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ON STEREOGRAPHIC CIRCULAR WEIBULL DISTRIBUTION

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Abstract – In this paper, we introduce Stereographic-l-axial Weibull and Stereographic Circular Weibull distributions for modeling l-axial and circular data by extending the Stereographic Semicircular Weibull distribution [8]. Discussed the estimation of parameters also and derived the first two trigonometric moments for the proposed Stereographic Circular Weibull model.

Keywords – *Semicircular models, l-axial data, Stereographic projection, Trigonometric moments.*

1 Introduction

In some of the cases the directional / angular data does not required full circular models for modeling, this fact is noted in Guardiola [3], Jones [5], Byoung et al [1] and Yedlapalli et at [8]. For example, when sea turtles emerge from the ocean in search of a nesting site on dry land, a random variable having values on a semicircle is well sufficient for modeling such data. Similarly, when an aircraft is lost but its departure and its initial headings are known, a semicircular random variable is sufficient for such angular data. And few more examples of semicircular data is available in Ugai et al [10].

Toshihiro Abe et al [11] constructed some Unimodal and symmetric distributions by applying Inverse Stereographic projection, Yedlapalli et al [8] constructed some semicircular distributions by applying Inverse Stereographic projection. In this paper we developed the Stereographic- l-axial Weibull distribution by extending the Stereographic Semicircular Weibull and observed that Stereographic Circular Weibull distribution is a

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special case to proposed model and also Stereographic Circular Exponential [9] and Stereographic Circular Rayleigh Distributions are special cases to Stereographic Circular Weibull Distribution, where the same is true in linear case also . We plotted the graphs of the density function for various values of parameters. First two trigonometric moments for proposed model are derived, we estimate parameters of the Stereographic Circular Weibull model by a maximum likelihood method.

2 Stereographic-l-axial Weibull Distribution

Here we recall the definition of Stereographic Semicircular Weibull Distribution [8].

Definition 2.1 (Stereographic Semicircular Weibull Distribution) A random variable θ on unit semicircle is said to have Stereographic Semicircular Weibull distribution with shape parameter $c>0$ and scale parameter $\sigma>0$ denoted by SCW$\left(c,\sigma\right)$, if the probability density and cumulative distribution functions are given by

1. $$g\left(\theta\right)=\frac{c}{2\sigma}\sec^2\left(\frac{\theta}{2}\right)\left(\tan\left(\frac{\theta}{2}\right)\right)^{c-1}\exp\left(\frac{1}{\sigma}\left(\tan\left(\frac{\theta}{2}\right)\right)^{c}\right), \text{ for } 0\le\theta<\pi, c>0 \text{ and } \sigma>0$$

2. $$G\left(\theta\right)=1-e^{-\frac{1}{\sigma}\left(\tan\left(\frac{\theta}{2}\right)\right)^{c}}$$

We extend the above Stereographic Semicircular Weibull model [8] to the l -axial distribution, which is applicable to any arc of arbitrary length say $2\pi/l$ for $l=1,2,...,$ so it is desirable to extend the Stereographic Semicircular Weibull distribution[8] to construct the Stereographic-l-axial Weibull distribution, we consider the density function of Stereographic Semicircular Weibull distribution and use the transformation $\phi=2\theta/l$, $l=1,2,...,$. The probability density function of ϕ is given by

$$g\left(\theta\right)=\frac{cl}{4\sigma}\sec^2\left(\frac{l\theta}{4}\right)\left(\tan\left(\frac{l\theta}{4}\right)\right)^{c-1}\exp\left(-\frac{1}{\sigma}\left(\tan\left(\frac{l\theta}{4}\right)\right)^{c}\right),$$

$$0<\theta<\frac{2\pi}{l},\ \sigma>0,\ c>0 \text{ and } l=1,2,.. \tag{2.1.1}$$

We call it as **Stereographic -l-axial Weibull distribution**

Case (1) When $l=1$, in the probability density function (2.1.1), we get the density function

$$g\left(\theta\right)=\frac{c}{4\sigma}\sec^2\left(\frac{\theta}{4}\right)\left(\tan\left(\frac{\theta}{4}\right)\right)^{c-1}\exp\left(-\frac{1}{\sigma}\left(\tan\left(\frac{\theta}{4}\right)\right)^{c}\right),$$

$$0 < \theta < 2\pi,\ \sigma > 0 \ \text{ and } c > 0 \qquad (2.1.2)$$

We call it as **Stereographic Circular Weibull distribution**.

