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ON SOFT e-OPEN SETS AND SOFT e-CONTINUITY IN SOFT TOPOLOGICAL SPACE

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Abstract: In this paper, a new class of generalized soft open sets in soft topological spaces, called soft *e-open* set is focused and investigated some properties of them. Then focused the relationships among soft δ-pre open sets, soft δ-semi open sets, soft pre-open sets and soft *e-open* sets. We also investigated the concepts of soft *e-open* functions, soft *e*-continuous, soft *e*-irresolute and soft *e*-homeomorphism on soft topological space and discussed their relations with existing soft continuous and other weaker forms of soft continuous functions. Further soft *e*-separation axioms have been introduced and investigated with the help of soft *e*-open sets. Finally, we observed that the collection S*e*r-h(X,τ,E) form a soft group.

Keywords: Soft e-open (S*e*-open) sets, Soft *e*-closed (S*e*-closed) sets, Soft *e*-continuous, soft *e*-irresolute and soft *e*-homomorphism.

1. Introduction

Molodtsov [1] initiated a novel concept of soft set theory, which is completely a new approach for modeling vagueness and uncertainty. He successfully applied the soft set theory into several directions such as smoothness of functions, game theory, Riemann Integration, theory of measurement, and so on. Soft set theory and its applications have shown great development in recent years. This is because of the general nature of parametrization expressed by a soft set. Shabir and Naz [2] introduced the notion of soft topological spaces which are defined over an initial universe with a fixed set of parameters. Later, Zorlutuna et al.[3], Aygunoglu and Aygun [4] and Hussain et al are continued to study the properties of soft topological space. They got many important results in soft topological spaces. Weak forms of soft open sets were first studied by Chen [5].He investigated soft semi-open sets in soft topological spaces and studied some properties of it. Yumak and Kaymakci [10] are defined soft β-open sets and continued to study weak forms

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of soft open sets in soft topological space. Later, Akdag and Ozkan [6] [7] defined soft b-open (soft b-closed) sets and soft α-open(soft α-closed) sets respectably.
In the present study, first of all, we have focused some new concepts such as soft *e*-open sets, soft *e*-closed sets, soft *e*-interior, soft *e*-closure in soft topological spaces and investigated some of their properties. Secondly, we have defined the concepts of soft *e*-continuous, soft *e*-open, soft *e*-irresolute mappings and soft *e*-homeomorphism on soft topological spaces and obtained some characterizations of these mappings. We have also studied the relationships among soft δ-semi-continuity, soft δ-pre-continuity and soft *e*-continuity and with the help of counter examples we have shown the non-coincidence of these various types of mappings. Further soft *e*-separation axioms have been introduced and investigated with the help of soft *e*-open sets. Finally, we have observed that the collection S*e*r-h(X, τ,E) form a soft group.

2. Preliminaries

Throughout the paper, the space X and Y stand for soft topological spaces with (X,τ, E) and (Y,ν, K) assumed unless otherwise stated. Moreover, throughout this paper, a soft mapping f : X $\rightarrow$Y stands for a mapping, where f : (X,τ, E) $\rightarrow$ (Y,ν, K), u : X $\rightarrow$Y and p : E $\rightarrow$K are assumed mappings unless otherwise stated.

Definition: 2.1[1]. Let X be an initial universe and E be a set of parameters. Let *P*(X) denotes the power set of X and *A* be a non-empty subset of E. A pair (F,*A*) is called a soft set over X, where F is a mapping given by F: $A\rightarrow P$(X) defined by F(e)$\in P$(X) $\forall$e$\in$A. In other words, a soft set over X is a parameterized family of subsets of the universe X. For e$\in$A, F(e) may be considered as the set of e-approximate elements of the soft set (F,A).

Definition 2.2[11]. A soft set (F,A) over X is called a null soft set, denoted by $\tilde{\varphi}$, if e$\in$A,F(e)=φ.

Definition 2.3[11]. A soft set (F,A) over X is called an absolute soft set, denoted by $\tilde{A}$, if e$\in$A, F(e)=X. If A=E, then the A-universal soft set is called a universal soft set, denoted by $\tilde{X}$.

Theorem 2.4[2]. Let Y be a non-empty subset of X, then $\tilde{Y}$ denotes the soft set (Y,E) over X for which Y(e)=Y, for all e$\in$E.

Definition 2.5 [11]. The union of two soft sets (F,A) and (G,B) over the common universe X is the soft set (H,C), where C=A$\cup$B and for all e$\in$C,

$$H(e)=\begin{cases} F(e), if e\in A-B \\ G(e), if e\in B-A \\ F(e)\cup G(e), if e\in A\cap B \end{cases}$$

We write (F,A) $\tilde{\cup}$ (G,B) = (H,C)

Definition 2.6 [11]. The intersection (H,C) of two soft sets (F,A) and (G,B) over a common universe X, denoted by (F,A) $\tilde{\cap}$ (G,B), is defined as C=A∩B and H(e)= F(e) ∩ G(e) for all e∈C.

Definition 2.7[2]. Let (F,A) be a soft set over a soft topological space (X,τ,E). We say that x∈(F,E) read as x belongs to the set (F,E) whenever x∈F(e) for all e∈E. Note that for any x∈X, x∉(F,E), if x∉F(e) for some e∈E.

Definition 2.8 [11]. Let (F,A) and (G,B) be two soft sets over a common universe X. Then (F,A) $\tilde{\subseteq}$(G,B) if A ⊆ B, and F(e) ⊆ G(e) for all e∈A.

Definition 2.9 [2]. Let τ be the collection of soft sets over X, then τ is said to be a soft topology on X if it satisfies the following axioms.

(1) $\tilde{\varphi}, \tilde{X}$ belong to τ.
(2) The union of any number of soft sets in τ belongs to τ.
(3) The intersection of any two soft sets in τ belongs to τ.

The triplet (X,τ,E) is called a soft topological space over X. Let (X,τ,E) be a soft topological space over X, then the members of τ are said to be soft open sets in X. A soft set (F,A) over X is said to be a soft closed set in X, if its relative complement $(F,A)^c$ belongs to τ.

Definition 2.10 [12]. For a soft set (F,A) over X, the relative complement of (F,A) is denoted by $(F,A)^c$ and is defined by $(F,A)^c = (F^c,A)$, where F^c : A→*P*(X) is a mapping given by $F^c(e)$ =X-F(e), for all e∈A.

