http://www.newtheory.org ISSN: 2149-1402

Journal of New Theory

Received: 04.04.2016 *Year*: **2016**, *Number*: **12**, *Pages*: **85-94**
Published: 16.04.2016 *Original Article*[**]

BRIEF DISCUSSION ON NEUTROSOPHIC h-IDEALS OF Γ-HEMIRINGS

Debabrata Mandal[*] <dmandaljumath@gmail.com>

Department of Mathematics, Raja Peary Mohan College, Uttarpara, Hooghly-712258, India

Abstaract − The concept of neutrosophic h-bi-ideals and neutrosophic h-quasi-ideals of a Γ-hemiring are introduced and some of their related properties are investigated. The notions of h-hemiregularity, h-intra-hemiregularity of a Γ-hemiring are studied and some of their characterizations in terms of neutrosophic h-ideals are also obtained.

Keywords − *Γ-hemiring, neutrosophic h-ideal, neutrosophic h-bi-ideal, neutrosophic h-quasi-ideal, h-hemiregular, h-intra-hemiregular.*

1 Introduction

Semiring is a well known universal algebra. This is a generalization of an associative ring $(R, +, .)$. If $(R, +)$ becomes a semigroup instead of a group then $(R, +, .)$ reduces to a semiring. Semiring has been found very useful for solving problems in different areas of applied mathematics and information sciences, since the structure of a semiring provides an algebraic framework for modelling and studying the key factors in these applied areas. Ideals of semiring play a central role in the structure theory and useful for many purposes. However they do not in general coincide with the usual ring ideals and for this reason, their use is somewhat limited in trying to obtain analogues of ring theorems for semiring. To ammend this gap Henriksen [12] defined a more restricted class of ideals, which are called k-ideals. A still more restricted class of ideals in hemirings are given by Iizuka [14], which are called h-ideals. LaTorre [18], investigated h-ideals and k-ideals in hemirings in an effort to obtain analogues of ring Results for hemiring and to amend the gap between ring ideals and semiring ideals. The theory of Γ-semiring was introduced by Rao [24]. These concepts are extended by Dutta and Sardar [10].

The theory of fuzzy sets, proposed by Zadeh [29], has provided a useful mathematical tool for describing the behavior of the systems that are too complex or illdefined to admit precise mathematical analysis by classical methods and tools.

[**] *Edited by* Said Broumi *(Area Editor) and* Naim Çağman *(Editor-in-Chief).*
[*] *Corresponding Author.*

The study of fuzzy algebraic structure has started by Rosenfeld [26]. Since then many researchers developed this ideas.

As a generalization of fuzzy sets, the intuitionistic fuzzy set was introduced by Atanassov [1] in 1986, where besides the degree of membership of each element there was considered a degree of non-membership with (membership value + non-membership value) ≤ 1.

There are also several well-known theories, for instances, rough sets, vague sets, interval-valued sets etc. which can be considered as mathematical tools for dealing with uncertainties. In 1995, inspired from the sport games (winning/tie/ defeating), votes, from (yes/NA/no), from decision making(making a decision/ hesitating/not making), from (accepted/pending/rejected) etc. and guided by the fact that the law of excluded middle did not work any longer in the modern logics, F. Smarandache [23] combined the non-standard analysis [8, 25] with a tri-component logic/set/probability theory and with philosophy and introduced *Neutrosophic set* which represents the main distinction between *fuzzy* and *intuitionistic fuzzy* logic/set. Here he included the middle component. i.e. the neutral/ indeterminate/ unknown part (besides the truth/membership and falsehood/non-membership components that both appear in fuzzy logic/set) to distinguish between 'absolute membership and relative membership' or 'absolute non-membership and relative non-membership'(see, [16, 27]). There are also several authors, for example [3, 4, 5, 6, 7] who have enriched the theory of neutrosophic sets.

Inspired from the above idea and motivated by the fact that 'semirings arise naturally in combinatorics, mathematical modelling, graph theory, automata theory, parallel computation system etc.', in the paper, we have used that to study the h-ideals, h-bi-ideals, h-quasi-ideals [13, 15, 19, 21, 22, 28, 30] of Γ-semirings [24] - a generalization of semirings [11] and obtain some of its characterizations.