Case (2) When $l = 2$, the probability density function (2.1.1) is the same as that of **Stereographic Semicircular Weibull Distribution** [8].

2.2 Stereographic Circular Weibull Distribution

A random variable X_S on a unit circle is said to have Stereographic Circular Weibull Distribution with scale parameters $\sigma > 0$, shape parameter $c > 0$ and location parameter μ denoted by **SCWD**(c,σ,μ). If its probability density and cumulative distribution functions are given by

$$g(\theta) = \frac{c}{4\sigma}\sec^2\left(\frac{\theta-\mu}{4}\right)\left(\tan\left(\frac{\theta-\mu}{4}\right)\right)^{c-1}\exp\left(-\frac{1}{\sigma}\left(\tan\left(\frac{\theta-\mu}{4}\right)\right)^{c}\right),$$
$$0 < \theta, \mu < 2\pi,\ c > 0 \text{ and } \sigma > 0 \qquad (2.2.1)$$

$$G(\theta) = 1 - e^{-\frac{1}{\sigma}\left(\tan\left(\frac{\theta-\mu}{4}\right)\right)^{c}} \qquad (2.2.2)$$

Special Cases

Case1: When $c=1$, in (2.1.1), we get the density function of **Stereographic-l- axial Exponential Distribution[9]**. i.e.,

$$g(\theta) = \frac{l}{4\sigma}\sec^2\left(\frac{l\theta}{4}\right)\exp\left(-\frac{1}{\sigma}\left(\tan\left(\frac{l\theta}{4}\right)\right)\right),\ 0 < \theta < \frac{2\pi}{l},\ \sigma > 0 \text{ and } l = 1, 2, .. \qquad (2.2.3)$$

Case2: When $c=1 \,\&\, l = 1$, in (2.1.1), we get the density function

$$g(\theta) = \frac{1}{4\sigma}\sec^2\left(\frac{\theta}{4}\right)\exp\left(-\frac{1}{\sigma}\left(\tan\left(\frac{\theta}{4}\right)\right)\right),\ 0 < \theta < 2\pi,\ \sigma > 0 \qquad (2.2.4)$$

We call it as **Stereographic Circular Exponential Distribution [9].**

Case3: When $c=1 \,\&\, l = 2$, in (2.1.1), we get the density function of **Stereographic Semicircular Exponential Distribution** [8].

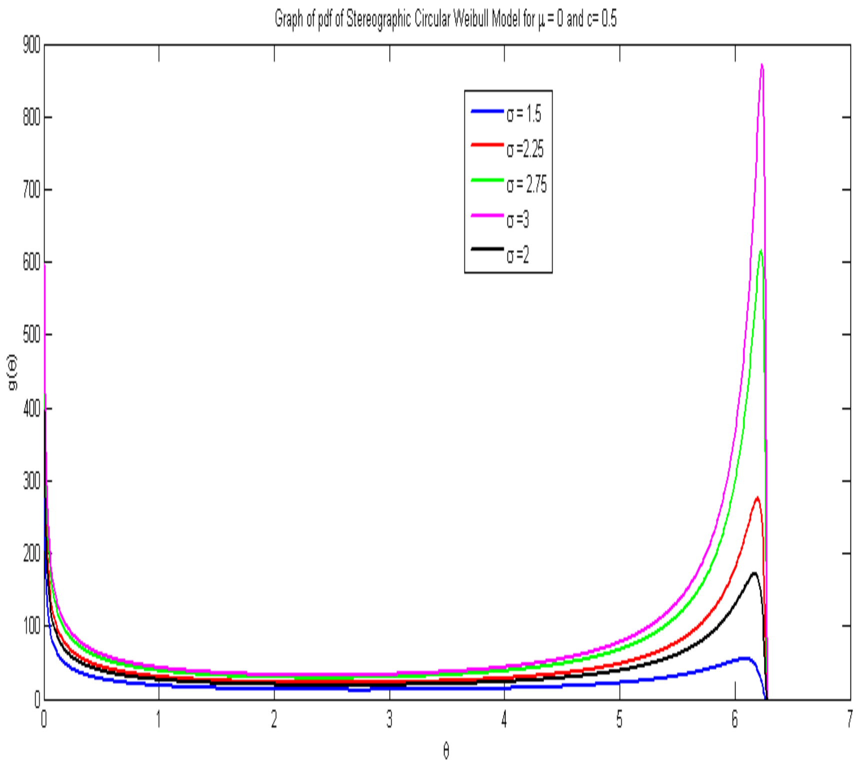
Case 4: When $c=2 \,\&\, l = 1$, in (2.1.1), we get the density function