Definition 2.11. A soft set (F,A) in a soft topological space X is called
(i) soft regular open (resp.soft regular closed) set [13] if (F,A) = Int(Cl(F,A)) [resp. (F,A) = Cl(Int (F,A))].
(ii) soft semi-open (resp.soft semi-closed) set [5] if (F,A)$\tilde{\subseteq}$Cl(Int(F,A)) [resp. Int(Cl(F,A)) $\tilde{\subseteq}$(F,A).
(iii) soft pre-open (resp.soft pre-closed)[13] if (F,A)$\tilde{\subseteq}$Int(Cl(F,A)) [resp. Cl(Int(F,A)) $\tilde{\subseteq}$(F,A).
(iv) soft α-open (resp.soft α-closed)[13] if (F,A) $\tilde{\subseteq}$Int(Cl(Int(F,A))) [resp. Cl(Int(Cl(F,A))) $\tilde{\subseteq}$(F,A)].
(v) soft β-open (resp.soft β-closed) set [13] if (F,A)$\tilde{\subseteq}$Cl(Int(Cl(F,A))) [resp. Int(Cl(Int(F,A))) $\tilde{\subseteq}$(F,A)].
(vi) soft γ-open (resp.soft γ-closed) set [6] if (F,A) $\tilde{\subseteq}$[Int(Cl(F,A))$\tilde{\cup}$Cl(Int(F,A))] [resp. Int(Cl(F,A))$\tilde{\cap}$Cl(Int(F,A))$\tilde{\subseteq}$(F,A)].

Definition 2.12[15]. The soft set (F,A) in a soft topological space (X,τ,E) is called a soft point in X, denoted by P_λ^F , if for λ∈A, F(λ) ≠ φ and F(β) = φ, for β∉A.

3. Soft *e*-open Sets and Soft *e*-closed Sets

In this section we introduce soft δ-open, soft δ-semi open, soft δ-pre open and soft *e*-open sets in soft topological spaces and study some of their properties.

Definition 3.1. A soft point P_λ^F in a soft topological space (X,τ,E) is called a soft δ-cluster point of a soft Set (G,A) if for each soft regular open set (U,A) containing P_λ^F, (G,A) $\tilde{\cap}$ (U,A) ≠ $\tilde{\varphi}$.

The set of all soft δ-cluster points of (G,A) is called soft δ-closure of (G,A) and is denoted by $[(G,A)]_\delta$ or SCl_δ(G,A).Soft δ-interior of a soft set (F,A) denoted by $SInt_\delta$(F,A) ={ $P_\lambda^F \in$ X: for some soft open subset (G,A) of X, $P_\lambda^F \in$ (G,A) $\tilde{\subseteq}$ Int(Cl(G,A)) $\tilde{\subseteq}$ (F,A)}.

Definition 3.2. A soft set (G,A) in a soft topological space (X,τ,E) is called soft δ-closed set iff (G,A) = $[(G,A)]_\delta$ and it's compliment $\tilde{X}$ - (G,A) is called soft δ-open sets in X.
Or, equivalently, if (G,A) is the union of soft regular open sets, then (G,A) is said to be soft δ-open sets in X.

The collection of all soft δ-open sets & soft δ-closed sets are respectably, denoted by SδOS(X) & SδCS(X).

Definition 3.3. A soft set (F,A) in a soft topological space (X,τ,E) is called
(i) soft δ-semi open (Sδ-semi open) set iff (F,A) $\tilde{\subseteq}$ Cl(Int_δ(F,A)).
(ii) soft δ-semi closed (Sδ-semi closed) set iff Int(Cl_δ(F,A)) $\tilde{\subseteq}$ (F,A).

The union of all soft δ-semi open sets contained in a soft set (F,A) in a soft topological space X is called the soft δ-semi interior of (F,A) and it is denoted by $SSInt_\delta$(F,A). The intersection of all soft δ-semi closed sets containing a soft set (F,A) in a soft topological space X is called the soft δ-semi closure of (F,A) and it is denoted by $SSCl_\delta$(F,A).

Definition 3.4. A soft set (F,A) in a soft topological space (X,τ,E) is called
(i) soft δ-pre open (Sδ-pre open) set iff (F,A) $\tilde{\subseteq}$ Int(Cl_δ(F,A)).
(ii) soft δ-pre closed (Sδ-pre closed) set iff Cl(Int_δ(F,A)) $\tilde{\subseteq}$ (F,A).

The union of all soft δ-pre open sets contained in a soft set (F,A) in a soft topological space X is called the soft δ-pre interior of (F,A) and it is denoted by $SPInt_\delta$(F,A). The intersection of all soft δ-pre closed sets containing a soft set (F,A) in a soft topological space X is called the soft δ-pre closure of (F,A) and it is denoted by $SPCl_\delta$(F,A).

Definition 3.5. A soft set (F,A) in a soft topological space (X,τ,E) is called
(i) soft *e*-open (s*e*-open) set iff (F,A) $\tilde{\subseteq}$ [Int(Cl_δ(F,A)) $\tilde{\cup}$ Cl(Int_δ(F,A))]
(ii) soft *e*-closed (s*e*-closed) set iff (F,A) $\tilde{\supseteq}$ [Int(Cl_δ(F,A)) $\tilde{\cap}$ Cl(Int_δ(F,A))]

Example 3.6. Let X ={x_1, x_2, x_3, x_4}, E ={e_1, e_2, e_3} and τ ={ $\tilde{\varphi}$, $\tilde{X}$, (G,E)} where, (G,A) = {(e_1,{ x_1}), (e_2,{ x_2,x_4}), (e_3,{x_2})}. Then, (X,τ,E) is a soft topological space and (G,A) = { (e_1,{ x_1}),(e_2,{x_2, x_4}),(e_3,{x_2})} is a soft *e*-open set.

Theorem 3.7. For a soft set (F,A) in a soft topological space (X,τ,E)

(i) (F,A) is a soft *e*-open set iff $(F,A)^c$ is a soft *e* -closed set.

(ii) (F,A) is a soft *e*-closed set iff $(F,A)^c$ is a soft *e*-open set.

Proof. Obvious from the Definition 3.5.

Definition 3.8. Let (X,τ,E) be a soft topological space and (F,A) be a soft set over X.
(i) Soft *e*-closure of a soft set (F,A) in X is denoted by S*e*-Cl(F,A)= $\tilde{\cap}$ {(H,A) $\tilde{\supseteq}$ (F,A): (H,A) is a soft *e*-closed set of X}.
(ii) Soft *e*-interior of a soft set (F,A) in X is denoted by S*e*-Int(F,A)= $\tilde{\cup}$ {(G,A) $\tilde{\subseteq}$ (F,A): (G,A) is a soft *e*-open set of X}.

Clearly, S*e*-Cl(F,A) is the smallest soft *e*-closed set over X which contains (F,A) and S*e*-Int(F,A) is the largest soft *e*-open set over X which is contained in (F,A).

Theorem 3.9. Let (F,A) be any soft set in a soft topological space X. Then,
(i) S*e*-Cl$(F,A)^c$ = $\tilde{X}$ -S*e*-Int(F,A).
(ii) S*e*-Int$(F,A)^c$ = $\tilde{X}$ -S*e*-Cl(F,A).

