http://www.newtheory.org ISSN: 2149-1402

Journal of New Theory

Received: 30.05.2015 *Year*: **2015**, *Number*: **8**, *Pages*: **29-40**
Published: 11.11.2015 *Original Article*[**]

PRE OPEN SOFT SETS VIA SOFT GRILLS

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Abstract – This paper aims to introduce and investigate a collection of pre open soft sets in a soft topological space via soft grill G, namely pre G-open soft sets. Detailed study on pre G-open soft sets through soft sets theory with examples is carried out. Suitable condition on the collection of pre G-open soft sets to coincide with soft topology is deduced.

Keywords – *Soft sets, Soft topological spaces, Soft grill, Pre open soft sets, Semi open soft sets, Regular open soft sets.*

1 Introduction

In [25], D. Molodtsov introduced the concept of soft set theory and it has received much attention since its inception. Molodtsov presented the fundamental results of new theory and successfully applied it into several directions such as smoothness of functions, game theory, operations research, Riemann-integration, Perron integration, theory of probability etc. A soft set is a collection of approximate description of an object. He also showed how soft set theory is free from parametrization inadequacy syndrome of fuzzy set theory, rough set theory, probability theory and game theory. Soft systems provide a very general framework with the involvement of parameters. Research works on soft set theory and its applications in various fields are progressing rapidly in these years. After presentation of the operations of soft sets [23], the properties and applications of soft set theory have been studied increasingly [2, 20, 26, 27]. In recent years, many interesting applications of soft set theory have been expanded by embedding the ideas of fuzzy sets [9, 18, 21, 22, 23, 24, 26]. To develop soft set theory, the operations of the soft sets are redefined and a uni-int decision making method was constructed by using these new operations [3]. It got some stability only after the introduction of soft topology [31] in 2011. In [10], Kandil et al. introduced some soft operations such as semi open soft, pre open soft, $\alpha-$open

**Edited by* Oktay Muhtaroğlu *(Area Editor) and* Naim Çağman *(Editor-in-Chief).*
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soft and $\beta-$open soft and investigated their properties in detail. The notion of soft ideal was initiated for the first time by Kandil et al. [13]. They also introduced the concept of soft local function. Applications to various fields were further investigated by Kandil et al. [11, 12, 17, 14, 15, 16, 19]. The notion of $b-$open soft sets was initiated for the first time by El-sheikh and Abd El-latif [6], which is generalized to the supra soft topological spaces in [1, 8]. Properties of $b-$open soft sets in [28] are discussed. The notions of soft grill G and soft operators φ_G, ψ_G were introduced in [29]. These concepts are discussed with a view to find new soft topologies τ_G from the original one τ via soft grill G. Pei and Miao [27] showed that soft sets are a class of special information systems. Nevertheless, the idea of pre G-open soft sets in soft topological spaces and suitable conditions for the collection of pre G-open soft sets to be soft topology are not investigated, which are the aims of the current paper.

2 Preliminary

Definition 2.1. [25] Let X be an initial universe and E be a set of parameters. Let $\mathcal{P}(X)$ denote the power set of X and A be a non-empty subset of E. A pair F denoted by F_A is called a soft set over X , where F is a mapping given by $F : A \rightarrow \mathcal{P}(X)$. In other words, a soft set over X is a parametrized family of subsets of the universe X. For a particular $e \in A$, $F(e)$ may be considered the set of $e-$approximate elements of the soft set (F, A) and if $e \notin A$, then $F(e) = \emptyset$ i.e $F_A = \{F(e) : e \in A \subseteq E,\ F : A \rightarrow \mathcal{P}(X)\}$.

Definition 2.2. [4] A soft set F over X is a set valued function from E to $\mathcal{P}(X)$. It can be written a set of ordered pairs $F = \big\{(e, F(e)) : e \in E\big\}$. Note that if $F(e) = \emptyset$, then the element $(e, F(e))$ is not appeared in F. The set of all soft sets over X is denoted by $S_E(X)$.

Definition 2.3. [4] Let $F, G \in S_E(X)$. Then,

(i). If $F(e) = \emptyset$ for each $e \in E$, F is said to be a null soft set, denoted by $\tilde{\emptyset}$.

(ii). If $F(e) = X$ for each $e \in E$, F is said to be absolute soft set, denoted by $\tilde{X}$.

(iii). F is soft subset of G, denoted by $F\tilde{\subseteq}G$, if $F(e) \subseteq G(e)$ for each $e \in E$.

(iv). $F = G$, if $F\tilde{\subseteq}G$ and $G\tilde{\subseteq}F$.

(v). Soft union of F and G, denoted by $F\tilde{\cup}G$, is a soft set over X and defined by $F\tilde{\cup}G : E \rightarrow \mathcal{P}(X)$ such that $(F\tilde{\cup}G)(e) = F(e) \cup G(e)$ for each $e \in E$.

