http://www.newtheory.org **ISSN: 2149-1402**

Journal of New Theory

Received: 16.05.2015 *Year*: **2015,** *Number*: **7**, *Pages*: **86-97**
Published: 06.10.2015 *Original Article*[**]

ON HYPERSPACES OF SOFT SETS

Metin Akdağ[1] <makdag@cumhuriyet.edu.tr>
Fethullah Erol[1,*] <feerol@cumhuriyet.edu.tr>

[1]*Cumhuriyet University, Department of Mathematics, 58200 Sivas, Turkey.*

Abstract – In this paper first, we introduce the soft sets families$(V,K)^{+}$, $(V,K)^{-}$ and we investigate basic properties of them. Second, by use these soft families, we introduce some hyperspaces of soft sets, called upper (lower) and Vietoris soft topological space, which defined on classes of soft sets $2^{S(Y,K)}$. Third, we define the upper and lower Vietoris continuity of soft multifunction and we give the relationship between Vietoris continuity of soft multifunction and continuity of soft mapping.

Keywords – *Soft hyperspaces, soft Vietoris topological spaces, soft sets, soft multifunction, soft continuity.*

1. Introduction

There are many complex problems in the several fields of sciences that involve uncertainties in data. Several set theories can be regarded as mathematical tools for dealing with these uncertainties, but these theories sometimes fail to handle uncertainty properly. This limitation was pointed by Molodtsov [7]. He introduced the concept of soft set theory. Çağman et al [4] defined a soft topological space. Shabir and Naz [10] introduced the notions of soft topological spaces. Then, Zorlutuna and et al. [11] studied the properties of soft topological spaces. After that, Kharal and Ahmad [5] defined a mapping on soft classes and studied properties of these mappings. Then Akdağ and Erol [1,2] introduced the concept of soft multifunction and studied their properties. In this paper we define and study the hyperspaces of soft sets.

2. Preliminaries

Definition 2.1. [7] Let X be an initial universe and E be a set of parameters. Let $P(X)$ denote the power set of X and A be a non-empty subset of E. A pair (F,A) is called a soft set over X, where F is a mapping given by $F: A \rightarrow P(X)$. In other words, a soft set over X is

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[**]*Edited by* Serkan Karataş *(Area Editor) and* Naim Çağman *(Editor-in-Chief).*
[*]*Corresponding Author.*

a parameterized family of subsets of the universe X. For $\varepsilon \in A$, $F(\varepsilon)$ may be considered as the set of ε-approximate elements of the soft set (F,A).

Definition 2.2. [6] A soft set (F,A) over X is called a null soft set, denoted by Φ_A, if $F(e)=\emptyset$ for all $e \in A$. If $A=E$, then the null soft set denoted by Φ.

Definition 2.3. [6] A soft set (F,A) over X is called an absolute soft set, denoted by $\widetilde{X_A}$, if $F(e)=X$ for all $e\in A$. If $A=E$, then the A-universal soft set is called a universal soft set, denoted by $\tilde{X}$.

Definition 2.4. [6] The union of two soft sets of (F,A) and (G,B) over the common universe X is the soft set (H,C), where $C=A\cup B$ and for all $e\in C$,

$$H(e)=\begin{cases} \mathrm{F(e)}, & \text{if } \mathrm{e} \in \mathrm{A}-\mathrm{B}, \\ \mathrm{G(e)}, & \text{if } \mathrm{e} \in \mathrm{B}-\mathrm{A}, \\ \mathrm{F(e)} \cup \mathrm{G(e)}, & \text{if } \mathrm{e} \in \mathrm{A} \cap \mathrm{B}. \end{cases}$$

We write $(F,A)\,\tilde{\cup}\,(G,B)=(H,C)$.

Definition 2.5. [6] Let (F,A) and (G,B) be two soft sets over a common universe X. The soft intersection (F,A) and (G,B) is also a soft set $(H,C)=(F,A)\,\tilde{\cap}\,(G,B)$ and defined as $H(e)=F(e)\cap G(e)$ for all $e\in C$, where $C=A\cap B$.

Definition 2.6. [6] Let (F,A) and (G,B) be two soft sets over a common universe X. (F,A) is soft subset of (G,B), if $A\subset B$ and $F(e)\subset G(e)$ for all $e\in A$. Then we write $(F,A)\,\tilde{\subseteq}\,(G,B)$.

Definition 2.7. [3] For a soft set (F,E) over X the relative complement of (F,E) is denoted by $(F,E)^c$ and is defined by $(F,E)^c=(F^c,E)$, where $F^c\colon E\to P(X)$ is a mapping given by $F^c(e)=X-F(e)$ for all $e\in E$.

Definition 2.8. [10] Let τ be the collection of soft sets over X, then τ is said to be a soft topology on X if satisfies the following axioms.

(1) $\Phi,\tilde{X}$ belong to τ,
(2) the union of any number of soft sets in τ belongs to τ,
(3) the intersection of any two soft sets in τ belongs to τ.

The triplet (X,τ,E) is called a soft topological space over X and the members of τ are said soft open sets in X. A soft set (F,E) over X is said soft closed set in X, if its relative complement $(F,E)^c$ belongs to τ.

Definition 2.9. [8] Let (X,τ,E) be a soft topological space. A sub-collection β of τ is said to be a soft open base of τ if every member of τ can be expressed as the union of some members of β.

