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## EFFECT OF DEFUZZIFICATION METHODS IN SOLVING FUZZY MATRIX GAMES

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**Abstract** – This paper deals with two-person matrix games whose elements of pay-off matrix are fuzzy numbers. Then the corresponding matrix game has been converted into crisp game using different defuzzification techniques. The value of the matrix game for each player is obtained by solving corresponding crisp game problems using the existing method. Finally, to illustrate the proposed methodology, a practical and realistic numerical example has been applied for different defuzzification methods and the obtained results have been compared.

**Keywords** – *Fuzzy number, Fuzzy payoff, Defuzzification, Matrix Game*

### 1. Introduction

In many real world practical problems with competitive situation, it is required to take the decision where there are two or more opposite parties with conflicting interests and the action of one depends upon the action which is taken by the opponent. A great variety of competitive situation is commonly seen in everyday life viz., in military battles, political campaign, elections, advertisement, etc. Game theory is a mathematical way out for finding of conflicting interests with competitive situations, which includes players or decision makers (DM) who select different strategies from the set of admissible strategies.

During the past, several researchers formulated and solved matrix game considering crisp/precise payoff. This means that every probable situation to select the payoff involved in the matrix game is perfectly known in advance. In this case, it is usually assumed that there exists some complete information about the payoff matrix. However, in real-life situations, there are not sufficient data available in most of the cases where the situation is known or it exists only a market situation. It is not always possible to observe the stability

from the statistical point of view. This means that only some partial information about the situations is known. In these cases, parameters are said to be imprecise.

To handle the problem with such types of imprecise parameters, generally stochastic, fuzzy and fuzzy-stochastic approaches are applied and the corresponding problems are converted into deterministic problems for solving them. In this paper, we have treated imprecise parameters considering fuzzy sets/fuzzy numbers. In the last few years, several attempts have been made in the existing literature for solving game problem with fuzzy payoff. Fuzziness in game problem has been well discussed by Campos [1]. Sakawa and Nishizaki [2] introduced max-min solution procedure for multi-objective fuzzy games. Based on fuzzy duality theory [3, 4, 5], Bector et al. [6, 7], and Vijay et al. [8] proved that a two person zero-sum matrix game with fuzzy goals and fuzzy payoffs is equivalent to a pair of linear programming problems. Nayak and Pal [9, 10] studied the interval and fuzzy matrix games. Chen and Larbani [11] used two persons zero-sum game approach to solve fuzzy multiple attributes decision making problem. Çevikel and Ahlatçioğlu [12] presented new concepts of solutions for multi-objective two person zerosum games with fuzzy goals and fuzzy payoffs using linear membership functions. Li and Hong [13] gave an approach for solving constrained matrix games with payoffs of triangular fuzzy numbers. Bandyopadhyay et al. [14] well studied a matrix game with payoff as triangular intuitionistic fuzzy number. Very recently, Mijanur et al. [15] introduced an alternative approach for solving fuzzy matrix games.

In this paper, two person matrix games have taken into consideration. The element of payoff matrix is considered to be fuzzy number [16]. Then the corresponding problem has been converted into crisp equivalent two person matrix game using different defuzzification methods [17]. The value of the matrix game for each player is obtained by solving corresponding crisp game problems using the existing method. Finally, to illustrate the methodology, a numerical example has been applied for different defuzzification methods and the computed results have been compared.

The rest of the paper is organized as follows. Sec. 2 presents the basic definition and preliminaries of Fuzzy Numbers. Defuzzification method is presented in Sec. 3. Mathematical model of matrix game is described in Sec. 4. Solution of matrix game is presented in Sec. 5. Numerical example and Computational results are reported in Sec. 6 and a conclusion has been drawn in Sec 7.

## 2. Definition and Preliminaries

**Definition 2.1.** Let  $X$  be a non empty set. A fuzzy set  $\tilde{A}$  is defined as the set of pairs  $\tilde{A} = \{(x, \mu_{\tilde{A}}(x)) : x \in X\}$ , where  $\mu_{\tilde{A}} : X \rightarrow [0,1]$  is a mapping and  $\mu_{\tilde{A}}(x)$  is called the membership function of  $\tilde{A}$  or grade of membership of  $x$  in  $\tilde{A}$ . The value  $\mu_{\tilde{A}}(x) = 0$  is used to represent for complete non-membership, whereas  $\mu_{\tilde{A}}(x) = 1$  is used to represent for complete membership. The values in between zero and one are used to represent intermediate degrees of membership.

