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FUZZY SOFT CONNECTEDNESS BASED ON FUZZY b-OPEN SOFT SETS

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Abstract – As a continuation of study fuzzy soft topological spaces, we are going to introduce and investigate fuzzy soft b-connected sets, fuzzy soft b-separated sets and fuzzy soft b s-connected sets and have established several interesting properties supported by examples. Moreover, we show that a fuzzy soft b-disconnectedness property is not hereditary property in general.

Keywords – *Soft sets, Fuzzy soft sets, Fuzzy soft connectedness, Fuzzy b-open soft sets.*

1 Introduction

Uncertain or imprecise data are inherent and pervasive in many important applications in the areas such as business management, computer science, engineering, environment, social science and medical science. Uncertain data in those applications could be caused by data randomness, information incompleteness, limitations of measuring instrument, delayed data updates, and so forth. Due to the importance of those applications and the rapidly increasing amount of uncertain data collected and accumulated, research on effective and efficient techniques that are dedicated to modeling uncertain data and tackling uncertainties has attracted much interest in recent years and yet remained challenging at large. There have been a great amount of research and applications in the literature concerning some special tools like probability theory, (intuitionistic) fuzzy set theory, rough set theory, vague set theory, and interval mathematics. However, all of these have theirs advantages as well as inherent limitations in dealing with uncertainties. One major problem shared by those theories is their incompatibility with the parameterizations tools. Soft set theory [33] was firstly proposed by a Russian researcher, Molodtsov, in 1999 to overcome these difficulties. At present, work on the extension of soft set theory is progressing rapidly. Fuzzy soft set is a hybridization of fuzzy sets and soft sets, in which soft set

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is defined over fuzzy set. Maji et al. proposed the concept of fuzzy soft set [31] and then gave its application. Roy and Maji presented a method of object recognition from an imprecise multi observer data [41]. Yuksel et al. [48] applied soft set theory to determine cancer risk. Yao et al. proposed the concept of fuzzy sets and defined some operations on fuzzy soft sets [46]. From the above discussion, we can see that all of these works are based on Zadeh$\grave{s}$ fuzzy sets theory [49]. Recently, many interesting applications of soft set theory have been expanded by embedding the ideas of fuzzy sets [4, 5, 8, 16, 25, 29, 30, 31, 32, 34, 35, 50]. To develop soft set theory, the operations of the soft sets are redefined and a uni-int decision making method was constructed by using these new operations [9]. In the year 2011, Shabir and Naz [42] introduced soft topology, soft relative topology and studied some introductory results. In the same year Cagman et al. [7] introduced soft topology in a different approach. Till then various researchers have studied various foundational results in soft topology. Min in [44] investigate some properties of these soft separation axioms. In [17], Kandil et al. introduced some soft operations such as semi open soft, pre open soft, α-open soft and β-open soft and investigated their properties in detail. Kandil et al. [24] introduced the notion of soft semi separation axioms. In particular they study the properties of the soft semi regular spaces and soft semi normal spaces. The notion of soft ideal was initiated for the first time by Kandil et al. [20]. They also introduced the concept of soft local function. These concepts are discussed with a view to find new soft topologies from the original one, called soft topological spaces with soft ideal $(X, \tau, E, \tilde{I})$. Applications to various fields were further investigated by Kandil et al. [18, 19, 21, 22, 23, 26]. The notion of supra soft topological spaces was initiated for the first time by El-sheikh and Abd El-latif [12]. They also introduced new different types of subsets of supra soft topological spaces and study the relations between them in detail. The notion of $b-$open soft sets was initiated by El-sheikh and Abd El-latif [11] and extended in [38]. An applications on $b-$open soft sets were introduced in [1, 13]. It is well known to us that fuzzy soft topology is playing crucial roles in mathematics, economics, data reduction, image processing, genotype-phenotype mapping of DNA etc. In 2001-2003 Maji et al. [29, 30, 31] who studied and worked on some mathematical aspects of soft sets and fuzzy soft sets. In [6] the notion of fuzzy soft set was introduced as a fuzzy generalization of soft sets and some basic properties of fuzzy soft sets are discussed in detail. Then, many scientists such as X. Yang et al. [47], improved the concept of fuzziness of soft sets. In [3], Karal and Ahmed defined the notion of a mapping on classes of fuzzy soft sets, which is fundamental important in fuzzy soft set theory, to improve this work and they studied properties of fuzzy soft images and fuzzy soft inverse images of fuzzy soft sets. Chang [10] introduced the concept of fuzzy topology on a set X by axiomatizing a collection $\mathfrak{T}$ of fuzzy subsets of X. Tanay et al. [43] proposed the definition of fuzzy soft topology over a subset of the initial universe set while Roy and Samanta [40] gave the definition of fuzzy soft topology over the initial universe set and discussed some properties of fuzzy topological spaces related fuzzy soft base. Some fuzzy soft topological properties based on fuzzy semi (resp. β-) open soft sets, were introduced in [2, 15, 16, 25]. Connectedness is one of the important notions of topology. F. Lin [27] defined the notions of soft connectedness in soft topological spaces. Mahanta et al.[28] introduced and studied the fuzzy soft connectedness in fuzzy soft topological spaces.

As a continuation of study fuzzy soft topological spaces, we are going to introduce and investigate the properties and behaviours of fuzzy soft b-connected sets, fuzzy soft b-separated sets and fuzzy soft bs-connected sets and have established several interesting properties supported by examples, which are basic for further research on fuzzy soft topology and will fortify the footing of the theory of fuzzy soft topological space. Moreover, we show that a fuzzy soft b-disconnectedness property is not hereditary property in general. Finally, we show that the fuzzy b-irresolute surjective soft image of fuzzy soft b-connected (resp. fuzzy soft bs-connected) is also fuzzy soft b-connected (resp. fuzzy soft b-s-connected).

2 Preliminary

In this section, for the sake of completeness, we first cite some useful definitions and results.
Throughout this paper, X refers to an initial universe, E is the set of all parameters for X and I^X is the set of all fuzzy sets on X (where, $I = [0, 1]$).

Definition 2.1. [29] Let $A \subseteq E$. Then the mapping $f : A \to I^X$ defined by $f_A(e) = \mu^e_{f_A}$ (a fuzzy subset of X) is called fuzzy soft set over (X, E), where $\mu^e_{f_A} = \overline{0}$ if $e \notin A$ and $\mu^e_{f_A} \neq \overline{0}$ if $e \in A$, where $\overline{0}(x) = 0\ \forall\ x \in X$. The family of all these fuzzy soft sets over X denoted by $FSS(X)_E$.

Several basic properties for fuzzy soft sets are given by [29, 39]

Definition 2.2. [39] Let $\mathfrak{T}$ be a collection of fuzzy soft sets over a universe X with a fixed set of parameters E, then $\mathfrak{T} \subseteq FSS(X)_E$ is called fuzzy soft topology on X if

(i) $\tilde{1}_E, \tilde{0}_E \in \mathfrak{T}$, where $\tilde{0}_E(e) = \overline{0}$ and $\tilde{1}_E(e) = \overline{1}$, $\forall e \in E$,

(ii) the union of any members of $\mathfrak{T}$ belongs to $\mathfrak{T}$,

(iii) the intersection of any two members of $\mathfrak{T}$ belongs to $\mathfrak{T}$.

