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A NOTE ON INTUITIONISTIC ANTI FUZZY BI-IDEALS OF SEMIGROUPS

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Abstaract − In this paper the notions of intuitionistic anti fuzzy bi-ideal, intuitionistic anti fuzzy interior ideal, intuitionistic anti fuzzy (1,2)-ideal in semigroups are introduced and some important characterizations have been obtained.

Keywords − *Semigroup, Regular semigroup, Intuitionistic anti fuzzy bi-ideal, Intuitionistic anti fuzzy interior ideal, Intuitionistic anti fuzzy (1,2)-ideal.*

1 Introduction

After the introduction of fuzzy set by Zadeh[12] in 1965, the researchers in mathematics were trying to introduce and study the concept of fuzzyness in different mathematical systems under study. In 1986, K. T. Atanassov[1] introduced the notion of intuitionistic fuzzy sets, which is the generalization of fuzzy sets. Fuzzy set gives the degree of membership of an element in a given set, but intuitionistic fuzzy set gives both degree of membership and degree of non-membership. The degree of membership and the degree of non-membership are the real numbers between 0 and 1, having sum not greater than 1. For more details on intuitionistic fuzzy sets, we refer [1, 2].

In [8] K. H. Kim and Y. B. Jun introduced the concept of intuitionistic fuzzy ideals of semigroups. In [7] K. H. Kim and J. G. Lee gave the notion of intuitionistic fuzzy bi-ideals of semigroups. In [6] Y. B. Jun introduced intuitionistic fuzzy bi-ideals of ordered semigroups and studied natural equivalence relation on the set of all intuitionistic fuzzy bi-ideals of an ordered semigroup. A. Iampan and M. Siripitukdet[5]

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studied the minimal and maximal left ideals in Po-Γ-semigroups and P. Dheena and B. Elavarasan[3] introduced the notion of right chain Po-Γ-semigroups. The concept of (1,2)-ideals in a semigroup was introduced by S. Lajos[10]. In [9] N. Kuroki gave some properties of fuzzy ideals and fuzzy bi-ideals in semigroups. In [4] K. Hila and E. Pisha introduced the notion of bi-ideals in ordered Γ-semigroups. T. Srinivas, T. Nagaiah and P. Narasimha Swamy[11] studied the various properties of Γ-nearrings in terms of their anti fuzzy ideals. In this paper we introduce the concept of intuitionistic anti fuzzy bi-ideals and intuitionistic anti fuzzy (1,2)-ideals in a semigroup. Here a regular semigroup has been characterized in terms of intuitionistic anti fuzzy bi-ideal.

2 Preliminary

Let S be a semigroup. Now recall the following from[1, 8].

- By a subsemigroup of S we mean a non-empty subset A of S such that $A^2 \subseteq A$.
- By a left(right) ideal of S we mean a non-empty subset A of S such that $SA \subseteq A(AS \subseteq A)$.
- By a two-sided ideal or simply ideal we mean a non-empty subset of S which is both a left and a right ideal of S.
- A subsemigroup A of a non empty subset of a semigroup S is called a bi-ideal of S if $ASA \subseteq A$.
- A subsemigroup A of S is called a (1,2)-ideal of S if $ASA^2 \subseteq A$.
- Let X be a non-empty set. A fuzzy set of X is a function $\Psi : X \to [0,1]$ and the complement of Ψ, denoted by $\overline{\Psi}$ is a fuzzy set in X given by $\overline{\Psi}(x) = 1 - \Psi(x)$ for all $x \in X$.
- A semigroup S is called regular if, for each $a \in S$, there exist $x \in S$ such that $a = axa$.
- A fuzzy set Ψ in a semigroup S is called an anti fuzzy subsemigroup of S if, $\Psi(xy) \leq \max\{\Psi(x), \Psi(y)\}$ for all $x, y \in S$
- An anti fuzzy subsemigroup Ψ of a semigroup S is called an anti fuzzy interior ideal of S if $\Psi(xay) \leq \Psi(a)$, for all $x, y, a \in S$.
- An intuitionistic fuzzy set (briefly, IFS) A in a non-empty set X is an object having the form

$$A = \{(x, \Psi_A(x), \Omega_A(x)) : x \in X\},$$

where the functions $\Psi_A : X \to [0,1]$ and $\Omega_A : X \to [0,1]$ denote the degree of membership and the degree of non-membership, respectively and

$$0 \leq \Psi_A(x) + \Omega_A(x) \leq 1$$

for all $x \in X$.

- An intuitionistic fuzzy set $A = \{(x, \Psi_A(x), \Omega_A(x)) : x \in X\}$ in X can be identified as an order pair (Ψ_A, Ω_A) in $I^X X I^X$. For the sake of simplicity, we use the symbol $A = (\Psi_A, \Omega_A)$ for IFS $A = \{(x, \Psi_A(x), \Omega_A(x)) : x \in X\}$.
- Let X be a non-empty set and let $A = (\Psi_A, \Omega_A)$ and $B = (\Psi_B, \Omega_B)$ be any two IFSs in X. Then the following operations and relations are valid [1].

(1) $A \subseteq B$ iff $\Psi_A \leq \Psi_B$ and $\Omega_A \geq \Omega_B$
(2) $A = B$ iff $A \subset B$ and $B \subset A$
(3) $\overline{A} = (\overline{\Psi}_A, \overline{\Omega}_A) = (\Omega_A, \Psi_A)$

(4) $A \cap B = (\Psi_A \wedge \Psi_B, \Omega_A \vee \Omega_B)$
(5) $A \cup B = (\Psi_A \vee \Psi_B, \Omega_A \wedge \Omega_B)$
(6) $\Box A = (\Psi_A, 1 - \Psi_A), \Diamond A = (1 - \Omega_A, \Omega_A)$.

• An IFS $A = (\Psi_A, \Omega_A)$ of a semigroup S is called an intuitionistic fuzzy subsemigroup of S if
(1) $\Psi_A(xy) \geq \min\{\Psi_A(x), \Psi_A(y)\} \forall x, y \in S$,
(2) $\Omega_A(xy) \leq \max\{\Omega_A(x), \Omega_A(y)\} \forall x, y \in S$.

• An IFS $A = (\Psi_A, \Omega_A)$ of a semigroup S is called an intuitionistic fuzzy left ideal of S if $\Psi_A(xy) \geq \Psi_A(y)$ and $\Omega_A(xy) \leq \Omega_A(y)$ for all $x, y \in S$. An intuitionistic fuzzy right ideal of S is defined in an analogous way.

• An IFS $A = (\Psi_A, \Omega_A)$ in S is called an intuitionistic fuzzy ideal of S if it is both an intuitionstic fuzzy left and intuitionistic fuzzy right ideal of S.