Example 2.10. [8] Let $X=\{x_1,x_2,x_3,x_4,x_5\}$, $E=\{e_1,e_2\}$ and
$\tau=\{\,\Phi,\tilde{X},\ \{(e_1,\{x_1\}),(e_2,\{x_2\})\},\ \{(e_1,\{x_4\}),(e_2,\{x_3\})\},\ \{(e_1,\{x_2,x_3\}),(e_2,\{x_1,x_4\})\},$
$\{(e_1,\{x_1,x_4\}),(e_2,\{x_2,x_3\})\},\ \{(e_1,\{x_1,x_2,x_3\}),(e_2,\{x_1,x_2,x_4\})\},$

$\{(e_1,\{x_2,x_3,x_4\}),(e_2,\{x_1,x_3,x_4\})\}$, $\{(e_1,\{x_1,x_2,x_3,x_4\}),(e_2,\{x_1,x_2,x_3,x_4\})\}$ }. Then τ is a soft topology over X.

Let $\beta=\{\Phi,\tilde{X},\{(e_1,\{x_1\}),(e_2,\{x_2\})\},\{(e_1,\{x_4\}),(e_2,\{x_3\})\},\{(e_1,\{x_2,x_3\}),(e_2,\{x_1,x_4\})\}\}$.

Then β forms an soft open base of τ.

Proposition 2.11. [8] A collection β of soft open sets of a soft topological space (X,τ,E) forms an open base of τ iff $\forall(F,E)\in\tau$ and $\forall E_e{}^x\,\tilde{\in}\,(F,E)$, $\exists(G,E)\in\beta$ such that $E_e{}^x\,\tilde{\in}\,(G,E)\,\tilde{\subset}\,(F,E)$.

Proposition 2.12. [8] A collection β of soft subsets over X forms an open base of a soft topology over X iff the following conditions are satisfied.

(i) $\Phi\in\beta$
(ii) $\tilde{X}$ is union of the members of β
(iii) If $(F,E),(G,E)\in\beta$ then $(F,E)\,\tilde{\cap}\,(G,E)$ is union of some members of β, i.e. $(F,E),(G,E)\in\beta$ and $E_e{}^x\,\tilde{\in}\,(F,E)\,\tilde{\cap}\,(G,E)$ then $\exists(H,E)\in\beta$ such that $E_e{}^x\,\tilde{\in}\,(H,E)\,\tilde{\subset}\,(F,E)\,\tilde{\cap}\,(G,E)$.

Definition 2.13. [9] A collection β of some soft subsets of (F,A) is called a soft open base or simply a base for some soft topology on (F,A) if the following conditions hold:

(i) $\Phi\in\beta$
(ii) $\cup\,\beta=(F,A)$ i.e., for each $e\in A$ and $x\in(F,A)(e)$, there exists $(G,B)\in\beta$ such that $x\in(G,B)(e)$, where $B\subseteq A$.
(iii) If $(G,B),(H,C)\in\beta$ then for each $e\in B\cap C$ and $x\in((G,B)\cap(H,C))(e)=(G,B)(e)\cap(H,C)(e)$, there exists $(I,D)\in\beta$ such that $(I,D)\,\tilde{\subset}\,(G,B)\,\tilde{\cap}\,(H,C)$ and $x\in(I,D)(e)$, where $D\subseteq B\cap C$.

Example 2.14. [9] Let $X=\{x_1,x_2,x_3,x_4,x_5,x_6,x_7,x_8,x_9,x_{10}\}$, $E=\{e_1,e_2,e_3,e_4,e_5\}$, $A=\{e_1,e_2,e_3,e_4\}$ and $(F,A)=\{(e_1,\{x_1,x_5,x_8\}),\ (e_2,\{x_2,x_6,x_9\}),\ (e_3,\{x_3,x_7,x_9\}),\ (e_4,\{x_4,x_7,x_{10}\})\}$ be a soft set.

Now let us consider the collection

$\tau=\{\Phi,(F,A),\{(e_2,\{x_2\})\},\{(e_4,\{x_4\})\},\{(e_1,\{x_1\}),(e_3,\{x_3\})\},\{(e_2,\{x_2\}),(e_4,\{x_4\})\},$
$\{(e_2,\{x_2,x_9\}),(e_4,\{x_4,x_7\})\},\{(e_1,\{x_1\}),(e_2,\{x_2\}),(e_3,\{x_3\})\},$
$\{(e_1,\{x_1\}),(e_3,\{x_3\}),(e_4,\{x_4\})\},\{(e_1,\{x_1,x_5\}),(e_2,\{x_2,x_6\}),(e_3,\{x_3,x_7\})\},$
$\{(e_1,\{x_1,x_8\}),(e_3,\{x_3,x_9\}),(e_4,\{x_4,x_{10}\})\},\{(e_1,\{x_1\}),(e_2,\{x_2\}),(e_3,\{x_3\}),(e_4,\{x_4\})\},$
$\{(e_1,\{x_1\}),(e_2,\{x_2,x_9\}),(e_3,\{x_3\}),(e_4,\{x_4,x_7\})\},$
$\{(e_1,\{x_1,x_8\}),(e_2,\{x_2\}),(e_3,\{x_3,x_9\}),(e_4,\{x_4,x_{10}\})\},$
$\{(e_1,\{x_1,x_5\}),(e_2,\{x_2,x_6\}),(e_3,\{x_3,x_7\}),(e_4,\{x_4\})\},$
$\{(e_1,\{x_1,x_8\}),(e_2,\{x_2,x_9\}),(e_3,\{x_3,x_9\}),(e_4,\{x_4,x_7,x_{10}\})\},$
$\{(e_1,\{x_1,x_5\}),(e_2,\{x_2,x_6,x_9\}),(e_3,\{x_3,x_7\}),(e_4,\{x_4,x_7\})\},$
$\{(e_1,\{x_1,x_5,x_8\}),(e_2,\{x_2,x_6\}),(e_3,\{x_3,x_7,x_9\}),(e_4,\{x_4,x_{10}\})\}\}$

of some soft subsets of (F,A). Then obviously, τ forms a soft topology on a soft set (F,A).