The triplet $(X, \mathfrak{T}, E)$ is called fuzzy soft topological space over X. Also, each member of $\mathfrak{T}$ is called fuzzy open soft in $(X, \mathfrak{T}, E)$. We denote the set of all open soft sets by $FOS(X, \mathfrak{T}, E)$, or $FOS(X)$. A fuzzy soft set f_A over X is said to be fuzzy closed soft set in X [39], if its relative complement f'_A is fuzzy open soft set. We denote the set of all fuzzy closed soft sets by $FCS(X, \mathfrak{T}, E)$, or $FCS(X)$.

Definition 2.3. [36] Let $(X, \mathfrak{T}, E)$ be a fuzzy soft topological space and $f_A \in FSS(X)_E$.

(i) The fuzzy soft closure of f_A, denoted by $Fcl(f_A)$ is the intersection of all fuzzy closed soft super sets of f_A. i.e.,

$$Fcl(f_A) = \sqcap\{h_D : h_D\ is\ fuzzy\ closed\ soft\ set\ and\ f_A \sqsubseteq h_D\}.$$

(ii) The fuzzy soft interior of f_A, denoted by $Fint(f_A)$ is the fuzzy soft union of all fuzzy open soft subsets of f_A.i.e.,

$$Fint(f_A) = \sqcup\{h_D : h_D\ is\ fuzzy\ open\ soft\ set\ and\ h_D \sqsubseteq f_A\}.$$

Definition 2.4. [28] The fuzzy soft set $f_A \in FSS(X)_E$ is called fuzzy soft point if there exist $x \in X$ and $e \in E$ such that $\mu^e_{f_A}(x) = \alpha$ $(0 < \alpha \leq 1)$ and $\mu^e_{f_A}(y) = \overline{0}$ for each $y \in X - \{x\}$, and this fuzzy soft point is denoted by x^e_α or f_e.

Definition 2.5. [28] The fuzzy soft point x^e_α is said to be belonging to the fuzzy soft set (g, A), denoted by $x^e_\alpha \tilde{\in} (g, A)$, if for the element $e \in A$, $\alpha \leq \mu^e_{g_A}(x)$.

Theorem 2.6. [28] Let $(X, \mathfrak{T}, E)$ be a fuzzy soft topological space and f_e be a fuzzy soft point. Then, the following properties hold:

(i) If $f_e \tilde{\in} g_A$, then $f_e \tilde{\notin} g'_A$;

(ii) $f_e \tilde{\in} g_A \not\Rightarrow f'_e \tilde{\in} g'_A$;

(iii) Every non-null fuzzy soft set f_A can be expressed as the union of all the fuzzy soft points belonging to f_A.

Definition 2.7. [28] A fuzzy soft set g_B in a fuzzy soft topological space $(X, \mathfrak{T}, E)$ is called fuzzy soft neighborhood of the fuzzy soft point x^e_α if there exists a fuzzy open soft set h_C such that $x^e_\alpha \tilde{\in} h_C \sqsubseteq g_B$. A fuzzy soft set g_B in a fuzzy soft topological space $(X, \mathfrak{T}, E)$ is called fuzzy soft neighborhood of the soft set f_A if there exists a fuzzy open soft set h_C such that $f_A \sqsubseteq h_C \sqsubseteq g_B$. The fuzzy soft neighborhood system of the fuzzy soft point x^e_α, denoted by $N_{\mathfrak{T}}(x^e_\alpha)$, is the family of all its fuzzy soft neighborhoods.

Definition 2.8. [28] Let $(X, \mathfrak{T}, E)$ be a fuzzy soft topological space and $Y \subseteq X$. Let h^Y_E be a fuzzy soft set over (Y, E) such that $h^Y_E : E \to I^Y$ such that $h^Y_E(e) = \mu^e_{h^Y_E}$,

$$\mu^e_{h^Y_E}(x) = \begin{cases} 1 & x \in Y, \\ 0, & x \notin Y. \end{cases}$$

Let $\mathfrak{T}_Y = \{h^Y_E \sqcap g_B : g_B \in \mathfrak{T}\}$, then the fuzzy soft topology $\mathfrak{T}_Y$ on $(Y, E)$is called fuzzy soft subspace topology for (Y, E) and $(Y, \mathfrak{T}_Y, E)$ is called fuzzy soft subspace of $(X, \mathfrak{T}, E)$. If $h^Y_E \in \mathfrak{T}$ (resp. $h^Y_E \in \mathfrak{T}'$), then $(Y, \mathfrak{T}_Y, E)$ is called fuzzy open (resp. closed) soft subspace of $(X, \mathfrak{T}, E)$.

Definition 2.9. [36] Let $FSS(X)_E$ and $FSS(Y)_K$ be families of fuzzy soft sets over X and Y, respectively. Let $u : X \to Y$ and $p : E \to K$ be mappings. Then, the map f_{pu} is called fuzzy soft mapping from X to Y and denoted by $f_{pu} : FSS(X)_E \to FSS(Y)_K$ such that,

(1) If $f_A \in FSS(X)_E$. Then, the image of f_A under the fuzzy soft mapping f_{pu} is the fuzzy soft set over Y defined by $f_{pu}(f_A)$, where $\forall\ k \in p(E)$, $\forall\ y \in Y$,

$$f_{pu}(f_A)(k)(y) = \begin{cases} \bigvee_{u(x)=y} \ [\vee_{p(e)=k}(f_A(e))](x) & if\ x \in u^{-1}(y), \\ 0 & otherwise. \end{cases}$$

(2) If $g_B \in FSS(Y)_K$, then the pre-image of g_B under the fuzzy soft mapping f_{pu} is the fuzzy soft set over X defined by $f^{-1}_{pu}(g_B)$, where $\forall\ e \in p^{-1}(K)$, $\forall\ x \in X$,

$$f^{-1}_{pu}(g_B)(e)(x) = \begin{cases} g_B(p(e))(u(x)) & for\ p(e) \in B, \\ 0 & otherwise. \end{cases}$$

The fuzzy soft mapping f_{pu} is called surjective (resp. injective) if p and u are surjective (resp. injective), also it is said to be constant if p and u are constant.

Definition 2.10. [36] Let $(X, \mathfrak{T}_1, E)$ and $(Y, \mathfrak{T}_2, K)$ be two fuzzy soft topological spaces and $f_{pu} : FSS(X)_E \rightarrow FSS(Y)_K$ be a fuzzy soft mapping. Then, f_{pu} is called

(i) Fuzzy continuous soft if $f_{pu}^{-1}(g_B) \in \mathfrak{T}_1 \; \forall \; (g_B) \in \mathfrak{T}_2$.

(ii) Fuzzy open soft if $f_{pu}(g_A) \in \mathfrak{T}_2 \forall \; (g_A) \in \mathfrak{T}_1$.

Several properties and characteristics for f_{pu} and f_{pu}^{-1} are reported in detail in [3].