(vi). Soft intersection of F and G, denoted by $F\tilde{\cap}G$, is a soft set over X and defined by $F\tilde{\cap}G : E \rightarrow \mathcal{P}(X)$ such that $(F\tilde{\cap}G)(e) = F(e) \cap G(e)$ for each $e \in E$.

(vii). Soft difference of F and G, denoted by $F\tilde{\setminus}G$, is a soft set over U whose approximate function is defined by $F\tilde{\setminus}G : E \rightarrow \mathcal{P}(X)$ such that $(F\tilde{\setminus}G)(e)$=$F(e)\setminus G(e)$.

(viii). Soft complement of F is denoted by $F^{\tilde{c}}$ and defined by $F^{\tilde{c}} : E \rightarrow \mathcal{P}(X)$ such that $F^{\tilde{c}}(e) = X \setminus F(e)$ for each $e \in E$.

Definition 2.4. [33] Let Δ be an arbitrary indexed set and $\Upsilon = \{F_i \mid i \in \Delta\}$ be a subfamily of $S_E(X)$, then *(i).* The soft union of Υ is the soft set H, however, $H(e) = \tilde{\cup}\{F_i(e) \mid i \in \Delta\}$ for all $e \in E$ that can be write as $\tilde{\cup}_{i\in\Delta} F_i = H$.

(ii). The soft intersection of Υ is the soft set K, however $K(e) = \tilde{\cap}\{F_i(e) \mid i \in \Delta\}$ for all $e \in E$ that can be write as $\tilde{\cup}_{i\in\Delta} F_i = K$.

Definition 2.5. [33] The soft set $F \in S_E(X)$ is called a soft point if there exist an $e \in E$ such that $F(e) \neq \emptyset$ and $F(e') = \emptyset$ for each $e' \in E \setminus \{e\}$, and the soft point F is denoted by e_F. The soft point e_F is said to be in the soft set G, denoted by $e_F \tilde{\in} G$, if $F(e) \tilde{\subseteq} G(e)$ for the element $e \in E$.

Definition 2.6. [17, 31] The soft set F over X such that $F(e) = \{x\}$ $\forall e \in E$ is called singleton soft point and denoted by x_E or (x, E).

Definition 2.7. [31] Let τ be a collection of soft sets over a universe X with a fixed set of parameters E, then $\tau \subseteq S_E(X)$ is called a soft topology on X if

(i). $\tilde{X}, \tilde{\emptyset} \in \tau$, where $\tilde{\emptyset}(e) = \emptyset$ and $\tilde{X}(e) = X$, for each $e \in E$,

(ii). The soft union of any number of soft sets in τ belongs to τ,

(iii). The soft intersection of any two soft sets in τ belongs to τ.

The triplet (X, τ, E) is called a soft topological space over X. A soft set F over X is said to be open soft set in X if $F \in \tau$, and it is said to be closed soft set in X, if its relative complement $F^{\tilde{c}}$ is an open soft set.

Definition 2.8. [31] Let (X, τ, E) be a soft topological space over X and $F \in S_E(X)$. Then, the soft interior and soft closure of F, denoted by $int(F)$ and $cl(F)$, respectively, are defined as,

$$\begin{aligned} int(F) &= \tilde{\cup}\big\{G : G \text{ is open soft set and } G\tilde{\subseteq}F\big\} \\ cl(F) &= \tilde{\cap}\big\{H : H \text{ is closed soft set and } F\tilde{\subseteq}H\big\}. \end{aligned}$$

Definition 2.9. [31] Let F be a soft set over X and $x \in X$. $x \in F$ whenever $x \in F(e)$ for all $e \in E$. Note that for any $x \in X$, $x \notin F$, if $x \notin F(e)$ for some $e \in E$.

Definition 2.10. [33] A soft set H of a soft topological space (X, τ, E) is known as a soft neighborhood (soft nbd.) of the soft point x, if there is a soft open set K such that $x \in K \tilde{\subseteq} F$.

Lemma 2.11. [23] Let (X, τ, E) be a soft topological space and F be a soft set. Then,

(i). $int(F\tilde{\cup}H)\tilde{\subseteq}int(F)\tilde{\cup}H$, if H is a τ-closed soft set.

(ii). $H\tilde{\cap}cl(F)\tilde{\subseteq}cl(H\tilde{\cup}F)$, if H is a τ-open soft set, then.

Definition 2.12. A soft set F of a soft topological space (X, τ, E) is called

(i). [10] Pre open soft, if $F\tilde{\subseteq}int\ clF$ (resp., pre closed soft, if $cl\ intF\tilde{\subseteq}F$).

(ii). [5] Semi open soft, if $F\tilde{\subseteq}cl\ intF$ (resp., semi closed soft, if $int\ clF\tilde{\subseteq}F$).

(iii). [32] Regular open soft, if $F = int\ clF$ (resp., regular closed soft, if $F = cl\ intF$).

(iv). [10] α-open soft, if $F\tilde{\subseteq}int\ cl\ intF$.