Proof. **(i)** Let soft *e*-open set (G,A) $\tilde{\subseteq}$ (F,A) and soft *e*-closed set (H,A) $\tilde{\supseteq}$ $(F,A)^c$. Then S*e*-Int(F,A)= $\tilde{\cup}$ {$(H,A)^c$: (H,A) is soft *e*-closed set and (H,A) $\tilde{\supseteq}$ $(F,A)^c$ } = $\tilde{X}$ - $\tilde{\cap}$ {(H,A): (H,A) is soft *e*-closed set and (H,A) $\tilde{\supseteq}$ $(F,A)^c$ }= $\tilde{X}$ - S*e*-Cl$(F,A)^c$.So,S*e*-Cl$(F,A)^c$ = $\tilde{X}$ -S*e*-Int(F,A).

(ii) Let (G,A) be a soft *e*-open set. Then for a soft *e*-closed set $(G,A)^c$ $\tilde{\supseteq}$ (F,A), (G,A) $\tilde{\subseteq}$ $(F,A)^c$. Now, S*e*-Cl(F,A)= $\tilde{\cap}$ {$(G,A)^c$: (G,A) is soft *e*-open set and (G,A) $\tilde{\subseteq}$ $(F,A)^c$} = $\tilde{X}$ - $\tilde{\cup}$ {(G,A): (G,A) is soft *e*-open set and (G,A) $\tilde{\subseteq}$ $(F,A)^c$ }= $\tilde{X}$ - S*e*-Int$(F,A)^c$.
So, S*e*-Int$(F,A)^c$ = $\tilde{X}$ -S*e*-Cl(F,A).

Theorem 3.10. In a soft topological space X, (F,A) be a soft *e*-closed (resp. soft *e*-open) if and only if (F,A) = S*e*-Cl(F,A) (resp. (F,A)= S*e*-Int(F,A).

Proof. Suppose (F,A) = S*e*-Cl(F,A)= $\tilde{\cap}$ {(H,A) $\tilde{\supseteq}$ (F,A): (H,A) is a soft *e*-closed set of X}.This means (F,A)∈ $\tilde{\cap}$ {(H,A) $\tilde{\supseteq}$ (F,A): (H,A) is a soft *e*-closed set of X} and hence (F,A) is soft *e*-closed set.
Conversely, suppose (F,A) be a soft *e*-closed in X. Then we have (F,A)∈ {(H,A) $\tilde{\supseteq}$ (F,A): (H,A) is a soft *e*-closed set of X}. Hence, (F,A) $\tilde{\subseteq}$ (H,A) implies (F,A) = $\tilde{\cap}$ {(H,A) $\tilde{\supseteq}$ (F,A): (H,A) is a soft *e*-closed set of X }= S*e*-Cl(F,A).
Similarly for (F,A)= S*e*-Int(F,A)).

Theorem 3.11. In a soft topological space X, the following holds for soft *e*-closure and soft *e*-interiors.

(i) S*e*-Cl ($\tilde{\varphi}$) = $\tilde{\varphi}$.

(ii) S*e*-Int($\tilde{\varphi}$) = $\tilde{\varphi}$.

(iii) S*e*-Cl(F,A) is a soft *e*-closed set in X.

(iv) S*e*-Int(F,A) is a soft *e*-open set in X.

(v) S*e*-Cl(F,A) $\tilde{\subseteq}$ S*e*-Cl(G,A) if (F,A) $\tilde{\subseteq}$ (G,A).

(vi) S*e*-Int(F,A) $\tilde{\subseteq}$ S*e*-Int(G,A) if (F,A) $\tilde{\subseteq}$ (G,A).

(vii) Se-Cl(Se-Cl(F,A)) = Se-Cl(F,A).
(viii) Se-Int(Se-Int(F,A)) = Se-Int(F,A).

Theorem 3.12. In a soft topological space X, we have
(i) Se-Cl ((F,A) $\tilde{\cup}$ (G,A)) $\tilde{\supseteq}$ Se-Cl(F,A) $\tilde{\cup}$ Se-Cl(G,A).
(ii) Se-Cl((F,A) $\tilde{\cap}$ (G,A)) $\tilde{\subseteq}$ Se-Cl(F,A) $\tilde{\cap}$ Se-Cl(G,A).

Proof. **(i)** (F,A) $\tilde{\subseteq}$ ((F,A) $\tilde{\cup}$ (G,A)) or (G,A) $\tilde{\subseteq}$ ((F,A) $\tilde{\cup}$ (G,A)) this implies Se-Cl(F,A) $\tilde{\subseteq}$ Se-Cl((F,A) $\tilde{\cup}$ (G,A)) or Se-Cl (G,A) $\tilde{\subseteq}$ Se-Cl((F,A) $\tilde{\cup}$ (G,A)).Therefore Se-Cl((F,A) $\tilde{\cup}$ (G,A)) $\tilde{\supseteq}$ Se-Cl(F,A) $\tilde{\cup}$ Se-Cl(G,A).

(ii) Similar to the proof of (i).

Theorem 3.13. In a soft topological space X, we have
(i) Se-Int((F,A) $\tilde{\cup}$ (G,A)) $\tilde{\supseteq}$ Se-Int(F,A) $\tilde{\cup}$ Se-Int(G,A)
(ii) Se-Int((F,A) $\tilde{\cap}$ (G,A)) $\tilde{\subseteq}$ Se-Int(F,A) $\tilde{\cap}$ Se-Int(G,A)

Proof. Same as the proof of theorem 3.12.

Theorem 3.14. Let (F,A) be soft e-open set,
(i) If Int_δ(F,A) = $\tilde{\varphi}$, then (F,A) is soft δ-preopen set.
(ii) If Cl_δ(F,A) = $\tilde{\varphi}$, then (F,A) is soft δ-semiopen set.

Proof. Ovious from definition 3.5.

Lemma 3.15. Let (F,A) be a soft subset of X, then
(i) $SSCl_\delta$(F,A) = (F,A) $\tilde{\cup}$ Int(Cl_δ(F,A)) and $SSInt_\delta$(F,A) = (F,A) $\tilde{\cap}$ Cl(Int_δ(F,A))
(ii) $SPCl_\delta$(F,A) = (F,A) $\tilde{\cup}$ Cl(Int_δ(F,A))and $SPInt_\delta$(F,A) = (F,A) $\tilde{\cap}$ Int(Cl_δ(F,A)).

Proof. **(i)** $SSCl_\delta$(F,A) $\tilde{\supseteq}$ Int(Cl_δ($SSCl_\delta$(F,A))) $\tilde{\supseteq}$ Int(Cl_δ(F,A))
⇒ (F,A) $\tilde{\cup}$ $SSCl_\delta$(F,A) $\tilde{\supseteq}$ (F,A) $\tilde{\cup}$ Int(Cl_δ(F,A))
So, (F,A) $\tilde{\cup}$ Int(Cl_δ(F,A)) $\tilde{\subseteq}$ $SSCl_\delta$(F,A)…………......(i)
Also, (F,A) $\tilde{\subseteq}$ $SSCl_\delta$(F,A)
⇒ Int(Cl_δ(F,A)) $\tilde{\subseteq}$ Int(Cl_δ($SSCl_\delta$(F,A))) $\tilde{\subseteq}$ $SSCl_\delta$(F,A)
⇒ (F,A) $\tilde{\cup}$ Int(Cl_δ(F,A)) $\tilde{\subseteq}$ $SSCl_\delta$(F,A) $\tilde{\cup}$ $SSCl_\delta$(F,A)= $SSCl_\delta$(F,A)……(ii)
Hence, from (i) and (ii), $SSCl_\delta$(F,A) = (F,A) $\tilde{\cup}$ Int(Cl_δ(F,A)).

