http://www.newtheory.org ISSN: 2149-1402

Journal of New Theory

Received: 04.04.2015 *Year*: **2015,** *Number*: **7**, *Pages*: **64-78**
Published: 19.10.2015 *Original Article*[**]

EXTENDED HAUSDORFF DISTANCE AND SIMILARITY MEASURES FOR NEUTROSOPHIC REFINED SETS AND THEIR APPLICATION IN MEDICAL DIAGNOSIS

Said Broumi[1,*] <broumisaid78@gmail.com>
Florentin Smarandache[2] <fsmarandache@gmail.com>

[1]*Faculty of Letters and Humanities, Hay El Baraka Ben M'sik Casablanca B.P. 7951, University of Hassan II - Casablanca, Morocco*
[2]*Department of Mathematics, University of New Mexico,705 Gurley Avenue, Gallup, NM 87301, USA*

Abstract - In this paper we present a new distance measure between neutrosophic refined sets on the basis of extended Hausdorff distance of neutrosophic set and we study some of their basic properties. Finally, using the extended Hausdorff distance and/or similarity measures, an application to medical diagnosis is presented.

Keywords - Neutrosophic sets, similarity measure, neutrosophic refined sets, extended Hausdorff distance.

1. Introduction

The neutrosophic set theory (NS) proposed in 1995 by Smarandache [6] was the generalization of the (FS for short) [21], intuituionitic fuzzy set (IFS for short) [20] and so on. In fuzzy set, the object, partially belong to a set with a membership degree (T) between 0 and 1 whereas in the IFS represent the uncertainty with respect to both membership ($T \in [0, 1]$) and non membership ($F \in [0, 1]$) such that $0 \leq T + F \leq 1$. Here, the number $I = 1 - T - F$ is called the hesitation degree or intuitionistic index. In neutrosophic set, indeterminacy is quantified explicitly and truth-membership, indeterminacy-membership and falsity-membership are independent. From scientific or engineering point of view, the

[**]*Edited by* NaimÇağman *(Editor-in- Chief) and* Irfan Deli *(Area Editor).*
[*]*Corresponding Author.*

neutrosophic set and set- theoretic view, operators need to be specified. Otherwise, it will be difficult to apply in the real applications. Therefore, H. Wang et al [8] defined a single valued neutrosophic set (SVNS) and then provided the set theoretic operations and various properties of single valued neutrosophic sets. Many researches on neutrosophic set on neutrosophic set theory and its applications in various fields are progressing rapidly [e.g., 2, 3, 4, 7, 8, 10, 11, 12, 15, 16, 17, 18, 19, 22, 23, 25, 31, 32, 34, 35, 36, 37, 38, 39].

The study of distance and similarity measure of NSs gives lots of measures, each representing specific properties and behavior in real-life decision making and pattern recognition works. For measuring the degree of similarity between neutrosophic sets, Broumi et al. [33] proposed several similarity measures and investigated some of their basic properties. Ye [16] presented the Hamming, Euclidean distance and similarity measures between interval neutrosophic sets. The generalized weighted distance and similarity measures between neutrosophic sets were given by J.Ye [15]. Also, the same author proposed three vector similarity measures for simplified neutrosophic sets (SNSs for short), including the Jaccard, Dice, and cosine similarity measures for SVNS and applied them to multicriteria decision-making problems with simplified neutrosophic information. Therefore, S.Broumi et al. [42] extended generalized weighted distance between neutrosophic sets (NSs) to the case of interval neutrosophic sets. Hanafy et al. [13, 14] presented the correlation measure neutrosophic sets.

The multi set introduced by Yager [30] allows the repeated occurrences of any element and hence the fuzzy multi set (FMS for short) can occur more than once with the possibly of the same or the different membership values.

Recently, based on [8], the new concept neutrosophic refined set(neutrosophic multisets) NRS was proposed by Broumi et al. [40] which allows the repeated occurrences of different truth membership, indeterminacy and non membership functions. Later on, Broumi et al. [40] studied correlation measure for neutrosophic refined sets and gave an application in decision making. The same author [41] defined the similarity measure between neutrosophic refined sets based on cosine function. The concept of NRS is a generalization of fuzzy multisets [42] and intuitionistic fuzzy multisets [43].

In this paper we extend the Hausdorff distance between neutrosophic sets to the case of neutrosophic refined sets (NRSs).

The organization of this paper is as follows: In section 2, the neutrosophic sets, neutrosophic refined sets and extended Hausdorff distance of neutrosophic sets are presented. The section 3 deals with the proposed extended Hausdorff distance and similarity measure for neutrosophic refined sets. Section 4, present an application of extended Hausdorff distance and similarity measure to medical diagnosis.

2. Preliminaries

This section gives a brief overview of concepts of neutrosophic set [6], and neutrosophic refined sets [8], Hausdorff distance and extended Hausdorff distance between NSs [33].

2.1 Neutrosophic Sets

Definition 2.1[6] Let X be an universe of discourse, with a generic element in X denoted by x, the neutrosophic (NS) set is an object having the form

$$A = \{< x: T_A(x), I_A(x), F_A(x)>, x \in X\}$$

where the functions T, I, F : X→ $]^-0, 1^+[$ define respectively the degree of membership (or Truth), the degree of indeterminacy, and the degree of non-membership (or Falsehood) of the element x ∈X to the set A with the condition.

$$^-0 \le T_A(x) + I_A(x) + F_A(x) \le 3^+ \quad (1)$$

From philosophical point of view, the neutrosophic set takes the value from real standard or non-standard subsets of $]^-0, 1^+[$. So instead of $]-0, 1^+[$ we need to take the interval [0, 1] for technical applications, because $]^-0, 1^+[$ will be difficult to apply in the real applications such as in scientific and engineering problems. For two NS,

$$A_{NS} = \{ <x, T_A(x), I_A(x), F_A(x)> \mid x \in X \} \quad (2)$$

And $B_{NS}=\{<x, T_B(x), I_B(x), F_B(x)> \mid x \in X \}$ the two relations are defined as follows:

(1) $A_{NS} \subseteq B_{NS}$ if and only if $T_A(x) \le T_B(x), I_A(x) \ge I_B(x), F_A(x) \ge F_B(x)$
(2) $A_{NS} = B_{NS}$ if and only if $T_A(x) = T_B(x), I_A(x) = I_B(x), F_A(x) = F_B(x)$

2.2 Neutrosophic Refined Sets

In 2013, Smarandache [8] extended the neutrosophic set to n-valued refined neutrosophic set.