$$g(\theta)=\frac{1}{2\sigma}\sec^2\left(\frac{\theta}{4}\right)\left(\tan\left(\frac{\theta}{4}\right)\right)\exp\left(-\frac{1}{\sigma}\left(\tan\left(\frac{\theta}{4}\right)\right)^2\right),$$
$$0<\theta<2\pi,\ \sigma>0 \tag{2.2.5}$$

We call it as **Stereographic Circular Rayleigh Distribution**

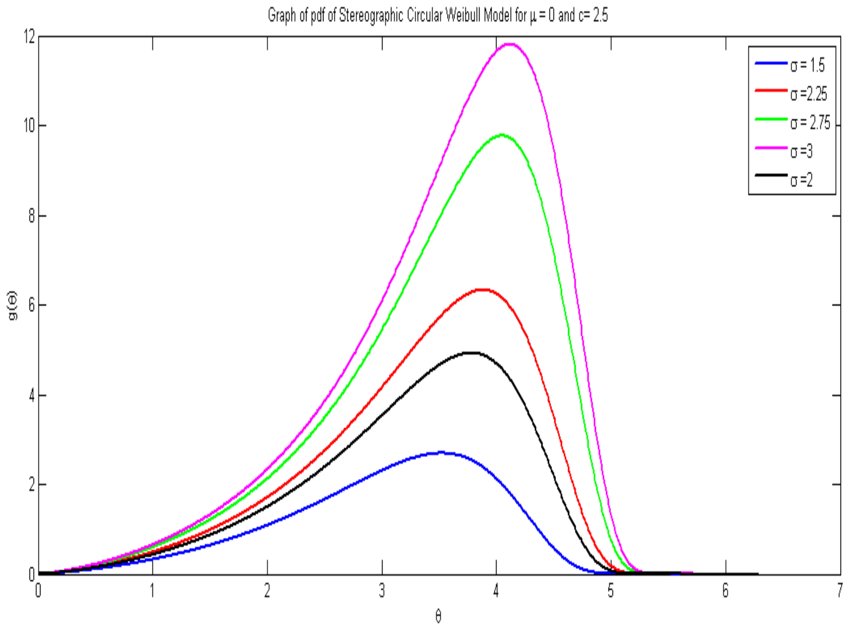
The same is true in linear case also.

2.3 Graphs of Density Function of Stereographic Circular Weibull Distribution for Various Values of the Parameters



3 Trigonometric moments of Stereographic Circular Weibull Model

Without loss of generality here we assume that $\mu=0$, in (2.1.2) .The trigonometric moments of the distribution are given by $\{\varphi_p : p=0,\pm1,\pm2,\pm3,...\}$, where $\varphi_p=\alpha_p+i\beta_p$, with $\alpha_p=E(\cos p\theta)$ and $\beta_p=E(\sin p\theta)$ being the p^{th} order cosine and sine moments of the random angle θ, respectively.