(ii) Follows immediately from (i) by taking the complements.

Theorem 3.16. For any soft subset (F,A) of a space X, (F,A) is soft e-open if and only if (F,A) = $SPInt_\delta$(F,A) $\tilde{\cup}$ $SSInt_\delta$(F,A).

Proof. Let (F,A) be soft e-open. Then (F,A) $\tilde{\subseteq}$ Int(Cl_δ(F,A)) $\tilde{\cup}$ Cl(Int_δ(F,A))).By above lemma 3.15, we have, $SPInt_\delta$(F,A) $\tilde{\cup}$ $SSInt_\delta$(F,A) = [(F,A) $\tilde{\cap}$ Int(Cl_δ(F,A))] $\tilde{\cup}$ [(F,A) $\tilde{\cap}$ Cl(Int_δ(F,A))]= (F,A) $\tilde{\cap}$ (Int(Cl_δ(F,A))] $\tilde{\cup}$ Cl(Int_δ(F,A))) = (F,A).

Conversely, if $(F,A) = SPInt_\delta(F,A) \tilde{\cup} SSInt_\delta(F,A)$, then by above lemma 3.15, $(F,A) = SPInt_\delta(F,A) \tilde{\cup} SSInt_\delta(F,A) = [(F,A) \tilde{\cap} Int(Cl_\delta(F,A))] \tilde{\cup} [(F,A) \tilde{\cap} Cl(Int_\delta(F,A))] = (F,A) \tilde{\cap} (Int(Cl_\delta(F,A))) \tilde{\cup} Cl(Int_\delta(F,A))) \tilde{\subseteq} (Int(Cl_\delta(F,A))) \tilde{\cup} Cl(Int_\delta(F,A)))$
and hence (F,A) is soft *e*-open set.

Theorem 3.17. Let (F,A) be a soft subset of a space X, then, $Se\text{-}Cl(F,A) = SPCl_\delta(F,A) \tilde{\cap} SSCl_\delta(F,A)$.

Proof. It is obvious that, $Se\text{-}Cl(F,A) \tilde{\subseteq} SPCl_\delta(F,A) \tilde{\cap} SSCl_\delta(F,A)$.
Conversely, from definition we have, $Se\text{-}Cl(F,A) \tilde{\supseteq} [Int(Cl_\delta(Se\text{-}Cl(F,A)) \tilde{\cap} Cl(Int_\delta(Se\text{-}Cl(F,A))] \tilde{\supseteq} Cl(Int_\delta(F,A)) \tilde{\cap} Int(Cl_\delta((F,A))$. Since $Se\text{-}Cl(F,A)$ is soft e-closed, by lemma 3.15, we have, $SPCl_\delta(F,A) \tilde{\cap} SSCl_\delta(F,A) = [(F,A) \tilde{\cup} Cl(Int_\delta(F,A))] \tilde{\cap} [(F,A) \tilde{\cup} Int(Cl_\delta(F,A))] = [(F,A) \tilde{\cup} (Cl(Int_\delta(F,A)) \tilde{\cap} Int(Cl_\delta(F,A)))] = (F,A) \tilde{\subseteq} Se\text{-}Cl(F,A)$.

Theorem 3.18. Let (F,A) be a soft subset of a space X, then, $Se\text{-}Int(F,A)= SPInt_\delta(F,A) \tilde{\cap} SSInt_\delta(F,A)$.

Proof. It is similar to the above proof.

Theorem 3.19. In a Soft topological space X, we have the followings:
(i) Arbitrary union of Soft *e*-open sets is a Soft *e*-open set, and
(ii) Arbitrary intersection of Soft *e*-closed sets is a Soft *e*-closed set.

Proof. **(i)** Let $\{(F,A)_\alpha : \alpha \in \Lambda, \text{an index set}\}$ be a collection of Soft *e*-open sets. Then for each α, $(F,A)_\alpha \tilde{\subseteq} [Int(Cl_\delta((F,A)_\alpha)) \tilde{\cup} Cl(Int_\delta((F,A)_\alpha))]$.
Taking union of all such relations we get,
$\tilde{\cup}\{(F,A)_\alpha\} \tilde{\subseteq} \tilde{\cup} [Int(Cl_\delta((F,A)_\alpha)) \tilde{\cup} Cl(Int_\delta((F,A)_\alpha))]$
$\tilde{\subseteq} [Int(Cl_\delta(\tilde{\cup} (F,A)_\alpha)) \tilde{\cup} Cl(Int_\delta(\tilde{\cup} (F,A)_\alpha))]$.
Thus $\tilde{\cup}(F,A)_\alpha$ is Soft *e*-open set.
(ii) Follows immediately from (i) by taking the complements.

Definition 3.20 [10]. Let τ_S be the collection of soft sets over X, then τ_S is said to be a soft supra topology on X if it satisfies the following axioms.
(1) $\tilde{\varphi}, \tilde{X}$ belong to τ_S.
(2) The union of any number of soft sets in τ_S belongs to τ_S.
The triplet (X,τ_S,E) is called a soft supratopological space over X.
Now we give the following property for soft *e*-open sets.

Proposition 3.21. The collection $\tau_S = S.e.OS(X)$ of all soft *e*-open sets form a soft supra topology over a soft space (X,τ,E).

Proof. **(1)** is obvious.

(2) Let $(F,E)_\alpha \in \tau_S$, $\forall \alpha \in \Lambda = \{1, 2, 3.....\}$. Then, for $\forall \alpha \in \Lambda$, $(F,E)_\alpha \tilde{\subseteq} Int(Cl_\delta((F,A)_\alpha)) \tilde{\cup} Cl(Int_\delta((F,A)_\alpha))$.Taking union of all such relations we get,
$\tilde{\cup}\{(F,A)_\alpha\} \tilde{\subseteq} \tilde{\cup} [Int(Cl_\delta((F,A)_\alpha)) \tilde{\cup} Cl(Int_\delta((F,A)_\alpha))]$
$\tilde{\subseteq} [Int(Cl_\delta(\tilde{\cup}(F,A)_\alpha)) \tilde{\cup} Cl(Int_\delta(\tilde{\cup}(F,A)_\alpha))]$.

This implies that $\tilde{\cup}(F,A)_\alpha$ is Soft *e*-open set and hence, $\tilde{\cup}(F,A)_\alpha \in \tau_S$.

Remark 3.22. In a soft topological space it is obvious that
(i) Every soft regular open set is soft δ-open set.
(iii) Every soft δ-open set is both soft δ-semi-open and soft δ-pre-open set.
(iii) Every soft δ-semi-open set and every soft δ-pre-open set is soft *e*-open set.

