y^*$, $k \in K$. Hence, $f(H, E)$ is a τ_2-semi compact.

5 Conclusion

Topology is an important and major area of mathematics and it can give many relationships between other scientific areas and mathematical models. Recently, many scientists have studied and improved the mset theory and easily applied to many problems having uncertainties from social life. This paper begins with the initiation of semi-open soft msets and semi-closed soft msets in soft mset topology. Then, we focus on the study of various set theoretic properties of semi-open and semi-closed soft msets. Further, we introduce the concept of semi-compactness in soft mset topological space along with certain characterizations. Also, we discuss some important results about semi-compact soft multi space.

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