If we take

$\beta = \{\Phi, \{(e_2, \{x_2\})\}, \{(e_4, \{x_4\})\}, \{(e_1, \{x_1\}), (e_3, \{x_3\})\}, \{(e_2, \{x_2, x_9\}),\ (e_4, \{x_4, x_7\})\},$
$\{(e_1, \{x_1, x_5\}), (e_2, \{x_2, x_6\}), (e_3, \{x_3, x_7\})\}, \{(e_1, \{x_1, x_8\}),\ (e_3, \{x_3, x_9\}),\ (e_4, \{x_4, x_{10}\})\}\}.$

Then obviously, β forms a soft base for the topology τ on (F, A).

Theorem 2.15. [9] Let β be a soft base for a soft topology on (F, A). Suppose τ_β consists of those soft subset (G, B) of (F, A) for which corresponding to each $e \in B$ and $x \in (G, B)(e)$, there exists $(H, C) \in \beta$ such that $(H, C) \tilde{\subseteq} (G, B)$ and $x \in (H, C)(e)$, where $C \subseteq B$. Then $\tau_\beta$is a soft topology on (F, A).

Definition 2.16. [9] Suppose β is a soft base for a soft topology on (F, A). Then τ_β, described in above theorem, is called the soft topology generated by β and β is called the soft base for τ_β.

Theorem 2.17. [9] Let β be a soft base for a soft topology on (F, A). Then $(G, B) \in \tau_\beta$ if and only if $(G, B) = \bigcup(G_i, B)$, where $(G_i, B) \in \beta$ for each $i \in \Lambda$ and Λ an index set.

Theorem 2.18. [9] Let $((F, A), \tau)$ be a soft topological space and β be a sub collection of τ such that every member of τ is a union of some members of β. Then β is a soft base for the soft topology τ on (F, A).

Definition 2.19. [9] A collection Ω the members of a soft topology τ is said to be subbase for τ if and only if the collection of all finite intersections of members of Ω is a base for τ.

Theorem 2.20. [9] A collection Ω of soft subsets of (F, A) is a subbase for a suitable soft topology τ on (F, A) if and only if
(i) $\Phi \in \Omega$ or Φ is the intersection of a finite number of members of Ω
(ii) $(F, A) = \bigcup\Omega$.

3. Hyperspaces of Soft Sets

Definition 3.1. Let (G, K) be a soft open set in a soft topological space (Y, τ, K) and $S(Y, K)$ is the family of soft set on X. Then the soft set families $(G, K)^+$ and $(G, K)^-$ are defined as follows:

$(G, K)^+ = \{(T, K) \in S(Y, K) : (T, K) \tilde{\subseteq} (G, K)\},$
$(G, K)^- = \{(T, K) \in S(Y, k) : (T, K) \tilde{\cap} (G, K) \neq \Phi\}.$

Proposition 3.2. Let (Y, τ, K) be a soft topological space. For a non null soft sets (G, K) and (H, K) the following statements are true;

(a) $(G, K)^+ \cap (H, K)^+ = ((G, K) \tilde{\cap} (H, K))^+$
(b) $(G, K)^+ \cup (H, K)^+ \subset ((G, K) \tilde{\cup} (H, K))^+$
(c) $((G, K) \tilde{\cap} (H, K))^- \subset (G, K)^- \cap (H, K)^-$
(d) $(G, K)^- \cup (H, K)^- = ((G, K) \tilde{\cup} (H, K))^-$
(e) $(G, K) \tilde{\subseteq} (H, K)$ if and only if $(G, K)^+ \subset (H, K)^+$

(f) $(G,K) \tilde{\subseteq} (H,K)$ if and only if $(G,K)^- \subset (H,K)^-$
(g) $(G,K) \tilde{\cap} (H,K) = \Phi$ if and only if $(G,K)^+ \cap (H,k)^+ = \Phi$
(h) $(G,K)^- \cap (H,K)^- \neq \Phi$.

Proof. (a) Let $(T,K) \in (G,K)^+ \cap (H,K)^+$. *Then* $(T,K) \in (G,K)^+$ *and* $(T,K) \in (H,K)^+$. *Thus* $(T,K) \tilde{\subseteq} (G,K)$ *and* $(T,K) \tilde{\subseteq} (H,K)$. *Hence* $(T,K) \tilde{\subseteq} (G,K) \tilde{\cap} (H,K)$ *and thus* $(T,K) \in ((G,K) \cap (H,K))^+$.

Conversely, let $(T,K) \in ((G,K) \cap (H,K))^+$. *Then* $(T,K) \tilde{\subseteq} (G,K) \tilde{\cap} (H,K)$. *Thus* $(T,K) \tilde{\subseteq} (G,K)$ *and* $(T,K) \tilde{\subseteq} (H,K)$. *Hence* $(T,K) \in (G,K)^+$ *and* $(T,K) \in (H,K)^+$. *Therefore* $(T,K) \in (G,K)^+ \cap (H,K)^+$.

(b) Let $(T,K) \in (G,K)^+ \cup (H,K)^+$, *then* $(T,K) \in (G,K)^+$ *and* $(T,K) \in (H,K)^+$. *Thus* $(T,K) \tilde{\subseteq} (G,K)$ *and* $(T,K) \tilde{\subseteq} (H,K)$. *Hence* $(T,K) \tilde{\subseteq} (G,K) \tilde{\cup} (H,K)$ *and* $(T,K) \in ((G,K) \tilde{\cup} (H,K))^+$.

(c) Let $(T,K) \in ((G,K) \tilde{\cap} (H,K))^-$, *then* $(T,K) \tilde{\cap} ((G,K) \tilde{\cap} (H,K)) \neq \Phi$. *Thus* $(T,K) \tilde{\cap} (G,K) \neq \Phi$ *and* $(T,K) \tilde{\cap} (H,K) \neq \Phi$. *Then* $(T,K) \in (G,K)^-$ *and* $(T,K) \in (H,K)^-$. *Therefore* $(T,K) \in (G,K)^- \cap (H,K)^-$.