Definition 2.13. [5] Let (X, τ, E) be a soft topological space, then a semi soft closure Scl_sF of a soft set F is the intersection of all semi-closed soft supersets of F. In other words, $Scl_sF = F\tilde{\cup}int\ clF$.

Definition 2.14. [30] A soft set F is called dense soft in H (resp., dense soft), if $H\tilde{\subseteq}clF$ (resp., $clF = \tilde{X}$).

Definition 2.15. [29] A non-empty collection $G\tilde{\subseteq}S_E(X)$ of soft sets over X is known as a soft grill, if these conditions hold:

(i). $\tilde{\emptyset} \notin G$

(ii). If $F \in G$ and $F\tilde{\subseteq}H$, then $H \in G$.

(iii). If $F\tilde{\cup}H \in G$, then $F \in G$ or $H \in G$.

Definition 2.16. [29] Let G be a soft grill over a soft topological space (X, τ, E). Now consider the soft operator $\varphi_G : S_E(X) \longrightarrow S_E(X)$, given by, for every soft set F, $\varphi_G(F) = \{x \mid U\tilde{\cap}F \in G$ for every soft open nbd. U of $x\}$. Then, the soft operator $\psi_G : S_E(X) \longrightarrow S_E(X)$, defined by for every soft set F, $\psi_G(F) = F\tilde{\cup}\varphi_G(F)$ is a kuratowski's soft closure operator and hence gives rise to a new soft topology over X with the same parameters, $\tau_G = \{H \mid \psi_G(\tilde{X} - H) = (\tilde{X} - H)\}$, which is finer than τ in general.

Lemma 2.17. [29] Let G be a soft grill over a soft topological space (X, τ, E). Then, for every soft set F the following statements hold:

(i). If $F \notin G$ then, $\varphi_G(F) = \tilde{\emptyset}$. Moreover, $\varphi_G(\tilde{\emptyset}) = \tilde{\emptyset}$.

(ii). $\varphi_G\varphi_G(F)\tilde{\subseteq}\varphi_G(F) = cl\varphi_G(F)\tilde{\subseteq}clF$. Moreover, $\varphi_G(F)$ is soft τ-closed.

(iii). $\varphi_G\psi_G(F) = \psi_G\varphi_G(F) = \varphi_G(F)$.

(iv). If a soft set F is τ-closed, then $\varphi_G(F)\tilde{\subseteq}F$. Moreover, $\psi_G(F)\tilde{\subseteq}F$.

(v). A soft set F is τ_G-closed if and only if $\varphi_G(F)\tilde{\subseteq}F$.

Lemma 2.18. [29] Let G be a soft grill over a soft topological space (X, τ, E). Then, for soft sets F, H the following statements hold:

(i). $F\tilde{\subseteq}H$ implies $\varphi_G(F)\tilde{\subseteq}\varphi_G(H)$.

(ii). $\varphi_G(F\tilde{\cup}H) = \varphi_G(F)\tilde{\cup}\varphi_G(H)$ and $\varphi_G(F\tilde{\cap}H)\tilde{\subseteq}\varphi_G(F)\tilde{\cap}\varphi_G(H)$.

(iii). $\varphi_G(F) - \varphi_G(H) = \varphi_G(F - H) - \varphi_G(H)$.

(iv). If $H \notin G$, then $\varphi_G(F\tilde{\cup}H) = \varphi_G(F) = \varphi_G(F - H)$.

Lemma 2.19. [29] Let G be a soft grill over a soft topological space (X, τ, E) with $(\tau - \{\tilde{\emptyset}\})\tilde{\subseteq}G$. Then, the following statements hold:

(i). $\varphi_G(\tilde{X}) = \tilde{X}$.

(ii). $H\tilde{\subseteq}\varphi_G(H)$, for any open soft set H.

Theorem 2.20. [29] Let G be a soft grill over a soft topological space (X, τ, E) and F be soft set such that $F\tilde{\subseteq}\varphi_G\ (F)$. Then, $clF = \psi_G(F) = \tau_G - clF = cl(\varphi_G(F)) = \varphi_G(F)$.

Theorem 2.21. [29] Let G be a soft grill over a soft topological space (X, τ, E). If H is a τ-open soft set, then $H\tilde{\cap}\varphi_G(F) = H\tilde{\cap}\varphi_G(H\tilde{\cap}F)$, for every soft set F.

Lemma 2.22. [29] Let (X, τ, E) be a soft topological space and F be a soft set. Then, for soft grills G_1, G_2 over X the following statements hold:

(i). If $G_1\tilde{\subseteq}G_2$, then $\varphi_{G1}(F)\tilde{\subseteq}\varphi_{G2}(F)$.

(ii). $\varphi_{G1\tilde{\cup}G2}(F) = \varphi_{G1}(F)\tilde{\cup}\varphi_{G2}(F)$.

Lemma 2.23. [29] Let (X, τ, E) be a soft topological space with $G = P(X) - \{\tilde{\emptyset}\}$, then for any soft set F, $\varphi_G(F) = F$. Moreover, $\psi_G(F) = F$.

