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Soft $\wedge_\beta$-CLOSED SETS IN SOFT TOPOLOGICAL SPACES

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Abstract – In this paper, we introduce the notions of soft β-kernel of soft sets, S$\wedge_\beta$-closed sets and S$\wedge_\beta$-open sets in soft topological spaces. The concept of S$\wedge_\beta$-sets, as a generalization to the class of soft β-open sets, is defined. Some of their fundamental properties that illustrated by examples are studied. Further, the soft topologies defined by families soft β-kernel of soft sets, S$\wedge_\beta$-closed sets are investigated. We show that, the collection of S$\wedge_\beta$-sets is Alexandroff soft space.

Keywords – *Soft sets, Soft topology, Soft β-open sets, Soft regular-closed sets, Soft $\wedge_\beta$-sets.*

1 Introduction

Molodtsov [13] initiated the concept of soft sets, which is a completely new approach for modeling vagueness and uncertainty. Also, it is free of the difficulties present in the theories of fuzzy sets, rough set, vague, etc. In recent years, researchers have been formulated the theoretical bases for further applications of topology on soft sets that lead to the development of information system and various fields in digital topology, engineering, economics, social science, medical science, etc. Maji et al. [11] proposed several operations on soft sets, and some basic properties of these operations have been revealed. Shabir et al. [15] introduced the notion of soft topological spaces that are defined to be over an initial universe with a fixed set of parameters. Hussain et al. [6] investigated the properties of soft open (closed) sets, soft neighbourhood and soft closure. In addition, Arockiarani et al. [3] introduced soft gβ-closed sets and soft gsβ-closed sets in soft topological spaces and they obtained some properties in the light of these defined sets. The concept of $\wedge_\beta$-sets has been introduced (known as $\wedge_{sp}$-sets) with some of their properties in [14].

In this paper, we generalize the concept $\wedge_\beta$-sets to soft setting. Soft $\wedge_\beta$-closed sets, soft $\wedge_\beta$-open sets on soft topological spaces will be introduced. Some of their properties on soft topology will be studied. The collection of S$\wedge_\beta$-sets is Alexandroff space will be presented. Furthermore, the soft topologies defined by families soft β-kernel of soft sets, S$\wedge_\beta$-closed sets will be investigated.

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2 Preliminary

For the definitions and results on soft set theory, we refer to [1, 3, 4, 6, 11, 12, 13, 15, 16, 17]. However, we recall some definitions and results on soft set theory and soft topology.

Let X be an initial universe set and E be a set of parameters. Let $\mathcal{P}$(X) be the power set of X and A be a non empty subset of E. F_A is called a soft set over X [13], where F is a mapping given by F: A$\longrightarrow\mathcal{P}$(X). For two soft sets F_A, H_B over common universe X, we say that F_A is a subset of H_B [11], if A$\subseteq$B and F(e)$\subseteq$H(e), for all e$\in$A. In this case, we write $F_A \sqsubseteq H_B$. Two soft sets F_A, H_B over a common universe X are said to be equal, if $F_A \sqsubseteq H_B$ and $H_B \sqsubseteq F_A$. A soft set F_A over X is called a null (resp., an absolute) soft set, denoted by $\tilde{\emptyset}_A$ (resp., $\tilde{X_A}$), if for each e$\in$A, F(e)=$\emptyset$ (resp., F(e)=X)[11]. In particular, (X, A) will be denoted by $\tilde{X_A}$.

The union of two soft sets of F_A and H_B over the common universe X [11] is the soft set K_C, where C=A$\cup$B and for all e$\in$C, K(e)=F(e), if e$\in$(A - B); K(e)=H(e), if e$\in$(B - A), and K(e)=F(e)$\cup$H(e), if e$\in$(A$\cap$B). We write $F_A \sqcup H_B = K_C$. Moreover, the intersection K_C of two soft sets F_A and H_B over a common universe X [15], denoted $F_A \sqcap H_B$, is defined as C=A$\cap$B, and K(e)=F(e)$\cap$H(e) for all e$\in$C. The difference between two soft sets F_A and H_A over X [15], denoted by K_A=(F_A - H_A), is defined as K(e)=F(e) - H(e) for all e$\in$A. The relative complement of a soft set F_A [1], is denoted by $(F_A)^c$ and is defined by $(F_A)^c = \tilde{X_A}$ - F_A, where F^c: A$\longrightarrow\mathcal{P}$(X) is a mapping given by $(F)^c$(e)= X - F(e) for all e$\in$A. Moreover, $((F_A)^c)^c = F_A$.