(d) Let $(T,K) \in ((G,K) \tilde{\cup} (H,K))^-$, *then we have* $(T,K) \tilde{\cap} ((G,K) \tilde{\cup} (H,K)) \neq \Phi$. *Thus* $((T,K) \tilde{\cap} (G,K)) \tilde{\cup} ((T,K) \tilde{\cap} (H,K)) \neq \Phi$. *Hence* $(T,K) \tilde{\cap} (G,k) \neq \Phi$ *or* $(T,K) \tilde{\cap} (H,K) \neq \Phi$. *Then* $(T,K) \in (G,K)^-$ *or* $(T,K) \in (H,K)^-$. *Thus we have* $(T,K) \in (G,K)^- \cup (H,K)^-$.

Conversely, let $(T,K) \in (G,K)^- \cup (H,K)^-$. *Then* $(T,K) \in (G,K)^-$ *or* $(T,K) \in (H,K)^-$ *and thus we have* $(T,K) \tilde{\cap} (G,K) \neq \Phi$ *or* $(T,K) \tilde{\cap} (H,K) \neq \Phi$. *Hence* $((T,K) \tilde{\cap} (G,K)) \tilde{\cup} ((T,K) \tilde{\cap} (H,K)) \neq \Phi$ *and* $(T,K) \tilde{\cap} ((G,K) \tilde{\cup} (H,K)) \neq \Phi$. *Thus* $(T,K) \in ((G,K) \tilde{\cup} (H,K))^-$.

(e) Let ${E_e}^x \tilde{\in} (G,K)$. *Then we have* ${E_e}^x \in (G,K)^+$. *Since* $(G,K)^+ \subset (H,K)^+$, *then we have* ${E_e}^x \in (H,K)^+$ *and thus* ${E_e}^x \tilde{\in} (H,K)$. *Therefore,* $(G,K) \tilde{\subseteq} (H,K)$.

Conversely, let $(T,K) \in (G,K)^+$. *Then* $(T,K) \tilde{\subseteq} (G,K)$. *Since* $(G,K) \tilde{\subseteq} (H,K)$ *then we have* $(T,K) \tilde{\subseteq} (H,K)$ *and thus* $(T,K) \in (H,K)^+$.

(f) Let $(T,K) \in (G,K)^-$. *Then* $(T,K) \tilde{\cap} (G,K) \neq \Phi$. *Since* $(G,K) \tilde{\subseteq} (H,K)$, *then* $(T,K) \tilde{\cap} (H,K) \neq \Phi$. *Thus* $(T,K) \in (H,K)^-$. *Therefore,* $(G,K)^- \subset (H,K)^-$.

Conversely, ${E_e}^x \tilde{\in} (G,K)$. *Then* ${E_e}^x \in (G,K)^-$. *Since* $(G,K)^- \subset (H,K)^-$, *then* ${E_e}^x \in (H,K)^-$ *and thus* ${E_e}^x \tilde{\cap} (H,K) \neq \Phi$. *Therefore,* ${E_e}^x \tilde{\in} (H,K)$ *and* $(G,K) \tilde{\subseteq} (H,K)$.

(g) Let $(G,K) \tilde{\cap} (H,K) = \Phi$, then $\left((G,K) \tilde{\cap} (H,K)\right)^+ = \Phi$. Since $(G,K)^+ \cap (H,K)^+ = \left((G,K) \tilde{\cap} (H,K)\right)^+$ thus we have $(G,K)^+ \cap (H,k)^+ = \Phi$.

Conversely, let $(G,K)^+ \cap (H,K)^+ = \Phi$. Then $\left((G,K) \tilde{\cap} (H,K)\right)^+ = \Phi$ and thus $(G,K) \tilde{\cap} (H,K) = \Phi$.

(h) Since $\tilde{X} \tilde{\cap} (G,K) = \Phi$ and $\tilde{X} \tilde{\cap} (H,K) = \Phi$ then $\tilde{X} \in (G,K)^-$ and $\tilde{X} \in (H,K)^-$. Thus we have $\tilde{X} \in (G,K)^- \cap (H,K)^-$. Therefore $(G,E)^- \cap (H,E)^- \neq \Phi$.

Theorem 3.3. Let (Y,τ,K) be a soft topological space. Then the soft set families

$\beta_{SV^+} = \{(G,K)^+ : (G,K)\ soft\ open\ set\}$,
$\mathcal{S}_{SV^-} = \{(G,K)^- : (G,K)\ soft\ open\ set\}$

are soft base and soft sub base for a different soft topological spaces on $2^{S(Y,K)}$, respectively.

Proof. For $\tilde{Y} \in \tau$, $\tilde{Y}^+ = 2^{S(Y,K)} \subset \beta_{SV^+}$ and $2^{S(Y,K)} = \bigcup_{(B,K)^+ \in \beta_{SV^+}} (B,K)^+$.

Also let $(G_1,K)^+, (G_2,K)^+ \in \beta_{SV^+}$ and $(H,K) \in (G_1,K)^+ \cap (G_2,K)^+$. Since (G_1,K) and (G_2,K) are soft open sets, then $(G_3,K) = (G_1,K) \tilde{\cap} (G_2,K)$ is soft open set. Since $(G_1,K)^+ \cap (G_2,K)^+ = \big((G_1,K) \tilde{\cap} (G_2,K)\big)^+ = (G_3,K)^+$ then we have $(H,K) \in (G_3,K)^+$, $(G_3,K)^+ \in \beta_{SV^+}$ and $(G_3,K)^+ \subset (G_1,K)^+ \cap (G_2,K)^+$. Thus β_{SV^+} is soft base for a soft topology.