Definition 2.24. [30] Let G be a soft grill over a soft topological space (X, τ, E). A soft set F is called

(i). τ_G-perfect (resp., G-dense) soft, if $\varphi_G(F) = F$ (resp., $\varphi_G(F) = \tilde{X}$)

(ii). G-dense in a soft set H (resp., G-dense in itself) soft, if $H\tilde{\subseteq}\varphi_G(F)$ (resp., $F\tilde{\subseteq}\varphi_G(F)$).

Lemma 2.25. [30] Every G-dense soft is dense soft set.

3 Pre G-Open Soft Sets

Definition 3.1. Let G be a soft grill over a soft topological space (X, τ, E). A soft set F is called pre G-open soft, if $F\tilde{\subseteq}int\varphi_G(F)$. The complement of such set will be called pre G-closed soft.

Remark 3.2. *(i).* G-dense soft set $\Longrightarrow$ pre G-open soft set $\Longrightarrow$ pre open soft set.

(ii). pre G-open soft set $\Longrightarrow$ G-dense in it self soft set.

These implications are irreversible as indicated in the next example.

Example 3.3. Let $X = \{a, b, c\}$, $A = \{e_1, e_2\}$, $\tau = \{\tilde{\emptyset}, \tilde{X}, F_1, F_2, F_3, F_4, F_5, F_6, F_7, F_8, F_9\}$ and $G = \{\tilde{X}, C, D, E, K\}$ where $F_1, F_2, F_3, F_4, F_5, F_6, F_7, F_8, F_9, C, D, M, K$ are soft sets over X, explained as follow
$F_1 = \{\{a\}, \emptyset\}, F_2 = \{\{b, c\}, \{a\}\}, F_3 = \{X, \{a\}\}, F_4 = \{\{b\}, \{a, c\}\}$,
$F_5 = \{\{a, b\}, \{a, c\}\}, F_6 = \{\{b, c\}, \{a, c\}\}, F_7 = \{X, \{a, c\}\}, F_8 = \{\{b\}, \{a\}\}$,
$F_9 = \{\{a, b\}, \{a\}\}, C = \{\{a\}, \emptyset\}, D = \{\{a, c\}, \emptyset\}, M = \{\{a, b\}, \emptyset\}, K = \{X, \emptyset\}$.
It is clear that

(i). $F_8 = \{\{b\}, \{a\}\}$ is pre open soft set, but it is not pre G-open soft set.

(ii). $F_1 = \{\{a\}, \emptyset\}$ is pre G-open soft set, but it is not G-dense soft set.

The following example indicates that the notions pre G-open soft and open soft sets are independent.

Example 3.4. Let $X = \{a, b\}, A = \{e_1, e_2\}$. Define $F_1 = \{\{a\}, \{b\}\}, F_2 = \{X, \{b\}\}, F_3 = \{\{a\}, X\}$ are all soft sets on universe set X. Soft topology $\tau = \{\tilde{\emptyset}, \tilde{X}, F_1, F_2\}$ and soft grill $G = \{\tilde{X}, F_3\}$. It is clear that F_3 is pre G-open soft set, but it is not open soft. Also, F_2 is open soft set, but it is not pre G-open soft.

Example 3.5. If $G = P(X) - \{\tilde{\emptyset}\}$, then in view of Lemma 2.6, the concepts pre G-open soft and open soft sets are equivalent.

Theorem 3.6. For any soft set F in a soft topological space (X, τ, E) with grill G, the following properties hold:

(i). If $(\tau - \{\tilde{\emptyset}\})\tilde{\subseteq}G$, then each τ-open soft set is pre G-open soft.

(ii). If F is pre G-open soft and τ_G-closed soft set, then it is τ-open soft set. Moreover, $\varphi_G(F)$ is τ-open soft set.

(iii). If F is pre G-open soft and τ_G-perfect soft set, then it is τ-open soft set.

Proof. We prove only (ii) and the rest of the proof may be done straightforward. Let F be a τ_G-closed soft set, then $\varphi_G(F)\tilde{\subseteq}F$, follows directly in view of Lemma 2.2. Since F is a pre G-open soft set, then $F\tilde{\subseteq}int\varphi_G(F)$. Hence, $F\tilde{\subseteq}int(F)$ and so F is a τ-open soft set. Also, $\varphi_G(F)\tilde{\subseteq}F\tilde{\subseteq}int\varphi_G(F)$. Hence, $\varphi_G(F)$ is a τ-open soft set.
The following corollary is immediate from (ii) of Theorem 3.1.

Corollary 3.7. If F is pre G-open soft and τ_G-closed soft set, then $int\varphi_G(F) = \varphi_G(intF)$.

Definition 3.8. A soft topological space (X, τ, E) is known as soft locally indiscrete, if each open soft set is closed soft.

Definition 3.9. A soft set F in a soft topological space (X, τ, E) with soft grill G is said to be semi G-open soft, if $F\tilde{\subseteq}\varphi_G(intF)$.