2 Preliminaries

We recall the following preliminaries for subsequent use.

Definition 2.1. [11] A hemiring [respectively semiring] is a non-empty set S on which operations addition and multiplication have been defined such that the following conditions are satisfied:

(i) $(S, +)$ is a commutative monoid with identity 0.
(ii) $(S, .)$ is a semigroup [respectively monoid with identity 1_S].
(iii) Multiplication distributes over addition from either side.
(iv) $0s = 0 = s0$ for all $s \in S$.
(v) $1_S \neq 0$

Definition 2.2. [21] Let S and Γ be two additive commutative semigroups with zero. Then S is called a $\Gamma-$hemiring if there exists a mapping
$S \times \Gamma \times S \rightarrow S$ ((a,α,b) $\mapsto$ aαb) satisfying the following conditions:

(i) $(a + b)\alpha c = a\alpha c + b\alpha c$,
(ii) $a\alpha(b + c) = a\alpha b + a\alpha c$,
(iii) $a(\alpha + \beta)b = a\alpha b + a\beta b$,
(iv) $a\alpha(b\beta c) = (a\alpha b)\beta c$.

(v) $0_S\alpha a = 0_S = a\alpha 0_S$,
(vi)$a0_\Gamma b = 0_S = b0_\Gamma a$

for all $a, b, c \in S$ and for all $\alpha, \beta \in \Gamma$.

For simplification we write 0 instead of 0_S and 0_Γ.

Throughout this paper, unless otherwise mentioned S denotes a $\Gamma-$hemiring and χ_S be its characteristic function.

A subset A of a Γ-hemiring S is called a left(resp. right) ideal of S if A is closed under addition and $S\Gamma A \subseteq A$ (resp. $A\Gamma S \subseteq A$). A subset$A$ of a hemiring S is called an ideal if it is both left and right ideal of S

A subset A of a Γ-hemiring S is called a quasi-ideal of S if A is closed under addition and $S\Gamma A \cap A\Gamma S \subseteq A$.

A subset A of a Γ-hemiring S is called a bi-ideal(resp. interior ideal) if A is closed under addition and $A\Gamma S\Gamma A \subseteq A$(resp. $S\Gamma A\Gamma S \subseteq A$).

A left ideal A of S is called a left h-ideal if $x, z \in S$, $a, b \in A$ and $x + a + z = b + z$ implies $x \in A$. A right h-ideal is defined analoguesly.

The h-closure $\overline{A}$ of A in S is defined as $\overline{A} = \{x \in S \mid x + a + z = b + z,$ for some $a, b \in A$ and $z \in S\}$.

Now if A is a left (right) ideal of S, then $\overline{A}$ is the smallest left (right) h-ideal containing A.

A quasi-ideal(resp. bi-ideal) A of S is called an h-quasi-ideal(resp. h-bi-ideal) of S if $\overline{S\Gamma A} \cap \overline{A\Gamma S}$(resp. $\overline{A\Gamma S\Gamma A}$) $\subseteq A$ and $x + a + z = b + z$ implies $x \in A$ for all $x, z \in S$ and $a, b \in A$.

Definition 2.3. [29] A fuzzy subset of a nonempty set X is defined as a function $\mu : X \to [0, 1]$.

Definition 2.4. [23] A neutrosophic set A on the universe of discourse X is defined as $A = \{< x, A^T(x), A^I(x), A^F(x) >, x \in X\}$, where $A^T, A^I, A^F : X \to]^-0, 1^+[$ and $^-0 \leq A^T(x) + A^I(x) + A^F(x) \leq 3^+$. From philosophical point of view, the neutrosophic set takes the value from real standard or non-standard subsets of $]^-0, 1^+[$. But in real life application in scientific and engineering problems it is difficult to use neutrosophic set with value from real standard or non-standard subset of $]^-0, 1^+[$. Hence we consider the neutrosophic set which takes the value from the subset of [0, 1].

3 Neutrosophic h-ideals in Γ-hemiring

Using the above concepts, we now define neutrosophic left(right) h-ideal, neutrosophic h-bi-ideal, neutrosophic h-quasi-ideal and several operations such as composition, cartesian product, intersection etc. on them and use these to study some of their related properties. At the time of investigation we may see that the obtained results are parallel to that of Γ-hemiring and by routine verification we can proof them. So, after giving one introductory proof, I omit all the proof.