Definition 3.4. Let (Y,τ,K) be a soft topological space and $2^{S(Y,K)}$ be family of all non null soft sets over Y.

i) The soft topological space, which accepts β_{SV^+} a base, defined on $2^{S(Y,K)}$ is called soft upper Vietoris and denoted by τ_{SV^+}.

ii) The soft topological space, which accepts $\mathcal{S}_{SV^-}$ a subbase, defined on $2^{S(Y,K)}$ is called soft lower Vietoris and denoted by τ_{SV^-}.

iii) The soft Vietoris topological space is denoted by τ_{SV} and defined as $\tau_{SV} = \tau_{SV^+} \cup \tau_{SV^-}$. Let $(U,K), (V_1,K), (V_2,K), \dots, (V_n,K)$ be soft open sets. Then a element of soft base for soft Vietoris topological space is denoted by $B\big((U,K), (V_1,K), (V_2,K), \dots, (V_n,K)\big) = \big\{(T,K) \in 2^{S(Y,K)} : (T,K) \tilde{\subseteq} (U,K), (T,K) \tilde{\cap} (V_i,K) \neq \Phi,\ i = 1,2,\dots,n\big\}$.

Example 3.5. Let (Y,τ,K) be a soft topological space with $Y = \{y_1, y_2\}$, $K = \{k_1, k_2\}$ and $\tau = \{\Phi, \tilde{Y}, (F,K), (G,K), (H,K)\}$. Where, $(F,K) = \{(k_1,\{y_1\})\}$, $(G,K) = \{(k_2,\{y_2\})\}$ and $(H,K) = \{(k_1,\{y_1\}), (k_2,\{y_2\})\}$.

Then

$(F,K)^+ = \{\{(k_1,\{y_1\})\}\}$
$(G,K)^+ = \{\{(k_2,\{y_2\})\}\}$
$(H,K)^+ = \{\{(k_1,\{y_1\})\},\ \{(k_2,\{y_2\})\},\ \{(k_1,\{y_1\}), (k_2,\{y_2\})\}\}$
$\tilde{Y}^+ = 2^{S(Y,K)} = S(Y,K) - \{\phi\}$.

Thus

$\beta_{SV^+} = \{(F,K)^+,\ (G,K)^+, (H,K)^+, \tilde{Y}^+\}$

is a base for a soft topological space on $2^{S(Y,K)}$. Because,

$\tilde{Y}^{+} = 2^{S(Y,K)}$ and
$(F,K)^{+} \cap (G,K)^{+} = \emptyset$,
$(F,K)^{+} \cap (H,K)^{+} = (F,K)^{+} \in \beta_{SV^{+}}$,
$(F,K)^{+} \cap \tilde{Y}^{+} = (F,K)^{+} \in \beta_{SV^{+}}$,
$(G,K)^{+} \cap (H,K)^{+} = (G,K)^{+} \in \beta_{SV^{+}}$,
$(G,K)^{+} \cap \tilde{Y}^{+} = (G,K)^{+} \in \beta_{SV^{+}}$,
$(H,K)^{+} \cap \tilde{Y}^{+} = (H,K)^{+} \in \beta_{SV^{+}}$.

This topology (called upper soft Vietoris topology) is

$$\tau_{SV^{+}} = \left\{(F,K)^{+},\ (G,K)^{+}, (H,K)^{+}, (F,K)^{+} \cup (G,K)^{+}, \tilde{Y}^{+}, \{\phi\}\right\}.$$

Example 3.6. Let (Y,τ,K) be a soft topological space in Example 3.5. Then,
$(F,K)^{-} = \{\{(k_1,\{y_1\})\},\ \{(k_1,Y)\},\ \{(k_1,\{y_1\}),(k_2,\{y_1\})\},\ \{(k_1,\{y_1\}),(k_2,\{y_2\})\},$
$\{(k_1,\{y_1\}),(k_2,Y)\},\ \{(k_1,Y),(k_2,\{y_1\})\},\ \{(k_1,Y),(k_2,\{y_2\})\},\tilde{Y}\}$

$(G,K)^{-} = \{\{(k_2,\{y_2\})\},\ \{(k_2,Y)\},\ \{(k_1,\{y_1\}),(k_2,\{y_2\})\},\ \{(k_1,\{y_1\}),(k_2,Y\},$
$\{(k_1,\{y_2\}),(k_2,\{y_2\})\},\ \{(k_1,\{y_2\}),(k_2,Y\},\ \{(k_1,Y),(k_2,\{y_2\})\},\ \tilde{Y}\}$

$(H,K)^{-} = \{\{(k_1,\{y_1\})\},\ \{(k_2,\{y_2\})\},\ \{(k_1,Y)\},\ \{(k_2,Y)\},\ \{(k_1,\{y_1\}),(k_2,\{y_1\})\},$
$\{(k_1,\{y_1\}),(k_2,\{y_2\})\},\ \{(k_1,\{y_1\}),(k_2,Y)\},\ \{(k_1,\{y_2\}),(k_2,\{y_2\})\},\ (k_1,\{y_2\}),(k_2,Y)\},$
$\{(k_1,Y),(k_2,\{y_1\})\},\ \{(k_1,Y),(k_2,\{y_2\})\},\ \tilde{Y}\}$

$\tilde{Y}^{-} = 2^{S(Y,K)} = S(Y,K) - \{\phi\}$.

Thus the family $\mathcal{S}_{SV^{-}} = \{(F,K)^{-},\ (G,K)^{-},(H,K)^{-},\tilde{Y}^{-}\}$ is a sub base for a soft topological space on $2^{S(Y,K)}$.