Definition 2.11. [16] Two fuzzy soft sets f_A, g_B are said to be disjoint, denoted by $f_A \sqcap g_B = \tilde{0}_E$, if $A \cap B = \emptyset$ and $\mu_{f_A}^e \cap \mu_{g_B}^e = \bar{0}$ for every $e \in E$

Definition 2.12. [28] Let $(X, \mathfrak{T}, E)$ be a fuzzy soft topological space. A fuzzy soft separation of $\tilde{1}_E$ is a pair of non null proper fuzzy open soft sets g_B, h_C such that $g_B \sqcap h_C = \tilde{0}_E$ and $\tilde{1}_E = g_B \sqcup h_C$.

Definition 2.13. [28] A fuzzy soft topological space $(X, \mathfrak{T}, E)$ is said to be fuzzy soft connected if and only if there is no fuzzy soft separations of $\tilde{X}$. Otherwise, $(X, \mathfrak{T}, E)$ is said to be fuzzy soft disconnected space.

3 Fuzzy Soft b-Connectedness

This section is devoted to define and discuss the fuzzy soft b-connectedness in fuzzy soft topological space and investigate its behaviours.

Definition 3.1. Let $(X, \mathfrak{T}, E)$ be a fuzzy soft topological space and $f_A \in FSS(X)_E$. If $f_A \sqsubseteq Fint(Fcl(f_A)) \sqcup Fcl(Fint(f_A))$, then f_A is called fuzzy b-open soft set. A fuzzy soft set f_A is fuzzy b-closed, if its complement is fuzzy b-open soft set.

We denote the set of all fuzzy b-open soft sets by $FBOS(X, \mathfrak{T}, E)$, or $FBOS(X)$ and the set of all fuzzy b-closed soft sets by $FBCS(X, \mathfrak{T}, E)$, or $FBCS(X)$.

Definition 3.2. Let $(X, \mathfrak{T}, E)$ be a fuzzy soft topological space, $f_A \in FSS(X)_E$ and $f_e \tilde{\in} FSS(X)_E$. Then,

(i) f_e is called fuzzy b-interior soft point of f_A if there exists $g_B \in FBOS(X)$ such that $f_e \tilde{\in} g_B \sqsubseteq f_A$. The set of all fuzzy b-interior soft points of f_A is called the fuzzy b-soft interior of f_A and is denoted by $FBint(f_A)$ consequently, $FBint(f_A) = \sqcup\{g_B : g_B \sqsubseteq f_A,\; g_B \in FBOS(X)\}$.

(ii) f_e is called fuzzy b-closure soft point of f_A if $f_A \sqcap h_D \neq \tilde{0}_E$ for every $h_D \in FBOS(X)$. The set of all fuzzy b-closure soft points of f_A is called fuzzy b-soft closure of f_A and denoted by $FBcl(f_A)$. Consequently, $FBcl(f_A) = \sqcap\{h_D : h_D \in FBCS(X),\; f_A \sqsubseteq h_D\}$.

Definition 3.3. Let $(X, \mathfrak{T}, E)$ be a fuzzy soft topological space. A fuzzy soft b-separation on $\tilde{1}_E$ is a pair of non null proper fuzzy b-open soft sets f_A, g_B such that $f_A \sqcap g_B = \tilde{0}_E$ and $\tilde{1}_E = f_A \sqcup g_B$.

Definition 3.4. A fuzzy soft topological space $(X, \mathfrak{T}, E)$ is said to be fuzzy soft b-connected if there is no fuzzy soft b-separations of $\tilde{1}_E$. Otherwise, $(X, \mathfrak{T}, E)$ is said to be fuzzy soft b-disconnected space.

Definition 3.5. A fuzzy soft topological space $(X, \mathfrak{T}, E)$ is said to be fuzzy soft b-disconnected if $(X, \mathfrak{T}, E)$ has a proper fuzzy b-open soft and fuzzy b-closed soft set in X.

Examples 3.6. (i) The discrete fuzzy soft topological space $(X, \mathfrak{T}, E)$ is not fuzzy soft b-connected i.e fuzzy soft b-disconnected .

(ii) Any space with indiscrete fuzzy soft topology is fuzzy soft connected but not it is fuzzy soft b-connected because soft b-open sets establish a discrete topology.

Example 3.7. Let $X = \{a, b\}$, $E = \{e_1, e_2, e_3\}$ and $A = \{e_1, e_2\} \subseteq E$. Let $\mathfrak{T} = \{\tilde{1}_E, \tilde{0}_E, f_A\}$ where f_A is fuzzy soft sets over X defined as follows:
$\mu^{e_1}_{f_A} = \{a_{0.4}, b_{0.2}\},\ \mu^{e_2}_{f_A} = \{a_{0.3}, b_{0.5}\},\ \mu^{e_3}_{f_A} = \{a_0, b_0\},$
Consider the fuzzy soft sets g_E, h_E
$\mu^{e_1}_{g_E} = \{a_1, b_0\},\ \mu^{e_2}_{g_E} = \{a_0, b_1\},\ \mu^{e_3}_{g_E} = \{a_1, b_0\},$
$\mu^{e_1}_{h_E} = \{a_0, b_1\},\ \mu^{e_2}_{h_E} = \{a_1, b_0\},\ \mu^{e_3}_{h_E} = \{a_0, b_1\},$
Then, g_E, h_E be a pair of non null proper fuzzy b-open soft sets such that $g_E \sqcap h_E = \tilde{0}_E$ and $\tilde{1}_E = g_E \sqcup h_E$. Hence, $(X, \mathfrak{T}, E)$ is not fuzzy soft b-connected.

Theorem 3.8. If $(X, \mathfrak{T}_2, E)$ is a fuzzy soft b-connected space and $\mathfrak{T}_1 \subseteq \mathfrak{T}_2$, then $(X, \mathfrak{T}_1, E)$ is also a fuzzy soft b-connected.

Proof. Let f_A, g_B be fuzzy soft b-separation on $(X, \mathfrak{T}_1, E)$. Then, $f_A, g_B \in \mathfrak{T}_1$. Since $\mathfrak{T}_1 \subseteq \mathfrak{T}_2$. Then, $f_A, g_B \in \mathfrak{T}_2$ such that f_A, g_B is fuzzy soft b-separation on $(X, \mathfrak{T}_2, E)$, which is a contradiction with the fuzzy soft b-connectedness of $(X, \mathfrak{T}_2, E)$. Hence, $(X, \mathfrak{T}_1, E)$ is fuzzy soft b-connected.

Remark 3.9. The converse of Theorem 3.8 is not true in general, as shown in the following example.

