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Barrow holographic dark energy models in Lyra and general relativity theories

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Barrow holographic dark energy, Lyra theory, Hubble parameter, General relativity

Abstract — This study investigates the Barrow holographic dark energy (BHDE) matter distribution in the Bianchi I universe model in Lyra and General Relativity Theories. To this end, it obtains exact solutions by Hubble parameter, conservation equation, and BHDE energy density equation and supports them with graphics. The results show that the solutions are in harmony with the functioning of the universe and the nature of dark energy. It finally discusses the need for further research.

Subject Classification (2020): 83C05, 83C15

1. Introduction

The accelerating expansion of the Universe is the expansion of the Universe observed from the fact that a galaxy at a certain distance is constantly moving away from the observer at an increasing rate over time. The idea of accelerating the universe's expansion was first proposed independently in 1998 by two research teams, the Supernova Cosmology Project and the High-Z Supernova Search Team [1,2]. By studying a type of supernova called Type Ia supernovae, these teams discovered that supernovae in distant galaxies appear fainter than those in nearby galaxies. This meant that the light from distant galaxies took longer to reach us. Therefore, the galaxies were moving away faster.

The universe's accelerating expansion is one of the most important discoveries of modern cosmology and has radically changed our understanding of how the universe works. This discovery also revealed the existence of a mysterious form of energy called dark energy. Dark energy is estimated to account for about %70 of the universe's total energy and is responsible for accelerating the universe's expansion [3]. Holographic dark energy models have also been frequently used in recent years to explain the universe's acceleration. Holographic dark energy is one of the theoretical models proposed to explain the universe's accelerating expansion and is related to the holographic principle. The holographic principle suggests that all system information can be expressed in a two-dimensional surface carried by its boundaries [4]. Based on this principle, the holographic dark energy model proposed by Li et

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al. [5] assumes that the dark energy density depends on a length scale determined by the universe's horizon.

Dark energy models and alternative theories of gravitation are frequently used to explain the universe's evolution. Various gravitation theories such as Lyra, Brans-Dicke, f(R), and f(R,T) are counted among alternative gravitation theories. In this study, we have considered the Barrow holographic dark energy (BHDE) model for the Bianchi I metric in the Lyra alternative gravitation theory. Similar studies are summarised below.

Chanda et al. [6] have analyzed the Barrow holographic dark energy matter distribution in the framework of the Brane world theory. Pankaj and Sharma [7] have studied the cosmological evolution of the Barrow holographic dark energy in a flat Friedmann-Lemaitre-Robertson-Walker (FLRW) universe using the Granda-Oliveros cutoff and a time-dependent scale factor. Devi and Kumar [8] have investigated the behavior of the Barrow holographic dark energy model in Brans-Dicke cosmology for the flat FRLW universe. Sobhanbabu et al. have researched the BHDE for the Kantowski-Sachs universe in the framework of the Saez-Ballester theory. In order to find the exact solution of the equations, they [9] assumed that the shear scalar is proportional to the expansion scalar. The cosmological properties of the Generalized BHDE were studied in the Finsler-Randers universe using a dark energy-density parameter, equation of state parameter, and deceleration parameter by Feng et al. [10]. The cosmological evolution of the BHDE model for a non-flat universe has been analyzed using the FRW metric with open and closed geometries. They studied. [11]. Sharma et al. [12] investigated the Barrow holographic dark energy universe model for the flat FRW metric in $f(R,T)$ theory. Saha et al. [13] Barrow presented an approach to explore the holographic dark energy inflationary cosmology in the FRW universe. An interactive scenario of Chaplygin gas and modified generalized Chaplygin gas were analyzed by Sultana et al. [14]. Shukla et al. [15] studied the flat FLRW cosmological model in $f(Q,T)$ theory. Moreover, Shukla et al. considered the field equations as an anisotropy parameter that can indicate the anisotropic behavior of the universe in the past and its approach to isotropy in the present and studied the Bianchi I universe model within the framework of $f(R,T)$ theory [16]. Devi et al. [17] have investigated the Barrow holographic dark energy universe model for FRW metric in $f(R,T)$ theory. Sing and Meitei [18] have studied dark energy models in general relativity theory for the Bianchi V universe using the linear deceleration parameter. Solutions for the FRW universe in fractal cosmology using special Hubble parameters have been made by Pawar et al. [19]. Ditta et al. [20] have compared the dark-energy compact stars in $f(T)$ and $f(T,\mathcal{T})$ alternative gravitational theories.