Definition 3.1. Let $\mu = (\mu^T, \mu^I, \mu^F)$ be a non empty neutrosophic subset of a Γ-semiring S (i.e. anyone of $\mu^T(x)$, $\mu^I(x)$ or $\mu^F(x)$ not equal to zero for some $x \in S$). Then μ is called a neutrosophic left ideal of S if

(i) $\mu^T(x+y) \geq \min\{\mu^T(x), \mu^T(y)\}$, $\mu^T(x\gamma y) \geq \mu^T(y)$

(ii) $\mu^I(x+y) \geq \frac{\mu^I(x)+\mu^I(y)}{2}$, $\mu^I(x\gamma y) \geq \mu^I(y)$

(iii) $\mu^F(x+y) \leq \max\{\mu^F(x), \mu^F(y)\}$, $\mu^F(x\gamma y) \leq \mu^F(y)$.

for all $x, y \in S$ and $\gamma \in \Gamma$.

A neutrosophic left ideal is called neutrosophic left h-ideal if for $x, a, b, z \in S$ with $x + a + z = b + z$ implies

(i) $\mu^T(x) \geq \min\{\mu^T(a), \mu^T(b)\}$,

(ii) $\mu^I(x) \geq \frac{\mu^I(a)+\mu^I(b)}{2}$,

(iii) $\mu^F(x) \leq \max\{\mu^F(a), \mu^F(b)\}$.

Similarly we can define neutrosophic right h-ideal of S.

Result 3.2. Intersection of a nonempty collection of neutrosophic left h-ideals is a neutrosophic left h-ideal of S.

Proof. Let $\{\mu_i : i \in I\}$ be a non-empty family of neutrosophic left h-ideals of S and $x, y \in S$ and $\gamma \in \Gamma$. Then

$$\begin{aligned}
(\bigcap_{i\in I}\mu_i^T)(x+y) &= \inf_{i\in I} \mu_i^T(x+y) \geq \inf_{i\in I} \{\min\{\mu_i^T(x), \mu_i^T(y)\}\} \\
&= \min\{\inf_{i\in I} \mu_i^T(x), \inf_{i\in I} \mu_i^T(y)\} \\
&= \min\{(\bigcap_{i\in I}\mu_i^T)(x), (\bigcap_{i\in I}\mu_i^T)(y)\}
\end{aligned}$$

$$\begin{aligned}
(\bigcap_{i\in I}\mu_i^I)(x+y) &= \inf_{i\in I} \mu_i^I(x+y) \geq \inf_{i\in I} \frac{\mu_i^I(x)+\mu_i^I(y)}{2} \\
&= \frac{\inf_{i\in I} \mu_i^I(x)+\inf_{i\in I} \mu_i^I(y)}{2} \\
&= \frac{\bigcap_{i\in I}\mu_i^I(x)+\bigcap_{i\in I}\mu_i^I(y)}{2}
\end{aligned}$$

$$\begin{aligned}
(\bigcap_{i\in I}\mu_i^F)(x+y) &= \sup_{i\in I} \mu_i^F(x+y) \leq \sup_{i\in I} \{\max\{\mu_i^F(x), \mu_i^F(y)\}\} \\
&= \max\{\sup_{i\in I} \mu_i^F(x), \sup_{i\in I} \mu_i^F(y)\} \\
&= \max\{(\bigcap_{i\in I}\mu_i^F)(x), (\bigcap_{i\in I}\mu_i^F)(y)\}
\end{aligned}$$

$$(\bigcap_{i\in I}\mu_i^T)(x\gamma y) = \inf_{i\in I} \mu_i^T(x\gamma y) \geq \inf_{i\in I} \mu_i^T(y) = (\bigcap_{i\in I}\mu_i^T)(y).$$

$$(\bigcap_{i\in I}\mu_i^I)(x\gamma y) = \inf_{i\in I} \mu_i^I(x\gamma y) \geq \inf_{i\in I} \mu_i^I(y) = (\bigcap_{i\in I}\mu_i^I)(y).$$

$$(\bigcap_{i\in I}\mu_i^F)(x\gamma y) = \sup_{i\in I} \mu_i^F(x\gamma y) \leq \sup_{i\in I} \mu_i^F(y) = (\bigcap_{i\in I}\mu_i^F)(y).$$