Example 3.10. Let $X = \{a, b, c\}$, $E = \{e_1, e_2, e_3, e_4\}$ and $A, B \subseteq E$ where $A = \{e_1, e_2\}$ and $B = \{e_3, e_4\}$. Let $\mathfrak{T}_1$ be the indiscrete fuzzy soft topology, then $\mathfrak{T}_1$ is fuzzy soft b-connected, on the other hand, let $\mathfrak{T}_2 = \{\tilde{1}_E, \tilde{0}_E, f_A, g_A, k_B, h_B, s_E, v_E\}$ where $f_A, g_A, k_B, h_B, s_E, v_E$ are fuzzy soft sets over X defined as follows:
$\mu^{e_1}_{f_A} = \{a_1, b_1, c_1\}, \mu^{e_2}_{f_A} = \{a_1, b_1, c_1\},$
$\mu^{e_1}_{g_A} = \{a_{0.2}, b_{0.5}, c_{0.8}\}, \mu^{e_2}_{g_A} = \{a_{0.1}, b_{0.6}, c_{0.7}\},$
$\mu^{e_3}_{k_B} = \{a_1, b_1, c_1\}, \mu^{e_4}_{k_B} = \{a_1, b_1, c_1\},$
$\mu^{e_3}_{h_B} = \{a_{0.5}, b_0, c_{0.3}\}, \mu^{e_4}_{h_B} = \{a_1, b_{0.8}, c_{0.3}\},$
$\mu^{e_1}_{s_E} = \{a_{0.2}, b_{0.5}, c_{0.8}\}, \mu^{e_2}_{s_E} = \{a_{0.1}, b_{0.6}, c_{0.7}\}, \mu^{e_3}_{s_E} = \{a_1, b_1, c_1\}, \mu^{e_4}_{s_E} = \{a_1, b_1, c_1\},$
$\mu^{e_1}_{v_E} = \{a_1, b_1, c_1\}, \mu^{e_2}_{v_E} = \{a_1, b_1, c_1\}, \mu^{e_3}_{v_E} = \{a_{0.5}, b_0, c_{0.3}\}, \mu^{e_4}_{v_E} = \{a_1, b_{0.8}, c_{0.3}\},$
Then, $\mathfrak{T}_2$ defines a fuzzy soft topology on X such that $\mathfrak{T}_1 \subseteq \mathfrak{T}_2$. Now, f_A and k_B are fuzzy b-open soft sets in which form a fuzzy soft b-separation of $(X, \mathfrak{T}_2, E)$ where $f_A \sqcap k_B = \tilde{0}_E$ and $\tilde{1}_E = f_A \sqcup k_B$. Hence, $(X, \mathfrak{T}_2, E)$ is fuzzy soft b-disconnected.

Theorem 3.11. A fuzzy soft topological space $(X, \mathfrak{T}, E)$ is a fuzzy soft b-connected space if and only if there exist no non-zero fuzzy b-open soft sets f_A and g_B such that $f_A = g^c_B$.

Proof. Let f_A and g_B be two fuzzy b-open soft sets in $(X, \mathfrak{T}, E)$ such that $f_A \neq \tilde{0}_E$, $g^c_B \neq \tilde{0}_E$ and $f_A = g^c_B$. Therefore g^c_B is a fuzzy b-closed soft set. Since $f_A \neq \tilde{0}_E$, $g_B \neq \tilde{1}_E$. This implies that g_B is a proper fuzzy soft set which is both fuzzy b-open soft and fuzzy b-closed soft in $(X, \mathfrak{T}, E)$. Hence $(X, \mathfrak{T}, E)$ is a fuzzy soft b-disconnected space. But this is a contradiction to our hypothesis, then there exist no non-zero fuzzy b-open soft sets f_A and g_B in $(X, \mathfrak{T}, E)$ such that $f_A = g_B^c$. Conversely, Let f_A be both fuzzy b-open soft and fuzzy b-closed soft set in $(X, \mathfrak{T}, E)$ such that $f_A \neq \tilde{0}_E$, $f_A \neq \tilde{1}_E$. Let $f_A^c = g_B$, then g_B is a fuzzy b-open soft set and $g_B^c \neq \tilde{1}_E$. This implies that $g_B = f_A^c \neq \tilde{0}_E$, which is a contradiction to our hypothesis. Hence $(X, \mathfrak{T}, E)$ is a fuzzy soft b-connected space.

Theorem 3.12. A fuzzy soft topological space $(X, \mathfrak{T}, E)$ is a fuzzy soft b-connected space if and only if there exist no non-zero fuzzy b-open soft sets f_A and g_B in such that $f_A = g_B^c$, $g_B = (FBcl(f_A))^c$ and $f_A = (FBcl(g_B))^c$.

Proof. Assume that there exists fuzzy soft sets f_A and g_B such that $f_A \neq \tilde{0}_E$, $g_B^c \neq \tilde{0}_E$, $f_A = g_B^c$, $g_B = (FBcl(f_A))^c$ and $f_A = (FBcl(g_B))^c$. Since $(FBcl(f_A))^c$ and $(FBcl(g_B))^c$ are fuzzy b-open soft sets in $(X, \mathfrak{T}, E)$, f_A and g_B are fuzzy b-open soft sets in $(X, \mathfrak{T}, E)$. This implies $(X, \mathfrak{T}, E)$ is a fuzzy soft b-disconnected space, which is a contradiction. Thus there exist no non-zero fuzzy b-open soft sets f_A and g_B in such that $f_A = g_B^c$, $g_B = (FBcl(f_A))^c$ and $f_A = (FBcl(g_B))^c$. Conversely, Let f_A be both fuzzy b-open soft and fuzzy b-closed soft set in $(X, \mathfrak{T}, E)$ such that $f_A \neq \tilde{0}_E$, $f_A \neq \tilde{1}_E$. Now by taking $f_A^c = g_B$, we obtain a contradiction to our hypothesis. Hence $(X, \mathfrak{T}, E)$ is a fuzzy soft b-connected space.

Definition 3.13. A fuzzy soft subspace $(Y, \mathfrak{T}_Y, E)$ of fuzzy soft topological space $(X, \mathfrak{T}, E)$ is said to be fuzzy b-open soft (resp. b-closed soft, soft b-connected) subspace if $h_E^Y \in FBOS(X)$ (resp. $h_E^Y \in FBCS(X)$, h_E^Y is fuzzy soft b-connected).

Theorem 3.14. Let $(Y, \mathfrak{T}_Y, E)$ be a fuzzy soft b-connected subspace of fuzzy soft topological space $(X, \mathfrak{T}, E)$ such that $h_E^Y \sqcap g_A \in FBOS(X)$ $g_A \in FBOS(X)$. If $\tilde{1}_E$ has a fuzzy soft b-separations f_A, g_B. Then, either $h_E^Y \sqsubseteq f_A$, or $h_E^Y \sqsubseteq g_B$.

Proof. Let f_A, g_B be fuzzy soft b-separation on $\tilde{1}_E$. By hypothesis, $f_A \sqcap h_E^Y \in FBOS(X)$, $g_B \sqcap h_E^Y \in FBOS(X)$ and $[g_B \sqcap h_E^Y] \sqcup [f_A \sqcap h_E^Y] = h_E^Y$. Since h_E^Y is fuzzy soft b-connected. Then, either $g_B \sqcap h_E^Y = \tilde{0}_E$, or $f_A \sqcap h_E^Y = \tilde{0}_E$. Therefore, either $h_E^Y \sqsubseteq f_A$, or $h_E^Y \sqsubseteq g_B$.