In the present paper, section 2 derives the equations of the Lyra gravitation theory for the Barrow holographic dark energy model and obtains the solutions with graphical support. Section 3 provides the results and a discussion. Section 4 discusses the need for further research.

2. Lyra Gravitation Theory and Solutions

The Einstein field equations in the Lyra theory, one of the alternative theories of gravitation, are as follows:

$$R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} + \frac{3}{2}\left(\Phi_\mu\Phi_\nu - \frac{1}{2}\ \mathrm{g}_{\mu\nu}\Phi_\xi\Phi^\xi\right) = -T_{\mu\nu} \tag{2.1}$$

Here, Φ_μ is the displacement vector. This study takes the displacement vector as $\Phi_\mu = (0,0,0,0,\beta(t))$. The homogeneous anisotropic Bianchi I metric in (x,y,z,t) coordinates is defined as follows.

$$ds^2 = A^2dx^2 + B^2dy^2 + C^2dz^2 - dt^2 \tag{2.2}$$

here A, B, and C are functions of time.

In this study, the universe is assumed to be filled with matter and a hypothetical fluid known as holographic dark energy. ρ_B is the energy density of the Barrow holographic dark energy, ρ_m is the energy density of matter, P is the pressure, and the total energy-momentum tensor is the sum of the energy-momentum tensors of matter, and dark energy

$$T_{\mu\nu} = (\rho_m + \rho_B + P)u_\mu u_\nu + Pg_{\mu\nu} \tag{2.3}$$

In the case of holographic dark energy, dark matter and dark energy are considered as a dual fluid [21]. The most common formulation of holographic dark energy takes the Hubble horizon as an IR cutcutoffd the Bekenstein-Hawking field law for the horizon degree of freedom of the universe [21]. Holographic dark energy based on Barrow entropy has attracted a great deal of attention in recent years [9]. The energy density of Barrow holographic dark energy using Barrow entropy :

$$\rho_B = LH^{2-\xi} \tag{2.4}$$

here L and ξ are constants [9, 21].

From (2.1)-(2.3), we obtain Einstein field equations in Lyra theory is

$$\frac{\ddot{C}}{C} + \frac{\ddot{B}}{B} + \frac{\dot{C}\dot{B}}{BC} + \frac{3\beta^2}{4} = -P \tag{2.5}$$

$$\frac{\ddot{A}}{A} + \frac{\ddot{B}}{B} + \frac{\dot{A}\dot{B}}{AB} + \frac{3\beta^2}{4} = -P \tag{2.6}$$

$$\frac{\ddot{C}}{C} + \frac{\ddot{A}}{A} + \frac{\dot{A}\dot{C}}{AC} + \frac{3\beta^2}{4} = -P \tag{2.7}$$

$$\frac{\dot{B}\dot{C}}{BC} + \frac{\dot{A}\dot{C}}{AC} + \frac{\dot{A}\dot{B}}{AB} - \frac{3\beta^2}{4} = \rho_m + \rho_B \tag{2.8}$$

The above system of equations contains 7 unknowns. In order to solve this system of equations, we need 3 more equations.

i. As is known, the Hubble parameter is an important time-varying parameter that plays a critical role in calculating the age of the universe and analyzing the expansion of the universe. For this reason, it can be used to solve field equations. The Hubble parameter is taken in the following form.

$$H = \frac{1}{3}\left(\frac{\dot{A}}{A} + \frac{\dot{B}}{B} + \frac{\dot{C}}{C}\right) = \frac{1}{\sqrt{2\alpha t + \gamma}} \tag{2.9}$$

here α and γ are constants.