Let (X,τ,E) be a soft topological space. Then, the family of all soft *e*-open set (resp. soft open,soft regular open, soft δ-open, soft δ-semi-open, soft δ-pre-open) sets in X may be denoted by S*e*-OS(X) (resp. SOS(X), SROS(X), SδOS(X), SδSOS(X), SδPOS(X)).The family of all soft regular closed (resp. soft δ-closed, soft δ-semi-closed, soft δ-pre-closed) sets in X may be denoted by S*e*-CS(X) (resp.SRCS(X), SδCS(X), SδSCS(X) , SδPCS(X). Thus we have implications as shown in Figure 1.

Soft regular open
↓
Soft δ-open → Soft open → soft semi-open → soft γ-open → soft β-open
↓ ↓
Soft δ-pre open Soft δ-semi open
↓ ↓
Soft *e*-open set

Figure-1

The examples given below show that the converses of these implications are not true.

Example 3.23. Let X={x_1, x_2, x_3, x_4},E ={e_1, e_2, e_3} and τ ={ $\tilde{\varphi}$, $\tilde{X}$, (F_1,E),(F_2,E), (F_3,E), (F_4,E), (F_5,E), (F_6,E), (F_7,E), (F_8,E), (F_9,E), (F_{10},E), (F_{11},E), (F_{12},E), (F_{13},E)} where, (F_1,E),(F_2,E), (F_3,E), (F_4,E), (F_5,E), (F_6,E), (F_7,E), (F_8,E), (F_9,E), (F_{10},E), (F_{11},E), (F_{12},E), and (F_{13},E) are soft sets over X, defined as follows:
(F_1,E) ={(e_1,{ x_1}),(e_2,{x_2,x_3}),(e_3,{x_1,x_3})};
(F_2,E) ={(e_1,{x_2,x_4}),(e_2,{ x_1,x_3,x_4}),(e_3,{ x_1, x_2, x_4})};
(F_3,E) ={ (e_1, φ),(e_2,{x_3}),(e_3,{ x_1})};
(F_4,E) ={ (e_1, {x_1,x_2,x_4}),(e_2, X),(e_3,X)};
(F_5,E) ={ (e_1,{x_1,x_3}),(e_2,{x_2, x_4}),(e_3,{x_2})};
(F_6,E) = {(e_1,φ),(e_2,{x_2}),(e_3, φ)};
(F_7,E) ={ (e_1,{ x_1,x_3}),(e_2,{ x_2, x_3, x_4}),(e_3,{ x_1, x_2,x_3})};
(F_8,E) ={ (e_1,{x_3}),(e_2,{ x_4}),(e_3,{x_2})};
(F_9,E) ={ (e_1,X),(e_2,X),(e_3,{ x_1, x_2, x_3})};
(F_{10},E) ={ (e_1,{ x_1,x_3}),(e_2,{ x_2, x_3, x_4}),(e_3,{ x_1, x_2})};
(F_{11},E) ={ (e_1,{ x_3}),(e_2,{ x_2, x_4}),(e_3,{x_2})};
(F_{12},E) ={ (e_1,{x_1}),(e_2,{x_2}),(e_3, φ)};
(F_{13},E) = { (e_1,{x_1}),(e_2,{ x_2, x_3}),(e_3,{x_1})}.

Then, τ defines a soft topology on X, and thus (X,τ, E) is a soft topological space over X. Clearly, soft closed sets are $\tilde{\varphi}$, $\tilde{X}$, $(F_1, E)^c$, $(F_2, E)^c$, $(F_3, E)^c$, $(F_4, E)^c$, $(F_5, E)^c$, $(F_6, E)^c$, $(F_7,E)^c$, $(F_8,E)^c$, $(F_9, E)^c$, $(F_{10},E)^c$, $(F_{11}, E)^c$, $(F_{12}, E)^c$ and $(F_{13}, E)^c$.

Now, consider the soft set (G,E)={ $(e_1,\{x_1,x_2\}),(e_2,\{x_2,x_3\}),(e_3,\{x_1,x_3\})$}, then, $[Int(Cl_\delta(G,E)) \tilde{\cup} Cl(Int_\delta(G,E))]=(F_8,E)^c \tilde{\supseteq} (G,E)$.So,$(G,E) \tilde{\subseteq} Int(Cl_\delta(G,E)) \tilde{\cup} Cl(Int_\delta(G,E))$. Thus, (G,E) is soft *e*-open set but since, $Int(Cl_\delta(G,E))=(F_1,E)$ which does not contain (G,E). So (G,E) is not soft δ-pre open set. Also it is clear that (G,E) is neither soft δ-open nor soft regular open nor soft open nor soft semi-open nor γ-open nor β-open set.

Again, consider the soft set (F,E) $=\{(e_1,\{x_4\}),(e_2,\{x_1, x_3\}),(e_3,\{x_1,x_2\})\}$, then $[Int(Cl_\delta(F,E)) \tilde{\cup} Cl(Int_\delta(F,E))]=\{(e_1,\{x_2,x_4\}), (e_2,\{x_1,x_3,x_4\}),(e_3,X)\}$ and so,$(F,A) \tilde{\subseteq} [Int(Cl_\delta(F,E)) \tilde{\cup} Cl(Int_\delta(F,E))]$. Thus, (F,E) is soft *e*-open set but since $Cl(Int_\delta(F,E)) = (F_5,E)^c$ which does not contain (F,E). So (F,E) is not soft δ-semi open set. Also it is clear that (F,E) is neither soft δ-open nor soft regular open nor soft open nor soft semi-open nor γ-open nor β-open set.

Remark 3.24. The intersection of two soft *e*-open sets need not be soft *e*-open set as is illustrated by the following example.

Example 3.25. Let (X,τ,E) be a soft topological space defined in Example 3.23. Now we consider two soft sets (G,E) and (H,E) in (X,τ,E) defined as follows:
$(G,E)=\{(e_1,\{x_1\}),(e_2,\{x_2\})\}$;$(H,E)=\{(e_1,\{x_1\}),(e_2,\{x_3\}),(e_3,\{x_3\})\}$.
Then, (G,E) and (H,E) are soft *e*-open sets over X, therefore, $(G,E) \tilde{\cap} (H,E) =\{(e_1,\{x_1\})\}=(K,E)$ and $Int(Cl_\delta(K,E)) \tilde{\cup} Cl(Int_\delta(K,E)) =\{(e_1,\{x_1\}), (e_2,\{x_2\})\} \tilde{\not\supseteq} (K,E)$. Hence, K,E) is not a soft *e*-open set.

Remark 3.26. The union of two soft *e*-closed sets need not be soft *e*-closed set as is illustrated by the following example.