Theorem 3.15. A fuzzy soft subspace $(Y, \mathfrak{T}_Y, E)$ of fuzzy soft b-disconnectedness space $(X, \mathfrak{T}, E)$ is fuzzy soft b-disconnected if $h_E^Y \sqcap g_A \in FBOS(X)$ $\forall$ $g_A \in FBOS(X)$.

Proof. Let $(Y, \mathfrak{T}_Y, E)$ be fuzzy soft b-connected space. Since $(X, \mathfrak{T}, E)$ is fuzzy soft b-disconnected. Then, there exist fuzzy soft b-separation f_A, g_B on $(X, \mathfrak{T}, E)$. By hypothesis, $f_A \sqcap h_E^Y \in FBOS(X)$, $g_B \sqcap h_E^Y \in FBOS(X)$ and $[g_B \sqcap h_E^Y] \sqcup [f_A \sqcap h_E^Y] = h_E^Y$, which is a contradiction with the fuzzy soft b-connectedness of $(Y, \mathfrak{T}_Y, E)$. Therefore, $(Y, \mathfrak{T}_Y, E)$ is fuzzy soft b-disconnected.

Remark 3.16. A fuzzy soft b-disconnectedness property is not hereditary property in general, as in the following example.

Example 3.17. In Example 3.10, let $Y = \{a, b\} \subseteq X$. We consider the fuzzy soft set h_E^Y over (Y, E) defined as follows:
$\mu_{h_E^Y}^{e_1} = \{a_1, b_1, c_0\}$, $\mu_{h_E^Y}^{e_2} = \{a_1, b_1, c_0\}$, $\mu_{h_E^Y}^{e_3} = \{a_1, b_1, c_0\}$, $\mu_{h_E^Y}^{e_4} = \{a_1, b_1, c_0\}$.
Then, we find $\mathfrak{T}_Y$ as follows, $\mathfrak{T}_Y = \{h_E^Y \sqcap z_E : z_E \in \mathfrak{T}\}$ where
$h_E^Y \sqcap \tilde{0}_E = \tilde{0}_E$, $h_E^Y \sqcap \tilde{1}_E = h_E^Y$, $h_E^Y \sqcap f_A = h_C$, where

$\mu^{e_1}_{h_C} = \{a_1, b_1, c_0\}$, $\mu^{e_2}_{h_C} = \{a_1, b_1, c_0\}$,

$h^Y_E \sqcap g_A = h_W$, where
$\mu^{e_1}_{h_W} = \{a_{0.2}, b_{0.5}, c_0\}$, $\mu^{e_2}_{h_W} = \{a_{0.1}, b_{0.6}, c_0\}$,

$h^Y_E \sqcap k_B = h_R$, where
$\mu^{e_3}_{h_R} = \{a_1, b_1, c_0\}$, $\mu^{e_4}_{h_R} = \{a_1, b_1, c_0\}$,

$h^Y_E \sqcap h_B = h_T$, where
$\mu^{e_3}_{h_T} = \{a_{0.5}, b_0, c_0\}$, $\mu^{e_4}_{h_T} = \{a_1, b_{0.8}, c_0\}$,

$h^Y_E \sqcap s_E = h_P$, where
$\mu^{e_1}_{h_P} = \{a_{0.2}, b_{0.5}, c_0\}$, $\mu^{e_2}_{h_P} = \{a_{0.1}, b_{0.6}, c_0\}$, $\mu^{e_3}_{h_P} = \{a_1, b_1, c_0\}$, $\mu^{e_4}_{h_P} = \{a_1, b_1, c_0\}$.
Thus, the collection $\mathfrak{T}_Y = \{h^Y_E \sqcap z_E : z_E \in \mathfrak{T}\}$ is a fuzzy soft topology on (Y, E) in which there is no fuzzy soft b-separation on $(Y, \mathfrak{T}_Y, E)$. Therefore, $(Y, \mathfrak{T}_Y, E)$ is fuzzy soft b-connected at the time that $(X, \mathfrak{T}, E)$ is fuzzy soft b-disconnected as shown in Example 3.10.

Definition 3.18. Let $(X, \mathfrak{T}_1, E)$, $(Y, \mathfrak{T}_2, K)$ be fuzzy soft topological spaces and $f_{pu} : FSS(X)_E \rightarrow FSS(Y)_K$ be a soft function. Then, f_{pu} is called;

(i) Fuzzy b-irresolute soft if $f^{-1}_{pu}(g_B) \in FBOS(X)\ \forall\ g_B \in FBOS(Y)$.

(ii) Fuzzy b-irresolute open soft if $f_{pu}(g_A) \in FBOS(Y)\ \forall\ g_A \in FBOS(X)$.

(iii) Fuzzy b-irresolute closed soft if $f_{pu}(f_A) \in FBCS(Y)\ \forall\ f_A \in FBCS(Y)$.

The following Theorem shows that the fuzzy soft b-connectedness is an invariant property under a fuzzy b-irresolute surjective soft function.

Theorem 3.19. Let $(X_1, \mathfrak{T}_1, E)$ and $(X_2, \mathfrak{T}_2, K)$ be fuzzy soft topological spaces and $f_{pu} : (X_1, \mathfrak{T}_1, E) \rightarrow (X_2, \mathfrak{T}_2, K)$ be a fuzzy b-irresolute surjective soft function. If $(X_1, \mathfrak{T}_1, E)$ is fuzzy soft b-connected, then $(X_2, \mathfrak{T}_2, K)$ is also a fuzzy soft b-connected.

Proof. Let $(X_2, \mathfrak{T}_2, K)$ be a fuzzy soft b-disconnected space. Then, there exist f_A, g_B pair of non null proper fuzzy b-open soft subsets of $\tilde{1}_K$ such that $f_A \sqcap g_B = \tilde{0}_K$ and $\tilde{1}_K = f_A \sqcup g_B$. Since f_{pu} is fuzzy b-irresolute soft function, then $f^{-1}_{pu}(f_A), f^{-1}_{pu}(g_B)$ are pair of non null proper fuzzy b-open soft subsets of $\tilde{1}_E$ such that $f^{-1}_{pu}(f_A) \sqcap f^{-1}_{pu}(g_B) = f^{-1}_{pu}(f_A \sqcap g_B) = f^{-1}_{pu}(\tilde{0}_K) = \tilde{0}_E$ and $f^{-1}_{pu}(f_A) \sqcup f^{-1}_{pu}(g_B) = f^{-1}_{pu}(f_A \sqcup g_B) = f^{-1}_{pu}(\tilde{1}_K) = \tilde{1}_E$. This means that, $f^{-1}_{pu}(f_A), f^{-1}_{pu}(g_B)$ forms a fuzzy soft b-separation of $\tilde{1}_E$, which is a contradiction with the fuzzy soft b-connectedness of $(X_1, \mathfrak{T}_1, E)$. Therefore, $(X_2, \mathfrak{T}_2, K)$ is fuzzy soft b-connected.

4 Fuzzy Soft b-s-Connected Spaces

In this section, we introduce the notions of fuzzy soft b-separated sets and use it to introduce the notions of fuzzy b-s-connectedness in fuzzy soft topological spaces and study some of its fundamental properties.