Theorem 3.1 Under the pdf of Stereographic Circular Weibull Model with $\mu = 0$, the first four $\alpha_{\mathrm{p}} = E(\cos \mathrm{p}\theta)$ and $\beta_{\mathrm{p}} = E(\sin \mathrm{p}\theta)$, $\mathrm{p} = 1, 2.$ are given as follows:

$$\alpha_1 = 1 - \frac{8}{\sigma}\sum_{n=0}^{\infty}(-1)^n (n+1)(\sigma)^{-\left(\frac{2n+c+2}{c}\right)}\Gamma\left(\frac{2n+c+2}{c}\right)$$

$$\beta_1 = \frac{4}{\sigma}\sum_{n=0}^{\infty}(-1)^n (n+1)(\sigma)^{\left(\frac{2n+c+2}{c}\right)}\Gamma\left(\frac{2n+c+2}{c}\right) - \frac{8}{\sigma}\sum_{n=0}^{\infty}(-1)^n (n+1)(\sigma)^{\left(\frac{2n+c+4}{c}\right)}\Gamma\left(\frac{2n+c+4}{c}\right)$$

$$\begin{aligned}\alpha_2 = 1 &- \frac{32}{\sigma}\sum_{n=0}^{\infty}(-1)^n (n+1)(\sigma)^{\left(\frac{2n+c+2}{c}\right)}\Gamma\left(\frac{2n+c+2}{c}\right)\\ &+ \frac{12}{\sigma}\sum_{n=0}^{\infty}(-1)^n C(n+3,3)(\sigma)^{\left(\frac{2n+c+4}{c}\right)}\Gamma\left(\frac{2n+c+4}{c}\right)\end{aligned}$$

$$\beta_2 = \frac{8}{\sigma}\sum_{n=0}^{\infty}(-1)^n (\sigma)^{\left(\frac{2n+c+1}{c}\right)}\Gamma\left(\frac{2n+c+1}{c}\right) - \frac{16}{\sigma}\sum_{n=0}^{\infty}(-1)^n (n+1)(\sigma)^{\left(\frac{2n+c+3}{c}\right)}\Gamma\left(\frac{2n+c+3}{c}\right)$$

$$-\frac{64}{\sigma}\sum_{n=0}^{\infty}(-1)^n C(n+3,3)(\sigma)^{\left(\frac{2n+c+4}{c}\right)}\Gamma\left(\frac{2n+c+4}{c}\right) + \frac{128}{\sigma}\sum_{n=0}^{\infty}(-1)^n C(n+3,3)(\sigma)^{\left(\frac{2n+c+5}{c}\right)}\Gamma\left(\frac{2n+c+5}{c}\right)$$

Proof:

$$\varphi_{\mathrm{p}} = \int_0^{\pi} e^{ip\theta} g(\theta)\, d\theta = \int_0^{\pi}\cos(\mathrm{p}\theta) g(\theta)\, d\theta + i\int_0^{\pi}\sin(\mathrm{p}\theta) g(\theta)\, d\theta$$

$$=E\left(\cos\left(\mathrm{p}\theta\right)\right)+i\,E\left(\sin\left(\mathrm{p}\theta\right)\right)=\alpha_{\mathrm{p}}+i\,\beta_{\mathrm{p}} \quad \text{for } \mathrm{p}=1,2.$$

For the first cosine and sine moments, use the transformation $x=\tan\left(\frac{\theta}{4}\right)$,

$$\cos\theta=1-\frac{8x^2}{\left(1+x^2\right)^2}$$

and $\sin\theta=\frac{4x}{\left(1+x^2\right)^2}-\frac{8x^3}{\left(1+x^2\right)^2}$,the results α_1 and β_1 follows by the integral formula