Because,

$(D,K)^{-} = \{\{(k_1,\{y_1\}),(k_2,\{y_2\})\},\ \{(k_1,\{y_1\}),(k_2,Y)\},\ \{(k_1,Y),(k_2,\{y_2\})\},\ \tilde{Y}\}$

the family

$\beta_{SV^{-}} = \{(F,K)^{-},\ (G,K)^{-},(H,K)^{-},(D,K)^{-},\tilde{Y}^{-}\}$

is a base for a soft topological space on $2^{S(Y,K)}$. This topology (called lower soft Vietoris topology) is

$$\tau_{SV^{-}} = \left\{(F,K)^{-},\ (G,K)^{-},\ (H,K)^{-},\ (D,K)^{-},\ (F,K)^{-} \cup (G,K)^{-},\ \tilde{Y}^{-},\{\phi\}\right\}.$$

Example 3.7. Let (Y,τ,K) be a soft topological space in Example 3.5. Then, the soft vietoris topology is

$\tau_{SV} = \Big\{(F,K)^+,\ (G,K)^+, (H,K)^+, (F,K)^+ \cup (G,K)^+, \tilde{Y}^+, (F,K)^-,\ (G,K)^-,\ (H,K)^-,$ $(D,K)^-,\ (F,K)^- \cup (G,K)^-,\ \tilde{Y}^-, \{\phi\}\Big\}.$

4. Vietoris Soft Continuous Multifunction

Definition 4.1. [2] Let $S(X,E)$ and $S(Y,K)$ be two classes of soft sets. Let $u: X \to Y$ be multifunction and $p: E \to K$ be mapping. Then a soft multifunction $F: S(X,E) \to S(Y,K)$ is defined as follows: for $k \in K$,

$$F(G)(k) = \begin{cases} \bigcup_{e \in p^{-1}(k) \cap E} u\big(G(e)\big), & p^{-1}(k) \cap E \neq \emptyset \\ \emptyset, & \text{otherwise} \end{cases}$$

For a soft set (G,E) in $S(X,E)$, $(F(G,E),K)$ is a soft set in $S(Y,K)$ and $(F(G,E),K)$ is called a soft image of a soft set (G,E). Moreover, $F(G,E) = \tilde{\cup} \{F({E_e}^x): {E_e}^x \tilde{\in} (G,E)\}$ for a soft subset (G,E) of X.

Definition 4.2. [2] Let $F: S(X,E) \to S(Y,K)$ be a soft multifunction. The soft upper inverse image of (H,K) denoted by $F^+(H,K)$ and the soft lower inverse image of (H,K) denoted by $F^-(H,K)$ defined as follows, respectively;

$F^+(H,K) = \{{E_e}^x \tilde{\in} \tilde{X}: F({E_e}^x) \tilde{\subseteq} (H,K)\}$
$F^-(H,K) = \{{E_e}^x \tilde{\in} \tilde{X}: F({E_e}^x) \tilde{\cap} (H,K) \neq \Phi\}.$

Also, $F(\tilde{X}) = \bigcup F({E_e}^x)$.

Definition 4.3. Let $F, G: X \to Y$ be two soft multifunctions. For ${E_e}^x \tilde{\in} \tilde{X}$, the union and intersection of F and G is denoted by

$(F \cup G)({E_e}^x) = F({E_e}^x) \cup G({E_e}^x),$
$(F \cap G)({E_e}^x) = F({E_e}^x) \cap G({E_e}^x).$

Definition 4.4. [2] Let $F: S(X,E) \to S(Y,K)$ and $G: S(X,E) \to S(Y,K)$ be two soft multifunctions. Then, F equal to G if $F({E_e}^x) = G({E_e}^x)$, for each ${E_e}^x \in X$.

Definition 4.5. [2] The soft multifunction $F: S(X,E) \to S(Y,K)$ is called surjective if p and u are surjective.

Theorem 4.6. [2] Let $F: S(X,E) \to S(Y,K)$ be a soft multifunction. Then, for soft sets (F,E), (G,E) and for a family of soft sets $(G_i,E)_{i \in I}$ in the soft class $S(X,E)$ the following statements are hold:

(a) $F(\Phi) = \Phi$
(b) $F(\tilde{X}) \tilde{\subseteq} \tilde{Y}$
(c) $F((G,A) \tilde{\cup} (H,B)) = F(G,A) \tilde{\cup} F(H,B)$ in general $F(\tilde{\cup}_i (G_i,E)) = \tilde{\cup}_i F(G_i,E)$
(d) $F((G,A) \tilde{\cap} (H,B)) \tilde{\subseteq} F(G,A) \tilde{\cap} F(H,B)$ in general $F(\cap_i (G_i,E)) \tilde{\subseteq} \tilde{\cap}_i F(G_i,E)$

(e) If $(G, E) \tilde{\subseteq} (H, E)$, then $F(G, E) \tilde{\subseteq} F(H, E)$.

Theorem 4.7. [2] Let $F: S(X, E) \to S(Y, K)$ be a soft multifunction. Then the following statements are true:

(a) $F^-(\Phi) = \Phi$ and $F^+(\Phi) = \Phi$
(b) $F^-(\tilde{Y}) = \tilde{X}$ and $F^+(\tilde{Y}) = \tilde{X}$
(c) $F^-((G, K) \tilde{\cup} (H, K)) = F^-(G, K) \tilde{\cup} F^-(H, K)$
(d) $F^+(G, K) \tilde{\cup} F^+(H, K) \tilde{\subseteq} F^+((G, K) \tilde{\cup} (H, K))$
(e) $F^-((G, K) \tilde{\cap} (H, K)) \tilde{\subseteq} F^-(G, K) \tilde{\cap} F^-(H, K)$
(f) $F^+(G, K) \tilde{\cap} F^+(H, K) = F^+((G, K) \tilde{\cap} (H, K))$
(g) If $(G, K) \tilde{\subseteq} (H, K)$, then $F^-(G, K) \tilde{\subseteq} F^-(H, K)$ and $F^+(G, K) \tilde{\subseteq} F^+(H, K)$.