Definition 4.1. A non null fuzzy soft subsets f_A, g_B of fuzzy soft topological space $(X, \mathfrak{T}, E)$ are said to be fuzzy soft b-separated sets if $FBcl(f_A) \sqcap g_B = FBcl(g_B) \sqcap f_A = \tilde{0}_E$.

Theorem 4.2. Let $f_A \sqsubseteq g_B$, $h_C \sqsubseteq k_D$ and g_B, k_D are soft fuzzy soft b-separated subsets of fuzzy soft topological space $(X, \mathfrak{T}, E)$. Then, f_A, h_C are fuzzy soft b-separated sets.

Proof. Let $f_A \sqsubseteq g_B$, then $FBcl(f_A) \sqsubseteq FBcl(g_B)$. It follows that, $FBcl(f_A) \sqcap h_C \sqsubseteq FBcl(f_A) \sqcap k_D \sqsubseteq FBcl(g_B) \sqcap k_D = \tilde{0}_E$. Also, since $h_C \sqsubseteq k_D$. Then, $FBcl(h_C) \sqsubseteq FBcl(k_D)$. Hence, $f_A \sqcap FBcl(h_C) \sqsubseteq FBcl(k_D) \sqcap g_B = \tilde{0}_E$. Thus, f_A, h_C are fuzzy soft b-separated sets.

Theorem 4.3. Two fuzzy b-closed soft subsets of fuzzy soft topological space $(X, \mathfrak{T}, E)$ are fuzzy soft b-separated sets if and only if they are disjoint.

Proof. Let f_A, g_B are fuzzy soft b-separated sets. Then, $FBcl(g_B) \sqcap f_A = g_B \sqcap FBcl(f_A) = \tilde{0}_E$. Since f_A, g_B are fuzzy b-closed soft sets. Then, $f_A \sqcap g_B = \tilde{0}_E$. Conversely, let f_A, g_B are disjoint fuzzy b-closed soft sets. Then, $g_B \sqcap FBcl(f_A) = f_A \sqcap g_B = \tilde{0}_E$ and $FBcl(g_B) \sqcap f_A = f_A \sqcap g_B = \tilde{0}_E$. It follows that, f_A, g_B are fuzzy soft b-separated sets.

Theorem 4.4. Let g_B and h_C be non null fuzzy b-open soft sets of a fuzzy soft topological space $(X, \mathfrak{T}, E)$. If $u_D = g_B \sqcap (\tilde{1}_E - h_C)$ and $v_A = h_C \sqcap (\tilde{1}_E - g_B)$, then u_D and v_A are fuzzy soft b-separated sets.

Proof. Let g_B and h_C be fuzzy b-open soft sets, then $(\tilde{1}_E - g_B)$ and $(\tilde{1}_E - h_C)$ are fuzzy b-closed soft sets. Let $u_D = g_B \sqcap (\tilde{1}_E - h_C)$ and $v_A = h_C \sqcap (\tilde{1}_E - g_B)$, then $u_D \sqsubseteq (\tilde{1}_E - h_C)$ and $v_A \sqsubseteq (\tilde{1}_E - g_B)$. Hence, $FBcl(u_D) \sqsubseteq (\tilde{1}_E - h_C) \sqsubseteq (\tilde{1}_E - v_A)$ and $FBcl(v_A) \sqsubseteq (\tilde{1}_E - g_B) \sqsubseteq (\tilde{1}_E - u_D)$. Consequently, $FBcl(u_D) \sqcap v_A = \tilde{0}_E$ and $v_A \sqcap FBcl(u_D) = \tilde{0}_E$ and so u_D and v_A are fuzzy soft b-separated.

Definition 4.5. A fuzzy soft topological space $(X, \mathfrak{T}, E)$ is said to be fuzzy soft b-s-connected if and only if $\tilde{1}_E$ can not expressed as the fuzzy soft union of two fuzzy soft b-separated sets in $(X, \mathfrak{T}, E)$.

Theorem 4.6. Let $(Z, \mathfrak{T}_Z, E)$ be a fuzzy soft subspace of fuzzy soft topological space $(X, \mathfrak{T}, E)$ and f_A, $g_B \sqsubseteq z_E \sqsubseteq \tilde{1}_E$. Then, f_A and g_B are fuzzy soft b-separated on $\mathfrak{T}_Z$ if and only if f_A and g_B are fuzzy soft b-separated on $\mathfrak{T}$, where $\mathfrak{T}_Z$ is the fuzzy soft subspace for z_E.

Proof. Suppose that f_A and g_B are fuzzy soft b-separated on $\mathfrak{T}_Z$, then $FBcl_{\mathfrak{T}_Z}(f_A) \sqcap g_B = \tilde{\phi}$ and $f_A \sqcap FBcl_{\mathfrak{T}_Z}(g_B) = \tilde{0}_E$. Hence, $[FBcl_{\mathfrak{T}}(f_A) \sqcap z_E] \sqcap g_B = FBcl_{\mathfrak{T}}(f_A) \sqcap g_B = \tilde{0}_E$ and $[FBcl_{\mathfrak{T}}(g_B) \sqcap z_E] \sqcap f_A = FBcl_{\mathfrak{T}}(g_B) \sqcap f_A = \tilde{0}_E$. Consequently, f_A and g_B are fuzzy soft b-separated sets on $\mathfrak{T}$.

Theorem 4.7. Let z_E be a fuzzy soft subset of fuzzy soft topological space $(X, \mathfrak{T}, E)$. Then, z_E is fuzzy soft b-s-connected with respect to $(X, \mathfrak{T}, E)$ if and only if z_E is fuzzy soft b-s-connected with respect to $(Z, \mathfrak{T}_Z, E)$.

Proof. Suppose that z_E is not fuzzy soft b-s-connected with respect to $(Z, \mathfrak{T}_Z, E)$. Then, $z_E = f_{1A} \sqcup f_{2B}$, where f_{1A} and f_{2B} are fuzzy soft b-separated sets on $\mathfrak{T}_Z$. So $z_E = f_{1A} \sqcup f_{2B}$, where f_{1A} and f_{2B} are fuzzy soft b-separated on $\mathfrak{T}_Z$ from Theorem 4.6 . Consequently, z_E is not fuzzy soft b-s-connected with respect to $(X, \mathfrak{T}, E)$.

Theorem 4.8. Let $(Z, \mathfrak{T}_Z, E)$ be a fuzzy soft b-s-connected subspace of fuzzy soft topological space $(X, \mathfrak{T}, E)$ and $f_A,\ g_B$ be fuzzy soft b-separated of $\tilde{1}_E$ with $z_E \sqsubseteq f_A \sqcup g_B$. Then, either $z_E \sqsubseteq f_A$, or $z_E \sqsubseteq g_B$.