Definition 2.2 [8] Let E be a universe, a neutrosophic sets on E can be defined as follows:

$$A = \{<x,(T_A^1(x),T_A^2(x),...,T_A^p(x)), (I_A^1(x),I_A^2(x),...,I_A^p(x)),(F_A^1(x),F_A^2(x),\ldots,F_A^p(x))>: x \in X\}$$

Where

$T_A^1(x),T_A^2(x),...,T_A^p(x)$: E→ [0 ,1],
$I_A^1(x),I_A^2(x),...,I_A^p(x)$: E→ [0 ,1], and
$F_A^1(x),F_A^2(x),...,F_A^p(x)$: E→ [0 ,1]

such that $0 \le T_A^i(x) + I_A^i(x) + F_A^i(x) \le 3$ for i=1 ,2,…,p for any x∈ X ,

$$(T_A^1(x),T_A^2(x),\ldots,T_A^p(x)),\ (I_A^1(x),I_A^2(x),\ldots,I_A^p(x)) \text{ and } (F_A^1(x),F_A^2(x),\ldots,F_A^p(x))$$

is the truth-membership sequence, indeterminacy-membership sequence and falsity-membership sequence of the element x, respectively. Also, P is called the dimension of neutrosophic refined sets (NRS) A.

2.3 Hausdorff distance

Definition 2.3 The Hausdorff distance defined as follows

$$d_H(A, B) = H(A, B) = \max\{|a_1 - b_1|, |a_2 - b_2|\} \quad (3)$$

For the two intervals A=$[a_1, a_2]$ and B=$[b_1, b_2]$ in areal space R.

Definition 2.4: The Hausdorff distance $d_H(A, B)$ between A and B satisfies the following properties (D1-D4):

(D1) $1 \leq d_H(A, B) \leq 1$.
(D2) $d_H(A, B) = 0$ if and only if A = B; for all A, B
(D3) $d_H(A, B) = d_H(B, A)$.
(D4) If $A \subseteq B \subseteq C$, for A, B, C, then $d_H(A, C) \geq d_H(A, B$ and $d_H(A, C) \geq d_H(B, C)$

2.4 Extended Hausdorff distance between neutrosophic sets

Definition2.5 [33] Based on the Hausdorff metric, Szmidt and Kacprzykdefined new distance between intuitionistic fuzzy sets and/or interval-valued fuzzy sets in [5], taking into account three parameter representation (membership, non-membership values, and the hesitation margins) of A-IFSs which fulfill the properties of the Hausdorff distances. Their definition is defined by

$$H_{A-IFS}(A, B) = \frac{1}{n}\sum_{i=1}^{n} max\{|\mu_A(x_i) - \mu_B(x_i)|, |\nu_A(x_i) - \nu_B(x_i)|, |\pi_A(x_i) - \pi_B(x_i)|\} \quad (4)$$

Where A = {< x, $\mu_A(x)$, $\nu_A(x)$, $\pi_A(x)$>} and B = {< x, $\mu_B(x)$, $\nu_B(x)$, $\pi_B(x)$>}.

The terms and symbols used in [5] are changed so that they are consistent with those in this section.

Let X= $\{x_1, x_2, \ldots, x_n\}$ be a discrete finite set. Consider a neutrosophic set A in X where $T_A(x_i)$, $I_A(x_i)$, $F_A(x_i) \in [0,1]$ for every $x_i \in X$, represent its membership, indeterminacy, and non-membership values respectively denoted by A = {< x, $T_A(x_i)$, $I_A(x_i)$, $F_A(x_i)$>}.

Then, the distance between A ∈NSs and B∈ NSs is defined as follows.

$$d_{HNS}(A, B) = \frac{1}{n}\sum_{i=1}^{n} \max\{|T_A(x_i) - T_B(x_i)|, |I_A(x_i) - I_B(x_i)|, |F_A(x_i) - F_B(x_i)|\} \quad (5)$$

Where $d_{HNS}(A, B)$= H(A, B)denote the extended Hausdorff distance between two neutrosophic sets(NS) A and B.

Let A, B and C be three neutrosophic sets

$$d_{HNS}(A, B) = H(A, B) = \max\{|T_A(x_i) - T_B(x_i)|, \left|I_A(x_i) - I_{B(x_i)}\right|, |F_A(x_i) - F_B(x_i)|\} \quad (6)$$

The same between A and C is written as:

$$H(A, C)= \max\{|T_A(x_i) - T_C(x_i)|, |I_A(x_i) - I_C(x_i)\ |, |F_A(x_i) - F_C(x_i)|\} \quad (7)$$

And between B and C is written as:

$$H(B, C)= \max\{|T_B(x_i) - T_C(x_i)|, |I_B(x_i) - I_C(x_i)\ |, |F_B(x_i) - F_C(x_i)|\} \quad (8)$$

Hung and yang [45] presented their similarity measures based on Hausdorff distance as follows

Definition2.6 The similarity measure based on the Hausdroff distance is

s_H^1(A,B)= 1-$d_H(A, B)$.
s_H^2(A,B)= $(e^{-d_H(A,B)} - e^{-1})/(1 - e^{-1})$.
s_H^3(A,B)= $(1 - d_H(A, B))/(1 - d_H(A, B))$.