Hence $\bigcap_{i\in I}\mu_i$ is a neutrosophic left ideal of S.
Now suppose $x, a, b, z \in S$ with $x + a + z = b + z$. Then

$$\begin{aligned}
(\bigcap_{i\in I}\mu_i^T)(x) &= \inf_{i\in I} \mu_i^T(x) \geq \inf_{i\in I} \min\{\mu_i^T(a), \mu_i^T(b)\} \\
&= \min\{\inf_{i\in I} \mu_i^T(a), \inf_{i\in I} \mu_i^T(b)\} = \min\{(\bigcap_{i\in I}\mu_i^T)(a), (\bigcap_{i\in I}\mu_i^T)(a)\}.
\end{aligned}$$

$$(\bigcap_{i\in I}\mu_i^I)(x) = \inf_{i\in I} \mu_i^I(x) \geq \inf_{i\in I} \frac{\mu_i^I(y)+\mu_i^I(b)}{2} = \frac{\inf_{i\in I}\mu_i^I(y)+\inf_{i\in I}\mu_i^I(b)}{2} = \frac{\bigcap_{i\in I}\mu_i^I(y)+\bigcap_{i\in I}\mu_i^I(b)}{2}.$$

$$(\bigcap_{i\in I}\mu_i^F)(x) = \sup_{i\in I} \mu_i^F(x) \leq \sup_{i\in I} \max\{\mu_i^F(a), \mu_i^F(b\} = \max\{\sup_{i\in I} \mu_i^F(a), \sup_{i\in I} \mu_i^F(b)\} = \max\{(\bigcap_{i\in I}\mu_i^F)(a), (\bigcap_{i\in I}\mu_i^F)(a)\}.$$

Therefore $\bigcap_{i\in I}\mu_i$ is a neutrosophic left h-ideal of S. □

Definition 3.3. Let μ and θ be two neutrosophic sets of a Γ-hemiring S. Now h-product of μ and θ denoted by $\mu o_h \theta$ and defined as

$$\mu^T o_h \theta^T(x) = \sup_{x+\sum_{i=1}^n a_i\gamma_i b_i + z = \sum_{i=1}^n c_i\delta_i d_i + z}[\min_i\{\mu^T(a_i), \mu^T(c_i), \theta^T(b_i),\ \theta^T(d_i)\}\}]$$
$= 0$, if x cannot be expressed as above

$$\mu^I o_h \theta^I(x) = \sup_{x+\sum_{i=1}^n a_i\gamma_i b_i + z = \sum_{i=1}^n c_i\delta_i d_i + z}[\min_i\{\tfrac{1}{4}[\mu^I(a_i) + \mu^I(c_i) + \theta^I(b_i)+\theta^I(d_i)]\}\}]$$
$= 0$, if x cannot be expressed as above

$$\mu^F o_h \theta^F(x) = \inf_{x+\sum_{i=1}^n a_i\gamma_i b_i + z = \sum_{i=1}^n c_i\delta_i d_i + z}[\max_i\{[\mu^F(a_i), \mu^F(c_i), \theta^F(b_i),\ \theta^F(d_i)]\}\}]$$
$= 0$, if x cannot be expressed as above

where $x, z, a_i, b_i, c_i, d_i \in S$ and $\gamma_i, \delta_i \in \Gamma$, for $i = 1, ..., n$.

Result 3.4. If μ and ν be two neutrosophic left h-ideals of S then $\mu o \nu$ is also a neutrosophic left h-ideal of S.

Result 3.5. Let μ_1, μ_2 be two neutrosophic h-ideal of a Γ-hemiring S. Then $\mu_1 o_h \mu_2 \subseteq \mu_1 \cap \mu_2 \subseteq \mu_1, \mu_2$.