Example 3.27. Let (X,τ,E) be a soft topological space defined in Example 3.23. Now we consider two soft sets (A,E) and (B,E) in (X,τ,E) defined as follows:

$(A,E)=\{(e_1,\{x_2,x_3,x_4\}),(e_2,\{x_1, x_3, x_4\}),(e_3,X)\}$;$(B,E)=\{(e_1,\{x_2, x_3, x_4\}),(e_3,\{x_1, x_2,x_4\}),(e_3, \{x_1, x_2,x_4\})\}$.Then, (A,E) and (B,E) are soft *e*-closed sets over X , therefore, $(A,E) \tilde{\cup} (B,E) = \{(e_1,\{x_2, x_3, x_4\}),(e_3,X), (e_3, X)\} = (C,E)$ and $Int(Cl_\delta(C,E)) \tilde{\cap} Cl(Int_\delta(C,E) = X \tilde{\not\subseteq} (C,E)$. Hence, (C,E) is not a soft *e*-closed set.

Theorem 3.28. In a soft topological space X,
(i) Every soft δ-pre-open set is soft *e*-open set.
(ii) Every soft δ-semi-open set is *e*-open set.

***Proof* . (i)** Let (F,A) be a soft δ-pre-open set in a soft topological space X.
Then, $(F,A) \tilde{\subseteq} Int(Cl_\delta(F,A))$ which implies that
$(F,A) \tilde{\subseteq} [Int(Cl_\delta(F,A)) \tilde{\cup} Int_\delta(F,A))] \tilde{\subseteq} [Int(Cl_\delta(F,A)) \tilde{\cup} Cl(Int_\delta(F,A))]$
Thus (F,A) is soft *e*-open set.

(ii) Let (F,A) be a soft δ-semi-open set in a soft topological space X.
Then, $(F,A) \tilde{\subseteq} Cl(Int_\delta(F,A))$ which implies that
$(F,A) \tilde{\subseteq} [Cl(Int_\delta(F,A)) \tilde{\cup} Int(F,A))] \tilde{\subseteq} [Cl(Int_\delta(F,A)) \tilde{\cup} Int(Cl_\delta(F,A))]$
Thus (F,A) is soft *e*-open set.

4. Soft *e*-continuity and Soft *e*-homeomorphisms

In this section, we introduce soft *e*-continuous maps, soft *e*-irresolute maps, soft e-closed maps, soft e-open maps and soft *e*-homeomorphisms. We also study some of their properties and separation axioms with the help of soft *e*-open sets.

Definition 4.1 [14]. Let (X,E) and (Y,K) be two soft classes. Let u : $X \to Y$ and p : E → K be mappings.Then a mapping *f* : (X,E) → (Y,K) is defined as follows: for a soft set (F,A) in (X,E) , (*f*(F,A),B), B = p(A) $\tilde{\subseteq}$ K is a soft set in (Y,K) given by

f(F,A)(β) = u$\begin{pmatrix} \cup F(\alpha) \\ \alpha \in p^{-1}(\beta) \cap A \end{pmatrix}$ β∈K and (*f*(F,A),B) is called a soft image of a soft set (F,A) . If B=K, then we will write (*f(F,A)*,K) as *f(F,A)*.

Definition 4.2 [14]. Let *f* : (X,E) → (Y,K) be a mapping from a soft class (X,E) to another soft class (Y,K), and (G,C) a soft set in soft class (Y,K), where, C ⊆ K. Let u : $X \to Y$ and p : E →K be mappings.Then (f^{-1}(G,C),D), D=p^{-1}(C), is a soft set in the soft classes(X,E), defined as (f^{-1}(G,C)(α) = u^{-1}(G(p(α))) for α∈D ⊆ E. (f^{-1}(G,C),D) is called a soft inverse image of (G,C). Hereafter, we shall write, (f^{-1}(G,C),E) as (f^{-1}(G,C).

Theorem 4.3 [14]. Let *f* : (X,E) → (Y,K) ; u : $X \to Y$ and p : E →K be mappings. Then for soft sets (F,A), (G,B) and a family of soft sets {(F_α,A_α):α∈Λ,an index set} in the soft class (X,E), we have:

(1) $f(\tilde{\varphi}) = \tilde{\varphi}$,

(2) $f(\tilde{X}) = \tilde{Y}$,

(3) f((F,A) $\tilde{\cup}$ (G,B)) = f(F,A) $\tilde{\cup}$ f(G,B), in general, $f(\bigcup_{\alpha\in\Lambda}(F_\alpha, A_\alpha)) = (\bigcup_{\alpha\in\Lambda} f(F_\alpha, A_\alpha))$,

(4) f((F,A) $\tilde{\cap}$ (G,B)) $\tilde{\subseteq}$ f(F,A) $\tilde{\cap}$ f(G,B), in general, $f(\bigcap_{\alpha\in\Lambda}(F_\alpha, A_\alpha)) \tilde{\cap} (\bigcap_{\alpha\in\Lambda} f(F_\alpha, A_\alpha))$,

(5) If (F,A) $\tilde{\subseteq}$ (G,B) then, f(F,A) $\tilde{\subseteq}$ f(G,B),

(6) $f^{-1}(\tilde{\varphi}) = \tilde{\varphi}$,

(7) $f^{-1}(\tilde{Y}) = \tilde{X}$,

(8) f^{-1}((F,A) $\tilde{\cup}$ (G,B))=f^{-1}(F,A) $\tilde{\cup}$ f^{-1}(G,B),ingeneral, $f^{-1}(\bigcup_{\alpha\in\Lambda}(F_\alpha, A_\alpha))=(\bigcup_{\alpha\in\Lambda} f^{-1}(F_\alpha, A_\alpha))$,

(9) f^{-1}((F,A) $\tilde{\cap}$ (G,B))=f^{-1}(F,A) $\tilde{\cap}$ f^{-1}(G,B),ingeneral, $f^{-1}(\bigcap_{\alpha\in\Lambda}(F_\alpha, A_\alpha)) \tilde{\subseteq} (\bigcap_{\alpha\in\Lambda} f^{-1}(F_\alpha, A_\alpha))$,

(10) If (F,A) $\tilde{\subseteq}$ (G,B) then, f^{1} (F,A) $\tilde{\subseteq}$ f^{1} (G,B),

Definition 4.4. A mapping f :(X,τ_1,E) → (Y,τ_2,K) is said to be a soft δ-pre continuous if f^{-1}(F,A) is soft δ-pre open in X for every soft open set (F,A) in Y.

Definition 4.5. A mapping f :(X,τ_1,E) → (Y,τ_2,K) is said to be a soft δ-semi continuous if f^{-1}(F,A) is soft δ-semi open in X for every soft open set (F,A) in Y.

Definition 4.6. A mapping f :(X,τ_1,E) $\rightarrow$ (Y,τ_2,K) is said to be a soft *e*-continuous if f^{-1}(F,A) is soft *e*-open in X for every soft open set (F,A) in Y.

Definition 4.7. A mapping f :(X,τ_1,E) $\rightarrow$ (Y,τ_2,K) is said to be a soft *e*-irresolute if f^{-1}(F,A) is soft *e*-open in X for every soft *e*-open set (F,A) in Y.