Where d_H (A, B) was the Hausdorff distance

3. Extended Hausdorff Distance and Similarity Measures for Neutrosophic Refined Sets

3.1 Extended Hausdorff Distance Measures for Neutrosophic Refined Sets

On the basis of the extended Hausdorff distance between two neutrosophic set defined by Broumi et al. in [33], defined as follows:

$$d_{HNS}(A, B) = \frac{1}{n}\sum_{i=1}^{n} \max\{|T_A(x_i) - T_B(x_i)|, |I_A(x_i) - I_B(x_i)|, |F_A(x_i)\ - F_B(x_i)|\} \quad (9)$$

Where $d_{HNS}(A, B)$ = H(A, B)denote the extended Hausdorff distance between two neutrosophic sets A and B.

We extend the above equation (9) distance to the case of neutrosophic refined set between A and B as follows:

when the sets A and B do not have the same number of subcomponents for T, I, F. If

A = {<x,(T_A^1(x),T_A^2(x),...,T_A^a(x)), (I_A^1(x),I_A^2(x),...,I_A^b(x)), (F_A^1(x),F_A^2(x),…,F_A^c(x))>: x ∈ X}

and

B = {<x,(T_B^1(x),T_B^2(x),...,T_B^p(x)), (I_B^1(x),I_B^2(x),...,I_B^r(x)), (F_B^1(x),F_B^2(x),…,F_B^q(x))>: x ∈ X}

then we take m = max{a, b, c, p, r, q} and transform A and B into refined neutrosophic sets where all components T, I, F have each of them m subcomponents, i.e.

A = {<x,($T_A^1(x)$,$T_A^2(x)$,...,$T_A^m(x)$), ($I_A^1(x)$,$I_A^2(x)$,...,$I_A^m(x)$), ($F_A^1(x)$,$F_A^2(x)$,…,$F_A^m(x)$)>: x ∈ X}

and

B = {<x,($T_B^1(x)$,$T_B^2(x)$,...,$T_B^m(x)$), ($I_B^1(x)$,$I_B^2(x)$,...,$I_B^m(x)$), ($F_B^1(x)$,$F_B^2(x)$,…,$F_B^m(x)$)>: x ∈ X}.

$$d_{HNRS}(A,B) = \frac{1}{m}\sum_{j=1}^{m}\left\{\frac{1}{n}\sum_{i=1}^{n}\max\left\{\left|T_A^j(x_i) - T_B^j(x_i)\right|, \left|I_A^j(x_i) - I_B^j(x_i)\right|, \left|F_A^j(x_i) - F_B^j(x_i)\right|\right\}\right\} \quad (10)$$

Proposition 3.1 The defined distance $d_{HNRS}(A,B)$ between NRSs A and B satisfies the following properties (D1-D4):

(D1) $d_{HNRS}(A,B) \geq 0$.
(D2) $d_{HNRS}(A,B) = 0$ if and only if A = B; for all A, B ∈ NRSs.
(D3) $d_{HNRS}(A,B) = d_{HNRS}(B,A)$.
(D4) If A⊆B⊆C, for A, B, C ∈ NRSs, then $d_{HNRS}(A, C) \geq d_{HNRS}(A,B)$ and $d_{HNRS}(A,C) \geq d_{HNRS}(B,C)$

Proof: (D1) $d_{HNRS}(A,B) \geq 0$.
As truth-membership, indeterminacy-membership and falsity-membership functions of the NRSs lies between 0 and 1, the distance measure based on these function also lies between 0 to 1.

(D2) $d_{HNRS}(A, B) = 0$ if and only if A = B.

(i) Let the two NRS A and B be equal(i.e) A=B

This implies for any $T_A^j(x_i) = T_B^j(x_i)$, $I_A^j(x_i) = I_B^j(x_i)$ and $F_A^j(x_i) = F_B^j(x_i)$ which states that

$$\left|T_A^j(x_i) - T_B^j(x_i)\right|, \left|I_A^j(x_i) - I_B^j(x_i)\right| \text{ and } \left|F_A^j(x_i) - F_B^j(x_i)\right|$$

Hence $d_{HNRS}(A,B) = 0$.

(ii) Let the $d_{HNRS}(A,B)=0$.

The zero distance measure is possible only if both

$$\left|T_A^j(x_i) - T_B^j(x_i)\right|, \left|I_A^j(x_i) - I_B^j(x_i)\right| \text{ and} \left|I_A^j(x_i) - I_B^j(x_i)\right| = 0$$

as the extended Hausdorff distance measure concerns with maximum truth-membership, indeterminacy-membership and falsity-membership differences.This refers that

$$T_A^j(x_i) = T_B^j(x_i),\ I_A^j(x_i) = I_B^j(x_i) \text{ and } F_A^j(x_i) = F_B^j(x_i)$$

for all i, j values. Hence A=B

(D3) $d_{\mathrm{HNRS}}(A, B)$ = $d_{\mathrm{HNRS}}(B, A)$

It is obvious that

$$T_A^j(x_i) - T_B^j(x_i) \neq T_B^j(x_i) - T_A^j(x_i)\ , I_A^j(x_i) - I_B^j(x_i) \neq I_B^j(x_i) - I_A^j(x_i)$$

and

$$F_A^j(x_i) - T_B^j(x_i) \neq T_B^j(x_i) - T_A^j(x_i)$$

But

$$\left|T_A^j(x_i) - T_B^j(x_i)\right| = \left|T_B^j(x_i) - T_A^j(x_i)\right|, \left|I_A^j(x_i) - I_B^j(x_i)\right| = \left|I_B^j(x_i) - I_A^j(x_i)\right|,$$

and

$$\left|F_A^j(x_i) - F_B^j(x_i)\right| = \left|F_A^j(x_i) - F_B^j(x_i)\right|$$

Hence

$$\begin{aligned} d_{\mathrm{HNRS}}(A, B) &= \frac{1}{m}\sum_{j=1}^{m}\left\{\frac{1}{\mathrm{n}}\sum_{\mathrm{i}=1}^{\mathrm{n}} \max\left\{\left|T_A^j(x_i) - T_B^j(x_i)\right|, \left|I_A^j(x_i) - I_B^j(x_i)\right|, \left|F_A^j(x_i) - F_B^j(x_i)\right|\right\}\right\} \\ &= \frac{1}{m}\sum_{j=1}^{m}\left\{\frac{1}{\mathrm{n}}\sum_{\mathrm{i}=1}^{\mathrm{n}} \max\left\{\left|T_B^j(x_i) - T_A^j(x_i)\right|, \left|I_B^j(x_i) - I_A^j(x_i)\right|, \left|F_B^j(x_i) - F_A^j(x_i)\right|\right\}\right\} \\ &= d_{\mathrm{HNRS}}(B, A). \end{aligned}$$