Definition 4.8 Let $F: S(X, E) \to S(Y, K)$ and $G: S(Y, K) \to S(Z, L)$ be two soft multifunction. The combination of F ad G denoted by $GoF: S(X, E) \to S(Z, L)$ is a soft multifunction and defined as $(GoF)(E_e{}^x) = G\big(F(E_e{}^x)\big)$.

Proposition 4.9. [2] Let $F: S(X, E) \to S(Y, K)$ be a soft multifunction. Then the following statements are true:

(a) $(G, A) \tilde{\subseteq} F^+(F(G, A)) \tilde{\subseteq} F^-(F(G, A))$ for a soft subset (G, A) in X. If F is surjectice then $(G, A) = F^+(F(G, A)) = F^-(F(G, A))$
(b) $F(F^+(H, B)) \tilde{\subseteq} (H, B) \tilde{\subseteq} F(F^-(H, B))$ for a soft subset (H, B) in Y.
(c) For two soft subsets (H, B) and (U, C) in Y such that $(H, B) \tilde{\cap} (U, C) = \Phi$ then $F^+(H, B) \tilde{\cap} F^-(U, C) = \Phi$.

Proposition 4.10. [2] Let $F: (X, \tau, E) \to (Y, \sigma, K)$ and $G: (Y, \sigma, K) \to (Z, \eta, L)$ be two soft multifunction. Then the follows are true:

(a) $(F^-)^- = F$
(b) For a soft subset (T, C) in Z, $(GoF)^-(T, C) = F^-(G^-(T, C))$ and $(GoF)^+(T, C) = F^+(G^+(T, C))$.

Proposition 4.11. Let $F, G: X \to Y$ be two soft multifunctions. For a soft set (H, K) in Y the following statements are hold:

(i) $(F \cup G)^-(H, K) = F^-(H, K) \cup G^-(H, K)$.
(ii) $(F \cup G)^+(H, K) = F^+(H, K) \cup G^+(H, K)$
(iii) $(F \cap G)^-(H, K) \subset F^-(H, K) \cap G^-(H, K)$
(iv) $F^+(H, K) \cap G^+(H, K) \subset (F \cap G)^+(H, K)$

Proof. (i) Let $E_e{}^x \in (F \cup G)^-(H, K)$, then $(F \cup G)(E_e{}^x) \tilde{\cap} (H, K) \neq \Phi$ implies that $(F(E_e{}^x) \cup G(E_e{}^x)) \tilde{\cap} (H, K) \neq \Phi$. Thus $(F(E_e{}^x) \tilde{\cap} (H, K)) \tilde{\cup} (G(E_e{}^x) \tilde{\cap} (H, K)) \neq \Phi$. Thus $F(E_e{}^x) \tilde{\cap} (H, K) \neq \Phi$ or $G(E_e{}^x) \tilde{\cap} (H, K) \neq \Phi$. Hence $E_e{}^x \in F^-(H, K)$ or $E_e{}^x \in G^-(H, K)$. Thus $E_e{}^x \in F^-(H, K) \cup G^-(H, K)$.

Conversely, the proof is similar.

(iv) Let ${E_e}^x \in F^+(H,K) \cap G^+(H,K)$ then $F({E_e}^x) \tilde{\subset} (H,K)$ and $G({E_e}^x) \tilde{\subset} (H,K)$. Thus $F({E_e}^x) \tilde{\cap} G({E_e}^x) \tilde{\subset} (H,K)$ and $(F \cap G)({E_e}^x) \tilde{\subset} (H,K)$. Thus ${E_e}^x \in (F \cap G)^+(H,K)$.

The proof is (ii) and (iii) is similar.

Proposition 4.12. Let $F: X \to Y$ be a soft multifunction. Then the following statements are hold:

i) $(\cup F_i)^-(H,K) = \cup {F_i}^-(H,K)$
ii) $(\cup F_i)^+(H,K) = \cup {F_i}^+(H,K)$
iii) $(\cap F_i)^-(H,K) \subset \cap {F_i}^-(H,K)$
iv) $\cap {F_i}^+(H,K) \subset (\cap F_i)^+(H,K)$

Proof. Obvious.

Proposition 4.13. [2] Let (G,K) be a soft set over Y. Then the followings are true for a soft multifunction $F: (X,\tau,E) \to (Y,\sigma,K)$:

(a) $F^+(\tilde{Y} - (G,K)) = \tilde{X} - F^-(G,K)$
(b) $F^-(\tilde{Y} - (G,K)) = \tilde{X} - F^+(G,K)$.

Definition 4.14. Let $(X,\tau,E), (Y,\sigma,K)$ be two soft topological space, ${E_e}^x$ be a soft point in X and $F: (X,\tau,E) \to (Y,\sigma,K)$ be a soft multifunction.

(i) F is Vietoris soft upper continuous at a ${E_e}^x$ if for each soft open set (H,K) with $F({E_e}^x) \tilde{\subset} (H,K)$, there exists (P,E) a soft open neighbourhood of ${E_e}^x$ such that $F({E_e}^z) \tilde{\subset} (H,K)$ for all ${E_e}^z \tilde{\in} (P,E)$.

(ii) F is Vietoris soft lower continuous at a ${E_e}^x$ if for each soft open set (H,K) with $F({E_e}^x) \tilde{\cap} (H,K) \neq \Phi$, there exists (P,E) a soft open neighbourhood of ${E_e}^x$ such that $F({E_e}^z) \tilde{\cap} (H,K) \neq \emptyset$ for all ${E_e}^z \tilde{\in} (P,E)$.