3.478.1 [2]. Now

$$\mathrm{E}\left(\cos\left(\mathrm{p}\theta\right)\right)=\frac{c}{4\sigma}\int_0^{2\pi}\cos\left(\mathrm{p}\theta\right)\sec^2\left(\frac{\theta}{4}\right)\left(\tan\left(\frac{\theta}{4}\right)\right)^{c-1}e^{-\frac{1}{\sigma}\left(\tan\left(\frac{\theta}{4}\right)\right)^c}d\theta$$

and

$$\mathrm{E}\left(\sin\left(\mathrm{p}\theta\right)\right)=\frac{c}{4\sigma}\int_0^{2\pi}\sin\left(\mathrm{p}\theta\right)\sec^2\left(\frac{\theta}{4}\right)\left(\tan\left(\frac{\theta}{4}\right)\right)^{c-1}e^{-\frac{1}{\sigma}\left(\tan\left(\frac{\theta}{4}\right)\right)^c}d\theta$$

$$\alpha_1=\frac{c}{4\sigma}\int_0^{2\pi}\cos\theta\sec^2\left(\frac{\theta}{4}\right)\left(\tan\left(\frac{\theta}{4}\right)\right)^{c-1}e^{-\frac{1}{\sigma}\left(\tan\left(\frac{\theta}{4}\right)\right)^c}d\theta$$

$$=\frac{c}{\sigma}\int_0^{\infty}\left[1-\frac{8x^2}{\left(1+x^2\right)^2}\right]x^{c-1}e^{-\frac{1}{\sigma}x^c}dx$$

$$=1-\frac{8c}{\sigma}\int_0^{\infty}x^{c+1}\left(1+x^2\right)^{-2}e^{-\frac{1}{\sigma}x^c}dx$$

$$=1-\frac{8c}{\sigma}\int_0^{\infty}x^{c+1}\sum_{n=0}^{\infty}(-1)^n\,(n+1)x^{2n}e^{-\frac{1}{\sigma}x^c}dx$$

$$=1-\frac{8c}{\sigma}\sum_{n=0}^{\infty}(-1)^n\,(n+1)\frac{1}{c}\left(\frac{1}{\sigma}\right)^{-\left(\frac{2n+c+2}{c}\right)}\Gamma\left(\frac{2n+c+2}{c}\right)$$

$$\alpha_1=1-\frac{8}{\sigma}\sum_{n=0}^{\infty}(-1)^n\,(n+1)(\sigma)^{-\left(\frac{2n+c+2}{c}\right)}\Gamma\left(\frac{2n+c+2}{c}\right)$$

$$\beta_1=\frac{c}{4\sigma}\int_0^{2\pi}\sin\theta\sec^2\left(\frac{\theta}{4}\right)\left(\tan\left(\frac{\theta}{4}\right)\right)^{c-1}e^{-\frac{1}{\sigma}\left(\tan\left(\frac{\theta}{4}\right)\right)^c}d\theta$$

$$=\frac{c}{\sigma}\int_0^{\infty}\left[\frac{4x}{\left(1+x^2\right)^2}-\frac{8x^3}{\left(1+x^2\right)^2}\right]x^{c-1}e^{-\frac{1}{\sigma}x^c}dx$$

$$=\frac{4c}{\sigma}\int_0^{\infty}\frac{x^{c+1}}{\left(1+x^2\right)^2}e^{-\frac{1}{\sigma}x^c}dx-\frac{8c}{\sigma}\int_0^{\infty}\frac{x^{c+3}}{\left(1+x^2\right)^2}e^{-\frac{1}{\sigma}x^c}dx$$

$$=\frac{4c}{\sigma}\int_0^{\infty}x^{c+1}\sum_{n=0}^{\infty}(-1)^n(n+1)x^{2n}e^{-\frac{1}{\sigma}x^c}dx-\frac{8c}{\sigma}\int_0^{\infty}x^{c+3}\sum_{n=0}^{\infty}(-1)^n(n+1)x^{2n}e^{-\frac{1}{\sigma}x^c}dx$$

$$=\frac{4c}{\sigma}\sum_{n=0}^{\infty}(-1)^n(n+1)\int_0^{\infty}x^{(2n+c+2)-1}e^{-\frac{1}{\sigma}x^c}dx-\frac{8c}{\sigma}\sum_{n=0}^{\infty}(-1)^n(n+1)\int_0^{\infty}x^{(2n+c+4)-1}e^{-\frac{1}{\sigma}x^c}dx$$

$$\beta_1=\frac{4}{\sigma}\sum_{n=0}^{\infty}(-1)^n(n+1)(\sigma)^{\left(\frac{2n+c+2}{c}\right)}\Gamma\left(\frac{2n+c+2}{c}\right)-\frac{8}{\sigma}\sum_{n=0}^{\infty}(-1)^n(n+1)(\sigma)^{\left(\frac{2n+c+4}{c}\right)}\Gamma\left(\frac{2n+c+4}{c}\right)$$