Remark: Let A, B ∈ NRSs , A⊆ B if and only if

$T_A^j(x_i) \leq T_B^j(x_i)$,$I_A^j(x_i) \geq I_B^j(x_i)$,$F_A^j(x_i) \geq F_B^j(x_i)$ for i=1,2,…,p, for every $x_i \in X$.

(D4) If A⊆B⊆C, for A, B, C ∈ NRSs, then $d_{\mathrm{HNRS}}(A, C) \geq d_{\mathrm{HNRS}}(A, B)$ and $d_{\mathrm{HNRS}}(A, C) \geq d_{\mathrm{HNRS}}(B, C)$

Let A⊆ B ⊆C,then the assumption is

$$T_A^j(x_i) \leq T_B^j(x_i) \leq T_C^j(x_i), I_A^j(x_i) \geq I_B^j(x_i) \geq I_C^j(x_i), F_A^j(x_i) \geq F_B^j(x_i) \geq F_C^j(x_i)$$

Case (i) we prove that $d_H(A, B) \leq d_H(A, C)$

$$\alpha\text{- } if \left|T_A^j(\boldsymbol{x_i}) - T_C^j(\boldsymbol{x_i})\right| \geq \left|I_A^j(\boldsymbol{x_i}) - I_C^j(\boldsymbol{x_i})\right| \geq \left|F_A^j(\boldsymbol{x_i}) - F_C^j(\boldsymbol{x_i})\right|$$

then

H(A,C)=$\left|T_A^j(\mathrm{x_i}) - T_C^j(\mathrm{x_i})\right|$but we have for all $x_i \in X$

(i) $\left|\mathrm{I}_\mathrm{A}^\mathrm{j}(\mathrm{x_i}) - \mathrm{I}_\mathrm{B}^\mathrm{j}(\mathrm{x_i})\right| \leq \left|\mathrm{I}_\mathrm{A}^\mathrm{j}(\mathrm{x_i}) - \mathrm{I}_\mathrm{C}^\mathrm{j}(\mathrm{x_i})\right|$for all $x_i \in X$

$$\leq \left|\mathrm{T}_\mathrm{A}^\mathrm{j}(\mathrm{x_i}) - \mathrm{T}_\mathrm{C}^\mathrm{j}(\mathrm{x_i})\right|$$

And $\left|F_A^j(x_i) - F_B^j(x_i)\right| \le \left|F_A^j(x_i) - F_C^j(x_i)\right|$ for all $x_i \in X$

$\le \left|T_A^j(x_i) - T_C^j(x_i)\right|$

(ii) $\left|I_B^j(x_i) - I_C^j(x_i)\right| \le \left|I_A^j(x_i) - I_C^j(x_i)\right|$ for all $x_i \in X$

$\le \left|T_A^j(x_i) - T_C^j(x_i)\right|$

And $\left|F_B^j(x_i) - F_C^j(x_i)\right| \le \left|F_A^j(x_i) - F_C^j(x_i)\right|$ for all $x_i \in X$

$\le \left|T_A^j(x_i) - T_C^j(x_i)\right|$ for all $x_i \in X$

On the other hand we have

(iii) $\left|T_A^j(x_i) - T_B^j(x_i)\right| \le \left|T_A^j(x_i) - T_C^j(x_i)\right|$

and $\left|T_B^j(x_i) - T_C^j(x_i)\right| \le \left|T_A^j(x_i) - T_C^j(x_i)\right|$ for all $x_i \in X$

Combining (i) and (iii) we obtain therefore

$$\frac{1}{m}\sum_{j=1}^{m}\left\{\frac{1}{n}\sum_{i=1}^{n}\max\{\left|T_A^j(x_i)-T_B^j(x_i)\right|,\left|I_A^j(x_i)-I_B^j(x_i)\right|,\left|F_A^j(x_i)-F_B^j(x_i)\right|\}\right\}$$
$$\le \frac{1}{m}\sum_{j=1}^{m}\left\{\frac{1}{n}\sum_{i=1}^{n}\max\{\left|T_A^j(x_i)-T_C^j(x_i)\right|,\left|I_A^j(x_i)-I_C^j(x_i)\right|,\left|F_A^j(x_i)-F_C^j(x_i)\right|\}\right\}$$
$$\frac{1}{m}\sum_{j=1}^{m}\left\{\frac{1}{n}\sum_{i=1}^{n}\max\{\left|T_B^j(x_i)-T_C^j(x_i)\right|,\left|I_B^j(x_i)-I_C^j(x_i)\right|,\left|F_B^j(x_i)-F_C^j(x_i)\right|\}\right\}$$
$$\le \frac{1}{m}\sum_{j=1}^{m}\left\{\frac{1}{n}\sum_{i=1}^{n}\max\{\left|T_A^j(x_i)-T_C^j(x_i)\right|,\left|I_A^j(x_i)-I_C^j(x_i)\right|,\left|F_A^j(x_i)-F_C^j(x_i)\right|\}\right\}$$

That is

$$d_{\mathrm{HNRS}}(A,B) \le d_{\mathrm{HNRS}}(A,C) \text{ AND } d_{\mathrm{HNRS}}(B,C) \le d_{\mathrm{HNRS}}(A,C)$$