• An intuitionistic fuzzy subsemigroup $A = (\Psi_A, \Omega_A)$ of a semigroup S is called an intuitionistic fuzzy bi-ideal of S if
(1) $\Psi_A(xay) \geq \min\{\Psi_A(x), \Psi_A(y)\} \forall x, y, a \in S$,
(2) $\Omega_A(xay) \leq \max\{\Omega_A(x), \Omega_A(y)\} \forall x, y, a \in S$.

• An intuitionistic fuzzy subsemigroup $A = (\Psi_A, \Omega_A)$ of a semigroup S is called an intuitionistic fuzzy (1,2)-ideal of S if
(1) $\Psi_A(xa(yz)) \geq \min\{\Psi_A(x), \Psi_A(y), \Psi_A(z)\} \forall x, y, z, a \in S$,
(2) $\Omega_A(xa(yz)) \leq \max\{\Omega_A(x), \Omega_A(y), \Omega_A(z)\} \forall x, y, z, a \in S$.

• Let f be a mapping from a set X to a set Y. If $A = (\Psi_A, \Omega_A)$ and $B = (\Psi_B, \Omega_B)$ are IFSs in X and Y respectively, then the pre-image of B under f, denoted by $f^{-1}(B)$, is an IFS in X defined by $f^{-1}(B) = (f^{-1}(\Psi_B), f^{-1}(\Omega_B))$.

2.1 Intuitionistic Anti Fuzzy Bi-ideal

In this section we introduce the concepts of intuitionistic anti fuzzy bi-ideal, intuitionistic anti fuzzy interior ideal and intuitionistic anti fuzzy $(1, 2)$-ideal of a semigroup S.

Definition 2.1. An IFS $A = (\Psi_A, \Omega_A)$ of a semigroup S is called an intuitionistic anti fuzzy subsemigroup of S if
(1) $\Psi_A(xy) \leq \max\{\Psi_A(x), \Psi_A(y)\} \forall x, y \in S$,
(2) $\Omega_A(xy) \geq \min\{\Omega_A(x), \Omega_A(y)\} \forall x, y \in S$.

Definition 2.2. An intuitionistic anti fuzzy subsemigroup $A = (\Psi_A, \Omega_A)$ of a semigroup S is called an intuitionistic anti fuzzy bi-ideal of S if
(1) $\Psi_A(xay) \leq \max\{\Psi_A(x), \Psi_A(y)\}, \forall x, y, a \in S$,
(2) $\Omega_A(xay) \geq \min\{\Omega_A(x), \Omega_A(y)\}, \forall x, y, a \in S$.

Example 2.3. Let $S = \{a, b, c, d, e\}$ be a semigroup with the following cayley table.

.	a	b	c	d	e
a	a	a	a	a	a
b	a	a	a	a	a
c	a	a	c	c	e
d	a	a	c	d	e
e	a	a	a	c	e

Define an IFS $A = (\Psi_A, \Omega_A)$ in S by $\Psi_A(a) = 0.3 = \Psi_A(b)$, $\Psi_A(c) = 0.4$, $\Psi_A(d) = 0.5$, $\Psi_A(e) = 0.6$ and $\Omega_A(a) = 0.6$, $\Omega_A(b) = 0.5$, $\Omega_A(c) = 0.4$, $\Omega_A(d) = \Omega_A(e) = 0.3$. The routine calculation shows that $A = (\Psi_A, \Omega_A)$ is an intuitionistic anti fuzzy bi-ideal of a semigroup S.

Definition 2.4. An intuitionistic anti fuzzy subsemigroup $A = (\Psi_A, \Omega_A)$ of a semigroup S is called an intuitionistic anti fuzzy interior ideal of S if
(1) $\Psi_A(xay) \leq \Psi_A(a) \forall x, y, a \in S$,
(2) $\Omega_A(xay) \geq \Omega_A(a)$ for all $x, y, a \in S$.

Definition 2.5. An intuitionistic anti fuzzy subsemigroup $A = (\Psi_A, \Omega_A)$ of a semigroup S is called an intuitionistic anti fuzzy (1,2)-ideal of S if
(1) $\Psi_A(xa(yz)) \leq \max\{\Psi_A(x), \Psi_A(y), \Psi_A(z)\} \forall x, y, z, a \in S$,
(2) $\Omega_A(xa(yz)) \geq \min\{\Omega_A(x), \Omega_A(y), \Omega_A(z)\} \forall x, y, z, a \in S$.

3 Main Results

Theorem 3.1. Every intuitionistic anti fuzzy bi-ideal of a regular semigroup S is an intuitionistic anti fuzzy subsemigroup of S.

Proof. Let $A = (\Psi_A, \Omega_A)$ be an intuitionistic anti fuzzy bi-ideal of a regular semigroup S and let $a, b \in S$. Since S is regular, there exists $x \in S$ such that $b = bxb$. Then we have

$$\Psi_A(ab) = \Psi_A(a(bxb)) = \Psi_A(a(bx)b) \leq \max\{\Psi_A(a), \Psi_A(b)\}$$

and

$$\Omega_A(ab) = \Omega_A(a(bxb)) = \Omega_A(a(bx)b) \geq \min\{\Omega_A(a), \Omega_A(b)\}.$$

Hence $A = (\Psi_A, \Omega_A)$ is an intuitionistic anti fuzzy subsemigroup of S. □

Lemma 3.2. If an IFS $A = (\Psi_A, \Omega_A)$ of a semigroup S is an intuitionistic anti fuzzy bi-deal of S, then so is $\Box A := (\Psi_A, \overline{\Psi}_A)$.

Proof. Suppose an IFS $A = (\Psi_A, \Omega_A)$ of a semigroup S is an intuitionistic anti fuzzy bi-ideal of S. Then by definition it is clear that Ψ_A is an intuitionistic anti fuzzy bi-ideal. It is sufficient to show that $\overline{\Psi}_A$ is an intuitionistic anti fuzzy bi-ideal of S. For any $x, y, a \in S$, we have

$$\begin{aligned} \overline{\Psi}_A(xy) &= 1 - \Psi_A(xy) \\ &\geq 1 - \max\{\Psi_A(x), \Psi_A(y)\} \\ &= \min\{1 - \Psi_A(x), 1 - \Psi_A(y)\} \\ &= \min\{\overline{\Psi}_A(x), \overline{\Psi}_A(y)\} \end{aligned}$$

and

$$\begin{aligned} \overline{\Psi}_A(xay) &= 1 - \Psi_A(xay) \\ &\geq 1 - \max\{\Psi_A(x), \Psi_A(y)\} \\ &= \min\{1 - \Psi_A(x), 1 - \Psi_A(y)\} \\ &= \min\{\overline{\Psi}_A(x), \overline{\Psi}_A(y)\}. \end{aligned}$$

Hence $\Box A$ is an intuitionistic anti fuzzy bi-ideal of S. This completes the proof. □

Similarly we can prove the following lemma.