**Definition 2.2.** A fuzzy set  $\tilde{A}$  is called convex iff for all  $x_1, x_2 \in X$   $\mu_{\tilde{A}}(\lambda x_1 + (1-\lambda)x_2) \geq \min\{\mu_{\tilde{A}}(x_1), \mu_{\tilde{A}}(x_2)\}$ , where  $\lambda \in [0,1]$ .

**Definition 2.3.** The set of elements that belong to the fuzzy set  $\tilde{A}$  at least to the degree  $\alpha$  is called the  $\alpha$ -level set or  $\alpha$ -cut and is given by  $\tilde{A}_\alpha = \{x \in X : \mu_{\tilde{A}}(x) \geq \alpha\}$ . If  $\tilde{A}_\alpha = \{x \in X : \mu_{\tilde{A}}(x) > \alpha\}$ , it is called strong  $\alpha$ -level set or strong  $\alpha$ -cut.

**Definition 2.4.** A fuzzy set  $\tilde{A}$  is called a normal fuzzy set if there exists at least one  $x \in X$  such that  $\mu_{\tilde{A}}(x) = 1$ .

**Definition 2.5.** A fuzzy number  $\tilde{A}$  is a fuzzy set on the real line  $R$ , must satisfy the following conditions.

- (i) There exists at least one  $x_0 \in R$  for which  $\mu_{\tilde{A}}(x_0) = 1$ .
- (ii)  $\mu_{\tilde{A}}(x)$  is pair wise continuous.
- (iii)  $\tilde{A}$  must be convex and normal.

**Definition 2.6.** A triangular fuzzy number (TFN)  $\tilde{A}$  is a normal fuzzy number represented by the triplet  $(a_1, a_2, a_3)$  where  $a_1 \leq a_2 \leq a_3$  are real numbers and its membership function  $\mu_{\tilde{A}}(x) : X \rightarrow [0, 1]$  is given below

$$\mu_{\tilde{A}}(x) = \begin{cases} \frac{x - a_1}{a_2 - a_1} & \text{if } a_1 \leq x \leq a_2 \\ 1 & \text{if } x = a_2 \\ \frac{a_3 - x}{a_3 - a_2} & \text{if } a_2 \leq x \leq a_3 \\ 0 & \text{otherwise} \end{cases}$$

**Definition 2.7.** A parabolic fuzzy number (PFN)  $\tilde{A}$  is a normal fuzzy number represented by the triplet  $(a_1, a_2, a_3)$  where  $a_1 \leq a_2 \leq a_3$  are real numbers and its membership function  $\mu_{\tilde{A}}(x) : X \rightarrow [0, 1]$  is given below

$$\mu_{\tilde{A}}(x) = \begin{cases} 1 - \left(\frac{a_2 - x}{a_2 - a_1}\right)^2 & \text{if } a_1 \leq x \leq a_2 \\ 1 & \text{if } x = a_2 \\ 1 - \left(\frac{x - a_2}{a_3 - a_2}\right)^2 & \text{if } a_2 \leq x \leq a_3 \\ 0 & \text{otherwise} \end{cases}$$

### 3. Defuzzification

Defuzzification is the process of producing a quantifiable result in fuzzy logic, given the fuzzy sets and the corresponding degrees of membership. There are several defuzzification techniques available in the existing literature. However, the common and useful techniques are as follows:

#### 3.1. Centre of Area of Fuzzy Number (COA of Fuzzy Number)

This defuzzification can be expressed as

$$x_{COA} = \frac{\int x \mu_A(x) dx}{\int \mu_A(x) dx}$$

where  $x_{COA}$  is the crisp output,  $\mu_A(x)$  is the membership function corresponding to the fuzzy number and  $x$  is the output variable. This method is also known as center of gravity or centroid defuzzification method.

#### 3.2. Bisector of Area of Fuzzy Number (BOA of Fuzzy Number)

The bisector of area is the vertical line that divides the region into two sub-regions of equal area. The formula for  $x_{BOA}$  is given by

$$\int_{a_1}^{x_{BOA}} \mu_A(x) dx = \int_{x_{BOA}}^{a_4} \mu_A(x) dx.$$

It is sometimes, but not always coincident with the centroid line.