β - If $\left|T_A^j(\mathbf{x_i}) - T_C^j(\mathbf{x_i})\right| \le \left|F_A^j(\mathbf{x_i}) - F_C^j(\mathbf{x_i})\right| \le \left|I_A^j(\mathbf{x_i}) - I_C^j(\mathbf{x_i})\right|$

Then

H(A,C)=$\left|I_A^j(x_i) - I_C^j(x_i)\right|$ But we have for all $x_i \in X$

(a) $\left|T_A^j(x_i) - T_B^j(x_i)\right| \le \left|T_A^j(x_i) - T_C^j(x_i)\right|$ for all $x_i \in X$

$\le \left|I_A^j(x_i) - I_C^j(x_i)\right|$

And $\left|F_A^j(x_i) - F_B^j(x_i)\right| \le \left|F_A^j(x_i) - F_C^j(x_i)\right|$ for all $x_i \in X$

$$\leq \left|I_A^j(x_i) - I_C^j(x_i)\right|$$

(b) $\left|T_B^j(x_i) - T_C^j(x_i)\right| \leq \left|T_A^j(x_i) - T_C^j(x_i)\right|$for all $x_i \in X$

$$\leq \left|I_A^j(x_i) - I_C^j(x_i)\right|$$

and$\left|F_B^j(x_i) - F_C^j(x_i)\right| \leq \left|F_A^j(x_i) - F_C^j(x_i)\right|$for all $x_i \in X$

$$\leq \left|I_A^j(x_i) - I_C^j(x_i)\right|$$

On the other hand we have

(c) $\left|I_A^j(x_i) - I_B^j(x_i)\right| \leq \left|I_A^j(x_i) - I_C^j(x_i)\right|$and

$\left|I_B^j(x_i) - I_C^j(x_i)\right| \leq \left|I_A^j(x_i) - I_C^j(x_i)\right|$ for all $x_i \in X$.

Combining (a) and (c) we obtain, therefore

$$\frac{1}{m}\sum_{j=1}^{m}\left\{\frac{1}{n}\sum_{i=1}^{n}\max\{\left|T_A^j(x_i) - T_B^j(x_i)\right|, \left|I_A^j(x_i) - I_B^j(x_i)\right|, \left|F_A^j(x_i) - F_B^j(x_i)\right|\}\right\}$$
$$\leq \frac{1}{m}\sum_{j=1}^{m}\left\{\frac{1}{n}\sum_{i=1}^{n}\max\{\left|T_A^j(x_i) - T_C^j(x_i)\right|, \left|I_A^j(x_i) - I_C^j(x_i)\right|, \left|F_A^j(x_i) - F_C^j(x_i)\right|\}\right\}$$
$$\frac{1}{m}\sum_{j=1}^{m}\left\{\frac{1}{n}\sum_{i=1}^{n}\max\{\left|T_B^j(x_i) - T_C^j(x_i)\right|, \left|I_B^j(x_i) - I_C^j(x_i)\right|, \left|F_B^j(x_i) - F_C^j(x_i)\right|\}\right\}$$
$$\leq \frac{1}{m}\sum_{j=1}^{m}\left\{\frac{1}{n}\sum_{i=1}^{n}\max\{\left|T_A^j(x_i) - T_C^j(x_i)\right|, \left|I_A^j(x_i) - I_C^j(x_i)\right|, \left|F_A^j(x_i) - F_C^j(x_i)\right|\}\right\}$$

That is

$$d_{\mathrm{HNRS}}(A,B) \leq d_{\mathrm{HNRS}}(A,C) \text{ AND } d_{\mathrm{HNRS}}(B,C) \leq d_{\mathrm{HNRS}}(A,C)$$

γ**. If** $\left|T_A^j(\mathbf{x_i}) - T_C^j(\mathbf{x_i})\right| \leq \left|I_A^j(\mathbf{x_i}) - I_C^j(\mathbf{x_i})\right| \leq \left|F_A^j(\mathbf{x_i}) - F_C^j(\mathbf{x_i})\right|$

Then

$H(A,C)=|F_A(x_i) - F_C(x_i)|$ But we have for all $x_i \in X$

(a) $\left|T_A^j(\mathbf{x_i}) - T_B^j(\mathbf{x_i})\right| \leq \left|T_A^j(\mathbf{x_i}) - T_C^j(\mathbf{x_i})\right|$for all $x_i \in X$

$$\leq \left|F_A^j(\mathbf{x_i}) - F_C^j(\mathbf{x_i})\right|$$

And $\left|I_A^j(x_i) - I_B^j(x_i)\right| \leq \left|I_A^j(x_i) - I_C^j(x_i)\right|$ for all $x_i \in X$

$$\leq \left|F_A^j(x_i) - F_C^j(x_i)\right|$$

(b) $\left|T_B^j(\mathbf{x_i}) - T_C^j(\mathbf{x_i})\right| \leq \left|T_A^j(\mathbf{x_i}) - T_C^j(\mathbf{x_i})\right|$for all $x_i \in X$

$$\leq \left|F_A^j(\mathbf{x_i}) - F_C^j(\mathbf{x_i})\right|$$

And $\left|I_B^j(\mathbf{x_i}) - I_C^j(\mathbf{x_i})\right| \leq \left|I_A^j(\mathbf{x_i}) - I_C^j(\mathbf{x_i})\right|$ for all $x_i \in X$

$$\leq \left|F_A^j(\mathbf{x_i}) - F_C^j(\mathbf{x_i})\right|$$

On the other hand we have

(c) $\left|F_A^j(\mathbf{x_i}) - F_B^j(\mathbf{x_i})\right| \leq \left|F_A^j(\mathbf{x_i}) - F_C^j(\mathbf{x_i})\right|$ and $\left|F_B^j(\mathbf{x_i}) - F_C^j(\mathbf{x_i})\right| \leq \left|F_A^j(\mathbf{x_i}) - F_C^j(\mathbf{x_i})\right|$ for all $x_i \in X$