ii. Secondly, in order to describe the nature of Barrow holographic dark energy, we can use (2.4), which gives the relation between the energy density of the BHDE and the Hubble parameter.

iii. One of the methods frequently used in Lyra alternative gravitation theory solutions is including the conservation equation. In Lyra gravitation theory, the conservation equation is provided as follows:

$$\left(R^\nu_\mu - \frac{1}{2}Rg^\nu_\mu\right)_{;\nu} + \frac{3}{2}\left(\Phi_\mu\Phi^\nu\right)_{;\nu} - \frac{3}{4}\left(\,\Phi_\eta\Phi^\eta g^\nu_\mu\right)_{;\nu} = 0 \tag{2.10}$$

From (2.2) and (2.10), the conservation equation of Lyra gravitation theory is obtained in terms of metric potentials as follows.

$$\frac{3\beta^2}{2}\left(\frac{\dot{A}}{A} + \frac{\dot{B}}{B} + \frac{\dot{C}}{C} + \frac{\dot{\beta}}{\beta}\right) = 0 \tag{2.11}$$

As shown from (2.11), two solutions exist for $\beta = 0$ and $\beta \neq 0$. If we use the solution $\beta = 0$, Lyra's theory of gravity reduces to Einstein's relativity theory. Since solutions are sought in Lyra gravity theory, we will first investigate solutions for the $\beta \neq 0$ state.

Using (2.5)-(2.9) and (2.11), the energy density (ρ_B) of the BHDE for the Bianchi I metric in the Lyra theory of gravitation is obtained as follows:

$$\rho_B = L\left(2\alpha t + \gamma\right)^{-2+\xi} \tag{2.12}$$

The density of matter (ρ_m) and pressure (P) are also obtained.

$$\rho_m = \frac{3}{2\alpha t + \gamma} - \left(c_2^2 - c_2 c_4 + c_4^2 + \frac{3}{4}c_6^2\right) e^{-\frac{6\sqrt{2\alpha t+\gamma}}{\alpha}} - L\left(2\alpha t + \gamma\right)^{-2+\xi} \tag{2.13}$$

and

$$P = \frac{2\alpha}{\left(2\alpha t + \gamma\right)^{\frac{3}{2}}} - \left(c_2^2 - c_2 c_4 + c_4^2 + \frac{3}{4}c_6^2\right) e^{-\frac{6\sqrt{2\alpha t+\gamma}}{\alpha}} - \frac{3}{2\alpha t + \gamma} \tag{2.14}$$

The metric potentials A, B, and C are obtained as follows.

$$A = \frac{c_1}{c_3 c_5 e^{\frac{2\left(\alpha t + \frac{\alpha\sqrt{2\alpha t+\gamma}}{6} + \frac{\gamma}{2}\right)\left(-3+\alpha(c_2-c_4)e^{-\frac{3\sqrt{2\alpha t+\gamma}}{\alpha}}\right)}{3\sqrt{2\alpha t+\gamma}\,\alpha}}} \tag{2.15}$$

$$B = c_5 e^{\frac{-c_4\alpha\left(\alpha+3\sqrt{2\alpha t+\gamma}\right)e^{-\frac{3\sqrt{2\alpha t+\gamma}}{\alpha}}+9\sqrt{2\alpha t+\gamma}}{9\alpha}} \tag{2.16}$$

and

$$C = c_3 e^{\frac{2\left(c_2\alpha\left(\alpha t + \frac{\alpha\sqrt{2\alpha t+\gamma}}{6} + \frac{\gamma}{2}\right)e^{-\frac{3\sqrt{2\alpha t+\gamma}}{\alpha}} + 3\alpha t - \frac{\alpha\sqrt{2\alpha t+\gamma}}{2} + \frac{3\gamma}{2}\right)}{3\sqrt{2\alpha t+\gamma}\,\alpha}} \tag{2.17}$$

Finally, the displacement vector β^2 is as follows.

$$\beta^2 = \frac{c_6^2}{e^{\frac{6\sqrt{2\alpha t+\gamma}}{\alpha}}} \tag{2.18}$$

Here, c_1-c_6 are constants. From (2.15)-(2.17), the time variation of the metric potentials A, B, and C is shown in Figure 1.