In order to efficiently discuss, we consider only soft sets F_A over a universe X in which all the parameters set A are the same.

In this paper for convenience, let $\mathcal{SS}$(X, A) be the family of soft sets over X with set of parameters A.

Definition 2.1. [15] Let F_A be a soft set over X and x$\in$X. x$\in F_A$ whenever x$\in$F(e) for all e$\in$A. Note that for any x$\in$X, x$\notin F_A$, if x$\notin$F(e) for some e$\in$A.

Definition 2.2. [15] Let x$\in$X, then x_A denotes the soft set over X for which x(e)={x} for all e$\in$A.

Definition 2.3. [15] Let τ be the collection of soft sets over X with the fixed set of parameters A. Then, τ is said to be a soft topology on X, if it satisfies the following axioms:
(i) $\tilde{\emptyset}_A$, $\tilde{X_A}$ belong to τ.
(ii) The union of any number of soft sets in τ belongs to τ.
(iii) The intersection of any two soft sets in τ belongs to τ.

The triplet (X, A, τ) is called a soft topological space over X or a soft space. Every member of τ is called a soft open set. The complement of a soft open set is called the soft closed set [6] in (X, A, τ).

Definition 2.4. Let (X, A, τ) be a soft topological space and F_A be a soft set. Then,

(i) [15] the soft closure of F_A, denoted by cl F_A, is the intersection of all soft closed sets that contain F_A, i.e., cl $F_A = \sqcap\{M_A \in \tau^c \mid F_A \sqsubseteq M_A\}$. Clearly, x$\in$cl F_A if and only if for every soft open nbd. U_A of x; $F_A \sqcap U_A \neq \tilde{\emptyset}_A$.

(ii) [6] the soft interior of F_A, denoted by int F_A, is the union of all soft open sets contained in F_A i.e, int $F_A = \sqcup\{M_A \in \tau \mid M_A \sqsubseteq F_A\}$.

Definition 2.5. A soft set F_A in a soft topological space (X, A, τ) is said to be
((i) [8] Soft regular closed, if cl(int(F_A))=F_A.

(ii)[7] Soft β-open, if $F_A \sqsubseteq cl(int(cl(F_A)))$ (resp., soft β-closed, if $int(cl(int(F_A))) \sqsubseteq F_A$).
(iii) [10] Soft g-closed set, if $cl(F_A) \sqsubseteq U_A$ whenever $F_A \sqsubseteq U_A$ and U_A is soft open set.
(iv) [3] Soft βg-closed set, if $cl(F_A) \sqsubseteq U_A$ whenever $F_A \sqsubseteq U_A$ and U_A is soft β-open set.

The family of all soft β-open sets in a soft topological space (X, A, τ) will be denote by $S\beta(X, A)$.

Lemma 2.6. [7] (i) Arbitrary intersection of soft β-closed sets is soft β-closed but the union of two soft β-closed sets need not be soft β-closed set.
(ii) Arbitrary union of soft β-open sets is soft β-open but the intersection of two soft β-open sets need not be soft β-open set.

Definition 2.7. [7] Let (X, A, τ) be a soft topological space and F_A be a soft set. Then,
(i) A soft β-closure of F_A, denoted by $Scl_\beta F_A$, is the intersection of all soft β-closed sets that contain F_A, i.e., $Scl_\beta F_A = \sqcap\{M_A \mid M_A$ is a soft β-closed and $F_A \sqsubseteq M_A\}$.
(ii) A soft β-interior of F_A, denoted by $Sint_\beta$, is the union of all soft β-open sets contained in F_A i.e, $Sint_\beta F_A = \sqcup\{M_A \mid M_A$ is a soft β-open and $M_A \sqsubseteq F_A\}$.