Proof. Let $z_E \sqsubseteq f_A \sqcup g_B$ for some fuzzy soft b-separated subsets $f_A,\ g_B$ of $\tilde{1}_E$. Since $z_E = (z_E \sqcap f_A) \sqcup (z_E \sqcap g_B)$. Then, $(z_E \sqcap f_A) \sqcap FBcl_{\mathfrak{T}}(z_E \sqcap g_B) \sqsubseteq (f_A \sqcap FBcl_{\mathfrak{T}} g_B) = \tilde{0}_E$. Also, $FBcl_{\mathfrak{T}}(z_E \sqcap f_A) \sqcap (z_E \sqcap g_B) \sqsubseteq FBcl_{\mathfrak{T}}(f_A) \sqcap g_B = \tilde{0}_E$. Since $(Z, \mathfrak{T}_Z, E)$ is fuzzy soft b-s-connected. Thus, either $z_E \sqcap f_A = \tilde{0}_E$ or $z_E \sqcap g_B = \tilde{0}_E$. It follows that, $z_E = z_E \sqcap f_A$ or $z_E = z_E \sqcap g_B$. This implies that, $z_E \sqsubseteq f_A$ or $z_E \sqsubseteq g_B$.

Theorem 4.9. Let $(Z, \mathfrak{T}_Z, N)$ and $(Y, \mathfrak{T}_Y, M)$ be fuzzy soft b-s-connected subspaces of fuzzy soft topological space $(X, \mathfrak{T}, E)$ such that none of them is fuzzy soft b-separated. Then, $z_N \sqcup y_M$ is fuzzy soft b-s-connected.

Proof. Let $(Z, \mathfrak{T}_Z, N)$ and $(Y, \mathfrak{T}_Y, M)$ be fuzzy soft b-s-connected subspaces of $\tilde{1}_E$ such that $z_N \sqcup y_M$ is not fuzzy soft b-s-connected. Then, there exist two non null fuzzy soft b-separated sets k_D and h_C of $\tilde{1}_E$ such that $z_N \sqcup y_M = k_D \sqcup h_C$. Since z_N, y_M are fuzzy soft b-s-connected, $z_N, y_M \sqsubseteq z_N \sqcup f_A = k_D \sqcup h_C$. By Theorem 4.8, either $z_N \sqsubseteq k_D$ or $z_N \sqsubseteq h_C$, also, either $y_M \sqsubseteq k_D$ or $y_M \sqsubseteq h_C$. If $z_N \sqsubseteq k_D$ or $z_N \sqsubseteq h_C$. Then, $z_N \sqcap h_C \sqsubseteq k_D \sqcap h_C = \tilde{0}_E$ or $z_N \sqcap k_D \sqsubseteq z_N \sqcap k_D = \tilde{0}_E$. Therefore, $[z_N \sqcup y_M] \sqcap k_D = [z_N \sqcap k_D] \sqcup [y_M \sqcup k_D] = [y_M \sqcap k_D] \sqcup \tilde{0}_E = y_M \sqcap k_D = y_M$ since $y_M \sqsubseteq k_D$. Similarly, if $y_M \sqsubseteq k_D$ or $y_M \sqsubseteq h_C$. we get $[z_N \sqcup y_M] \sqcap h_C = z_N$.

Now, $[(z_N \sqcup y_M) \sqcap h_C] \sqcap FBcl[(z_N \sqcup y_M) \sqcap k_D] \sqsubseteq [(z_N \sqcup y_M) \sqcap h_C] \sqcap [FBcl(z_N \sqcup y_M) \sqcap FBcl(k_D)] = [z_N \sqcup y_M] \sqcap [h_C \sqcap FBcl(k_D)] = \tilde{0}_E$ and $FBcl[(z_N \sqcup y_M) \sqcap h_C] \sqcap [(z_N \sqcup y_M) \sqcap k_D] \sqsubseteq [FBcl(z_N \sqcup y_M) \sqcap FBcl(h_C)] \sqcap [(z_N \sqcup y_M) \sqcap k_D] = [z_N \sqcup y_M] \sqcap [FBcl(h_C) \sqcap k_D] = \tilde{0}_E$. It follows that, $[z_N \sqcup y_M] \sqcap k_D = z_N$ and $[z_N \sqcup y_M] \sqcap h_C = y_M$ are fuzzy soft b-separated, which is a contradiction. Hence, $z_N \sqcup y_M$ is fuzzy soft b- s-connected.

Theorem 4.10. Let $(Z, \mathfrak{T}_Z, N)$ be a fuzzy soft b-s-connected subspace of fuzzy soft topological space $(X, \mathfrak{T}, E)$ and $S_M \in SS(X)_E$. If $z_N \sqsubseteq S_M \sqsubseteq FBcl(z_N)$. Then, $(S, \mathfrak{T}_S, M)$ is fuzzy soft b-s-connected subspace of $(X, \mathfrak{T}, E)$.

Proof. Suppose that $(S, \mathfrak{T}_S, M)$ is not fuzzy soft b-s-connected subspace of $(X, \mathfrak{T}, E)$. Then, there exist fuzzy soft b-separated sets f_A and g_B on $\mathfrak{T}$ such that $S_M = f_A \sqcup g_B$. So, we have z_N is fuzzy soft b-s-connected subset of fuzzy soft b-s-disconnected space. By Theorem 4.8, either $z_N \sqsubseteq f_A$ or $z_N \sqsubseteq g_B$. If $z_N \sqsubseteq f_A$. Then, $FBcl(z_N) \sqsubseteq FBcl(f_A)$. It follows $FBcl(z_N) \sqcap g_B \sqsubseteq FBcl(f_A) \sqcap g_B = \tilde{0}_E$. Hence, $g_B = FBcl(z_N) \sqcap g_B = \tilde{0}_E$ which is a contradiction. If $z_N \sqsubseteq g_B$. By a similar way, we can get $f_A = \tilde{0}_E$, which is a contradiction. Hence, $(S, \mathfrak{T}_S, M)$ is fuzzy soft b- s-connected subspace of $(X, \mathfrak{T}, E)$.

Corollary 4.11. If $(Z, \mathfrak{T}_Z, N)$ is fuzzy soft b-s-connected subspace of fuzzy soft topological space $(X, \mathfrak{T}, E)$. Then, $FBcl(z_N)$ is fuzzy soft b-s-connected.

Proof. It is obvious from Theorem 4.10.

Theorem 4.12. If for all pair of distinct fuzzy soft point f_e, g_e, there exists a fuzzy soft b-s-connected set $z_N \sqsubseteq \tilde{1}_E$ with $f_e, g_e \tilde{\in} z_N$, then $\tilde{1}_E$ is fuzzy soft b-s-connected.

Proof. Suppose that $\tilde{1}$ is fuzzy soft b-s-disconnected. Then, $\tilde{1}_E = f_A \sqcup g_B$, where f_A, g_B are fuzzy soft b-separated sets. It follows $f_A \sqcap g_B = \tilde{0}_E$. So, $\exists\ f_e \tilde{\in} f_A$ and $g_e \tilde{\in} g_B$. Since $f_A \sqcap g_B = \tilde{0}_E$. Then, f_e, g_e are distinct fuzzy soft point in $\tilde{1}_E$. By hypothesis, there exists a fuzzy soft b-s-connected set z_N such that $f_e, g_e \tilde{\in} z_N \sqsubseteq \tilde{1}_E$ and $f_e, g_e \tilde{\in} z_N$. Moreover, we have z_N is fuzzy soft b-s-connected subset of a a fuzzy soft b-s-disconnected space. It follows by Theorem 4.8, either $z_N \sqsubseteq f_A$ or $z_N \sqsubseteq g_B$ and both cases is a contradiction with the hypothesis. Therefore, $\tilde{1}_E$ is fuzzy soft b-s-connected.