Combining (a) and (c) we obtain

$$\frac{1}{m}\sum_{j=1}^{m}\left\{\frac{1}{n}\sum_{i=1}^{n}\max\{\left|T_A^j(x_i) - T_B^j(x_i)\right|, \left|I_A^j(x_i) - I_B^j(x_i)\right|, \left|F_A^j(x_i) - F_B^j(x_i)\right|\}\right\}$$
$$\leq \frac{1}{m}\sum_{j=1}^{m}\left\{\frac{1}{n}\sum_{i=1}^{n}\max\{\left|T_A^j(x_i) - T_C^j(x_i)\right|, \left|I_A^j(x_i) - I_C^j(x_i)\right|, \left|F_A^j(x_i) - F_C^j(x_i)\right|\}\right\}$$

$$\frac{1}{m}\sum_{j=1}^{m}\left\{\frac{1}{n}\sum_{i=1}^{n}\max\{\left|T_B^j(x_i) - T_C^j(x_i)\right|, \left|I_B^j(x_i) - I_C^j(x_i)\right|, \left|F_B^j(x_i) - F_C^j(x_i)\right|\}\right\}$$
$$\leq \frac{1}{m}\sum_{j=1}^{m}\left\{\frac{1}{n}\sum_{i=1}^{n}\max\{\left|T_A^j(x_i) - T_C^j(x_i)\right|, \left|I_A^j(x_i) - I_C^j(x_i)\right|, \left|F_A^j(x_i) - F_C^j(x_i)\right|\}\right\}$$

That is

$$d_{\mathrm{HNRS}}(A,B) \leq d_{\mathrm{HNRS}}(A,C) \text{ AND } d_{\mathrm{HNRS}}(B,C) \leq d_{\mathrm{HNRS}}(A,C).$$

From γ, α and β, we can obtain the property (D4).

3.2 Weighted Extended Hausdorff Distance between Neutrosophic Refined Sets

In many situation the weight of the element $x_i \in X$ should be taken into account. Usually the elements have different importance .We need to consider the weight of the element so that we have the following weighted distance between NRSs. Assume that the weight of $x_i \in X$ is w_i where $X=\{x_1, x_2,..,x_n\}$,$w_i \in [0, 1]$, i= 1, 2, 3, .., n and $\sum_1^n w_i = 1$. Then the weighted extended hausdorff distance between NRSs A and B is defined as:

$$d_{\mathrm{WHNRS}}(A,B) = \sum_1^n w_i\, d_{\mathrm{HNRS}}(A(x_i), B(x_i)) \qquad (11)$$

It is easy to check that $d_{\mathrm{WHNRS}}(A,B)$ satisfies the four properties D1-D4 defined above.

It is well known that similarity measure can be generated from distance measure. Therefore we may use the proposed distance measure to define similarity measures.

Based on the relationship of similarity measure and distance we can define some similarity measures between NRSs A and B as follows.

Definition The similarity measure based on the extended Hausdorff distance is

$s^1_{\mathrm{HNRS}}(A, B) = 1 - d_{\mathrm{HNRS}}(A, B)$
$s^2_{\mathrm{HNRS}}(A, B) = (e^{-d_{\mathrm{HNRS}}(A,B)} - e^{-1})/(1 - e^{-1})$
$s^3_{\mathrm{HNRS}}(A, B) = (1 - d_{\mathrm{HNRS}}(A, B))/(1 + d_{\mathrm{HNRS}}(A, B))$

$s_{\mathrm{HNRS}}(A, B)$ is said to be the similarity measure between A an B, where A ,B∈ NRS, as $s_{\mathrm{HNRS}}(A, B)$ satisfies the following properties

(P1) S(A,B) = S(B, A).
(P2) S(A,B) = (1, 0, 0)=$\underline{1}$ if A=B for all A,B ∈ NVNSs.
(P3) $S(A, B) \in [0, 1]$
(P4) If A⊆B⊆C for all A, B, C ∈ NRSs then S(A, B)≥S(A, C) and S(B, C) ≥ S(A, C)

4. Medical Diagnosis Using Extended Hausdorff Distance of NRS.

In what follows, let us consider an illustrative example adopted from [40].

"As Medical diagnosis contains lots of uncertainties and increased volume of information available to physicians from new medical technologies, the process of classifying different set of symptoms under a single name of disease becomes difficult. In some practical situations, there is the possibility of each element having different truth membership, indeterminate and false membership functions. The proposed correlation measure among the patients Vs symptoms and symptoms Vs diseases gives the proper medical diagnosis. The unique feature of this proposed method is that it considers multi truth membership, indeterminate and false membership. By taking one time inspection, there may be error in diagnosis. Hence, this multi time inspection, by taking the samples of the same patient at different times gives best diagnosis".

Now, an example of a medical diagnosis will be presented.

Example 1 Let P={P_1, P_2, P_3} be a set of patients, D={Viral Fever, Tuberculosis, Typhoid, Throat disease} be a set of diseases and S={Temperature, cough, throat pain, headache, body pain} be a set of symptoms. Our solution is to examine the patient at different time intervals (three times a day), which in turn give arise to different truth membership, indeterminate and false membership function for each patient

Table I -3-valued neutrosophic set: The Relation between Patient and Symptoms

Q	Temperature	Cough	Throat	Head Ache	Body Pain
P_1	(0.4, 0.3, 0.4) (0.2, 0.5, 0.5) (0.3, 0.4, 0.6)	(0.5, 0.4, 0.4) (0.4, 0.1, 0.3) (0.3, 0.4, 0.5)	(0.3, 0.5, 0.5) (0.2, 0.6, 0.4) (0.1, 0.6, 0.3)	(0.5, 0.3, 0.4) (0.5, 0.4, 0.7) (0.3, 0.3, 0.6)	(0.5, 0.2, 0.4) (0.2, 0.3, 0.5) (0.1, 0.4, 0.3)
P_2	(0.6, 0.3, 0.5) (0.5, 0.5, 0.2) (0.4, 0.4, 0.5)	(0.6, 0.3, 0.7) (0.4, 0.4, 0.2) (0.2, 0.4, 0.5)	(0.6, 0.3, 0.3) (0.3, 0.5, 0.4) (0.1, 0.4, 0.5)	(0.6, 0.3, 0.1) (0.4, 0.5, 0.8) (0.2, 0.4, 0.3)	(0.4, 0.4, 0.5) (0.3, 0.2, 0.7) (0.1, 0.5, 0.5)
P_3	(0.8, 0.3, 0.5) (0.7, 0.5, 0.4) (0.6, 0.4, 0.4)	(0.5, 0.5, 0.3) (0.1, 0.6, 0.4) (0.3, 0.4, 0.3)	(0.3, 0.3, 0.6) (0.2, 0.5, 0.7) (0.1, 0.4, 0.5)	(0.6, 0.2, 0.5) (0.5, 0.3, 0.6) (0.2, 0.2, 0.6)	(0.6, 0.4, 0.5) (0.3, 0.3, 0.4) (0.2, 0.2, 0.6)