Definition 2.8. [5] Let τ be the collection of soft sets over X; then, τ is said to be a supra soft topology on X, if it satisfies the following axioms
(i) $\tilde{\emptyset}_A$, $\tilde{X}_A$ belong to τ.
(ii) the union of any number of soft sets in τ belongs to τ.

3 Soft $\wedge_\beta$-closed Sets

Definition 3.1. The soft β-kernel of a soft set F_A, denoted by $S\wedge_\beta(F_A)$, is the intersection of all soft β-open superset of F_A i.e $S\wedge_\beta(F_A) = \sqcap\{U_A \in S\beta(X, A) \mid F_A \sqsubseteq U_A\}$.

Example 3.2. Let X={x, y, z}, A={e_1, e_2} and the soft topology τ over (X, A) is

$\tau = \{F_{iA} \mid i = 1, 2, \ldots, 7\} \sqcup \{\tilde{\emptyset}_A, \tilde{X}_A\}$.

We consider the following soft sets F_{iA}, i = 1,2, . . . ,7 over X defined as follows:

F_{1A}={{x}, {x}},
F_{2A}={{y}, {z}},
F_{3A}={{y}, $\emptyset$},
F_{4A}={{x, y}, {x}},
F_{5A}={{x, y}, {x, z}},
F_{6A}={X, {x}},
F_{7A}={X, {x, z}}.

A soft set {{z}, {y}} is $S\wedge_\beta$-set, but {{x, z}, X} is not.

Some of fundamental properties of soft β-kernel of soft sets will be shown in the next theorem.

Lemma 3.3. For soft sets F_A, H_A and F_{iA} ($i \in \Gamma$) of a soft topological space (X, A, τ), the following properties hold:
(i) $S\wedge_\beta(\tilde{\emptyset}_A) = \tilde{\emptyset}_A$ and $S\wedge_\beta(\tilde{X}_A) = \tilde{X}_A$.
(ii) $F_A \sqsubseteq S\wedge_\beta(F_A)$.
(iii) $F_A \sqsubseteq H_A$, then $S\wedge_\beta(F_A) \sqsubseteq S\wedge_\beta(H_A)$.
(iv) $S\wedge_\beta(S\wedge_\beta(F_A)) = S\wedge_\beta(F_A)$.
(v) $F_A \in S\beta(X, A)$, then $F_A = S\wedge_\beta(F_A)$.
(vi) $S\wedge_\beta(\sqcup_{i\in\Gamma} F_{iA}) = \sqcup_{i\in\Gamma} S\wedge_\beta(F_{iA})$.
(vii) $S\wedge_\beta(\sqcap_{i\in\Gamma} F_{iA}) \sqsubseteq \sqcap_{i\in\Gamma} S\wedge_\beta(F_{iA})$.

Proof. We prove only (vi) and the rest of the proof follows directly from Definition 3.1.
(vi) It is obvious that $S\wedge_\beta(F_{iA})\sqsubseteq S\wedge_\beta(\sqcup_{i\in\Gamma}F_{iA})$ for each $i\in\Gamma$ and so $\sqcup_{i\in\Gamma}(S\wedge_\beta(F_{iA}))\sqsubseteq S\wedge_\beta(\sqcup_{i\in\Gamma}F_{iA})$. Conversely, suppose $x\notin\sqcup_{i\in\Gamma}(S\wedge_\beta(F_{iA}))$, then $x\notin(S\wedge_\beta(F_{iA}))$ for each $i\in\Gamma$. Therefore, for each $i\in\Gamma$there exists $U_{iA}\in S\beta(X, A)$ such that $x\notin U_{iA}$ and $F_{iA}\sqsubseteq U_{iA}$. Thus $\sqcup_{i\in\Gamma}(F_{iA})\sqsubseteq\sqcup_{i\in\Gamma}(U_{iA})$ and $\sqcup_{i\in\Gamma}(U_{iA})\in S\beta(X, A)$ which does not contain x. Which implies that $x\notin S\wedge_\beta(\sqcup_{i\in\Gamma}F_{iA})$. Consequently, $S\wedge_\beta(\sqcup_{i\in\Gamma}F_{iA})\sqsubseteq\sqcup_{i\in\Gamma}S\wedge_\beta(F_{iA})$.