(iii) If F is Vietoris soft upper continuous and Vietoris soft lower continuous at ${E_e}^x$ then F is called Vietoris soft continuous at ${E_e}^x$.

(iv) F is Vietoris soft upper continuous (resp. Vietoris soft lower continuous, Vietoris soft continuous) if F has this property at every ${E_e}^x$ soft point of X.

Theorem 4.15. Let $(X,\tau,E), (Y,\sigma,K)$ be two soft topological space and $F: (X,\tau,E) \to (Y,\sigma,K)$ be soft multifunction. We define a soft mapping $f: (X,\tau,E) \to (2^{S(Y,K)}, \tau_{SV^+}, K)$, $f({E_e}^x) = F({E_e}^x)$ for each soft point ${E_e}^x$ in X. Then the soft multifunction F is Vietoris soft upper continuous if and only if the soft mapping f is soft continuous.

Proof. ($\Rightarrow$) Let $F: X \to Y$ be Vietoris soft upper continuous at ${E_e}^{x_0}$ and let $f({E_e}^{x_0}) \in (G,K)^+ \in \beta_1$. Since $(G,K)^+ = \{(H,K): (H,K) \tilde{\subset} (G,K)\}$ then we have $f({E_e}^{x_0}) \tilde{\in} (G,K)$ and thus $F({E_e}^{x_0}) \tilde{\subset} (G,K)$. Since $F: X \to Y$ be Vietoris soft upper continuous, then there exists (P,E) is soft open neighborhood of ${E_e}^{x_0}$ such that $F({E_e}^x) \tilde{\subset} (G,K)$ for every ${E_e}^x \tilde{\in} (P,E)$. Therefore $f({E_e}^x) \tilde{\in} (G,K)$. Hence $f: X \to (2^{S(Y,K)}, \tau_{SV^+}, K)$ is soft continuous at ${E_e}^{x_0}$.

(⇐) Let $f: X \to (2^{S(Y,K)}, {\tau_{SV}}^{+}, K)$ is soft continuous at ${E_e}^{x_0}$ and let (G, K) be soft open set such that $F({E_e}^{x_0}) \tilde{\subset} (G, K)$. Then $f({E_e}^{x_0}) \in (G, K)^{+} \in \beta_1$. Since f is soft continuous to ${\tau_{SV}}^{+}$ there exists (P, E) a soft open neigborhood of ${E_e}^{x_0}$ such that $F(P, E) \tilde{\subset} (G, K)$. Then we have $f({E_e}^{x}) \in (G, K)^{+}$ for ${E_e}^{x} \tilde{\in} (P, E)$. Thus $F({E_e}^{x}) \tilde{\subset} (G, K)$. This implies that $F: X \to Y$ is Vietoris soft upper continuous at ${E_e}^{x_0}$.

Theorem 4.16. Let $(X, \tau, E), (Y, \sigma, K)$ be two soft topological space and $F: (X, \tau, E) \to (Y, \sigma, K)$ be soft multifunction. We define a soft mapping $f: (X, \tau, E) \to (2^{S(Y,K)}, \tau_{SV^{-}}, K)$, $f({E_e}^{x}) = F({E_e}^{x})$ for each soft point ${E_e}^{x}$ in X. Then the soft multifunction F is Vietoris soft lower continuous if and only if the soft mapping f is soft continuous.

Proof. It can be show that similarly to Theorem 4.15.

5. Conclusions

Recently, many researcher have studied the soft set theory, which is applied to many problems having uncertainties. In this paper, we define the concept of Vietoris soft topological space one of the hyperspaces of soft sets. Then we define the Vietoris continuity of soft multifunction. Finally, we give the relationship between Vietoris continuity of soft multifunction and continuity of soft mapping. We hope that this paper is going to help researcher to enhance the further study on soft set theory.

Acknowledgements

Authors are grateful to the reviewers and the editor in chief for their helpful comments and suggestions for improving the paper.

References

[1] Akdağ M. and Erol F., Upper and Lower Continuity of Soft Multifunctions, Appl. Math. Inf. Sci.,7, 1-8 (2013).
[2] Akdağ M., Erol F., Multifunction Between Soft Topological Spaces, International Journal of Mathematics Trends and Technology, 62-69, 20(1) (2015).
[3] Ali, M.I., Feng, F., Liu, X., Min, W.K., Shabir, M.: On some new operations in soft set theory. Comput. Math. Appl. 57, 1547--1553 (2009)
[4] Çağman N., Karataş S., Enginoğlu S., Soft topology, Computer Math. Appl. 62, 351-358, 2011.
[5] Kharal, A., Ahmad, B.: Mappings on soft classes. New Math. Nat. Comput. 7(3), 471-481 (2011).
[6] Maji, P.K., Biswas, R., Roy, A.R.: Soft set theory. Comput. Math. Appl. 45, 555--562 (2003).
[7] Molodtsov D., Soft set theory - First results, Comput. Math.with Appl. 37 (4/5) (1999) 19-31.
[8] Nazmul S. K., Samanta S.K., Neighbourhood properties of soft topological spaces, Annals of Fuzzy Mathematics and Informatics, (2012).
[9] Roy S., T. K. Samanta, A note on Soft Topological Spaces, Punjab University Journal of Mathematics, 46(1), 2014, pp. 19-24.

[10] Shabir M., Naz M., On soft topological spaces, Comput. Math. Appl. 61 (2011) 1786-1799.
[11] Zorlutuna I., M. Akdağ, W. K. Min and S. Atmaca, Remarks on soft topological spaces, Ann. Fuzzy Math. Inform. 3(2) (2012) 171-185.