#### 3.3. Largest of Maxima of Fuzzy Number (LOM of Fuzzy Number)

Largest of maximum  $x_{LOM}$  takes the largest amongst all  $x$  that belong to  $[a_2, a_3]$  as the crisp value.

#### 3.4. Smallest of Maxima of Fuzzy Number (SOM of Fuzzy Number)

It takes the smallest output with the maximum membership function as the crisp value and it is denoted by  $x_{SOM}$ .

#### 3.5. Mean of Maxima of Fuzzy Number (MOM of Fuzzy Number)

In this method only active rules with the highest degree of fulfillment are taken into account. The output is computed as:

$$x_{MOM} = \frac{1}{2}(x_{LOM} + x_{SOM}).$$

**3.6. Regular Weighted Point of Fuzzy Number (RWP of Fuzzy Number)**

For the fuzzy number  $A=(a_1, a_2, a_3)$ , the  $\alpha$ -cut is  $A_\alpha = [L_A(\alpha), R_A(\alpha)]$  and the regular weighted point for  $\tilde{A}$  is given by Saneifard [18].

$$RWP(\tilde{A}) = \frac{\int_0^1 \left( \frac{L_A(\alpha) + R_A(\alpha)}{2} \right) f(\alpha) d\alpha}{\int_0^1 f(\alpha) d\alpha} = \int_0^1 (L_A(\alpha) + R_A(\alpha)) f(\alpha) d\alpha$$

where

$$f(\alpha) = \begin{cases} 1 - 2\alpha & \text{when } \alpha \in [0, 1/2] \\ 2\alpha - 1 & \text{when } \alpha \in [1/2, 1]. \end{cases}$$

**3.7. Graded Mean Integration Value of Fuzzy Number (GMIV of Fuzzy Number)**

For the generalized fuzzy number  $\tilde{A}$  with membership function  $\mu_{\tilde{A}}(x)$ , according to Chen et al. [19], the Graded Mean Integral Value  $P_{dGw}(\tilde{A})$  of  $\tilde{A}$  is given by

$$P_{dGw}(\tilde{A}) = \frac{\int_0^1 x \{ (1-w)L^{-1}(x) + wR^{-1}(x) \} dx}{\int_0^1 x dx} = 2 \int_0^1 x \{ (1-w)L^{-1}(x) + wR^{-1}(x) \} dx$$

where the pre-assigned parameter  $w \in [0, 1]$  refers the degree of optimism.  $w = 1$  represents an optimistic point of view,  $w = 0$  represents a pessimistic point of view and  $w = 0.5$  indicates a moderately optimistic decision makers' point of view.

**3.8. Centre of the Approximated Interval of Fuzzy Number (COAI of Fuzzy Number)**

Let  $\tilde{A}$  be a fuzzy number with interval of confidence at the level  $\alpha$ , then the  $\alpha$ -cut is  $[A_L(\alpha), A_R(\alpha)]$ . The nearest interval approximation of  $\tilde{A}$  with respect to the distance metric  $d$  is

$$C_d(\tilde{A}) = \left[ \int_0^1 A_L(\alpha) d\alpha, \int_0^1 A_R(\alpha) d\alpha \right],$$

where

$$d(\tilde{A}, \tilde{B}) = \sqrt{\int_0^1 \{A_L(\alpha) - B_L(\alpha)\}^2 d\alpha + \int_0^1 \{A_R(\alpha) - B_R(\alpha)\}^2 d\alpha}.$$

The interval approximation for the triangular fuzzy number  $\tilde{A} = (a_1, a_2, a_3)$  is  $\left[\frac{(a_1 + a_2)}{2}, \frac{(a_2 + a_3)}{2}\right]$  and for the parabolic fuzzy number  $\tilde{A} = (a_1, a_2, a_3)$  is  $\left[\frac{1}{3}(2a_1 + a_2), \frac{1}{3}(a_2 + 2a_3)\right]$ . The defuzzified value for triangular fuzzy number is  $\frac{1}{4}(a_1 + 2a_2 + a_3)$  and for the parabolic fuzzy number is  $\frac{1}{3}(a_1 + a_2 + a_3)$ . The defuzzification values for different fuzzy numbers are listed in Table 1.