The equality of (vii) of Lemma 3.3 does not hold as indicated in the following example.

Example 3.4. Let (X, A, τ) be a soft topological space as in Example 3.2. Let U_A={{z}, X}, V_A={X, {x, y}}, then $S\wedge_\beta(U_A)=\tilde{X}_A$, $S\wedge_\beta(V_A)$={X, {x, y}} and so
$S\wedge_\beta(U_A)\sqcap S\wedge_\beta(V_A)$={X, {x, y}}, but $S\wedge_\beta(U_A\sqcap V_A)$={{z}, {x, y}}.

Corollary 3.5. (i) $Sint_\beta(F_A)\sqsubseteq S\wedge_\beta(F_A)$.
(ii) If $F_A\in S\beta(X, A)$, then $Sint_\beta(F_A)=S\wedge_\beta(F_A)$.
(iii) If $F_A\in S\beta(X, A)$, then $S\wedge_\beta(F_A)\sqsubseteq Scl_\beta(F_A)\sqsubseteq cl(F_A)$.
(iv) If F_A is soft β-closed, then $Scl_\beta(F_A)\sqsubseteq S\wedge_\beta(F_A)$.

In view of Lemma 3.3, the next theorem hold.

Theorem 3.6. The collection of all soft β-kernel of soft sets is a supra soft topological space.

Definition 3.7. The concept of $S\wedge_\beta$-sets in a soft topological space (X, A, τ) is the soft sets that coincide with their soft β-kernel. In other words, a soft set F_A is called $S\wedge_\beta$-set, if $F_A=S\wedge_\beta(F_A)$.

Theorem 3.8. Let (X, A, τ) be a soft topological space. Then, for a soft set F_A the following statements hold:
(i) $\tilde{\emptyset}_A$, $\tilde{X}_A$ are $S\wedge_\beta$-sets.
(ii) $S\wedge_\beta(F_A)$ is $S\wedge_\beta$-sets.
(iii) Every soft β-open is $S\wedge_\beta$-set.
(iv) The union of $S\wedge_\beta$-sets is $S\wedge_\beta$-set.
(v) The intersection of $S\wedge_\beta$-sets is $S\wedge_\beta$-set.

Proof. Follows directly from Lemma 3.3 and Definition 3.7.

Corollary 3.9. The class of all $S\wedge_\beta$-sets is stronger than soft β-open sets.

The converse of this corollary is not true as shown of the next example.

Example 3.10. Let (X, A, τ) be a soft topological space as in Example 3.2. Then, a soft set {{z}, {y}} is $S\wedge_\beta$-set and it is not soft β-open.

Definition 3.11. A soft topological space (X, A, τ) is an Alexandroff soft space, if arbitrary intersections of soft sets in τ belongs to τ.

Theorem 3.12. Let (X, A, τ) be a soft topological space, we put $\tau^{\wedge_\beta}$={F_A | F_A is $S\wedge_\beta$-set}. Then, (X, A, $\tau^{\wedge_\beta}$) is an Alexandroff soft space.

Proof. Immediate from Theorem 3.8 (i), (iv).

Corollary 3.13. (i) $S\beta(X, A)\sqsubseteq\tau^{\wedge_\beta}$.
(ii) $\tau\sqsubseteq\tau^{\wedge_\beta}$.

Definition 3.14. A soft topological space (X, A, τ) is called
(i) β-disconnected, if there exist nonempty soft β-open sets G_A, H_A such that $G_A\sqcup H_A=\tilde{X}_A$, $G_A\sqcap H_A=\tilde{\emptyset}_A$.
(ii) $S\wedge_\beta$-disconnected, if there exist $\tilde{\emptyset}_A\neq G_A$, $H_A\in\tau^{\wedge_\beta}$ such that $G_A\sqcup H_A=\tilde{X}_A$, $G_A\sqcap H_A=\tilde{\emptyset}_A$.