Theorem 3.10. Let G be a soft grill on a soft locally indiscrete topological space (X, τ, E). Therefore, F is pre G-open soft, if it is a semi G-open soft set.

Proof. Let F be a semi G-open soft set, then $F\tilde{\subseteq}\varphi_G int(F)\tilde{\subseteq}\varphi_G(F)$. Since (X, τ, E) is soft locally indiscrete space and in view of Lemma 2.2, then $F\tilde{\subseteq}\varphi_G int(F)\tilde{\subseteq}cl\ intF = intF\tilde{\subseteq}int\varphi_G(F)$. Thus, F is a pre G-open soft set.

Theorem 3.11. Let G be a soft grill on a soft topological space (X, τ, E) with $(\tau - \{\tilde{\emptyset}\})\tilde{\subseteq}G$. Then, F is a G-dense soft set with respect to τ if and only if it is a dense soft set with respect to τ_G.

Proof. Let F be a G-dense soft set, then $\tilde{X} = \varphi_G(F)\tilde{\subseteq}\psi_G(F)$. Hence, F is a dense soft set with respect to τ_G. Conversely, let F be a dense soft set with respect to τ_G, then $\tilde{X} = \psi_G(F)$. Then, $\tilde{X} = \varphi_G(F)$ follows directly since $(\tau - \{\tilde{\emptyset}\})\tilde{\subseteq}G$ and by using (iii) of Lemma 2.2, (i) of Lemma 2.3 and (i) of Lemma 2.4. Consequently, F is a G-dense soft set with respect to τ.

Theorem 3.12. Let G be a soft grill on a soft topological space (X, τ, E) with $(\tau - \{\tilde{\emptyset}\})\tilde{\subseteq}G$. Then, F is a pre G-open soft set with respect to τ, if and only if it is a pre open soft set with respect to τ_G.

Proof. It is clear that, every pre G-open soft set with respect to τ is a pre open soft set with respect to τ_G. Conversely, let F be a pre open soft set with respect to τ_G, then $F\tilde{\subseteq}int_{\tau G}\psi_G(F)$. Hence, x is a τ_G-interior soft point of $\psi_G(F)$ for $x \in F$. Consequently, there exists a τ_G-open soft set H of x which is a subset of $\psi_G(F)$ and then further there exists a soft open base member $(U-K)$ where U is τ-open soft set containing x and $K \notin G$ such that $(U-K)\tilde{\subseteq}H\tilde{\subseteq}\psi_G(F)$. Hence, $\varphi_G(U-K)\tilde{\subseteq}\varphi_G\psi_G(F)$ follows directly from Lemma 2.3. Therefore, $\varphi_G(U)\tilde{\subseteq}\varphi_G(F)$ by using Lemmas 2.2 and 2.3. Since $(\tau-\{\tilde{\emptyset}\})\tilde{\subseteq}G$, U is τ-open soft set containing x and $F\tilde{\subseteq}U$, then $U\tilde{\subseteq}\varphi_G(U)\tilde{\subseteq}\varphi_G(F)$ follows from Lemma 2.4. Consequently, $U\tilde{\subseteq}int\varphi_G(F)$. Then, $F\tilde{\subseteq}int\varphi_G(F)$ and so F is a pre G-open soft set with respect to τ.

Theorem 3.13. Let G be a soft grill on a soft topological space (X,τ,E). Then,

(i). If H is a pre G-open soft set and $F\tilde{\subseteq}H\tilde{\subseteq}\varphi_G(F)$, then F is a pre G-open soft set.

(ii). Let F be a G-dense in an open soft set H and $F\tilde{\subseteq}H$, then F is a pre G-open soft set.

Proof. We prove only (i) and the rest of the proof is obvious. Since $F\tilde{\subseteq}H\tilde{\subseteq}\varphi_G(F)$ and H is a pre G-open soft set, then $H\tilde{\subseteq}int\varphi_G(H)$ and so $F\tilde{\subseteq}H\tilde{\subseteq}int\varphi_G(H)\tilde{\subseteq}int\varphi_G\varphi_G(F)$. Then, $F\tilde{\subseteq}int\varphi_G(F)$ is obtained by using Lemma 2.2. Consequently, F is a pre G-open soft set.

The following theorem is immediate in view of Lemma 2.5.

Theorem 3.14. Let (X,τ,E) be a soft topological space with soft grills G_1, G_2 over X. Then,

(i). G_1-pre open soft set is G_2-pre open soft, if $G_1\tilde{\subseteq}G_2$.

(ii). If a soft set F is both G_1-pre open soft and G_2-pre open soft, then it is a $(G_1\tilde{\cup}G_2)$-pre open soft set.

Theorem 3.15. Let G be a soft grill on a soft topological space (X,τ,E). Then, an arbitrary union (resp., intersection) of pre G-open (resp., pre G-closed) soft sets is a pre G-open (resp., pre G-closed) soft.