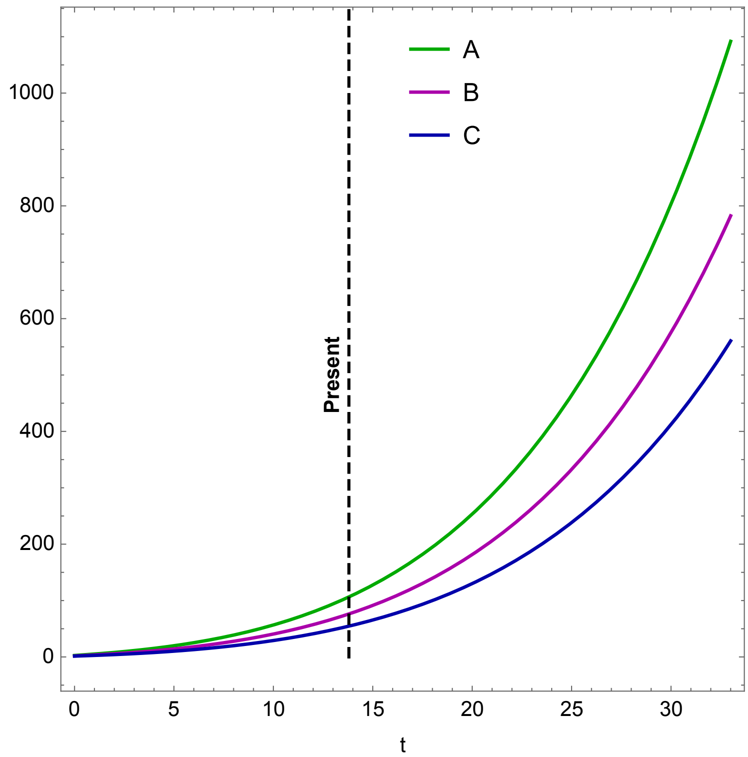

Figure 1. Variation of metric potential versus cosmic time t

The change of BHDE with time is shown in Figure 2.

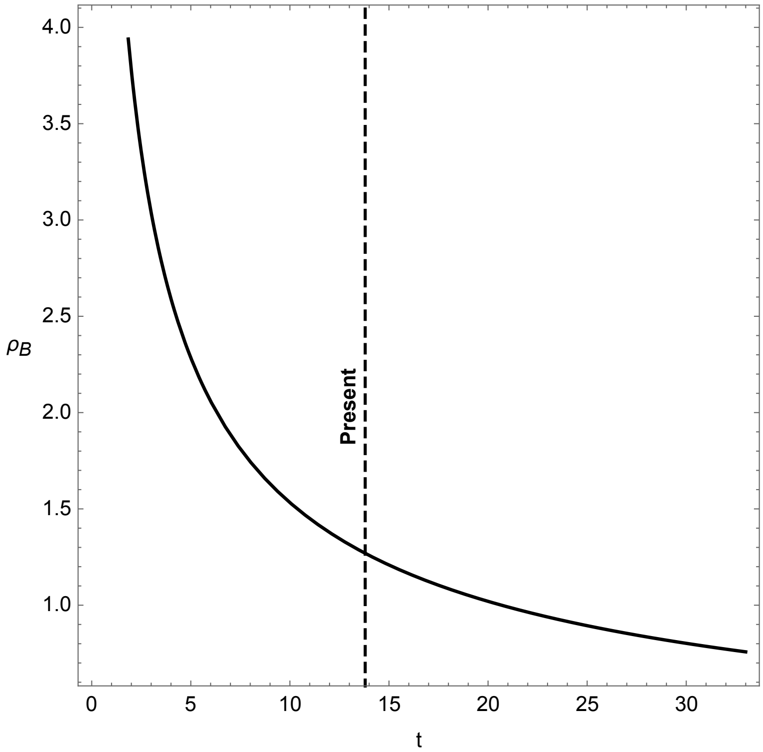

Figure 2. Variation of BHDE density versus cosmic time t

The graph of the change of (2.14) concerning time is shown in Figure 3.