Theorem 3.15. If (X, A, τ) is soft β-disconnected, then (X, A, $\tau^{\wedge_\beta}$) is soft disconnected.

Proof. Immediate.

Lemma 3.16. A soft set F_A of a soft topological space (X, A, τ) is a soft βg-closed if and only if $cl(F_A)\sqsubseteq S\wedge_\beta(F_A)$.

Proof. Let $x\notin S\wedge_\beta(F_A)$, then there is soft β-open set U_A such that $F_A\sqsubseteq U_A$ and $x\notin U_A$. Since F_A is a soft βg-closed, then $clF_A\sqsubseteq U_A$. Hence, $x\notin cl\ F_A$. On the other hand, let $F_A\sqsubseteq U_A$ and U_A be soft β-open set. Then, $clF_A\sqsubseteq S\wedge_\beta(F_A)\sqsubseteq S\wedge_\beta(U_A)=U_A$. Hence, F_A is soft βg-closed set.

Lemma 3.17. A soft set F_A of a soft topological space (X, A, τ) is soft βg-closed and soft β-open set if and only if it is soft regular closed.

Proof. Immediate.

Corollary 3.18. (i) $cl(F_A)=S\wedge_\beta(F_A)$, if F_A is soft regular closed set.
(ii) $Scl_\beta(F_A)\sqsubseteq S\wedge_\beta(F_A)$, if F_A is a soft βg-closed .
(iii) $cl(F_A)$ is $S\wedge_\beta$-set, if $F_A\in S\beta(X, A)$.

Proof. Immediate from Lemmas 3.3, 3.16, 3.17 and Corollary 3.5.

Lemma 3.19. A soft set F_A of a soft topological space (X, A, τ) is soft βg-closed if and only if $cl(F_A)$ - F_A does not contain any nonempty soft β-closed set.

Proof. Let F_A be soft βg-closed set over X and suppose that U_A is soft β-closed set such that $U_A\sqsubseteq cl(F_A) - F_A$, then $U_A\sqsubseteq cl(F_A) - F_A\sqsubseteq\tilde{X}_A - F_A$. Which implies that $F_A\sqsubseteq\tilde{X}_A - U_A$ and $(\tilde{X}_A - U_A)$ is soft β-open set. Since F_A is soft βg-closed set, then $cl(F_A)\sqsubseteq(\tilde{X}_A - U_A)$. Hence, $U_A\sqsubseteq\tilde{X}_A - cl(F_A)$, which is a contradiction. Consequently, $U_A=\tilde{\emptyset}_A$. On the other hand, let $F_A\sqsubseteq U_A$ and U_A be soft β-open set. Suppose $cl(F_A)\sqcap(\tilde{X}_A - U_A)\neq\tilde{\emptyset}_A$, since $cl(F_A)\sqcap(\tilde{X}_A - U_A)\sqsubseteq cl(F_A) - F_A$. Then, $cl(F_A)\sqcap(\tilde{X}_A - U_A)$ is nonempty soft β-closed set, which is a contradiction. Hence, $clF_A\sqsubseteq U_A$.

Definition 3.20. A soft set F_A of a soft topological space (X, A, τ) is called $S\wedge_\beta$-closed set, if $F_A=G_A\sqcap H_A$, where G_A is a $S\wedge_\beta$-set and H_A is a soft closed set. A soft set is said to be a $S\wedge_\beta$-open set, if its complement is a $S\wedge_\beta$-closed.

In view of $\tilde{X}_A$ is both $S\wedge_\beta$-set and soft closed set, then proof of next lemma is immediate.

Lemma 3.21. (i) Every $S\wedge_\beta$-set is $S\wedge_\beta$-closed set.
(ii) Every soft closed set is $S\wedge_\beta$-closed set.