Remark 4.8. It is clear that every soft δ-pre continuous map and soft δ-semi continuous map is soft *e*-continuous. Thus we have implications as shown in Figure 2.
The converses of these implications are not necessarily true, which is clear from the following examples.

Soft δ-pre continuous   Soft δ-semi continuous
↓   ↓
Soft *e*-continuous

Figure-2

Example 4.9. Let X={x_1, x_2, x_3, x_4}, Y={y_1, y_2, y_3, y_4}, E ={e_1, e_2, e_3}, K={k_1, k_2, k_3}and (X,τ,E) and (Y,ν,K) be soft topological spaces. Let f_{up}: (X,τ,E) $\rightarrow$ (Y,ν,K) be a soft mapping. Define u : X $\rightarrow$ Y and P : E $\rightarrow$ K as u(x_1) = y_2, u(x_2) = y_3, u(x_3) = y_4, u(x_4) = y_1, and p(e_1) = k_2, p(e_2) = k_1, p(e_3) = k_3;

Let us consider the soft topology τ in X given in Example 3.23; that is,τ ={ $\tilde{\varphi}, \tilde{X}$, (F_1,E), (F_2, E), (F_3,E), (F_4,E), (F_5,E), (F_6,E), (F_7,E), (F_8,E), (F_9,E), (F_{10},E), (F_{11},E), (F_{12},E), (F_{13},E)} and soft topology ν ={ $\tilde{\varphi}, \tilde{Y}$, (G,K) = { (k_1,{ y_1,y_2}),(k_2,{ y_2, y_3}),(k_3,{y_1, y_3})}} in Y. Then (G,K) is a soft open in Y and f_{up}^{-1}(G,K) = {(e_1,{x_1,x_2}), (e_2,{x_2, x_3}),(e_3,{ x_1, x_3})} is soft *e*-open but not soft δ-pre open in X. Therefore, f_{up} is a soft *e*-continuous but not soft δ-pre continuous function.

Example 4.10. Let X={x_1, x_2, x_3, x_4}, Y={y_1, y_2, y_3, y_4}, E ={e_1, e_2, e_3}, K={k_1, k_2, k_3}and (X,τ,E) and (Y,ν,K) be soft topological spaces. Let f_{up}: (X,τ,E) $\rightarrow$ (Y,ν,K) be a soft mapping. Define u : X $\rightarrow$ Y and P : E $\rightarrow$ K as u(x_1) = y_3, u(x_2) = y_1, u(x_3) = y_4, u(x_4) = y_2, and p(e_1) = k_2, p(e_2) = k_1, p(e_3) = k_3;

Let us consider the soft topology τ in X given in Example 3.23; that is,τ ={ $\tilde{\varphi}, \tilde{X}$, (F_1,E), (F_2,E), (F_3,E), (F_4,E), (F_5,E), (F_6,E), (F_7,E), (F_8,E), (F_9,E), (F_{10},E), (F_{11},E), (F_{12},E), (F_{13},E)} and soft topology ν={ $\tilde{\varphi}, \tilde{Y}$, (H,K) = {(k_1,{ y_3,y_4}),(k_2,{ y_2}), (k_3,{ y_1,y_3})}} in Y. Then (H,K) is a soft open in Y and f_{up}^{-1}(H,K)= {(e_1,{ x_4}),(e_2,{ x_1, x_3}),(e_3,{ x_1, x_2})} is soft *e*-open but not soft δ-semi open in X. Therefore, f_{up} is a soft *e*-continuous but not soft δ-semi continuous function.

Theorem 4.11. For a mapping f :(X,τ_1,E) $\rightarrow$ (Y,τ_2,K), the following statements are equivalent
(i) f is a soft *e*-continuous.

(ii) For every soft singleton $P_\lambda^F \in$ X and every soft open set (F,A) in Y such that f(P_λ^F) $\tilde{\subseteq}$(F,A), ∃ a soft e-open set (G,A) in X such that $P_\lambda^F \in$(G,A) and f((G,A))$\tilde{\subseteq}$ (F,A).
(iii) f^{-1}(F,A) = Int(Cl$_\delta$(f^{-1}(F,A)))$\tilde{\cup}$Cl(Int$_\delta$(f^{-1}(F,A))) for each soft open set (F,A) in Y.
(iv) The inverse image of each soft closed set in Y is soft *e*-closed.
(v) Int(Cl$_\delta$(f^{-1}(F,A)))$\tilde{\cap}$Cl(Int$_\delta$(f^{-1}(F,A))) $\tilde{\subseteq}$ f^{-1}((Cl(F,A))) for each soft set (F,A) $\tilde{\subseteq}$Y.
(vi) f[Cl(Int$_\delta$(G,A))$\tilde{\cap}$Int(Cl$_\delta$(G,A))]$\tilde{\subseteq}$Cl(f(G,A)) for every soft set (G,A) in X.

Proof. **(i)⇒(ii):** Let the singleton set P_λ^F in X and every soft open set (F,A) in Y such that f(P_λ^F)∈ (F,A). Since f is soft *e*-continuous. Then $P_\lambda^F \in f^{-1}$(f(P_λ^F))$\tilde{\subseteq}$ f^{-1}(F,A) . Let (G,A) = f^{-1}(F,A) which is a soft *e*-open set in X. So, we have $P_\lambda^F \in$ (G,A). Now f(G,A) = f(f^{-1}(F,A)) $\tilde{\subseteq}$ (F,A).

(ii)⇒(iii): Let (F,A) be any soft open set in Y. Let P_λ^F be any soft point in X such that f(P_λ^F)$\tilde{\subseteq}$(F,A). Then $P_\lambda^F \in f^{-1}$(F,A). By(ii), there exists a soft e-open set (G,A) in X such that $P_\lambda^F \in$ (G,A) and f((G,A)) $\tilde{\subseteq}$ (F,A). Therefore, $P_\lambda^F \in$(G,A)$\tilde{\subseteq}$ f^{-1}(f((G,A)))$\tilde{\subseteq}$$f^{-1}$(F,A) $\tilde{\subseteq}$ Int(Cl$_\delta$(f^{-1}(F,A)) $\tilde{\cup}$ Cl(Int$_\delta$(f^{-1}(F,A)).