Lemma 3.3. If $A = (\Psi_A, \Omega_A)$ is an intuitionistic anti fuzzy bi-ideal of S then so is $\Diamond A = (\overline{\Omega}_A, \Omega_A)$.

Combining Lemmas 3.2 and 3.3 we obtain the following theorem.

Theorem 3.4. $A = (\Psi_A, \Omega_A)$ is an intuitionistic anti fuzzy bi-ideal of S, if and only if $\Box A$ and $\Diamond A$ are intuitionistic anti fuzzy bi-ideals of S.

Theorem 3.5. An IFS $A = (\Psi_A, \Omega_A)$ of a semigroup S is an intuitionistic anti fuzzy bi-deal of S if and only if $\overline{A}$ is an intuitionistic fuzzy bi-ideal of S.

Proof. Let $A = (\Psi_A, \Omega_A)$ be an intuitionistic anti fuzzy bi-ideal of S. To show that $\overline{A} = (\overline{\Psi}_A, \overline{\Omega}_A) = (\Omega_A, \Psi_A)$ is an intuitionistic fuzzy bi-ideal. For any $x, y \in S$, we have

$$\begin{aligned}
\overline{\Psi}_A(xy) &= 1 - \Psi_A(xy) \\
&\geq 1 - \max\{\Psi_A(x), \Psi_A(y)\} \\
&= \min\{1 - \Psi_A(x), 1 - \Psi_A(y)\} \\
&= \min\{\overline{\Psi}_A(x), \overline{\Psi}_A(y)\}
\end{aligned}$$

and

$$\begin{aligned}
\overline{\Omega}_A(xy) &= 1 - \Omega_A(xy) \\
&\leq 1 - \min\{\Psi_A(x), \Omega_A(y)\} \\
&= \max\{1 - \Omega_A(x), 1 - \Omega_A(y)\} \\
&= \max\{\overline{\Omega}_A(x), \overline{\Omega}_A(y)\}.
\end{aligned}$$

Hence $\overline{A} = (\overline{\Psi}_A, \overline{\Omega}_A)$ is an intuitionistic fuzzy subsemigroup of S. Let $x, y, a \in S$. Then

$$\begin{aligned}
\overline{\Psi}_A(xay) &= 1 - \Psi_A(xay) \\
&\geq 1 - \max\{\Psi_A(x), \Psi_A(y)\} \\
&= \min\{1 - \Psi_A(x), 1 - \Psi_A(y)\} \\
&= \min\{\overline{\Psi}_A(x), \overline{\Psi}_A(y)\}
\end{aligned}$$

and

$$\begin{aligned}
\overline{\Omega}_A(xay) &= 1 - \Omega_A(xay) \\
&\leq 1 - \min\{\Omega_A(x), \Omega_A(y)\} \\
&= \max\{1 - \Omega_A(x), 1 - \Omega_A(y)\} \\
&= \max\{\overline{\Omega}_A(x), \overline{\Omega}_A(y)\}.
\end{aligned}$$

Hence $\overline{A}$ is an intuitionistic fuzzy bi-ideal of S.

Conversely suppose that $\overline{A}$ is an intuitionistic fuzzy bi-ideal of S. To show that $A = (\Psi_A, \Omega_A)$ is an intuitionistic anti fuzzy bi-ideal of S. Let $x, y \in S$. Then

$$\begin{aligned}
\Psi_A(xy) &= 1 - \overline{\Psi}_A(xy) \\
&\leq 1 - \min\{\overline{\Psi}_A(x), \overline{\Psi}_A(y)\} \\
&= \max\{1 - \overline{\Psi}_A(x), 1 - \overline{\Psi}_A(y)\} \\
&= \max\{\Psi_A(x), \Psi_A(y)\}
\end{aligned}$$

and

$$\begin{aligned}\Omega_A(xy) &= 1-\overline{\Omega}_A(xy)\\ &\geq 1-\max\{\overline{\Omega}_A(x),\overline{\Omega}_A(y)\}\\ &= \min\{1-\overline{\Omega}_A(x),1-\overline{\Omega}_A(y)\}\\ &= \min\{\Omega_A(x),\Omega_A(y)\}.\end{aligned}$$

Hence A is an intuitionistic anti fuzzy subsemigroup of S. Let $x, y, a \in S$. Then

$$\begin{aligned}\Psi_A(xay) &= 1-\overline{\Psi}_A(xay)\\ &\leq 1-\min\{\overline{\Psi}_A(x),\overline{\Psi}_A(y)\}\\ &= \max\{1-\overline{\Psi}_A(x),1-\overline{\Psi}_A(y)\}\\ &= \max\{\Psi_A(x),\Psi_A(y)\}\end{aligned}$$

and

$$\begin{aligned}\Omega_A(xay) &= 1-\overline{\Omega}_A(xay)\\ &\geq 1-\max\{\overline{\Omega}_A(x),\overline{\Omega}_A(y)\}\\ &= \min\{1-\overline{\Omega}_A(x),1-\overline{\Omega}_A(y)\}\\ &= \min\{\Omega_A(x),\Omega_A(y)\}.\end{aligned}$$

Hence A is an intuitionistic anti fuzzy bi-ideal of S. This completes the proof. □

Theorem 3.6. If an IFS $A = (\Psi_A, \Omega_A)$ in S is an intuitionistic anti fuzzy interior ideal of S, then so is $\Box A := (\Psi_A, \overline{\Psi}_A)$ where $\overline{\Psi}_A = 1 - \Psi_A$.

Proof. Since A is an intuitionistic anti fuzzy interior ideal of S then $\Psi_A(xy) \leq \max\{\Psi_A(x), \Psi_A(y)\}$ and $\Psi_A(xay) \leq \Psi_A(a) \forall x, y, a \in S$. Now

$$\begin{aligned}\overline{\Psi}_A(xay) &= 1-\Psi_A(xay)\\ &\geq 1-\max\{\Psi_A(x),\Psi_A(y)\}\\ &= \min\{1-\Psi_A(x),1-\Psi_A(y)\}\\ &= \min\{\overline{\Psi}_A(x),\overline{\Psi}_A(y)\}\end{aligned}$$

and $\overline{\Psi}_A(xay) = 1 - \Psi_A(xay) \geq 1 - \Psi_A(a) = \overline{\Psi}_A(a)$.

Hence A is an intuitionistic anti fuzzy interior ideal of S. This completes the proof. □

Theorem 3.7. An IFS $A = (\Psi_A, \Omega_A)$ in S is an intuitionistic anti fuzzy interior ideal of S if and only if the fuzzy sets Ψ_A and $\overline{\Omega}_A$ are anti fuzzy interior ideals of S.