Proof. Let $\{F_i \mid i \in \Gamma\}$ be a class of pre G-open soft sets, then for each $i \in \Gamma$, $F_i\tilde{\subseteq}int\varphi_G(F_i)$. Hence, $\tilde{\cup}_{i\in\Gamma}(F_i)\tilde{\subseteq}\tilde{\cup}_{i\in\Gamma}(int\varphi_G(F_i))\tilde{\subseteq}int\varphi_G(\tilde{\cup}_{i\in\Gamma}(F_i))$ and so $\tilde{\cup}_{i\in\Gamma}(F_i)$ is a pre G-open soft set. The other result follows immediately by taking complements.

The next example shows that intersection of two pre G-open soft sets may not be a pre G-open soft.

Example 3.16. Let $X=\{a,b\}$, $A=\{e_1,e_2\}$. Define $F_1=\{\{a\},\{b\}\}, F_2=\{X,\{b\}\}, F_3=\{\{a\},X\}, F_4=\{\{a\},\emptyset\}, F_5=\{X,\emptyset\}, F_6=\{X,\{a\}\}$ are all soft sets on universe set X and $\tau=\{\tilde{\emptyset},\tilde{X},F_1,F_2\}$ is soft topology over X If $G=\{\tilde{X},F_1,F_2,F_3,F_4,F_5,F_6\}$ is a soft grill over X, then it is clear that F_3, F_6 are pre G-open soft sets, but their intersection F_8 is not a pre G-open soft set.

Lemma 3.17. Let G be a soft grill on a soft topological space (X,τ,E). Then,

(i). The intersection of pre G-open soft set and open soft set is a pre G-open soft.

(ii). The intersection of G-dense soft set and open soft set is a pre G-open soft.

Proof. *(i).* Let F be a pre G-open soft set and H be an open soft set, then $F\tilde{\subseteq}int\varphi_G(F)$ and $H = intH$. Therefore, $(H\tilde{\cap}F)\tilde{\subseteq}intH\tilde{\cap}int\varphi_G(F) = int(H\tilde{\cap}\varphi_G(F))$ $\tilde{\subseteq}int\varphi_G(H\tilde{\cap}F)$, in view of Theorem 2.2. This shows that $(H\tilde{\cap}F)$ is a pre G-open soft set.

(ii). Let F be a G-dense soft set and H be an open soft set, then $\varphi_G(F) = \tilde{X}$ and $H = intH$. Hence, $(H\tilde{\cap}F)\tilde{\subseteq}H = intH = int(H\tilde{\cap}\varphi_G(F))\tilde{\subseteq}int\varphi_G(H\tilde{\cap}F)$, by using Theorem 2.2. Then, $(H\tilde{\cap}F)$ is pre G-open soft set.

Theorem 3.18. Let G be a soft grill on a soft topological space (X, τ, E) with $(\tau - \{\tilde{\emptyset}\})\tilde{\subseteq}G$. Then, A soft set is pre G-open soft if and only if it is the intersection of G-dense soft set and open soft set.

Proof. Let H be G-dense soft set and U be open soft set, then by using (ii) of Lemma 3.1 $H\tilde{\cap}U$ is pre G-open soft set. On the other hand, let F be pre G-open soft set, then $F\tilde{\subseteq}int\varphi_G(F)$. $\tilde{X} = \varphi_G\varphi_G(F)\tilde{\cup}(\tilde{X} - \varphi_G\varphi_G(F))\tilde{\subseteq}\varphi_G(F)\tilde{\cup}(\tilde{X} - \varphi_G\varphi_G(F)) = \varphi_G(F)\tilde{\cup}(\varphi_G(\tilde{X}) - \varphi_G\varphi_G(F))\tilde{\subseteq}\varphi_G(F)\tilde{\cup}\varphi_G(\tilde{X} - \varphi_G(F))\tilde{\subseteq}\varphi_G[F\tilde{\cup}(\tilde{X} - \varphi_G(F))]$, in view of Lemmas 2.2, 2.3 and 2.4. Consequently, $[F\tilde{\cup}(\tilde{X} - \varphi_G(F))]$ is G-dense soft set. It is obvious that $F = [F\tilde{\cup}(\tilde{X} - \varphi_G(F))]\tilde{\cap}int\varphi_G(F)$.

Definition 3.19. A soft topological space (X, τ, E) is called soft G-sub maximal, if each soft G-dense set is a soft open set.

In the following theorem, conditions for the collection of soft pre G-open sets to be soft topology will be deduced.

Theorem 3.20. Let G be a soft grill on a soft topological space (X, τ, E) with $(\tau - \{\tilde{\emptyset}\})\tilde{\subseteq}G$. Then, τ is the collection of pre G-open soft sets if and only if a soft topological space (X, τ, E) is soft G-sub maximal.