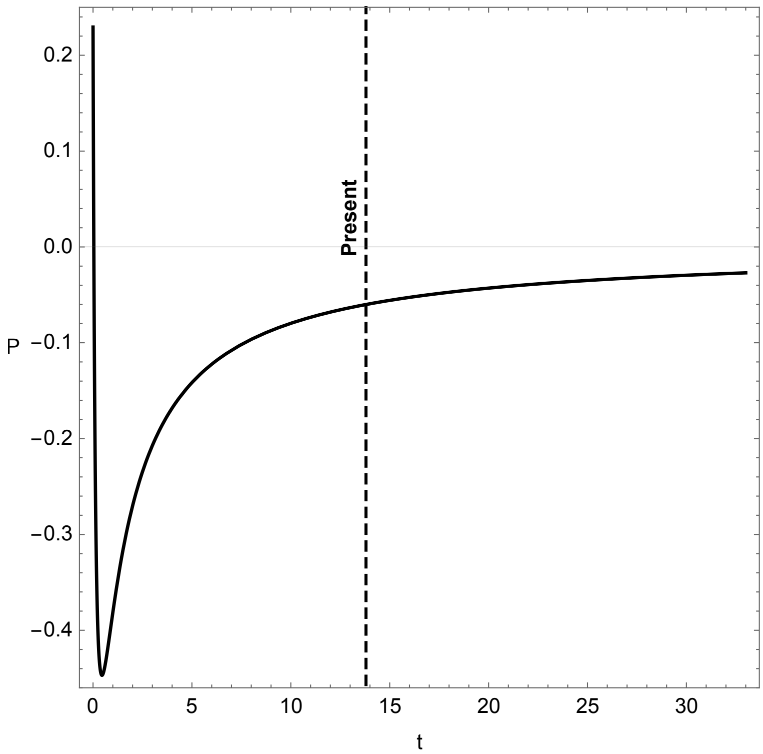

Figure 3. Variation of pressure versus cosmic time t

Figure 4 shows the change of the displacement vector concerning time.