To obtain second cosine and sine moments α_2 and β_2, we use the transformation

$$x=\tan\left(\frac{\theta}{4}\right),\quad \cos 2\theta=1+\frac{12x^4}{\left(1+x^2\right)^4}-\frac{32x^2}{\left(1+x^2\right)^2}$$

and

$$\sin 2\theta=\frac{8x}{\left(1+x^2\right)^2}-\frac{16x^3}{\left(1+x^2\right)^2}-\frac{64x^3}{\left(1+x^2\right)^4}+\frac{128x^5}{\left(1+x^2\right)^4},$$

the results of α_2 and β_2 follows by the same integral formula of α_1.

$$\alpha_2=\frac{c}{4\sigma}\int_0^{2\pi}\cos 2\theta\sec^2\left(\frac{\theta}{4}\right)\left(\tan\left(\frac{\theta}{4}\right)\right)^{c-1}e^{-\frac{1}{\sigma}\left(\tan\left(\frac{\theta}{4}\right)\right)^c}d\theta$$

$$=\frac{c}{\sigma}\int_0^{\infty}\left[1+\frac{12x^4}{\left(1+x^2\right)^4}-\frac{32x^2}{\left(1+x^2\right)^2}\right]x^{c-1}e^{-\frac{1}{\sigma}x^c}dx$$

$$=1+\frac{12c}{\sigma}\int_0^{\infty}x^{c+3}\left(1+x^2\right)^{-4}e^{-\frac{1}{\sigma}x^c}dx-\frac{32c}{\sigma}\int_0^{\infty}x^{c+1}\left(1+x^2\right)^{-2}e^{-\frac{1}{\sigma}x^c}dx$$

$$=1-\frac{32c}{\sigma}\sum_{n=0}^{\infty}(-1)^n(n+1)\int_0^{\infty}x^{(2n+c+2)-1}e^{-\frac{1}{\sigma}x^c}dx+\frac{12c}{\sigma}\sum_{n=0}^{\infty}C(n+3,3)(-1)^n\int_0^{\infty}x^{(2n+c+4)-1}e^{-\frac{1}{\sigma}x^c}dx$$

$$\alpha_2=1-\frac{32}{\sigma}\sum_{n=0}^{\infty}(-1)^n(n+1)(\sigma)^{\left(\frac{2n+c+2}{c}\right)}\Gamma\left(\frac{2n+c+2}{c}\right)$$
$$+\frac{12}{\sigma}\sum_{n=0}^{\infty}(-1)^nC(n+3,3)(\sigma)^{\left(\frac{2n+c+4}{c}\right)}\Gamma\left(\frac{2n+c+4}{c}\right),$$

$$\beta_2=\frac{c}{4\sigma}\int_0^{2\pi}\sin 2\theta\sec^2\left(\frac{\theta}{4}\right)\left(\tan\left(\frac{\theta}{4}\right)\right)^{c-1}e^{-\frac{1}{\sigma}\left(\tan\left(\frac{\theta}{4}\right)\right)^c}d\theta$$

$$=\frac{c}{\sigma}\int_0^\infty\left[\frac{8x}{\left(1+x^2\right)^2}-\frac{16x^3}{\left(1+x^2\right)^2}-\frac{64x^3}{\left(1+x^2\right)^4}+\frac{128x^5}{\left(1+x^2\right)^4}\right]x^{c-1}e^{-\frac{1}{\sigma}x^c}dx$$

$$=\frac{8c}{\sigma}\int_0^\infty x^c\left(1+x^2\right)^{-2}e^{-\frac{1}{\sigma}x^c}dx-\frac{16c}{\sigma}\int_0^\infty x^{c+2}\left(1+x^2\right)^{-2}e^{-\frac{1}{\sigma}x^c}dx$$