#### 4. Mathematical Model of a Matrix Game

Let  $A_i \in \{A_1, A_2, \dots, A_m\}$  be a pure strategy available for player  $A$  and  $B_j \in \{B_1, B_2, \dots, B_n\}$  be a pure strategy available for player  $B$ . When player  $A$  chooses a pure strategy  $A_i$  and the player  $B$  chooses a pure strategy  $B_j$ , then  $g_{ij}$  is the payoff for player  $A$  and  $-g_{ij}$  be a payoff for player  $B$ . The two-person zero-sum matrix game  $G$  can be represented as a payoff matrix  $G = [g_{ij}]_{m \times n}$ .

##### 4.1 Fuzzy Payoff matrix:

Let players  $A$  has  $m$  strategies, say,  $A_1, A_2, \dots, A_m$  and player  $B$  has  $n$  strategies, say,  $B_1, B_2, \dots, B_m$ .

**Table 1.** Defuzzified values for different fuzzy numbers.

Defuzzification technique	Defuzzified value for TFN	Defuzzified value for PFN
COA	$\frac{1}{3}(a_1 + a_2 + a_3)$	$\frac{1}{8}(3a_1 + 2a_2 + 3a_3)$
BOA	$\frac{1}{4}(a_1 + 2a_2 + a_3)$	$\frac{1}{3}(a_1 + a_2 + a_3)$
MOM	$a_2$	$a_2$
SOM	$a_2$	$a_2$
LOM	$a_2$	$a_2$
RWP	$\frac{1}{4}(a_1 + 2a_2 + a_3)$	$a_2 + \frac{2(\sqrt{2}+1)}{15}(a_3 - 2a_2 + a_1)$
GMIV (with $w=0.5$ )	$\frac{1}{6}(a_1 + 4a_2 + a_3)$	$\frac{1}{15}(4a_1 + 7a_2 + 4a_3)$
COAI	$\frac{1}{4}(a_1 + 2a_2 + a_3)$	$\frac{1}{3}(a_1 + a_2 + a_3)$

Here, it is assumed that each player has his/her choices from amongst the pure strategies. Also, it is assumed that player A is always the gainer and player B is always the loser. That is, all payoffs are assumed in terms of player A. Let  $\tilde{g}_{ij}$  be the fuzzy payoff which is the gain of player A from player B if player A chooses strategy  $A_i$  where as player B chooses  $B_j$ . Then the fuzzy payoff matrix of player A and A and B is  $\tilde{G} = [\tilde{g}_{ij}]_{m \times n}$ .

**4.2 Mixed strategy**

Let us consider the fuzzy matrix game whose payoff matrix is  $\tilde{G} = [\tilde{g}_{ij}]_{m \times n}$ . The mixed strategy for the player-A, is denoted by  $\xi = (x_1, \dots, x_m)'$ , where  $x_i \geq 0, i = 1, 2, \dots, m$  and  $\sum_{i=1}^m x_i = 1$ .

It is to be noted that  $e_i^m = (0, \dots, 0, 1, 0, \dots, 0)'$ ,  $i = 1, 2, \dots, m$  represent the pure strategy for the player-A and  $\xi = \sum_{i=1}^m e_i^m x_i$ . If  $S_m = \left\{ \xi : x_i \geq 0, \sum_{i=1}^m x_i = 1 \right\}$  then  $S_m \in E_m$ .

Similarly, a mixed strategy for the player-B is denoted by  $\eta = (y_1, y_2, \dots, y_n)'$  where  $y_j \geq 0, j = 1, 2, \dots, n$  and  $\sum_{j=1}^n y_j = 1$ . It is to be note that  $e_j^n = (0, 0, \dots, 0, 1, 0, \dots, 0)'$ ,

$j = 1, 2, \dots, n$  represent the pure strategy of the player-B and  $\eta = \sum_{j=1}^n e_j^n y_j$ . If

$S_n = \left\{ \eta : y_j \geq 0, \sum_{j=1}^n y_j = 1 \right\}$ , then  $S_n \in E_n$ . Where  $S_m$  and  $S_n$  are the spaces of mixed strategies for the player-A and player-B respectively.