Result 3.6. Let S be a Γ-hemiring and $A, B \subseteq S$. Then we have

(i) $A \subseteq B$ if and only if $\chi_A \subseteq \chi_B$.
(ii) $\chi_A \cap \chi_B = \chi_{A\cap B}$
(iii) $\chi_A o_h \chi_B = \chi_{A\Gamma B}$

Definition 3.7. Let μ and ν be two neutrosophic subsets of S. The cartesian product of μ and ν is defined by

$$(\mu^T \times \nu^T)(x, y) = \min\{\mu^T(x), \nu^T(y)\}$$
$$(\mu^I \times \nu^I)(x, y) = \frac{\mu^I(x) + \nu^I(y)}{2}$$
$$(\mu^F \times \nu^F)(x, y) = \max\{\mu^F(x), \nu^F(y)\}$$

for all $x, y \in S$.

Result 3.8. Let μ and ν be two neutrosophic left h-ideals of S. Then $\mu \times \nu$ is a neutrosophic left h-ideal of $S \times S$.

Definition 3.9. A neutrosophic subset μ of a Γ-hemiring S is called neutrosophic h-bi-ideal if for all $x, y, z, a, b \in S$ and $\alpha, \beta \in \Gamma$ we have

(i) $\mu^T(x+y) \geq \min\{\mu^T(x), \mu^T(y)\}$

(ii) $\mu^T(x\alpha y) \geq \min\{\mu^T(x), \mu^T(y)\}$

(iii) $\mu^T(x\alpha y\beta z) \geq \min\{\mu^T(x), \mu^T(z)\}$

(iv) $x + a + z = b + z \;\Rightarrow \mu^T(x) \geq \min\{\mu^T(a), \mu^T(b)\}$

(v) $\mu^I(x+y) \geq \frac{\mu^I(x)+\mu^I(y)}{2}$

(vi) $\mu^I(x\alpha y) \geq \frac{\mu^I(x)+\mu^I(y)}{2}$

(vii) $\mu^I(x\alpha y\beta z) \geq \frac{\mu^I(x)+\mu^I(z)}{2}$

(viii) $x + a + z = b + z \;\Rightarrow \mu^T(x) \geq \frac{\mu^I(a)+\mu^I(b)}{2}$

(ix) $\mu^F(x+y) \leq \max\{\mu^F(x), \mu^F(y)\}$

(x) $\mu^F(x\alpha y) \leq \max\{\mu^F(x), \mu^F(y)\}$

(xi) $\mu^F(x\alpha y\beta z) \leq \max\{\mu^F(x), \mu^F(z)\}$

(xii) $x + a + z = b + z \;\Rightarrow \mu^F(x) \leq \max\{\mu^F(a), \mu^F(b)\}$

Definition 3.10. A neutrosophic subset μ of a Γ-hemiring S is called neutrosophic h-quasi-ideal if for all $x, y, z, a, b \in S$ we have

(i) $\mu^T(x+y) \geq \min\{\mu^T(x), \mu^T(y)\}$

(ii) $\mu^I(x+y) \geq \frac{\mu^I(x)+\mu^I(y)}{2}$

(iii) $\mu^F(x+y) \leq \max\{\mu^F(x), \mu^F(y)\}$

(iv) $(\mu^T o_h \chi_S^T) \cap (\chi_S^T o_h \mu^T) \subseteq \mu^T$

(v) $(\mu^I o_h \chi_S^I) \cap (\chi_S^I o_h \mu^I) \subseteq \mu^I$

(vi) $(\mu^F o_h \chi_S^F) \cap (\chi_S^F o_h \mu^F) \supseteq \mu^T$

(vii) $x + a + z = b + z \;\Rightarrow \mu^T(x) \geq \min\{\mu^T(a), \mu^T(b)\}$

(viii) $x + a + z = b + z \;\Rightarrow \mu^T(x) \geq \frac{\mu^I(a)+\mu^I(b)}{2}$

(ix) $x + a + z = b + z \;\Rightarrow \mu^F(x) \leq \max\{\mu^F(a), \mu^F(b)\}$

For any neutrosophic subset in a set X and any $t \in [0,1]$, define level subsets of μ by $\{\mu_t^T := \{x \in S : \mu^T(x) \geq t,\ t \in [0,1]\}$, $\mu_t^I := \{x \in S : \mu^I(x) \geq t,\ t \in [0,1]\}$ and $\mu_t^F := \{x \in S : \mu^F(x) \leq t,\ t \in [0,1]\}\}$. In [17], Kondo et al. introduced the Transfer Principle in fuzzy set theory, from which a neutrosophic set can be characterized by its level subsets. For any algebraic system $\mathcal{U} = (X, F)$, where F is a family of operations defined on X, the Transfer Principle can be formulated as follows:

Result 3.11. A fuzzy subset defined on $\mathcal{U}$ has the property $\mathcal{P}$ if and only if all non-empty level subset μ_t have the property $\mathcal{P}$.