Proof. Let $A = (\Psi_A, \Omega_A)$ be an intuitionistic anti fuzzy interior ideal of S. Then Ψ_A is an anti fuzzy interior ideal of S. Let $x, y, a \in S$. Then

$$\begin{aligned}\overline{\Omega}_A(xy) &= 1-\Omega_A(xy)\\ &\leq 1-\min\{\Omega_A(x),\Omega_A(y)\}\\ &= \max\{1-\Omega_A(x),1-\Omega_A(y)\}\\ &= \max\{\overline{\Omega}_A(x),\overline{\Omega}_A(y)\}.\end{aligned}$$

also $\overline{\Omega}_A(xay) = 1 - \Omega_A(xay) \leq 1 - \Omega_A(a) = \overline{\Omega}_A(a)$.

Hence $\overline{\Omega}_A$ is an anti fuzzy interior ideals of S.

Conversely, suppose that Ψ_A and $\overline{\Omega}_A$ are anti fuzzy interior ideals of S. Let $x, y, a \in S$. Then we have to show that $A = (\Psi_A, \Omega_A)$ intuitionistic anti fuzzy interior ideal of S. Since Ψ_A is an anti fuzzy interior ideal then $\Psi_A(xy) \leq \max\{\Psi_A(x), \Psi_A(y)\}$ and $\Psi_A(xay) \leq \Psi_A(a)$. Now

$$\begin{aligned}
1 - \Omega_A(xy) &= \overline{\Omega}_A(xy) \\
&\leq \max\{\overline{\Omega}_A(x), \overline{\Omega}_A(y)\} \\
&= \max\{1 - \Omega_A(x), 1 - \Omega_A(y)\} \\
&= 1 - \min\{\Omega_A(x), \Omega_A(y)\} \\
\Rightarrow 1 - \Omega_A(xy) &\leq 1 - \min\{\Omega_A(x), \Omega_A(y)\} \\
\Rightarrow \Omega_A(xy) &\geq \min\{\Omega_A(x), \Omega_A(y)\}
\end{aligned}$$

Also $1 - \Omega_A(xay) = \overline{\Omega}_A(xay) \leq \overline{\Omega}_A(a) = 1 - \Omega_A(a)$. This implies $\Omega_A(xay) \geq \Omega_A(a)$. Hence $A = (\Psi_A, \Omega_A)$ is an intuitionistic anti fuzzy interior ideals of S. This completes the proof. □

Theorem 3.8. Let $\{A_i : i \in I\}$ be a collection of intuitionistic anti fuzzy bi-ideals(intuitonistic anti fuzzy (1,2)-ideals, intuitionistic anti fuzzy interior ideals) of a semigroup S then their union $\bigcup_{i\in I} A_i = (\bigvee_{i\in I} \Psi_{A_i}, \bigwedge_{i\in I} \Omega_{A_i})$ is an intuitionistic anti fuzzy bi-ideal(intuitionistic anti fuzzy $(1,2)$-ideal, intuitionistic anti fuzzy interior ideal) of S, where

$\bigvee_{i\in I} \Psi_{A_i}(x) = \sup\{\Psi_{A_i}(x) : i \in I, x \in S\}$ and $\bigwedge_{i\in I} \Omega_{A_i}(x) = \inf\{\Omega_{A_i}(x) : i \in I, x \in S\}$.

Proof. As $\bigcup_{i\in I} A_i = (\bigvee_{i\in I} \Psi_{A_i}, \bigwedge_{i\in I} \Omega_{A_i})$, where $\bigvee_{i\in I} \Psi_{A_i}(x) = \sup\{\Psi_{A_i}(x) : i \in I, x \in S\}$ and $\bigwedge_{i\in I} \Omega_{A_i}(x) = \inf\{\Omega_{A_i}(x) : i \in I, x \in S\}$. For $x, y \in S$, we have

$$\begin{aligned}
(\bigvee_{i\in I} \Psi_{A_i})(xy) &= \sup\{\Psi_{A_i}(xy) : i \in I, xy \in S\} \\
&\leq \sup\{\max\{\Psi_{A_i}(x), \Psi_{A_i}(y)\} : i \in I, x, y \in S\} \\
&\leq \max\{\sup\{\Psi_{A_i}(x) : i \in I, x \in S\}, \sup\{\Psi_{A_i}(y) : i \in I, y \in S\}\} \\
&= \max\{\bigvee_{i\in I} \Psi_{A_i}(x), \bigvee_{i\in I} \Psi_{A_i}(y)\} \\
(\bigvee_{i\in I} \Psi_{A_i})(xy) &\leq \max\{\bigvee_{i\in I} \Psi_{A_i}(x), \bigvee_{i\in I} \Psi_{A_i}(y)\}
\end{aligned}$$

and

$$\begin{aligned}
(\bigwedge_{i\in I} \Omega_{A_i})(xy) &= \inf\{\Omega_{A_i}(xy) : i \in I, xy \in S\} \\
&\geq \inf\{\min\{\Omega_{A_i}(x), \Omega_{A_i}(y)\} : i \in I, x, y \in S\} \\
&\geq \min\{\inf\{\Omega_{A_i}(x) : i \in I, x \in S\}, \inf\{\Omega_{A_i}(y) : i \in I, y \in S\}\} \\
&= \min\{\bigwedge_{i\in I} \Omega_{A_i}(x), \bigwedge_{i\in I} \Omega_{A_i}(y)\} \\
(\bigvee_{i\in I} \Omega_{A_i})(xy) &\geq \min\{\bigwedge_{i\in I} \Omega_{A_i}(x), \bigwedge_{i\in I} \Omega_{A_i}(y)\}
\end{aligned}$$

Hence $\bigcup_{i\in I} A_i$ is an intuitionistic anti fuzzy subsemigroup of S. Also for any $x, y, a \in S$

$$\begin{aligned}(\bigvee_{i\in I}\Psi_{A_i})(xay) &= \sup\{\Psi_{A_i}(xay) : i\in I, xay\in S\}\\ &\leq \sup\{\max\{\Psi_{A_i}(x), \Psi_{A_i}(y)\} : i\in I, x, y\in S\}\\ &\leq \max\{\sup\{\Psi_{A_i}(x) : i\in I, x\in S\}, \sup\{\Psi_{A_i}(y) : i\in I, y\in S\}\}\\ &= \max\{\bigvee_{i\in I}\Psi_{A_i}(x), \bigvee_{i\in I}\Psi_{A_i}(y)\}\end{aligned}$$

and

$$\begin{aligned}(\bigwedge_{i\in I}\Omega_{A_i})(xay) &= \inf\{\Omega_{A_i}(xay) : i\in I, xay\in S\}\\ &\geq \inf\{\min\{\Omega_{A_i}(x), \Omega_{A_i}(y)\} : i\in I, x, y\in S\}\\ &\geq \min\{\inf\{\Omega_{A_i}(x) : i\in I, x\in S\}, \inf\{\Omega_{A_i}(y) : i\in I, y\in S\}\}\\ &= \min\{\bigvee_{i\in I}\Omega_{A_i}(x), \bigvee_{i\in I}\Omega_{A_i}(y)\}.\end{aligned}$$

Hence $\bigcup_{i\in I} A_i$ is an intuitionistic anti fuzzy bi-ideal of S. Similarly we can prove the other cases also. This completes the proof. □

Definition 3.9. For any $t \in [0,1]$ and a fuzzy subset Ψ of a semigroup S, the set $U(\Psi;t) = \{x \in S : \Psi(x) \geq t\}$(resp. $L(\Psi;t) = \{x \in S : \Psi(x) \leq t\}$) is called an upper(resp. lower) t-level cut of Ψ.