**4.3. Maximin-Minimax principle or Maximin-Minimax criteria of optimality for Fuzzy Payoff matrix**

Let the player A's payoff matrix be  $[\tilde{g}_{ij}]_{m \times n}$ . If player A takes the strategy  $A_i$  then surely he/she will get at least  $i = 1, 2, \dots, m$  for taking any strategy by the opponent player B. Thus by the maximin-minimax criteria of optimality, the player A will choose that strategy which corresponds to the best of these worst outcomes

$$\min_j DFV(\tilde{g}_{1j}), \min_j DFV(\tilde{g}_{2j}), \dots, \min_j DFV(\tilde{g}_{mj})$$

Thus the maximin value for player A is given by  $\max_i \left( \min_j DFV(\tilde{g}_{ij}) \right)$

Similarly, player B will choose that strategy which corresponds to the best (minimum) of the worst outcomes (maximum losses)

$$\max_i DFV(\tilde{g}_{i1}), \max_i DFV(\tilde{g}_{i2}), \dots, \max_i DFV(\tilde{g}_{in})$$

Thus the minimax value for player  $B$  is given by  $\min_j \left( \max_i DFV(\tilde{g}_{ij}) \right)$

Here,  $DFV(\tilde{g}_{ij})$  represents defuzzified value of the fuzzy number  $\tilde{g}_{ij}$ .

**Theorem 4.1.** If a matrix game possesses a saddle point, it is necessary and sufficient that

$$\max_i \min_j DFV(\tilde{g}_{ij}) = \min_j \max_i DFV(\tilde{g}_{ij})$$

**Definition 4.2.** A pair  $(\xi, \eta)$  of mixed strategies for the players in a matrix game is called a situation in mixed strategies. In a situation  $(\xi, \eta)$  of mixed strategies each usual situation  $(i, j)$  in pure strategies becomes a random event occurring with probabilities  $x_i, y_j$ . Since in the situation  $(i, j)$ , player-A receives a payoff  $DFV(\tilde{g}_{ij})$ , the mathematical expectation of his payoff under  $(\xi, \eta)$  is equal to

$$E(\xi, \eta) = \sum_{i=1}^m \sum_{j=1}^n DFV(\tilde{g}_{ij}) x_i y_j$$

**Theorem 4.2.** Let  $E(\xi, \eta)$  be such that both  $\min_{\eta \in S_n} \max_{\xi \in S_m} E(\xi, \eta)$  and  $\max_{\xi \in S_m} \min_{\eta \in S_n} E(\xi, \eta)$  exist, then

$$\min_{\eta \in S_n} \max_{\xi \in S_m} E(\xi, \eta) \geq \max_{\xi \in S_m} \min_{\eta \in S_n} E(\xi, \eta)$$

#### 4.4 Saddle point of a function

Let  $E(\xi, \eta)$  be a function of two variables (vectors)  $\xi$  and  $\eta$  in  $S_m$  and  $S_n$  respectively. The point  $(\xi_0, \eta_0)$ ,  $\xi_0 \in S_m, \eta_0 \in S_n$  is said to be the saddle point of the function  $E(\xi, \eta)$  if

$$E(\xi, \eta_0) \leq E(\xi_0, \eta_0) \leq E(\xi_0, \eta)$$

**Theorem 4.3.** Let  $E(\xi, \eta)$  be a function of two variables  $\xi \in S_m$  and  $\eta \in S_n$  such that  $\max_{\xi} \min_{\eta} E(\xi, \eta)$  and  $\min_{\eta} \max_{\xi} E(\xi, \eta)$  exist. Then the necessary and sufficient condition for the existence of a saddle point  $(\xi_0, \eta_0)$  of  $E(\xi, \eta)$  is that

$$E(\xi_0, \eta_0) = \max_{\eta} \min_{\xi} E(\xi, \eta) = \min_{\eta} \max_{\xi} E(\xi, \eta)$$

### 4.4 Value of a Matrix Game

The common value of  $\max_{\xi} \left\{ \min_{\eta} E(\xi, \eta) \right\}$  and  $\min_{\eta} \left\{ \max_{\xi} E(\xi, \eta) \right\}$  is called the value of the matrix game with payoff matrix  $\tilde{G} = [\tilde{g}_{ij}]$  and denoted by  $v(G)$  or simply  $v$ .