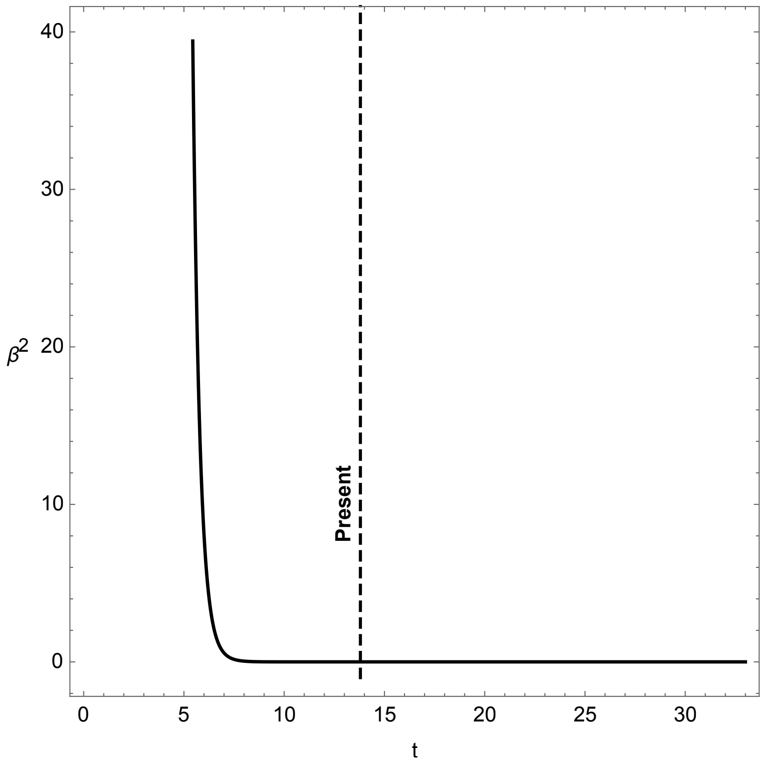

Figure 4. Variation of displacement vector versus cosmic time t

3. Results and Discussions

The Bianchi I universe model, known for its homogeneous and anisotropic properties, was studied in Lyra gravitation theory, accompanied by Barrow holographic dark energy. Hubble parameter, position equation, and density equation of BHDE were used to obtain exact solutions. As can be seen from (2.18), the displacement vector β^2 is obtained depending on time and decreases over time (see figure 4). The fact that the β^2 function is non-zero indicates the existence of Lyra gravitation theory. If $c_6 = 0$ is taken in (2.18), $\beta^2 = 0$. As a result, our solutions are reduced to the General Relativity Theory in the BHDE matter distribution for the Bianchi I metric. From (2.15)-(2.17), we observe that the metric potentials of A, B and C increase over time (see figure 1). Again, the same equations show that the constants c_1, c_3, and c_5 must differ from zero. We can say that the increase in metric potentials over time may indicate the universe's expansion. Also, the present value in the graphs shows $t = 13.8$ Gyr, which is the universe's current age.

As is known, the BHDE model was discussed in this study. In dark energy models, pressure takes negative values. In our research, when we look at figure 3, it is seen that the pressure behaves by the dark energy model. As can be seen from (2.14), there is singularity for $t = -\frac{\gamma}{2\alpha}$ in our model. As seen from (2.12) and (2.13), Barrow's dark energy density and matter density decrease over time. This situation can be seen in figure 2.

As it is known, $p = w\rho_B$ is called the equation of state (EoS) that gives the relationship between pressure and density. In dark energy models, w is a parameter that takes negative values. Using (2.12) and (2.14), the w value and graph for BHDE are as follows:

$$w = \frac{\frac{2\alpha}{(2\alpha t+\gamma)^{\frac{3}{2}}} - \left(c_2^2 - c_2c_4 + c_4^2 + \frac{3}{4}c_6^2\right) e^{-\frac{6\sqrt{2\alpha t+\gamma}}{\alpha}} - \frac{3}{2\alpha t+\gamma}}{L\,(2\alpha t + \gamma)^{-2+\xi}} \tag{3.1}$$

Figure 5 shows the change of the equation of state concerning time.