Theorem 3.10. If $A = (\Psi_A, \Omega_A)$ is an intuitionistic anti fuzzy bi-ideal of a semigroup S, then the upper and lower level cuts $L(\Psi_A;t)$ and $U(\Omega_A;t)$ are bi-ideals of S, for every $t \in Im(\Psi_A) \cap Im(\Omega_A)$.

Proof. Let $t \in Im(\Psi_A) \cap Im(\Omega_A)$. Let $A = (\Psi_A, \Omega_A)$ be an intuitionistic anti fuzzy bi-ideal of S. Then $A = (\Psi_A, \Omega_A)$ is an intuitionistic anti fuzzy subsemigroup of S. Let $x, y \in L(\Psi_A;t)$. Then $\Psi_A(x) \leq t$ and $\Psi_A(y) \leq t$ whence $\max\{\Psi_A(x), \Psi_A(y)\} \leq t$. Now $\Psi_A(xy) \leq \max\{\Psi_A(x), \Psi_A(y)\}$. Hence $\Psi_A(xy) \leq t, i.e., xy \in L(\Psi_A;t)$. Consequently, $L(\Psi_A;t)$ is a subsemigroup of S.

Now let $x, z \in L(\Psi_A;t)$. Then $\Psi_A(x) \leq t$ and $\Psi_A(z) \leq t$ whence $\max\{\Psi_A(x), \Psi_A(z)\} \leq t$. Now for $y \in S$, $\Psi_A(xyz) \leq \max\{\Psi_A(x), \Psi_A(z)\}$. Hence $\Psi_A(xyz) \leq t$ whence $xyz \in L(\Psi_A;t)$. Consequently, $L(\Psi_A;t)$ is a bi-ideal of S. Similarly we can prove the other case also. □

Theorem 3.11. If $A = (\Psi_A, \Omega_A)$ is an intuitionistic fuzzy subset of S such that the non-empty sets $L(\Psi_A;t)$ and $U(\Omega_A;t)$ are bi-ideals of S, for $t \in [0,1]$, then $A = (\Psi_A, \Omega_A)$ is an intuitionistic anti fuzzy bi-ideal of S.

Proof. Let $L(\Psi_A;t)$ and $U(\Omega_A;t)$ be bi-ideals of S, for all $t \in [0,1]$. Then $L(\Psi_A;t)$ and $U(\Omega_A;t)$ are subsemigroups of S. Let $x, y \in S$. Let $\Psi_A(x) = t_1$ and $\Psi_A(y) =$

t_2. Without any loss of generality suppose $t_1 \geq t_2$. Then $x, y \in L(\Psi_A; t_1)$. Then by hypothesis $xy \in L(\Psi_A; t_1)$. Hence $\Psi_A(xy) \leq t_1 = \max\{\Psi_A(x), \Psi_A(y)\}$. Now let $\Omega_A(x) = t_3$ and $\Omega_A(y) = t_4$. Without any loss of generality suppose $t_3 \geq t_4$. Then $x, y \in U(\Omega_A; t_4)$. Then by hypothesis $xy \in U(\Omega_A; t_4)$. Hence $\Omega_A(xy) \geq t_4 = \min\{\Omega_A(x), \Omega_A(y)\}$. Hence $A = (\Psi_A, \Omega_A)$ is an intuitionistic anti fuzzy subsemigroup of S.

Now let $x, y, z \in S$. Let $\Psi_A(x) = t_1, \Psi_A(z) = t_2$. Without any loss of generality suppose $t_1 \geq t_2$. Then $x, z \in L(\Psi_A; t_1)$. Hence by hypothesis $xyz \in L(\Psi_A; t_1)$ whence $\Psi_A(xyz) \leq t_1 = \max\{\Psi_A(x), \Psi_A(z)\}$. Again let $\Omega_A(x) = t_3, \Omega_A(z) = t_4$. Without any loss of generality suppose $t_3 \geq t_4$. Then $x, z \in U(\Omega_A; t_4)$. Hence by hypothesis $xyz \in U(\Omega_A; t_4)$ whence $\Omega_A(xyz) \geq t_4 = \min\{\Omega_A(x), \Omega_A(z)\}$. Hence $A = (\Psi_A, \Omega_A)$ is an intuitionistic anti fuzzy bi-ideal of S. This completes the proof. □

Theorem 3.12. Let I be a non-empty subset of a semigroup S. If two fuzzy subsets Ψ and Ω are defined on S by

$$\Psi(x) := \begin{cases} \alpha_0 & \text{if } x \in I \\ \alpha_1 & \text{if } x \in S - I \end{cases}$$

and

$$\Omega(x) := \begin{cases} \beta_0 & \text{if } x \in I \\ \beta_1 & \text{if } x \in S - I \end{cases}$$

where $0 \leq \alpha_0 < \alpha_1, 0 \leq \beta_1 < \beta_0$ and $\alpha_i + \beta_i \leq 1$ for $i = 0, 1$. Then $A = (\Psi, \Omega)$ is an intuitionistic anti fuzzy bi-ideal of S and $U(\Omega; \alpha_0) = I = L(\Psi; \beta_0)$.

Proof. Let I be a bi-ideal of S. Then I is a subsemigroup of S. Let $x, y \in S$. Then following four cases may arise:

(1) : If $x \in I$ and $y \in I$, (2) : If $x \notin I$ and $y \in I$, (3) : If $x \in I$ and $y \notin I$, (4) : If $x \notin I$ and $y \notin I$.

In Case 4, $\Psi(x) = \Psi(y) = \alpha_1$ and $\Omega(x) = \Omega(y) = \beta_1$. Then $\max\{\Psi(x), \Psi(y)\} = \alpha_1$ and $\min\{\Omega(x), \Omega(y)\} = \beta_1$. Now $\Psi(xy) = \alpha_0$ and α_1 or $\Omega(xy) = \beta_0$ and β_1 according as $xy \in I$ or $xy \notin I$. Again $\alpha_0 < \alpha_1$ and $\beta_1 < \beta_0$. Hence we see that $\Psi(xy) \leq \max\{\Psi(x), \Psi(y)\}$ and $\Omega(xy) \geq \min\{\Omega(x), \Omega(y)\}$. Hence $A = (\Psi, \Omega)$ is an intuitionistic anti fuzzy subsemigroup of S. For other cases, by using a similar argument we can deduce that $A = (\Psi, \Omega)$ is an intuitionistic anti fuzzy subsemigroup of S.