**Definition 4.3.** Thus if  $(\xi^*, \eta^*)$  is an equilibrium situation in mixed strategies of the game  $\langle S_m, S_n, E \rangle$ , then  $\xi^*, \eta^*$  are the optimal strategies for the players A and B respectively in the matrix game with fuzzy payoff matrix  $\tilde{G} = [\tilde{g}_{ij}]_{m \times n}$ . Hence  $\xi^*, \eta^*$  are optimal strategies for the players A and B respectively iff

$$E(\xi, \eta^*) \leq E(\xi^*, \eta^*) \leq E(\xi^*, \eta) \quad \forall \xi \in S_m, \eta \in S_n$$

**Definition 4.2.**

(i)  $\min_{\xi} E(\xi, \eta) = E(\xi, \eta^*) \Rightarrow \therefore \max_{\xi} \left\{ \min_{\eta} E(\xi, \eta) \right\} = \max_{\xi} E(\xi, \eta^*) = E(\xi^*, \eta^*)$

(ii)  $\max_{\eta} E(\xi, \eta) = E(\xi^*, \eta) \Rightarrow \min_{\eta} \left\{ \max_{\xi} E(\xi, \eta) \right\} = \min_{\eta} E(\xi^*, \eta) = E(\xi^*, \eta^*)$

**Theorem 4.4.**  $v = \max_{\xi} \left\{ \min_j E(\xi, j) \right\} = \min_{\eta} \left\{ \max_i E(i, \eta) \right\}$  and the outer extrema are attained at optimal strategies of players.

**Theorem 4.5.**  $\max_i \left\{ \min_j DFV(\tilde{g}_{ij}) \right\} \leq v \leq \min_i \left\{ \max_j DFV(\tilde{g}_{ij}) \right\}$

**Proof:** By the theorem 4.4, we have  $v = \max_{\xi} \left\{ \min_j E(\xi, j) \right\} \quad \forall \xi \in S_m$ . But  $\max_{\xi} \left\{ \min_j E(\xi, j) \right\} \geq \min_j E(\xi, j) \quad \forall \xi \in S_m$ . Therefore  $v \geq \min_j E(\xi, j) \quad \forall \xi \in S_m$ . Letting  $\xi = e_i^m$  we have  $v \geq \min_j E(e_i^m, j) = \min_j E(i, j) = \min_j DFV(\tilde{g}_{ij})$  and we get  $v \geq \min_j DFV(\tilde{g}_{ij})$ . The left side  $v$  is independent of  $i$  so that taking maximum with respect to  $i$ , we obtain  $v \geq \max_i \left\{ \min_j DFV(\tilde{g}_{ij}) \right\}$ . Proof of the second part is similar.

**Theorem 4.6.**

(i). If player-A possesses a pure optimal strategy  $i^*$ , then

$$v = \max_i \left( \min_j DFV(\tilde{g}_{ij}) \right) = \min_j DFV(\tilde{g}_{i^*j})$$

(ii). If player-B possesses a pure optimal strategy  $j^*$ , then

$$v = \min_j \left( \max_i DFV(\tilde{g}_{ij}) \right) = \max_i DFV(\tilde{g}_{ij^*})$$

**Proof:**  $v = \max_{\xi} \min_j E(\xi, j) = \min_j E(e_i^m, j)$  as  $\xi^* = e_i^m$  is optimal. Proof of the rest is similar.

### 5. Solution of Matrix Game

Let us consider a  $2 \times 2$  Matrix game whose fuzzy payoff matrix  $\tilde{G}$  is given by

$$\tilde{G} = \begin{bmatrix} \tilde{g}_{11} & \tilde{g}_{12} \\ \tilde{g}_{21} & \tilde{g}_{22} \end{bmatrix}$$

If  $\tilde{G}$  has a saddle point, solution is obvious.

Let  $\tilde{G}$  have no saddle point. Let the player-A has the strategy  $\xi = (x_1, x_2)' \equiv (x, 1-x) (0 \leq x \leq 1)$  and the player-B has the strategy  $\eta = (y, 1-y)' (0 \leq y \leq 1)$ . Then

$$E(\xi, \eta) = \sum_{i=1}^2 \sum_{j=1}^2 DFV(\tilde{g}_{ij}) x_i y_j$$

If  $\xi^* = (x^*, 1-x^*)'$ ,  $\eta^* = (y^*, 1-y^*)'$  be optimal strategies, then from

$$E(\xi, \eta^*) \leq E(\xi^*, \eta^*) \leq E(\xi^*, \eta) \quad \forall \xi \in S_2, \eta \in S_2$$

we have  $E(x, y^*) \leq E(x^*, y^*) \leq E(x^*, y) \quad \forall x \in (0,1), y \in (0,1)$ .