As a direct consequence of the above Result, the following two results can be obtained.

Result 3.12. Let S be a Γ-hemiring. Then the following conditions hold:

(i) μ is a neutrosophic left(resp. right) h-ideal of S if and only if all non-empty level subsets μ_t are left (resp. right) h-ideals of S.

(ii) μ is a neutrosophic h-bi-ideal of S if and only if all non-empty level subsets μ_t are h-bi-ideals of S.

(iii) μ is a neutrosophic h-quasi-ideal of S if and only if all non-empty level subsets μ_t are h-quasi-ideals of S.

Result 3.13. Let S be a Γ-hemiring and $A \subseteq S$. Then the following conditions hold:

(i) A is a left(resp. right) h-ideal of S if and only if χ_A is a neutrosophic left (resp. right) h-ideal of S.

(ii) A is an h-bi-ideal of S if and only if χ_A is a neutrosophic h-bi-ideal of S.

(iii) A is an h-quasi-ideal of S if and only if χ_A is a neutrosophic h-quasi-ideal of S.

Result 3.14. Any neutrosophic h-quasi-ideal of S is a neutrosophic h-bi-ideal of S.

Definition 3.15. [19] A Γ-hemiring S is said to be h-hemiregular if for each $x \in S$, there exist $a, b \in S$ and $\alpha, \beta, \gamma, \delta \in \Gamma$ such that $x + x\alpha a\beta x + z = x\gamma b\delta x + z$.

Result 3.16. A Γ-hemiring S is h-hemiregular if and only if for any neutrosophic right h-ideal μ and any neutrosophic left h-ideal ν of S we have $\mu o_h \nu = \mu \cap \nu$.

Now we obtain the following characterizations of h-hemiregular Γ-hemirings. Note that for any two neutrosophic subsets μ and ν of S, $\mu \sqsubseteq \nu$ implies $\mu^T \subseteq \nu^T$, $\mu^I \subseteq \nu^I$ and $\mu^F \supseteq \nu^F$.

Result 3.17. Let S be a Γ-hemiring. Then the following conditions are equivalent.
(i) S is h-hemireglar.
(ii) $\mu \sqsubseteq \mu o_h \chi_S o_h \mu$ for every neutrosophic h-bi-ideal μ of S.
(iii) $\mu \sqsubseteq \mu o_h \chi_S o_h \mu$ for every neutrosophic h-quasi-ideal μ of S.

Result 3.18. Let S is a Γ-hemiring. Then the following conditions are equivalent.
(i) S is h-hemiregular.
(ii) $\mu \cap \nu \sqsubseteq \mu o_h \nu o_h \mu$ for every neutrosophic h-bi-ideal μ and every neutrosophic h-ideal ν of S.
(iii) $\mu \cap \nu \sqsubseteq \mu o_h \nu o_h \mu$ for every neutrosophic h-quasi-ideal μ and every neutrosophic h-ideal ν of S.

Result 3.19. Let S is a Γ-hemiring. Then the following conditions are equivalent.