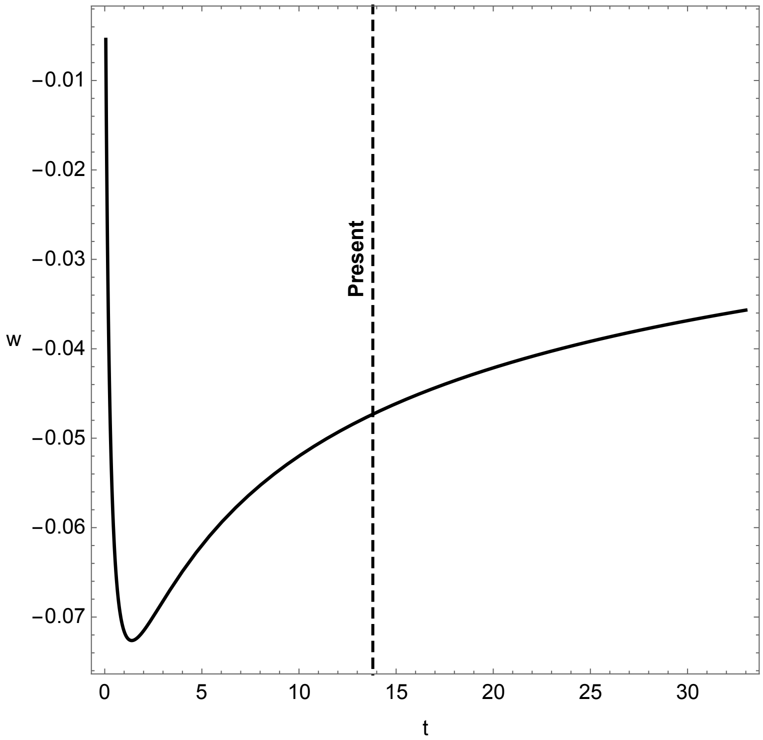

Figure 5. Variation of equation of state versus cosmic time t

As seen in Figure 5, the w value was obtained as negative in our solutions by the nature of dark energy.

As mentioned above, for the case $c_6 = 0$, $\beta^2 = 0$. In this case, the dynamics that are different in the Bianchi I universe model for BHDE in the General Relativity Theory are as follows.

$$\rho_m = \frac{3}{2\alpha t + \gamma} - \left(c_2^2 - c_2 c_4 + c_4^2\right) e^{-\frac{6\sqrt{2\alpha t+\gamma}}{\alpha}} - L\,(2\alpha t + \gamma)^{-2+\xi} \tag{3.2}$$

$$P = \frac{2\alpha}{(2\alpha t + \gamma)^{\frac{3}{2}}} - \left(c_2^2 - c_2 c_4 + c_4^2\right) e^{-\frac{6\sqrt{2\alpha t+\gamma}}{\alpha}} - \frac{3}{2\alpha t + \gamma} \tag{3.3}$$

Other dynamics are as in the Lyra theory. No change occurs.

4. Conclusion

This study investigates the homogeneous and anisotropic Bianchi I universe model in Lyra gravitation theory with Barrow holographic dark energy matter distribution. To obtain the exact solutions of the field equations in Lyra gravitation theory, Hubble parameter, conservation equation, and variation of the energy density of the BHDE were used. In future studies, it will be worthwhile to investigate universe models with BHDE matter distribution within the framework of other alternative gravity theories such as $f(Q)$, $f(R,T)$, $f(Q,T)$, and $f(R)$.

Author Contributions

All the authors equally contributed to this work. They all read and approved the final version of the paper.

Conflicts of Interest

All the authors declare no conflict of interest

Ethical Review and Approval

No approval from the Board of Ethics is required.

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References

[1] A. G. Riess, A. V. Filippenko, P. Challis, A. Clocchiatti, A. Diercks, P. M. Garnavich, R. L. Gilliland, C. J. Hogan, S. Jha, R. P. Kirshner, B. Leibundgut, M. M. Phillips, D. Reiss, B. P. Schmidt, R. A. Schommer, R. C. Smith, J. Spyromilio, C. Stubbs, N. B. Suntzeff, J. Tonry, *Observational evidence from supernovae for an accelerating universe and a cosmological constant*, The Astronomical Journal 116 (3) (1998) 1009.

[2] S. Perlmutter, G. Aldering, G. Goldhaber, R. A. Knop, P. Nugent, P. G. Castro, S. Deustua, S. Fabbro, A. Goobar, D. E. Groom, I. M. Hook, A. G. Kim, M. Y. Kim, J. C. Lee, N. J. Nunes, R. Pain, C. R. Pennypacker, R. Quimby, C. Lidman, R. S. Ellis, M. Irwin, R. G. McMahon, P. Ruiz-Lapuente, N. Walton, B. Schaefer, B. J. Boyle, A. V. Filippenko, T. Matheson, A. S. Fruchter, N. Panagia, H. J. M. Newberg, W. J. Couch, T. S. C. Project, *Measurements of* Ω *and* Λ *from 42 high-redshift supernovae*, The Astrophysical Journal 517 (2) (1999) 565–586.

[3] P. J. E. Peebles, B. Ratra, *The cosmological constant and dark energy*, Reviews of Modern Physics 75 (2003) 559–606.