From the first part of the inequality, we set that  $E(x, y^*)$  regarded as a function of  $x$  has a maximum at  $x^*$  thus,

$$\left. \frac{\partial E}{\partial x} \right|_{(x^*, y^*)} = 0 \Rightarrow y^* = \frac{DFV(\tilde{g}_{22}) - DFV(\tilde{g}_{12})}{(DFV(\tilde{g}_{11}) + DFV(\tilde{g}_{22})) - (DFV(\tilde{g}_{12}) + DFV(\tilde{g}_{21}))}$$

Provided that  $(DFV(\tilde{g}_{11}) + DFV(\tilde{g}_{22})) - (DFV(\tilde{g}_{12}) + DFV(\tilde{g}_{21})) \neq 0$

Similarly, from the second part of the inequality, it is seen that  $E(x^*, y)$  regard as a function of  $y$  has a minimum at  $y^*$  i.e.,

$$\left. \frac{\partial E}{\partial y} \right|_{(x^*, y^*)} = 0 \Rightarrow x^* = \frac{DFV(\tilde{g}_{22}) - DFV(\tilde{g}_{21})}{(DFV(\tilde{g}_{11}) + DFV(\tilde{g}_{22})) - (DFV(\tilde{g}_{12}) + DFV(\tilde{g}_{21}))}$$

Provided that  $(DFV(\tilde{g}_{11})+DFV(\tilde{g}_{22}))-(DFV(\tilde{g}_{12})+DFV(\tilde{g}_{21})) \neq 0$ . And

$$v^* = E(x^*, y^*) = \frac{DFV(\tilde{g}_{11})DFV(\tilde{g}_{22}) - DFV(\tilde{g}_{12})DFV(\tilde{g}_{21})}{(DFV(\tilde{g}_{11}) + DFV(\tilde{g}_{22})) - (DFV(\tilde{g}_{12}) + DFV(\tilde{g}_{21}))}$$

It can be proved that  $(DFV(\tilde{g}_{11})+DFV(\tilde{g}_{22}))-(DFV(\tilde{g}_{12})+DFV(\tilde{g}_{21}))=0$  implies that  $\tilde{G}$  has a saddle point.

### 6. Numerical Example

To illustrate the proposed methodology, we have solved one numerical example. In this example, the elements of payoff matrix are fuzzy valued (taken from Mijanur et al. [15]). Using eight different defuzzification methods, the matrix game has been converted into eight matrix games which are shown in Table 2. Finally, we have solved all the matrix games and computed results have been presented in Table 3.

#### Example-1

Suppose that there are two companies A and B to enhance the market share of a new product by competing in advertising. The two companies are considering two different strategies to increase market share: strategy I (adv. by TV), II (adv. by Newspaper). Here it is assumed that the targeted market is fixed, i.e. the market share of the one company increases while the market share of the other company decreases and also each company puts all its advertisements in one. The above problem may be regarded as matrix game. Namely, the company A and B are considered as players A and B respectively.

**Table 2.** Converted matrix games

Defuzzification Methods	Defuzzified Pay of Matrix for TFN	Defuzzified Pay of Matrix for PFN
COA	$\begin{pmatrix} 181.67 & 154.67 \\ 90 & 181.67 \end{pmatrix}$	$\begin{pmatrix} 181.88 & 154.50 \\ 90 & 181.88 \end{pmatrix}$
BOA	$\begin{pmatrix} 181.25 & 155.00 \\ 90 & 181.25 \end{pmatrix}$	$\begin{pmatrix} 181.65 & 154.67 \\ 90 & 181.65 \end{pmatrix}$
MOM	$\begin{pmatrix} 180.00 & 156.00 \\ 90 & 180.00 \end{pmatrix}$	$\begin{pmatrix} 180.00 & 156.00 \\ 90 & 180.00 \end{pmatrix}$
SOM	$\begin{pmatrix} 180.00 & 156.00 \\ 90 & 180.00 \end{pmatrix}$	$\begin{pmatrix} 180.00 & 156.00 \\ 90 & 180.00 \end{pmatrix}$
LOM	$\begin{pmatrix} 180.00 & 156.00 \\ 90 & 180.00 \end{pmatrix}$	$\begin{pmatrix} 180.00 & 156.00 \\ 90 & 180.00 \end{pmatrix}$
RWP	$\begin{pmatrix} 181.25 & 155.00 \\ 90 & 181.25 \end{pmatrix}$	$\begin{pmatrix} 181.61 & 154.71 \\ 90 & 181.61 \end{pmatrix}$
GMIV (with $\omega=0.5$ )	$\begin{pmatrix} 180.83 & 155.33 \\ 90 & 180.83 \end{pmatrix}$	$\begin{pmatrix} 181.33 & 154.93 \\ 90 & 181.33 \end{pmatrix}$
COAI	$\begin{pmatrix} 181.25 & 155.00 \\ 90 & 181.25 \end{pmatrix}$	$\begin{pmatrix} 181.25 & 154.67 \\ 90 & 181.25 \end{pmatrix}$

The marketing research department of company A establishes the following pay-off matrix.