Proof. Let τ be a collection of soft pre G-open sets i.e every soft pre G-open soft set is open soft, then in view of Remark 3.1, the space (X, τ, E) is soft G-sub maximal. Conversely, since $(\tau - \{\tilde{\emptyset}\})\tilde{\subseteq}G$ and F is open soft set, then F is a pre G-open soft set follows from (i) of Theorem 3.1. On the other hand, let F be a pre G-open soft set; then in view of Theorem 3.8, there exist G-dense soft set H and open soft set U such that $F = H\tilde{\cap}U$. Since (X, τ, E) be a soft G-sub maximal, then H is an open soft set. Consequently, F is an soft open soft set.

Theorem 3.21. Let G be a soft grill on a soft topological space (X, τ, E). Then, for every soft set F the following statements are equivalent

(i). F is pre G-open soft set.

(ii). F is G-dense in itself soft and pre open soft set.

(iii). F is G-dense in itself soft set and $Scl_s(F) = int\ cl(F)$.

Proof. $(i) \implies (ii)$ In view of Remark 3.1, every pre G-open soft set is G-dense in itself soft and pre open soft.

$(ii) \implies (iii)$ Let F be G-dense in itself soft and pre open soft set, then in view of Definition 2.12, $Scl_s(F) = F\tilde{\cup}int\ clF = intcl(F)$.

$(iii) \Longrightarrow (i)$ Straightforward.

Corollary 3.22. Let G be a soft grill on a soft topological space (X, τ, E). If F is a semi-closed soft and pre G-open soft set, then it is a regular open soft.

Lemma 3.23. Let G be a soft grill on a soft topology space (X, τ, E). If F is a pre G-closed soft set, then $\varphi_G(intF)\tilde{\subseteq}cl\ intF\tilde{\subseteq}F$.

Proof. Let F be pre G-closed soft set, then $(F)^c$ is pre G-open soft i.e $(F)^c\tilde{\subseteq}int\varphi_G((F)^c)\tilde{\subseteq}\varphi_G((F)^c)$. $\varphi_G((F)^c) = cl((F)^c) = (int(F))^c$ follows directly by using Theorem 2.1. Therefore, $(F)^c\tilde{\subseteq}int(intF)^c = (cl\ intF)^c$. Thus, $cl\ intF\tilde{\subseteq}F$. Moreover, $\varphi_G(intF)\tilde{\subseteq}cl\ intF\tilde{\subseteq}F$ follows from Lemma 2.2.

Theorem 3.24. Let G be a soft grill on a soft topological space (X, τ, E). If F is a pre G-closed soft and α-open soft set, then it is regular clopen soft.

Proof. Let F be a soft α-open set, then it is semi open soft and pre open soft set. Therefore, $F\tilde{\subseteq}cl\ intF$ and $F\tilde{\subseteq}int\ clF$. Since F is pre G-closed soft set, then $cl\ intF\tilde{\subseteq}F$ follows from Lemma 3.2. Hence, $cl\ intF = F$ and so $F\tilde{\subseteq}int\ clF\tilde{\subseteq}clF = cl\ cl\ intF = cl\ intF = F$. Consequently, $F = cl\ intF = int\ clF$ and so F is a soft regular clopen set.

Theorem 3.25. Let G be a soft grill on a soft topological space (X, τ, E) with $(\tau - \{\tilde{\emptyset}\})\tilde{\subseteq}G$. Then, for every soft set F the following statements are equivalent

(i). F is pre G-open soft set.

(ii). There is regular open soft set U such that $F\tilde{\subseteq}U$ and $\varphi_G(F) = \varphi_G(U)$.

(iii). $F = U\tilde{\cap}H$, where U be regular open soft set and H be G-dense soft set.

(iv). $F = U\tilde{\cap}H$, where U be open soft set and H be G-dense soft set.

Proof. $(i) \Longrightarrow (ii)$ Let F be pre G-open soft set, then $F\tilde{\subseteq}int\varphi_G(F)\tilde{\subseteq}\varphi_G(F)$. Hence, $\varphi_G(F)\tilde{\subseteq}\varphi_G(int\varphi_G(F))\tilde{\subseteq}\varphi_G\varphi_G(F)\tilde{\subseteq}\varphi_G(F)$ follows from Lemmas 2.2 and 2.3. Thus, $\varphi_G(int\varphi_G(F)) = \varphi_G(F)$. Put $U = int\varphi_G(F)$, then $F\tilde{\subseteq}U$, $\varphi_G(U) = \varphi_G(F)$ and $int\varphi_G(U) = int\varphi_G(F)$. In view of Theorem 2.1, $int\ cl(U) = int\varphi_G(U) = int\varphi_G(F) = U$. Hence, U is regular open soft set.

$(ii) \Longrightarrow (iii)$ Let $F\tilde{\subseteq}U$ and U be a soft regular open set such that $\varphi_G(F) = \varphi_G(U)$. Suppose $H = F\tilde{\cup}(U)^c$, then $\varphi_G(H) = \varphi_G(F\tilde{\cup}(U)^c) = \varphi_G(F)\tilde{\cup}\varphi_G((U)^c) = \varphi_G(U)\tilde{\cup}\varphi_G((U)^c) = \varphi_G(U\tilde{\cup}(U)^c) = \varphi_G(\tilde{X}) = \tilde{X}$, follows by using Lemmas 2.3, 2.4. Hence, H is a soft G-dense set and $U\tilde{\cap}H = F$. The rest of the proof are immediate.