[4] L. Susskind, *The world as a hologram*, Journal of Mathematical Physics 36 (11) (1995) 6377–6396.

[5] M. Li, *A model of holographic dark energy*, Physics Letters B 603 (1-2) (2004) 1–5.

[6] A. Chanda, A. K. Mitra, S. Dey, S. Ghose, B. C. Paul, *Barrow holographic dark energy in Brane world cosmology*, Classical and Quantum Gravity 41 (3) (2024) 035004.

[7] Pankaj, A. Remya, U. K. Sharma, *Cosmological model in Barrow holographic dark energy with Granda-Oliveros cut-off*, International Journal of Modern Physics A 38 (2023) 2350185–6.

[8] K. Devi, P. Kumar, *Recent acceleration and future deceleration in Brans-Dicke theory*, General Relativity and Gravitation 56 (1) (2023) 4.

[9] Y. Sobhanbabu, R. S. Rao, Y. J. Prasuna, G. Satyanarayana, *Kantowski-Sachs Barrow holographic dark energy model in Saez-Ballester theory of gravitation*, New Astronomy 104 (2023) 102066.

[10] W. Feng, W. Yang, B. Jiang, Y. Wang, T. Han, Y. Wu, *Theoretical analysis on the Barrow holographic dark energy in the Finsler-Randers cosmology*, International Journal of Modern Physics D 32 (5) (2023) 2350029.

[11] A. Remya, U. K. Pankaj, Sharma, N. M. Ali, *Cosmological evolution and stability analysis in non-flat universe and Barrow holographic model of dark energy*, Astrophysics and Space Science 368 (3) (2023) 15.

[12] U. K. Sharma, M. Kumar, G. Varshney, *Scalar field models of Barrow holographic dark energy in f(R,T) Gravity*, Universe 8 (12) (2022) 642.

[13] S. Saha, S. Chattopadhyay, E. Güdekli, *A coupled-fluid approach to explore bounce and inflationary cosmology with Barrow holographic as the driving dark fluid*, European Physical Journal C 84 (3) (2024) 314.

[14] S. Sultana, E. Güdekli, S. Chattopadhyay, *Some versions of Chaplygin gas model in modified gravity framework and validity of generalized second law of thermodynamics*, Zeitschrift Naturforschung Teil A 79 (1) (2024) 51–70.

[15] B. K. Shukla, D. Sofuoğlu, H. Chaudhary, F. Atamurotov, G. Mustafa, *Cosmic evolution in f(Q,T) gravity with observational constraints: A comparative analysis with* ΛCDM, Journal of High Energy Astrophysics 43 (2024) 1–14.

[16] B. K. Shukla, R. K. Tiwari, A. Beesham, D. Sofuoğlu, *Cosmology with the late time isotropy of the Bianchi type I model in f(R,T) theory*, International Journal of Geometric Methods in Modern Physics 21 (3) (2024) 2450060–134.

[17] K. Devi, A. Kumar, P. Kumar, *Barrow holographic dark energy model in f(R,T) theory*, Astrophysics and Space Science 369 (7) (2024) 73.

[18] K. P. Singh, A. J. Meitei, *Late time acceleration in Bianchi type-V dark energy cosmological models with linear deceleration parameter*, Physica Scripta 99 (7) (2024) 075046.

[19] D. D. Pawar, D. K. Raut, A. P. Kale, *Two forms of dark energy in fractal cosmological model using specific Hubble parameter*, New Astronomy 110 (2024) 102214.

[20] A. Ditta, G. Mustafa, S. K. Maurya, D. Sofuoğlu, A. Mahmood, *Comparative analysis of dark energy compact stars in* $f(T,\mathcal{T})$ *and* $f(T)$ *gravity theories via conformally flat condition*, Classical and Quantum Gravity 41 (15) (2024) 155004.

[21] M. Srivastava, M. Kumar, S. Srivastava, *Barrow holographic dark energy with hybrid expansion law*, Gravitation and Cosmology 28 (1) (2022) 70–80.