Lemma 3.22. For a soft set F_A of a soft topological space (X, A, τ), the following statements are equivalent
(i) F_A is $S\wedge_\beta$-closed set.
(ii) $F_A = G_A \sqcap cl(F_A)$, where G_A is a $S\wedge_\beta$-set.
(iii) $F_A = S\wedge_\beta(F_A) \sqcap cl(F_A)$.

Proof. (i)$\Longrightarrow$(ii) Let F_A be $S\wedge_\beta$-closed set, then there exist $S\wedge_\beta$-set G_A and soft closed set H_A such that $F_A = G_A \sqcap H_A$. Since $F_A \sqsubseteq H_A$, then $cl(F_A) \sqsubseteq H_A$ and so $F_A \sqsubseteq G_A \sqcap cl(F_A) \sqsubseteq G_A \sqcap H_A = F_A$. Hence, $F_A = G_A \sqcap cl(F_A)$.

(ii)$\Longrightarrow$(iii) Let $F_A = G_A \sqcap cl\,(F_A)$ and G_A is a $S\wedge_\beta$-set. Since $F_A \sqsubseteq G_A$, then $S\wedge_\beta(F_A) \sqsubseteq S\wedge_\beta(G_A) = G_A$ follows from Lemma 3.3. Therefore, $F_A \sqsubseteq S\wedge_\beta(F_A) \sqcap cl(F_A) \sqsubseteq G_A \sqcap cl(F_A) = F_A$ and so $F_A = S\wedge_\beta(F_A) \sqcap cl(F_A)$.

(iii)$\Longrightarrow$(i) Straightforward.

Corollary 3.23. A soft set F_A in a soft topological space (X, A, τ) is soft closed if and only if it is $S\wedge_\beta$-closed and soft βg-closed.

Proof. Immediate from Lemmas 3.16, 3.22.

Theorem 3.24. The intersection of $S\wedge_\beta$-closed sets is a $S\wedge_\beta$-closed set.

Proof. Let F_{iA} be $S\wedge_\beta$-closed sets and $i\in\Gamma$, then for each $i\in\Gamma$ there exist $S\wedge_\beta$-sets G_{iA} and soft closed sets H_{iA} such that $F_{iA} = G_{iA} \sqcap H_{iA}$. Hence, $\sqcap_{i\in\Gamma} F_{iA} = \sqcap_{i\in\Gamma}(G_{iA} \sqcap H_{iA}) = (\sqcap_{i\in\Gamma} G_{iA}) \sqcap (\sqcap_{i\in\Gamma} H_{iA})$. In view of Theorem 3.8, $\sqcap_{i\in\Gamma} G_{iA}$ is $S\wedge_\beta$-set and $\sqcap_{i\in\Gamma} H_{iA}$ is a soft closed set. Consequently, $\sqcap_{i\in\Gamma} F_{iA}$ is $S\wedge_\beta$-closed set.

Corollary 3.25. The union of $S\wedge_\beta$-open sets is a $S\wedge_\beta$-open set.

The union of $S\wedge_\beta$-closed sets need not to be a $S\wedge_\beta$-closed set as shown by the following example.

Example 3.26. The soft topological space (X, A, τ) is the same as in Example 3.2. Then, $H_A = \{\{y, z\}, \{x, y\}\}$, $K_A = \{\{z\}, \{z\}\}$ are $S\wedge_\beta$-closed sets but $H_A \sqcup K_A = \{\{y, z\}, X\}$ does not $S\wedge_\beta$-closed set.

Theorem 3.27. The class of $S\wedge_\beta$-closed sets is a supra soft topology over X, which is finer than $\tau^{\wedge_\beta}$.

Definition 3.28. For a soft set F_A of a soft topological space (X, A, τ)
(i) $S\vee_\beta(F_A) = \sqcup\{H_A \mid (H_A)^c \in S\beta(X, A) \text{ and } H_A \sqsubseteq F_A\}$.
(ii) $S\coprod_\beta(F_A) = S\wedge_\beta(F_A) \sqcap S\wedge_\beta(F_A)^c$.