Theorem 4.13. Let $\{(Z_j, \mathfrak{T}_{Z_j}, N) : j \in J\}$ be a non null family of fuzzy soft b-s-connected subspaces of fuzzy soft topological space $(X, \mathfrak{T}, E)$. If $\sqcap_{j\in J}(z_j, N) \neq \tilde{0}_E$, then $(\sqcup_{j\in J} Z_j, \mathfrak{T}_{\sqcup_{j\in J} Z_j}, N)$ is also a fuzzy soft b-s-connected fuzzy subspace of $(X, \mathfrak{T}, E)$.

Proof. Suppose that $(Z, \mathfrak{T}_Z, N) = (\sqcup_{j\in J} Z_j, \mathfrak{T}_{\sqcup_{j\in J} Z_j}, N)$ is fuzzy soft b-s-disconnected. Then, $z_N = f_A \sqcup g_B$ for some fuzzy soft b-separated subsets f_A, g_B of $\tilde{1}_E$. Since $\sqcap_{j\in J}(z_j, N) \neq \tilde{0}_E$. Then, $\exists\ f_e \tilde{\in} \sqcap_{j\in J}(z, N)_j$. It follows that, $f_e \tilde{\in} z_N$. So, either $f_e \tilde{\in} f_A$ or $f_e \tilde{\in} g_B$. Suppose that $f_e \tilde{\in} f_A$. Since $f_e \tilde{\in} (z, N)_j\ \forall j \in J$ and $(z, N)_j \sqsubseteq z_N$. So, we have $(z, N)_j$ is fuzzy soft b-s-connected subset of fuzzy soft b-s-disconnected set z_N. By Theorem 4.8, either $(z, N)_j \sqsubseteq f_A$ or $(z, N)_j \sqsubseteq g_B\ \forall j \in J$. If $(z, N)_j \sqsubseteq f_A\ \forall j \in J$. Then, $z_N \sqsubseteq f_A$. This implies that, $g_B = \tilde{0}_E$, which is a contradiction. Also, if $(z, N)_j \sqsubseteq g_B\ \forall j \in J$. Also, if $f_e \tilde{\in} g_B$, by a similar way, we get $f_A = \tilde{0}_E$, which is a contradiction. Therefore, $(Z, \mathfrak{T}_Z, N) = (\sqcup_{j\in J} Z_j, \mathfrak{T}_{\sqcup_{j\in J} Z_j}, N)$ is fuzzy soft b-s-connected.

Theorem 4.14. Let $\{(Z_j, \mathfrak{T}_{Z_j}, N) : j \in J\}$ be a family of fuzzy soft b-s-connected subspaces of fuzzy soft topological space $(X, \mathfrak{T}, E)$ such that one of the members of the family intersects every other members, then $(\sqcup_{j\in J} Z_j, \mathfrak{T}_{\sqcup_{j\in J} Z_j}, N)$ is fuzzy subspace of $(X, \mathfrak{T}, E)$.

Proof. Let $(Z, \mathfrak{T}_Z, N) = (\sqcup_{j\in J} Z_j, \mathfrak{T}_{\sqcup_{j\in J} Z_j}, N)$ and $(z, N)_{jo} \in \{(z, N)_j : j \in J\}$ such that $(z, N)_{jo} \sqcap (z, N)_j \neq \tilde{0}_E\ \ \forall j \in J$. Then, $(z, N)_{jo} \sqcup (z, N)_j$ is fuzzy soft b-s-connected $\forall j \in J$ by Theorem 4.9. Hence, the collection $\{(z, N)_{jo} \sqcup (z, N)_j : j \in J\}$ is a collection of fuzzy soft b-s-connected subsets of $\tilde{1}$, which having a non null fuzzy soft intersection. Therefore, $(Z, \mathfrak{T}_Z, N) = (\sqcup_{j\in J} Z_j, \mathfrak{T}_{\sqcup_{j\in J} Z_j}, N)$ is fuzzy soft b-s-connected subspace of $(X, \mathfrak{T}, E)$ by Theorem 4.9.

Theorem 4.15. Let $(X_1, \mathfrak{T}_1, E)$ and $(X_2, \mathfrak{T}_2, K)$ be fuzzy soft topological spaces and $f_{pu} : (X_1, \mathfrak{T}_1, E) \rightarrow (X_2, \mathfrak{T}_2, K)$ be a fuzzy b-irresolute surjective soft function. If $(X_1, \mathfrak{T}_1, E)$ is fuzzy soft b-s-connected, then $(X_2, \mathfrak{T}_2, K)$ is also a fuzzy soft b-s-connected.

Proof. Let $(X_2, \mathfrak{T}_2, K)$ be fuzzy soft b-disconnected space. Then, there exist f_A, g_B pair of non null proper fuzzy soft b-separated sets such that $\tilde{1}_K = f_A \sqcup g_B$, $FBcl(f_A) \sqcap g_B = FBcl(g_B) \sqcap f_A = \tilde{0}_E$. Since f_{pu} is fuzzy b-irresolute soft function, then $f_{pu}^{-1}(f_A), f_{pu}^{-1}(g_B)$ are pair of non null proper fuzzy b-open soft subsets of $\tilde{1}_E$ such that $FBcl(f_{pu}^{-1}(f_A)) \sqcap f_{pu}^{-1}(g_B) \sqsubseteq f_{pu}^{-1}(FBcl(f_A)) \sqcap f_{pu}^{-1}(g_B) = f_{pu}^{-1}(f_A \sqcap g_B) = f_{pu}^{-1}(\tilde{0}_K) = \tilde{0}_E$, $f_{pu}^{-1}(f_A) \sqcap FBcl(f_{pu}^{-1}(g_B)) \sqsubseteq f_{pu}^{-1}(f_A) \sqcap f_{pu}^{-1}(FBcl(g_B)) = f_{pu}^{-1}(f_A \sqcap g_B) = f_{pu}^{-1}(\tilde{0}_K) = \tilde{0}_E$ and $f_{pu}^{-1}(f_A) \sqcup f_{pu}^{-1}(g_B)) = f_{pu}^{-1}(f_A \sqcup g_B) = f_{pu}^{-1}(\tilde{1}_K) = \tilde{1}_E$ from [[25], Theorem 4.2]. This means that, $f_{pu}^{-1}(f_A), f_{pu}^{-1}(g_B)$ are pair of non null proper fuzzy soft b-separated sets of $\tilde{1}_E$, which is a contradiction of the fuzzy soft b-s-connectedness of $(X_1, \mathfrak{T}_1, E)$. Therefore, $(X_2, \mathfrak{T}_2, K)$ is fuzzy soft b-s-connected.

5 Conclusion

As a continuation of study fuzzy soft topological spaces, fuzzy soft b-connected sets, fuzzy soft b-separated sets and fuzzy soft b s-connected sets have been introduced and several interesting properties supported by examples have been established. Moreover, a fuzzy soft b-disconnectedness property is not hereditary property in general have been showed.

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