Let the samples be taken at three different timing in a day (morning, noon and night)

Table II – neutrosophic refined set:The Relation between Patient and Symptoms

Q	Viral Fever	Tuberculosis	Typhoid	Throat disease
Temperature	(0.2, 0.5, 0.6)	(0.4, 0.6, 0.5)	(0.4, 0.3, 0.5)	(0.3, 0.7, 0.8)
Cough	(0.6, 0.4, 0.6)	(0.8, 0.2, 0.3)	(0.3, 0.2, 0.6)	(0.2, 0.4, 0.1)
Throat	(0.5, 0.2, 0.3)	(0.4, 0.5, 0.3)	(0.4, 0.5, 0.5)	(0.2, 0.6, 0.2)
Head Ache	(0.6, 0.8, 0.2)	(0.2, 0.3, 0.6)	(0.1, 0.6, 0.3)	(0.2, 0.5, 0.5)
Body Pain	(0.7, 0.5, 0.4)	(0.2, 0.3, 0.4)	(0.2, 0.3, 0.4)	(0.2, 0.2, 0.3)

Distance measure (d)

Proposed distance measure	Viral Fever	Tuberculosis	Typhoid	Throat disease
P_1	0.34667	0.2533	0.22	0.2733
P_2	0.366	0.3	0.266	0.346
P_3	0.426	0.326	0.3067	0.34

S=1-d

Proposed similarity measure(s^1_{HNVNS}(A,B))	Viral Fever	Tuberculosis	Typhoid	Throat disease
P_1	0.6534	0.7467	**0.78**	0.7267
P_2	0.634	0.7	**0.733**	0.654
P_3	0.574	0.674	**0.693**	0.66

Proposed similarity measure(s^3_{HNVNS}(A,B)	Viral Fever	Tuberculosis	Typhoid	Throat disease
P_1	0.4852	0.5957	**0.6393**	0.5707
P_2	0.4641	0.5384	**0.5797**	0.4858
P_3	0.4025	0.5082	**0.5306**	0.4925

The highest similarity measure from table gives the proper medical diagnosis

Patient P1 suffers from Tuberculosis and Typhoid, Patient P2 suffers from Typhoid and Patient P3 suffers Typhoid

5 Conclusions

In this paper we have presented a new distance measure between NRS on the basis of extended Hausdorff distance of neutrosophic set, then we proved their properties. Finally we used this distance measure in an application of medical diagnosis. It's hoped that our findings will help enhancing this study on neutrosophic set for researchers. In future work, we will extended this distance to the case of interval neutrosophic refined sets.

References

[1] Q. Ansari, R. Biswas and S. Aggarwal, "Neutrosophic classifier: An extension of Fuzzy Classifer," Applied Soft Computing, 13,pp.563-573, 2013.

[2] A. Kharal, "A Neutrosophic Multicriteria Decision Making Method," New Mathematics and Natural Computation, Creighton University, USA, 2013.

[3] C. Ashbacher, "Introduction to Neutrosophic Logic," American Research Press, Rehoboth, 2002.

[4] D. Rabounski, F. Smarandache, L. Borissova, "Neutrosophic Methods in General Relativity,"Hexis, 10, 2005.

[5] E. Szmidt and J. Kacprzyk, "A Note on the Hausdorff Distance between Atanassov's Intuitionistic Fuzzy Sets," NIFS ,vol. 15,no. 1, pp.1-12, 2009.

[6] F. Smarandache, "A Unifying Field in Logics, "Neutrosophy: Neutrosophic Probability," Set and Logic, Rehoboth: American Research Press, 1998.

[7] F. G. Lupiáñez, "On Neutrosophic Topology, "Kybernetes, 37(6),pp.797-800, 2008.

[8] Florentin Smarandache, n-Valued Refined Neutrosophic Logic and Its Applications to Physics, Annual Fall Meeting of the American Physical Society Ohio-Region Section, Friday–Saturday, October 4–5, 2013; Cincinnati, Ohio. http://meetings.aps.org/Meeting/OSF13/Event/205641

[9] H. Wang, F. Smarandache, Y. Q. Zhang and R. Sunderraman, "Single Valued Neutrosophic Sets," Multispace and Multistructure 4,pp.410-413, 2010.

[10] Deli, S. Broumi and F. Smarandache, "Neutrosophic Soft Matrices and NSM-Decision Making," Journal of Intelligent & Fuzzy Systems, DOI:10.3233/IFS-141505,2015, in press

[11] I. Deli and S. Broumi, "Neutrosophic Soft Relations and Some Properties, "AFMI, vol.9, no.1, pp.169-182, 2015.

[12] I. Deli, Y. Toktas, and S. Broumi, "Neutrosophic Parameterized Soft Relations and Their Application," NSS, vol. 4, pp.25-34, 2014.