Several results which are similar to Lemma 3.3 for $S\vee_\beta(\ )$ will be obtained.

Lemma 3.29. For soft sets F_A, H_A and F_{iA} ($i\in\Gamma$) of a soft topological space (X, A, τ), the following properties hold:
(i) $S\vee_\beta(\tilde{\emptyset}_A) = \tilde{\emptyset}_A$ and $S\vee_\beta(\tilde{X}_A) = \tilde{X}_A$.
(ii) $S\vee_\beta(F_A) \sqsubseteq F_A$.
(iii) $F_A \sqsubseteq H_A$, then $S\vee_\beta(F_A) \sqsubseteq S\vee_\beta(H_A)$.
(iv) $S\vee_\beta(S\vee_\beta(F_A)) = S\vee_\beta(F_A)$.
(v) If F_A is soft β-closed, then $F_A = S\vee_\beta(F_A)$.
(vi) $S\vee_\beta(\sqcap_{i\in\Gamma} F_{iA}) = \sqcap_{i\in\Gamma} S\vee_\beta(F_{iA})$.

(vii) $\sqcup_{i\in\Gamma}$S$\vee_\beta$(F$_{iA}$)$\sqsubseteq$S$\vee_\beta$($\sqcup_{i\in\Gamma}$F$_{iA}$).

The proof of next theorems are obvious and omitted.

Theorem 3.30. For a soft set F$_A$ of a soft topological space (X, A, τ), the following statements hold:
(i) S$\vee_\beta$(F$_A$)^c=$\tilde{X_A}$ - S$\wedge_\beta$(F$_A$).
(ii) S$\wedge_\beta$(F$_A$)^c=$\tilde{X_A}$ - S$\vee_\beta$(F$_A$).
(iii) S$\coprod_\beta$(F$_A$)=S$\coprod_\beta$(F$_A$)^c.
(iv) S$\vee_\beta$(F$_A$)$\sqcup$S$\coprod_\beta$(F$_A$)=S$\wedge_\beta$(F$_A$).
(v) S$\vee_\beta$(F$_A$)$\sqcap$S$\coprod_\beta$(F$_A$)=$\tilde{\emptyset_A}$.
(vi) S$\coprod_\beta$(F$_A$)=S$\wedge_\beta$(F$_A$) - S$\vee_\beta$(F$_A$).
(vii) S$\vee_\beta$(F$_A$)=F$_A$ - S$\coprod_\beta$(F$_A$).
(viii) S$\wedge_\beta$S$\coprod_\beta$(F$_A$)=S$\coprod_\beta$(F$_A$).
(ix) S$\coprod_\beta$S$\coprod_\beta$(F$_A$)$\sqsubseteq$S$\coprod_\beta$(F$_A$).
(x) S$\coprod_\beta$(S$\vee_\beta$(F$_A$))$\sqsubseteq$S$\coprod_\beta$(F$_A$).

Theorem 3.31. For soft sets F$_A$, H$_A$ of a soft topological space (X, A, τ), the following statements hold:
(i) S$\wedge_\beta$(F$_A$ - H$_A$)$\sqsubseteq$S$\wedge_\beta$(F$_A$) - S$\vee_\beta$(H$_A$).
(ii) S$\vee_\beta$(F$_A$ - H$_A$)=S$\vee_\beta$(F$_A$) - S$\wedge_\beta$(H$_A$).
(iii) S$\coprod_\beta$(F$_A$$\sqcup$H$_A$)$\sqsubseteq$S$\coprod_\beta$(F$_A$)$\sqcup$S$\coprod_\beta$(H$_A$).

4 Conclusion

In this paper, we generalized the concept $\wedge_\beta$-sets to soft setting and introduced soft $\wedge_\beta$-closed sets, soft $\wedge_\beta$-open sets on soft topological spaces. Some of their properties were studied. The collection of S$\wedge_\beta$-sets is Alexandroff space was presented. Furthermore, the soft topologies defined by families, soft β-kernel of soft sets, S$\wedge_\beta$-closed were investigated.

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