(i) S is h-hemiregular.
(ii) $\mu \cap \nu \sqsubseteq \mu o_h \nu$ for every neutrosophic h-bi-ideal μ and every neutrosophic left h-ideal ν of S.
(iii) $\mu \cap \nu \sqsubseteq \mu o_h \nu$ for every neutrosophic h-quasi-ideal μ and every neutrosophic left h-ideal ν of S.
(iv) $\mu \cap \nu \sqsubseteq \mu o_h \nu$ for every neutrosophic right h-ideal μ and every neutrosophic h-bi-ideal ν of S.
(v) $\mu \cap \nu \sqsubseteq \mu o_h \nu$ for every neutrosophic right h-ideal μ and every neutrosophic h-quasi-ideal ν of S.
(vi) $\mu \cap \nu \cap \omega \sqsubseteq \mu o_h \nu o_h \omega$ for every neutrosophic right h-ideal μ, for every neutrosophic h-bi-ideal ν and for every neutrosophic left h-ideal ω of S.
(vii) $\mu \cap \nu \cap \omega \sqsubseteq \mu o_h \nu o_h \omega$ for every neutrosophic right h-ideal μ, for every neutrosophic h-quasi-ideal ν and for every neutrosophic left h-ideal ω of S.

Result 3.20. If a Γ-hemiring S is h-hemiregular then any neutrosophic right h-ideal μ and neutrosophic left h-ideal ν are idempotent and $\mu o_h \nu$ is an quasi-ideal of S.

Definition 3.21. A Γ-hemiring S is said to be h-intra-hemiregular if for each $x \in S$, there exist $z, a_i, a_i^{'}, b_i, b_i^{'} \in$S, and $\alpha_i, \beta_i, \gamma_i, \delta_i, \eta \in \Gamma$, i$\in$ **N**, the set of natural numbers, such that $x + \sum_{i=1}^{n} a_i \alpha_i x \eta x \beta_i a_i^{'} + z = \sum_{i=1}^{n} b_i \gamma_i x \eta x \delta b_i^{'} + z$.

Result 3.22. Let S be a Γ-hemiring. Then S is h-intra-hemiregular if and only if $\mu \cap \nu \sqsubseteq \mu o_h \nu$ for every neutrosophic left h-ideal μ and every neutrosophic right h-ideal ν of S.

Result 3.23. Let S be a Γ-hemiring and $x \in S$. Then S is h-intra-hemiregular if and only if $\mu(x) = \mu(x\gamma x)$, for all neutrosophic h-ideal μ of S and for all $x \in S$ and $\gamma \in \Gamma$.

Result 3.24. Let S be a Γ-hemiring. Then the following conditions are equivalent.

(i) S is both h-hemiregular and h-intra-hemiregular.
(ii) $\mu = \mu o_h \mu$ for every h-bi-ideal μ of S.
(iii) $\mu = \mu o_h \mu$ for every h-quasi-ideal μ of S.

Result 3.25. Let S be a Γ-hemiring. Then the following conditions are equivalent.

(i) S is both h-hemiregular and h-intra-hemiregular.
(ii) $\mu \cap \nu \sqsubseteq \mu o_h \nu$ for all neutrosophic h-bi-ideals μ and ν of S.
(iii) $\mu \cap \nu \sqsubseteq \mu o_h \nu$ for every neutrosophic h-bi-ideals μ and every neutrosophic h-quasi-ideal ν of S.
(iv) $\mu \cap \nu \sqsubseteq \mu o_h \nu$ for every neutrosophic h-quasi-ideals μ and every neutrosophic h-bi-ideal ν of S.
(v) $\mu \cap \nu \sqsubseteq \mu o_h \nu$ for all neutrosophic h-quasi-ideals μ and ν of S.

Conclusion: Since I have studied the results in case of Γ-hemiring $-$ a general setting of hemiring, the obtained results are also true for hemiring along with some parallel changes. In a similar way, neutrosophic k-ideals of Γ-semiring can be studied.

References

[1] K. Atanassov, *Intuitionistic fuzzy sets*, Fuzzy Sets and Systems, 20 (1986) 87 - 96.

[2] S. Broumi, F. Smarandache and P. K. Maji, Intuitionistic neutrosophic soft set over rings, Mathematics and Statistics 2(3): (2014) 120-126, DOI: 10.13189/ms.2014.020303.

[3] S. Broumi, M. Talea, A. Bakali, F. Smarandache, "Single Valued Neutrosophic Graphs," Journal of New Theory, N 10, 2016, pp. 86-101.

[4] S. Broumi, M. Talea, A. Bakali, F. Smarandache, "On Bipolar Single Valued Neutrosophic Graphs," Journal of New Theory, N11, 2016, pp.84-102.