Now, let $x, y, z \in S$. Then following four cases may arise:

(1) : If $x \in I$ and $z \in I$, (2) : If $x \notin I$ and $z \in I$, (3) : If $x \in I$ and $z \notin I$, (4) : If $x \notin I$ and $z \notin I$.

In Case 4, $\Psi(x) = \Psi(z) = \alpha_1$ and $\Omega(x) = \Omega(z) = \beta_1$. Then $\max\{\Psi(x), \Psi(z)\} = \alpha_1$ and $\min\{\Omega(x), \Omega(z)\} = \beta_1$. Now $\Psi(xyz) = \alpha_0$ and α_1 or $\Omega(xyz) = \beta_0$ and β_1 according as $xyz \in I$ or $xyz \notin I$. Again $\alpha_0 < \alpha_1$ and $\beta_1 < \beta_0$. Hence we see that $\Psi(xyz) \leq \max\{\Psi(x), \Psi(z)\}$ and $\Omega(xyz) \geq \min\{\Omega(x), \Omega(z)\}$. Hence $A = (\Psi, \Omega)$ is an intuitionistic anti fuzzy bi-ideal of S. For other cases, by using a similar argument we can deduce that $A = (\Psi, \Omega)$ is an intuitionistic anti fuzzy bi-ideal of S.

In order to prove the converse, we first observe that by definition of Ψ and Ω, $U(\Omega; \alpha_0) = I = L(\Psi; \beta_0)$. Then the proof follows from Theorem 3.11. □

Definition 3.13. Let S be a semigroup. Let $A = (\Psi_A, \Omega_A)$ and $B = (\Psi_B, \Omega_B)$ be two intuitionistic fuzzy subsets of S. Then the anti intuitionistic fuzzy product $A \circ B$ of A and B is defined as

$$(\Psi_{A\circ B})(x) = \begin{cases} \inf\limits_{x=uv}[\max\{\Psi_A(u), \Psi_B(v)\} : u, v \in S] \\ 1, \text{ if for any } u, v \in S, x \neq uv \end{cases}$$

and

$$(\Omega_{A\circ B})(x) = \begin{cases} \sup\limits_{x=uv}[\min\{\Omega_A(u), \Omega_B(v)\} : u, v \in S] \\ 0, \text{ if for any } u, v \in S, x \neq uv \end{cases}$$

Theorem 3.14. A non-empty intuitionistic fuzzy subset $A = (\Psi_A, \Omega_A)$ of a semigroup S is an intuitionistic anti fuzzy subsemigroup of S if and only if $A \subseteq A \circ A$.

Proof. Let $A \subseteq A \circ A$. Then for $x, y \in S$ we obtain

$$\Psi_A(xy) \leq \Psi_{A\circ A}(xy) \leq \max\{\Psi_A(x), \Psi_A(y)\}$$

and

$$\Omega_A(xy) \geq \Omega_{A\circ A}(xy) \geq \min\{\Omega_A(x), \Omega_A(y)\}.$$

So $A = (\Psi_A, \Omega_A)$ is an intuitionistic anti fuzzy subsemigroup of S.

Conversely, let $A = (\Psi_A, \Omega_A)$ be an intuitionistic anti fuzzy subsemigroup of S and $x \in S$. Suppose there exist $y, z \in S$ such that $x = yz$ then $\Psi_{A\circ A}(x) \neq 1$ and $\Omega_{A\circ A}(x) \neq 0$. By hypothesis, $\Psi_A(yz) \leq \max\{\Psi_A(y), \Psi_A(z)\}$ and $\Omega_A(yz) \geq \min\{\Omega_A(y), \Omega_A(z)\}$. Hence

$$\Psi_A(yz) \leq \inf_{x=yz} \max\{\Psi_A(y), \Psi_A(z)\} = \Psi_{A\circ A}(x)$$

and

$$\Omega_A(yz) \geq \sup_{x=yz} \min\{\Omega_A(y), \Omega_A(z)\} = \Omega_{A\circ A}(x).$$

Again if there does not exist $y, z \in S$ such that $x = yz$ then $\Psi_{A\circ A}(x) = 1 \geq \Psi_A(x)$ and $\Omega_{A\circ A}(x) = 0 \leq \Omega_A(x)$. Consequently, $\Psi_A \subseteq \Psi_{A\circ A}$ and $\Omega_A \supseteq \Omega_{A\circ A}$. Hence $A \subseteq A \circ A$. This completes the proof. □

Theorem 3.15. In a semigroup S for a non-empty intuitionistic fuzzy subset $A = (\Psi_A, \Omega_A)$ of S the following are equivalent: (1) $A = (\Psi_A, \Omega_A)$ is an intuitionistic anti fuzzy bi-ideal of S, (2) $A \subseteq A \circ A$ and $A \subseteq A \circ S \circ A$, where $S = (\Psi_S, \Omega_S)$ and Ψ_S is the characteristic function of S.

Proof. Suppose (1) holds, *i.e.*, $A = (\Psi_A, \Omega_A)$ is an intuitionistic anti fuzzy bi-ideal of S. Then $A = (\Psi_A, \Omega_A)$ is an intuitionistic anti fuzzy subsemigroup of S. So by Theorem 3.14, $A \subseteq A \circ A$. Let $a \in S$. Suppose there exists $x, y, p, q \in S$ such that $a = xy$ and $x = pq$. Since $A = (\Psi_A, \Omega_A)$ is an intuitionistic anti fuzzy bi-ideal of S, we obtain $\Psi_A(pqy) \leq \max\{\Psi_A(p), \Psi_A(y)\}$ and $\Omega_A(pqy) \geq \min\{\Omega_A(p), \Omega_A(y)\}$. Then

$$\begin{aligned}
\Psi_{A\circ S\circ A}(a) &= \inf_{a=xy}[\max\{\Psi_{A\circ S}(x), \Psi_A(y)\}] \\
&= \inf_{a=xy}[\max\{\inf_{x=pq}\{\max\{\Psi_A(p), \Psi_S(q)\}\}, \Psi_A(y)\}] \\
&= \inf_{a=xy}[\max\{\inf_{x=pq}\{\max\{\Psi_A(p), 0\}\}, \Psi_A(y)\}] \\
&= \inf_{a=xy}[\max\{\Psi_A(p), \Psi_A(y)\}] \\
&\geq \Psi_A(pqy) = \Psi_A(xy) = \Psi_A(a).
\end{aligned}$$