$$-\frac{64c}{\sigma}\int_0^\infty x^{c+3}\left(1+x^2\right)^{-4}e^{-\frac{1}{\sigma}x^c}dx+\frac{128c}{\sigma}\int_0^\infty x^{c+4}\left(1+x^2\right)^{-4}e^{-\frac{1}{\sigma}x^c}dx$$

$$=\frac{8c}{\sigma}\sum_{n=0}^\infty(-1)^n(n+1)\int_0^\infty x^{(2n+c+1)-1}e^{-\frac{1}{\sigma}x^c}dx-\frac{16c}{\sigma}\sum_{n=0}^\infty(-1)^n(n+1)\int_0^\infty x^{(2n+c+3)-1}e^{-\frac{1}{\sigma}x^c}dx$$

$$-\frac{64c}{\sigma}\sum_{n=0}^\infty(-1)^n C(n+3,3)\int_0^\infty x^{(2n+c+4)-1}e^{-\frac{1}{\sigma}x^c}dx+\frac{128c}{\sigma}\sum_{n=0}^\infty(-1)^n C(n+3,3)\int_0^\infty x^{(2n+c+5)-1}e^{-\frac{1}{\sigma}x^c}dx$$

$$\beta_2=\frac{8}{\sigma}\sum_{n=0}^\infty(-1)^n(\sigma)^{\left(\frac{2n+c+1}{c}\right)}\Gamma\left(\frac{2n+c+1}{c}\right)-\frac{16}{\sigma}\sum_{n=0}^\infty(-1)^n(n+1)(\sigma)^{\left(\frac{2n+c+3}{c}\right)}\Gamma\left(\frac{2n+c+3}{c}\right)$$

$$-\frac{64}{\sigma}\sum_{n=0}^\infty(-1)^n C(n+3,3)(\sigma)^{\left(\frac{2n+c+4}{c}\right)}\Gamma\left(\frac{2n+c+4}{c}\right)+\frac{128}{\sigma}\sum_{n=0}^\infty(-1)^n C(n+3,3)(\sigma)^{\left(\frac{2n+c+5}{c}\right)}\Gamma\left(\frac{2n+c+5}{c}\right)$$

In the similar process we can obtain the higher order moments also .

4 Estimation of Parameter

The minus log-likelihood for a random sample of size n, $\theta_1,\theta_2,\theta_3,...,\theta_n$, from the Circular Weibull distribution is given by

$$L(\mu,\sigma;\theta)=\prod_{i=1}^n g(\theta_i)=\prod_{i=1}^n\left[\frac{c}{4\sigma}\sec^2\left(\frac{\theta_i-\mu}{4}\right)\left(\tan\left(\frac{\theta_i-\mu}{4}\right)\right)^{c-1}\exp\left\{-\frac{1}{\sigma}\left(\tan\left(\frac{\theta_i-\mu}{4}\right)\right)^c\right\}\right] \tag{4.1}$$

$$-l\left((\mu,\sigma,c;\theta)\right)=-\log\left(L(\mu,\sigma,c;\theta)\right)=n\log(4\sigma)-n\log(c)+2\sum_{i=1}^n\log\left(\cos\left(\frac{(\theta_i-\mu)}{4}\right)\right)$$
$$+\left(\frac{1}{\sigma}-c+1\right)\sum_{i=1}^n\log\left(\tan\left(\frac{(\theta_i-\mu)}{4}\right)\right) \tag{4.2}$$

To find maximum likelihood estimates, we can use any minimization subroutine for direct minimization of the minus log-likelihood itself.

5 Conclusions

In this paper, we discussed circular distribution resulting from extending Stereographic Semicircular Weibull distribution on unit semicircle which is obtained by inducing Inverse Stereographic Projection on the real line. The density and distribution functions of

Stereographic Circular Weibull distribution admit explicit forms, as do trigonometric moments and observed that in similar to linear case, Stereographic Circular Exponential and Stereographic Circular Rayleigh distributions are Special cases to proposed Stereographic Circular Weibull Distribution. As this distribution is asymmetric, promising for modeling asymmetrical directional data.

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