The collection $G_\delta = \{F \mid int\ clF \neq \tilde{\emptyset}\}$ is a soft grill on X.

Theorem 3.26. Let G_δ be a soft grill on a soft topological space (X, τ, E). F is a soft pre G_δ-open set if and only if it is a pre open soft.

Proof. Clearly, pre G_δ-open soft set is a pre open soft. Conversely, let F be a soft pre open set, subsequently $F\tilde{\subseteq}int\ clF$. Let $x \notin \varphi_{G\delta}(F)$, then there exists soft open nbd. U of x such that $(U\tilde{\cap}F) \notin G_\delta$. Hence, $int\ cl(U\tilde{\cap}F) = \tilde{\emptyset}$ and $(U\tilde{\cap}F)\tilde{\subseteq}(U\tilde{\cap}int\ clF) = int(U\tilde{\cap}clF)\tilde{\cap}int\ cl(U\tilde{\cap}F) = \tilde{\emptyset}$ follows from Lemma 1.10. Therefore, $x \notin F$ and so $F\tilde{\subseteq}\varphi_{G\delta}(F)$. $F\tilde{\subseteq}int\ cl(F) = int\varphi_{G\delta}(F)$ follows directly by using Theorem 1.19. Consequently, F is soft pre G_δ-open set.

Definition 3.27. A soft GP-interior of F, denoted by $Sint_{GP}(F)$, is defined as the largest soft pre G-open sets contained in F.

Theorem 3.28. Let G be a soft grill on a soft topological space (X, τ, E). Then, for any soft set F the following statements hold:

(i). $(F \tilde{\cap} int\varphi_G(F))$ is a soft pre G-open set.

(ii). $Sint_{GP}(F) = F \tilde{\cap} int\varphi_G(F)$.

(iii). $F \notin G$, then $Sint_{GP}(F) = \tilde{\emptyset}$.

(iv). If F is soft pre G-open set and $Sint_{GP}(F) = \tilde{\emptyset}$, then $F = \tilde{\emptyset}$

(v). $(F \tilde{\cap} \varphi_G(F)) \tilde{\subseteq} Sint_{GP}(F)$.

Proof. *(i).* Since int $\varphi_G(F) = \varphi_G(F) \tilde{\cap} int\varphi_G(F)$, then in view of Theorem 2.2 int $\varphi_G(F) \tilde{\subseteq} \varphi_G(F \tilde{\cap} int\varphi_G(F))$. Hence, $(F \tilde{\cap} int\varphi_G(F)) \tilde{\subseteq} int\varphi_G(F) \tilde{\subseteq} int\varphi_G(F \tilde{\cap} int\varphi_G(F))$. Thus, $(F \tilde{\cap} int\varphi_G(F))$ is soft pre G-open set.

(ii). $(F \tilde{\cap} int\varphi_G(F))$ is soft pre G-open set contained in F, From (i). Suppose H is soft pre G-open set contained in F, then $H \tilde{\subseteq} int\varphi_G(H)$ and $int\varphi_G(H) \tilde{\subseteq} int\varphi_G(F)$. Therefore, $H \tilde{\subseteq} int\varphi_G(F)$ and so $H \tilde{\subseteq} (F \tilde{\cap} int\varphi_G(F))$. Consequently, $(F \tilde{\cap} int\varphi_G(F))$ is largest soft pre G-open sets contained in F. Then, $Sint_{GP}(F) = F \tilde{\cap} int\varphi_G(F)$.

(iii). In view of Lemma 2.2 and $F \notin G$, then $Sint_{GP}(F) = F \tilde{\cap} int\varphi_G(F) = \tilde{\emptyset}$.

(iv). Straightforward.

(v). Since $(F \tilde{\cap} \varphi_G(F)) - Sint_{GP}(F)) = [(F \tilde{\cap} \varphi_G(F)) - (F \tilde{\cap} int\varphi_G(F))] = [(F \tilde{\cap} \varphi_G(F)) - int\varphi_G(F)]$, then by (ii) $Sint_{GP}[(F \tilde{\cap} \varphi_G(F)) - int\varphi_G(F)] = [(F \tilde{\cap} \varphi_G(F)) - int\varphi_G(F)] \tilde{\cap} int\varphi_G[(F \tilde{\cap} \varphi_G(F)) - int\varphi_G(F)] = \tilde{\emptyset}$.

Corollary 3.29. $Sint_{GP}(F) = F \tilde{\cap} \varphi_G(F)$

Acknowledgement

The authors express their sincere thanks to the reviewers for their valuable suggestions. The authors are also thankful to the editors-in-chief and managing editors for their important comments which helped to improve the presentation of the paper.

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