So we have $\Psi_A \subseteq \Psi_{A\circ S\circ A}$. Otherwise $\Psi_{A\circ S\circ A}(a) = 1 \geq \Psi_A(a)$. Thus $\Psi_A \subseteq \Psi_{A\circ S\circ A}$. Now

$$\begin{aligned}
\Omega_{A\circ S\circ A}(a) &= \sup_{a=xy}[\min\{\Omega_{A\circ S}(x), \Omega_A(y)\}] \\
&= \sup_{a=xy}[\min\{\sup_{x=pq}\{\min\{\Omega_A(p), \Omega_S(q)\}\}, \Omega_A(y)\}] \\
&= \sup_{a=xy}[\min\{\sup_{x=pq}\{\min\{\Omega_A(p), 1\}\}, \Omega_A(y)\}] \\
&= \sup_{a=xy}[\min\{\Omega_A(p), \Omega_A(y)\}] \\
&\leq \Omega_A(pqy) = \Omega_A(xy) = \Omega_A(a).
\end{aligned}$$

So we have $\Omega_A \supseteq \Omega_{A\circ S\circ A}$. Otherwise $\Omega_{A\circ S\circ A}(a) = 0 \leq \Omega_A(a)$. Thus $\Omega_A \supseteq \Omega_{A\circ S\circ A}$. Hence $A \subseteq A \circ S \circ A$.

Conversely, let us assume that (2) holds. Since $\Psi_A \subseteq \Psi_{A\circ A}$ and $\Omega_A \supseteq \Omega_{A\circ A}$ so $A = (\Psi_A, \Omega_A)$ is an intuitionistic anti fuzzy subsemigroup of S. Let $x, y, z \in S$ and $a = xyz$. Since $\Psi_A \subseteq \Psi_{A\circ S\circ A}$ and $\Omega_A \supseteq \Omega_{A\circ S\circ A}$, we have

$$\begin{aligned}
\Psi_A(xyz) = \Psi_A(a) &\leq \Psi_{A\circ S\circ A}(a) \\
&= \inf_{a=xyz}[\max\{\Psi_{A\circ S}(xy), \Psi_A(z)\}] \\
&\leq \max\{\Psi_{A\circ S}(p), \Psi_A(z)\}(\text{let } p = xy) \\
&= \max[\inf_{p=xy}\{\max\{\Psi_A(x), \Psi_S(y)\}\}, \Psi_A(z)] \\
&\leq \max[\max\{\Psi_A(x), 0\}, \Psi_A(z)] \\
&= \max\{\Psi_A(x), \Psi_A(z)\}
\end{aligned}$$

and

$$\begin{aligned}
\Omega_A(xyz) = \Omega_A(a) &\geq \Omega_{A\circ S\circ A}(a) \\
&= \sup_{a=xyz}[\min\{\Omega_{A\circ S}(xy), \Omega_A(z)\}] \\
&\geq \min\{\Omega_{A\circ S}(p), \Omega_A(z)\}(\text{let } p = xy) \\
&= \min[\sup_{p=xy}\{\min\{\Omega_A(x), \Omega_S(y)\}\}, \Omega_A(z)] \\
&\geq \min[\min\{\Omega_A(x), 1\}, \Omega_A(z)] \\
&= \min\{\Omega_A(x), \Omega_A(z)\}
\end{aligned}$$

Hence $A = (\Psi_A, \Omega_A)$ is an intuitionistic anti fuzzy bi-ideal of S. This completes the proof. □

Theorem 3.16. Every intuitionistic anti fuzzy bi-ideal is an intuitionistic anti fuzzy $(1, 2)$-ideal.

Proof. Let $A = (\Psi_A, \Omega_A)$ be an intuitionistic anti fuzzy bi-ideal of a semigroup S and let $x, y, z, a \in S$. Then

$$\begin{aligned}\Psi_A(xa(yz)) &= \Psi_A((xay)z)) \\ &\leq \max\{\Psi_A(xay), \Psi_A(z)\} \\ &\leq \max\{\max\{\Psi_A(x), \Psi_A(y)\}, \Psi_A(z)\} \\ &= \max\{\Psi_A(x), \Psi_A(y), \Psi_A(z)\}\end{aligned}$$

and

$$\begin{aligned}\Omega_A(xa(yz)) &= \Omega_A((xay)z)) \\ &\geq \min\{\Omega_A(xay), \Omega_A(z)\} \\ &\geq \min\{\min\{\Omega_A(x), \Omega_A(y)\}, \Omega_A(z)\} \\ &= \min\{\Omega_A(x), \Omega_A(y), \Omega_A(z)\}.\end{aligned}$$

Hence $A = (\Psi_A, \Omega_A)$ is an intuitionistic anti fuzzy (1,2)-ideal of S. □

For the converse of Theorem 3.16 we need to strengthen the condition of a semigroup S.

Theorem 3.17. If S is a regular semigroup then every intuitionistic anti fuzzy $(1,2)$-ideal of S is an intuitionistic anti fuzzy bi-ideal of S.

Proof. Let us assume that the semigroup S is regular and let $A = (\Psi_A, \Omega_A)$ be an intuitionistic anti fuzzy (1,2)-ideal of S. Let $x, y, a \in S$. Since S is regular, we have $xa \in (xsx)s \subseteq xsx$, which implies that $xa = xsx$ for some $s \in S$.

$$\begin{aligned}\Psi_A(xay) &= \Psi_A((xa)y) \\ &= \Psi_A((xsx)y) \\ &= \Psi_A(xs(xy)) \\ &\leq \max\{\Psi_A(x), \Psi_A(x), \Psi_A(y)\} \\ &= \max\{\Psi_A(x), \Psi_A(y)\}\end{aligned}$$

and

$$\begin{aligned}\Omega_A(xay) &= \Omega_A((xa)y)) \\ &= \Omega_A((xsx)y)) \\ &= \Omega_A(xs(xy)) \\ &\geq \min\{\Omega_A(x), \Omega_A(x), \Omega_A(y)\} \\ &= \min\{\Omega_A(x), \Omega_A(y)\}.\end{aligned}$$

Hence $A = (\Psi_A, \Omega_A)$ is an intuitionistic anti fuzzy bi–ideal of S. □

Theorem 3.18. Every intuitionistic anti fuzzy bi-ideal of a group S is constant.