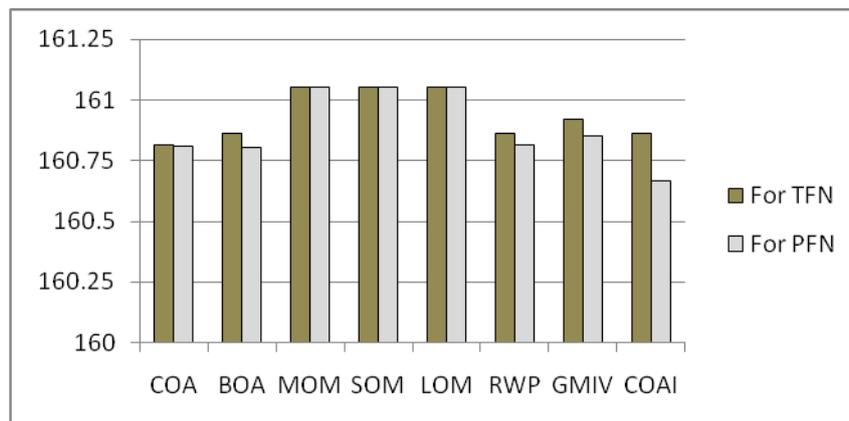
$$\tilde{G} = \begin{matrix} & \text{Adv. by TV} & \text{Adv. by Newspaper} \\ \text{Adv. by TV} & (175,180,190) & (150,156,158) \\ \text{Adv. by Newspaper} & (80,90,100) & (175,180,190) \end{matrix}$$

Where the element (175, 180, 190) in the matrix  $\tilde{G}$  indicates that the sales amount of the company A increase by “about 180” units when the company A and B use the strategy I (adv. by TV) simultaneously. The other elements in the matrix  $\tilde{G}$  can be explained similarly.

**Table 3.** Solutions of matrix games

Defuzzification Methods	Player-A (For TFN)			For PFN (Player-A)		
	$x^*$	$1-x^*$	$V^*$	$x^*$	$1-x^*$	$V^*$
COA	0.227522	0.772478	160.81309	0.229324	0.770676	160.80972
BOA	0.223404	0.776596	160.86436	0.227430	0.772570	160.80606
MOM	0.210526	0.789474	161.05263	0.210526	0.789474	161.05263
SOM	0.210526	0.789474	161.05263	0.210526	0.789474	161.05263
LOM	0.210526	0.789474	161.05263	0.210526	0.789474	161.05263
RWP	0.223404	0.776596	160.86436	0.226985	0.773015	160.81590
GMIV(with $\omega=0.5$ )	0.219204	0.780796	160.91970	0.224242	0.775758	160.84999
COAI	0.223404	0.776596	160.86436	0.225579	0.774421	160.66590

The computational results have been shown in Table 3 for different parametric values. From Table 3, it follows that in the case of TFN values, the best game value is obtained in the cases of MOM, SOM and LOM. In case of PFN values, the best game value is obtained in cases of MOM, SOM and LOM.



**Fig. 1.** Value of the game for different defuzzification methods

All the results have been shown in Fig. 1. The optimal solution sets, as obtained by the defuzzification approach, are consistent with those obtained by standard existing approach

under fuzzy set up. Thus, it can be claimed that the defuzzification approach attempted in this work well to handle the matrix game with fuzzy payoff.

## 7. Conclusion

In this paper, a method of solving fuzzy game problem using several fuzzy defuzzification techniques of fuzzy numbers has been considered. A Numerical example is presented to illustrate the proposed methodology. Due to the choices of decision makers', the payoff value in a zero sum game might be imprecise rather than precise value. This impreciseness may be represented by various ways. In this paper, we have represented this by fuzzy number. Then the fuzzy game problem has been converted into crisp game problem after defuzzification in which all the payoff values are crisp valued. Here, several defuzzification techniques have been used to solve the fuzzy game and the corresponding crisp games with their strategies and value of the game have been presented and compared.

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