[13] M. Hanafy, A. A. Salama and K. Mahfouz, “Correlation of Neutrosophic Data,” International Refereed Journal of Engineering and Science, 1(2), pp.39- 43, 2012.
[14] M. Hanafy, A. A. Salama and K. Mahfouz, “Correlation coefficients of neutrosophic sets by centroid Method,” International Journal of Probability and Statistics, 2(1), pp.9-12, 2013.
[15] J. Ye,”Multicriteria Decision-Making Method using the Correlation Coefficient under Single-Valued Neutrosophic Environment,” International Journal of General Systems, 42(4), pp.386-394, 2013.
[16] J. Ye, “Similarity Measure between Interval Neutrosophic Sets and Their Applications in Multiciteria Decision Making, ”Journal of intelligent and fuzzy systems 26, pp-165-172, 2014.
[17] J. Ye, “Single Valued Neutrosophic Cross-Entropy for Multicriteria Decision Making Problems,” Appl.Math.Model., vol.38, pp.1170-1175, 2014.
[18] J. Ye,”Cosine Similarity Measures for Intuitionistic Fuzzy Sets and Their Applications,” Math. Comput.Model.,vol.53, pp. 91-97, 2011.
[19] J. Ye, “Vector Similarity Measures of Simplified Neutrosophic Sets and Their Application in Multicriteria decision making,” International Journal of Fuzzy Systems, vol. 16, no. 2, pp.204-215, 2014.
[20] K. T. Atanassov, “Intuitionistic Fuzzy Set,” Fuzzy Sets and Systems, 20(1), pp.87-86, 1986.
[21] L. A. Zadeh, “Fuzzy Sets,” Inform. and Control, 8 pp-338-353, 1965.
[22] M. Arora and R. Biswas, “Deployment of Neutrosophic Technology to Retrieve Answers for Queries Posed in Natural Language,” 3. Int. Conf. on Comp. Sci. and Inform. Tech., pp.435-439. 2010.
[23] M. Arora, R. Biswas and U. S. Pandy, “Neutrosophic Relational Database Decomposition,” Int. J. of Advanced Computer Science and Applications, 2(8), pp.121-125, 2011.
[24] M. Bora, B. Bora, T. J. Neog, and D. K. Sut, “Intuitionistic Fuzzy Soft Matrix Theory and it's Application in Medical Diagnosis,” AFMI, 7(1) pp.143-153, 2014.
[25] M. Dhar, S. Broumi and F. Smarandache. “A Note on Square Neutrosophic Fuzzy Matrices,” NSS, vol 3. pp.37-41, 2014.
[26] P. A. Ejegwa, J. A. Awolola, Intuitionistic Fuzzy Multiset (IFMS) In Binomial Distributions, Int. J. Of Scientific and Technology Research, 3(4) pp.335-337, 2014.
[27] P. Rajarajeswari and N. Uma,” On Distance and Similarity Measures of Intuitionistic Fuzzy Multi Set,” IOSR Journal of Mathematics, 5(4),pp.19- 23, 2013
[28] P. Rajarajeswari and N. Uma, “A Study of Normalized Geometric and Normalized Hamming Distance Measures In Intuitionistic Fuzzy Multi Sets,” Int. J. of Science and Research, Engineering and Technology, 2(11) ,pp.76-80, 2013.
[29] P. Rajarajeswari, N. Uma, “Correlation Measure for Intuitionistic Fuzzy Multi Sets,” Int. J. of Research in Engineering and Technology, 3(1), pp. 611-617, 2014.
[30] R. R. Yager, “On the Theory of Bags (Multi sets), “Int. J. of General System, 13 pp.23-37, 1986.
[31] S. Broumi, “Generalized Neutrosophic Soft Set,” Int. J. of Comput. Sci., Eng. and Inform. Tech, 3(2), pp.17- 30, 2013.
[32] S. Broumi, F. Smarandache, “Correlation Coefficient of Interval Neutrosophic Set,” Appl. Mech. Mater. 436 511–517, 2013.
[33] S. Broumi, F. Smarandache, “Several Similarity Measures of Neutrosophic Sets,” NSS, 1 pp. 54-62, 2013.
[34] S. Broumi, F. Smarandache,” More on Intuitionistic Neutrosophic Soft Sets,” Computer Science and Information Technology, 1(4), pp.257-268, 2013.
[35] S. Broumi, I. Deli and F. Smarandache, “Relations on Interval Valued Neutrosophic Soft Sets,” JNRS, 5, pp.1-20, 2014.

[36] S. Broumi, I. Deli, and F. Smarandache," Neutrosophic parametrized Soft Set Theory and Its Decision Making Problem," International Frontier Science Letters, 1 (1), pp.01-11. 2014.
[37] S.Broumi and F Smarandache, "On neutrosophic implications," NSS, vol. 2, pp. 9-17, 2014.
[38] S. Broumi, and F. Smarandache, "Lower and Upper Soft Interval Valued Neutrosophic Rough Approximations of An IVNSS-Relation," Proceedings of SISOM ACOUSTICS, International Conference, Bucharest, 22-23 May 2014, pp. 204-211.
[39] S. Broumi, M. Dhar, and F. Smarandache, "Rough Neutrosophic Sets," Italian Journal of Pure and Applied Mathematics, no. 32, pp.493-502, 2014.
[40] S. Broumi and I. Deli, "Correlation Measure for Neutrosophic Refined Sets and Its Application In Medical Diagnosis," Palestine Journal of Mathematics, Vol. 5(1) (2016) , 135–143
[41] S. Broumi, and F. Smarandache, "Neutrosophic Refined Similarity Measure Based on Cosine Function," NSS,vol.6, pp.42-48, 2014.
[42] S. Broumi, I. Deli and F. Smarandache, "Distance and Similarity Measures of Interval Neutrosophic Soft Sets," Critical Review, Center for Mathematics of Uncertainty, Creighton University,USA,8 pp.14–31, 2014.
[43] S. Sebastian and T. V. Ramakrishnan, "Multi-Fuzzy Sets," International Mathematical Forum, 5(50), pp. 2471-2476, 2010.
[44] T. K. Shinoj and S. J. John, "Intuitionistic Fuzzy Multisets and Its Application in Medical Diagnosis," WASET, vol.6, pp.01-28, 2012.