[5] S. Broumi, M. Talea, A. Bakali, F. Smarandache, Interval Valued Neutrosophic Graphs, SISOM (2016) in press.

[6] S. Broumi, M. Talea, F. Smarandache, A. Bakali, Single Valued Neutrosophic Graphs: Degree, Order and Size, FuzzIEEE, 8 pages (2016) accepted

[7] S Broumi, F. Smarandache, M. Talea , A. Bakali, An Introduction to Bipolar Single Valued Neutrosophic Graph Theory, Applied Mechanics and Materials, Vol. 841, pp 184-191, doi:10.4028/www.scientific.net/AMM.841.184

[8] N. J. Cutland, *Nonstandard Analysis and its Applications*, Cambridge University Press, Cambridge, 1988.

[9] T.K.Dutta, B.K. Biswas, *Ph.D. dissertation*, University of Calcutta, India

[10] T.K. Dutta, S.K. Sardar, "On Operator Semiring of a $\Gamma-$semiring", *Southest Asian Bull of Mathematics, Springer-Vergal*, 26(2002)203-213

[11] J.S.Golan, *Semirings and their applications*, Kluwer Academic Publishers,1999.

[12] M.Henriksen, "Ideals in Semirings with Commutative Addition", *Am.Math.Soc.Notices* 6(1958)321

[13] X.Huang, H.Li and Y.Yin, "The h-hemiregular Fuzzy Duo Hemirings", *Int. J. Fuzzy Systems*, vol. 9, no. 2(2007) 105-109

[14] K.Iizuka, "On the Jacobson Radical of Semiring", *Tohoku Math.J.*11(2) (1959)409-421

[15] Y.B.Jun, M.A.Öztürk, S.Z.Song, "On Fuzzy h-ideals in hemiring", *Information sciences* 162(2004)211-226.

[16] G. J. Klir and B. Yuan, *Fuzzy Sets and Fuzzy logic- Theory and Applications*, Prentice Hail P T R Upper Saddle River, New Jersey, 1995.

[17] M.Kando, W.A.Dudek, "On Transfer Principle in Fuzzy Theory", *Mathware Soft Comput.* 12(2005)41-55.

[18] D.R.La Torre, "On h-ideals and k-ideals in hemirings", *Publ. Math. Debrecen* 12(1965)219-226.

[19] X.Ma, J.Zahn, "Fuzzy h-ideals in h-hemiregular and h-semisimple Γ-hemirings", *Neural Comput and Applic* (2010)19:477-485

[20] D.Mandal, Neutrosophic Ideals of Γ-Semirings, *Journal of New Results in Science*, No. 6(2014), 51-61

[21] S.K.Sardar, D.Mandal, "On fuzzy h-ideal in Γ-hemiring", *Int.J.Pure.Appl.Math*, Vol. 56, No. 3(2009),439-450

[22] S.K.Sardar and D.Mandal, On fuzzy h-ideals in h-regular Γ-hemiring and h-duo Γ-hemiring, *Gen. Math. Notes*, Vol. 2, No. 1(2011), 64-85

[23] F. Smarandache, *Neutrosophic set- a generalization of the intuitionistic fuzzy sets*, Int. J. Pure Appl. Math., 24 (2005) 287 - 297.

[24] M.M.K.Rao, "$\Gamma-$semirings-1", *Southeast Asian Bull. of Math.*, 19(1995)49-54

[25] A. Robinson, *Nonstandard Analysis*, Princeton University Press, Princeton, 1996.

[26] A. Rosenfeld, "Fuzzy groups", *J. Math. Anal. Appl.* 35(1971) 512-517

[27] T. J. Ross, J. M. Booker, V. J. Parkinson, *Fuzzy logic and probability : Bridiging the gap*, ASA-SIAM Series on Statistics and Applied Probability, Alexandria, 2002.

[28] Y.Yin and H.Li, "The characterization of h-hemiregular hemirings and h-intra-hemiregular hemirings", *Information Sciences*, 178(2008)3451-3464

[29] L.A.Zadeh, "Fuzzy Sets", *Information and Control* 8(1965)338-353.

[30] J.Zhan and W.A.Dudek, "Fuzzy h-ideals of hemirings",*Information sciences* 177(2007)876-886.