Proof. Let $A = (\Psi_A, \Omega_A)$ be an intuitionistic anti fuzzy bi-ideal of a group S and e be the identity of S. Let $x, y \in S$. Then

$$\begin{aligned}\Psi_A(x) &= \Psi_A(exe) \\ &\leq \max\{\Psi_A(e), \Psi_A(e)\} = \Psi_A(e) \\ &= \Psi_A(ee) \\ &= \Psi_A((xx^{-1})(x^{-1}x)) \\ &= \Psi_A(x(x^{-1}x^{-1})x) \\ &\leq \max\{\Psi_A(x), \Psi_A(x)\} = \Psi_A(x)\end{aligned}$$

and

$$\begin{aligned}
\Omega_A(x) &= \Omega_A(exe) \\
&\geq \min\{\Omega_A(e), \Omega_A(e)\} = \Omega_A(e) \\
&= \Omega_A(ee) \\
&= \Omega_A((xx^{-1})(x^{-1}x)) \\
&= \Omega_A(x(x^{-1}x^{-1})x) \\
&\geq \min\{\Omega_A(x), \Omega_A(x)\} = \Omega_A(x).
\end{aligned}$$

It follows that $\Psi_A(x) = \Psi_A(e)$ and $\Omega_A(x) = \Omega_A(e)$ which means that $A = (\Psi_A, \Omega_A)$ is constant. This completes the proof. □

Theorem 3.19. Let $f : S \to T$ be an homomorphism of semigroups. If $B = (\Psi_B, \Omega_B)$ is an intuitionistic anti fuzzy bi-ideal(intuitionistic anti fuzzy $(1, 2)$-ideal) of T, then the pre-image $f^{-1}(B) = (f^{-1}(\Psi_B), f^{-1}(\Omega_B))$ of B under f is an intuitionistic anti fuzzy bi-ideal(resp. intuitionistic anti fuzzy $(1, 2)$-ideal) of S.

Proof. Let $B = (\Psi_B, \Omega_B)$ be an intuitionistic anti fuzzy bi-ideal of T and let $x, y \in S$. Then

$$\begin{aligned}
f^{-1}(\Psi_B(xy)) &= \Psi_B(f(xy)) \\
&= \Psi_B(f(x)f(y)) \\
&\leq \max\{\Psi_B(f(x)), \Psi_B(f(y))\} \\
&= \max\{f^{-1}(\Psi_B(x)), f^{-1}(\Psi_B(y))\}
\end{aligned}$$

and

$$\begin{aligned}
f^{-1}(\Omega_B(xy)) &= \Omega_B(f(xy)) \\
&= \Omega_B(f(x)f(y)) \\
&\geq \min\{\Omega_B(f(x)), \Omega_B(f(y))\} \\
&= \max\{f^{-1}(\Omega_B(x)), f^{-1}(\Omega_B(y))\}.
\end{aligned}$$

Hence $f^{-1}(B) = (f^{-1}(\Psi_B), f^{-1}(\Omega_B))$ is an intuitionistic anti fuzzy subsemigroup of S. For any $a, x, y \in S$ we have

$$\begin{aligned}
f^{-1}(\Psi_B(xay)) &= \Psi_B(f(xay)) \\
&= \Psi_B(f(x)f(a)f(y)) \\
&\leq \max\{\Psi_B(f(x)), \Psi_B(f(y))\} \\
&= \max\{f^{-1}(\Psi_B(x)), f^{-1}(\Psi_B(y))\}
\end{aligned}$$

and

$$\begin{aligned}
f^{-1}(\Omega_B(xay)) &= \Omega_B(f(xay)) \\
&= \Omega_B(f(x)f(a)f(y)) \\
&\geq \min\{\Omega_B(f(x)), \Omega_B(f(y))\} \\
&= \min\{f^{-1}(\Omega_B(x)), f^{-1}(\Omega_B(y))\}.
\end{aligned}$$

Therefore $f^{-1}(B) = (f^{-1}(\Psi_B), f^{-1}(\Omega_B))$ is an intuitionistic anti fuzzy bi-ideal of S. Similarly we can prove the other case also. This completes the proof. □

Proposition 3.20. Let θ be an endomorphism and $A = (\Psi_A, \Omega_A)$ be an intuitionistic anti fuzzy subsemigroup(intuitionistic anti fuzzy bi-ideal, intuitionistic anti fuzzy $(1, 2)$-ideal) of a semigroup S. Then $A[\theta]$ is also an intuitionistic anti fuzzy subsemigroup(resp. intuitionistic anti fuzzy bi-ideal, intuitionistic anti fuzzy $(1, 2)$-ideal) of S, where $A[\theta](x) := (\Psi_A[\theta](x), \Omega_A[\theta](x)) = (\Psi_A(\theta(x)), \Omega_A(\theta(x)))\forall x \in S$.

Proof. Let $x, y \in S$. Then

$$\begin{aligned}\Psi_A[\theta](xy) &= \Psi_A(\theta(xy)) = \Psi_A(\theta(x)\theta(y)) \\ &\leq \max\{\Psi_A(\theta(x)), \Psi_A(\theta(y))\} \\ &= \max\{\Psi_A[\theta](x), \Psi_A[\theta](y)\}\end{aligned}$$

and

$$\begin{aligned}\Omega_A[\theta](xy) &= \Omega_A(\theta(xy)) = \Omega_A(\theta(x)\theta(y)) \\ &\geq \min\{\Omega_A(\theta(x)), \Omega_A(\theta(y))\} \\ &= \min\{\Omega_A[\theta](x), \Omega_A[\theta](y)\}.\end{aligned}$$

Hence $A[\theta]$ is an intuitionistic anti fuzzy subsemigroup of S. Similarly we can prove other cases also. This completes the proof. □

Proposition 3.21. Let $\alpha \geq 0$ be a real number and $A = (\Psi_A, \Omega_A)$ be an intuitionistic anti fuzzy subsemigroup(intuitionistic anti fuzzy bi-ideal, intuitionistic anti fuzzy $(1, 2)$-ideal) of a semigroup S. Then so is $A^\alpha = (\Psi_A^\alpha, \Omega_A^\alpha)$, where $\Psi_A^\alpha(x) = (\Psi_A(x))^\alpha$ and $\Omega_A^\alpha(x) = (\Omega_A(x))^\alpha$ for all $x \in S$.

Proof. Let $x, y \in S$. Without any loss of generality, suppose $\Psi_A(x) \geq \Psi_A(y)$ and $\Omega_A(x) \leq \Omega_A(y)$. Then $\Psi_A^\alpha(x) \geq \Psi_A^\alpha(y)$ and $\Omega_A^\alpha(x) \leq \Omega_A^\alpha(y)$. Now $\Psi_A(xy) \leq \max\{\Psi_A(x), \Psi_A(y)\} = \Psi_A(x)$ and $\Omega_A(xy) \geq \min\{\Omega_A(x), \Omega_A(y)\} = \Omega_A(x)$. Then

$$\Psi_A^\alpha(xy) = (\Psi_A(xy))^\alpha \leq (\Psi_A(x))^\alpha = \Psi_A^\alpha(x) = \max\{\Psi_A^\alpha(x), \Psi_A^\alpha(y)\}$$

and

$$\Omega_A^\alpha(xy) = (\Omega_A(xy))^\alpha \geq (\Omega_A(x))^\alpha = \Omega_A^\alpha(x) = \min\{\Omega_A^\alpha(x), \Omega_A^\alpha(y)\}.$$

Consequently, $A^\alpha = (\Psi_A^\alpha, \Omega_A^\alpha)$ is an intuitionistic anti fuzzy subsemigroup of S. Similarly we can prove other cases also. This completes the proof. □

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