(iii)⇒(iv): Let (F,A) be any soft closed set in Y. Then $\tilde{Y}$ -(F,A) be a soft open set in Y. By (iii), (f^{-1}($\tilde{Y}$- (F,A))) $\tilde{\subseteq}$ Int(Cl$_\delta$(f^{-1}($\tilde{Y}$- (F,A)))) $\tilde{\cup}$ Cl(Int$_\delta$(f^{-1}($\tilde{Y}$-(F,A)))).This implies $\tilde{X}$ -(f^{-1}(F,A)) $\tilde{\subseteq}$ Int(Cl$_\delta$($\tilde{X}$ -f^{-1}(F,A)))$\tilde{\cup}$Cl(Int$_\delta$($\tilde{X}$ -f^{-1}(F,A))) $\tilde{\subseteq}$ Int($\tilde{X}$ - Cl$_\delta$(f^{-1}(F,A))) $\tilde{\cup}$ Cl($\tilde{X}$ -Int$_\delta$(f^{-1}(F,A))) $\tilde{\subseteq}$ [$\tilde{X}$ -Int(Cl$_\delta$(f^{-1}(F,A))] $\tilde{\cup}$ [$\tilde{X}$ -Cl(Int$_\delta$(f^{-1}(F,A))] and hence $\tilde{X}$ -(f^{-1}(F,A)) $\tilde{\subseteq}$ $\tilde{X}$ -[Int(Cl$_\delta$(f^{-1}(F,A))$\tilde{\cap}$Cl(Int$_\delta$(f^{-1}(F,A))].Hence (f^{-1}(F,A)) $\tilde{\supseteq}$[Int (Cl$_\delta$(f^{-1}(F,A))$\tilde{\cap}$Cl(Int$_\delta$(f^{-1}(F,A))] and this implies that f^{-1}(F,A) is soft *e*-closed in X.

(iv)⇒(v): Let (F,A) $\tilde{\subseteq}$Y. Then f^{-1}(Cl(F,A)) is soft *e*-closed in X. Now, [Int(Cl$_\delta$(f^{-1} (F,A)))$\tilde{\cap}$Cl(Int$_\delta$(f^{-1}(F,A)))]$\tilde{\subseteq}$[Int(Cl$_\delta$(f^{-1}(Cl(F,A))))$\tilde{\cap}$Cl(Int$_\delta$(f^{-1}(Cl(F,A))))]$\tilde{\subseteq}$$f^{-1}$(Cl (F,A)).

(v)⇒(vi): Let (G,A) $\tilde{\subseteq}$ X. Put (F,A) = f(G,A) in **(v)**. Then, [Int(Cl$_\delta$(f^{-1}(f(G,A)))) $\tilde{\cap}$ Cl (Int$_\delta$(f^{-1}(f(G,A))))] $\tilde{\subseteq}$$f^{-1}$(Cl($f$(G,A))).This implies that [Int(Cl$_\delta$(G,A))$\tilde{\cap}$Cl(Int$_\delta$(G,A))] $\tilde{\subseteq}$ f^{-1} (Cl(f((G,A)))) ⇒ f[Int(Cl$_\delta$(G,A))$\tilde{\cap}$Cl(Int$_\delta$(G,A))] $\tilde{\subseteq}$ Cl(f(G,A)).

(vi)⇒(i): Let (G,A) $\tilde{\subseteq}$ Y be soft open set. Put (G,A) = f^{-1}(F,A) and (F,A) =$\tilde{Y}$ -(G,A) then f[Int(Cl$_\delta$(f^{-1}(F,A)))$\tilde{\cap}$Cl(Int$_\delta$(f^{-1}(F,A)))] $\tilde{\subseteq}$ Cl(f(f^{-1}(F,A))) $\tilde{\subseteq}$ Cl(F,A) = (F,A). That is, f^{-1}(F,A) is soft *e*-closed in X, so f is soft *e*-continuous.

Theorem 4.12. Every soft *e*-irresolute mapping is soft *e*-continuous mapping.

Proof. Let f :(X,τ_1,E) → (Y,τ_2,K) is soft *e*-irresolute mapping. Let (F,K) be a soft closed set in Y, then (F,K) is soft *e*-closed set in Y. Since f is soft *e*-irresolute mapping, f^{1}(F,K) is a soft *e*-closed set in X. Hence, f is soft *e*-continuous mapping.

Theorem 4.13. If f : $(X,\tau_1,E) \to (Y,\tau_2,K)$ be soft e-continuous function and g : $(Y,\tau_2,K) \to (Z,\tau_3,L)$ be soft continuous function. Then gof: $(X,\tau_1,E) \to (Z,\tau_3,L)$ is also soft e-continuous function.

Proof. Let (F,A) be a soft open set in Z. Now, $(gof)^{-1}(F,A) = (f^{-1}og^{-1})(F,A) = (f^{-1}(g^{-1}(F,A))$.Since g is soft continuous, $g^{-1}(F,A)$ is soft open & then $(gof)^{-1}(F,A) = f^{1}$(soft open in Y). But f being soft e-continuous $(gof)^{-1}(F,A)$ is soft e-open set in X. Thus gof is soft e-continuous function.

Theorem 4.14. If f :$(X,\tau_1,E) \to (Y,\tau_2,K)$ be soft e-irresolute function and g : $(Y,\tau_2,K) \to (Z,\tau_3,L)$ be soft e-continuous function. Then gof: $(X,\tau_1,E) \to (Z,\tau_3,L)$ is also soft e-continuous function.

Proof. Let (F,A) be a soft open set in Z. Now, $(gof)^{-1}(F,A) = (f^{-1}og^{-1})(F,A) = (f^{-1}(g^{-1}(F,A))$.Since g is soft e-continuous, $g^{-1}(F,A)$ is soft e-open & then $(gof)^{-1}(F,A) = f^{-1}$(soft e-open in Y). But f being e-irresolute, $(gof)^{-1}(F,A)$ is soft e-open set in X. Thus gof is soft e-continuous function.

Theorem 4.15. Composition of two soft e-irresolute function is again a soft e-irresolute function.

Proof. Straight forward.

Definition 4.16. A mapping $f : X \to Y$ is said to be soft e-open (briefly se-open) map if the image of every soft open set in X is soft e-open set in Y.

Definition 4.17. A mapping $f : X \to Y$ is said to be soft e-closed (briefly se-closed) map if the image of every soft closed set in X is soft e-closed set in Y.

Theorem 4.18. If $f : X \to Y$ is soft closed function and $g : Y \to Z$ is soft e-closed function, then gof is soft e-closed function.

Proof. For a soft closed set (F,A) in X, f (F,A) is soft closed set in Y. Since g : Y → Z is soft e-closed function, $g(f$ (F,A)) is soft e-closed set in Z. $g(f$ (F,A)) = (gof) (F,A) is soft e-closed set in Z. Therefore, gof is soft e-closed function.

Theorem 4.19. A map $f : X \to Y$ is soft e-closed if and only if for each soft set (H,K) of Y and for each soft open set (F,A) such that $f^{1}(H,K) \tilde{\subseteq} (F,A)$, there is a soft e-open set (G,K) of Y such that $(H,K) \tilde{\subseteq} (G,K)$ and $f^{1}(G,K) \tilde{\subseteq} (F,A)$.

Proof. Suppose f is soft e-closed map. Let (H,K) be a soft set of Y, and (F,A) be a soft open set of X, such that $f^{1}(H,K) \tilde{\subseteq} (F,A)$. Then $(G,K) = (f((F,A)^c))^c$ is a soft e-open set in Y such that $(H,K) \tilde{\subseteq} (G,K)$ and $f^{1}(G,K) \tilde{\subseteq} (F,A)$.

Conversely, suppose that (F,B) is a soft closed set of X.Then $f^{1}(f((F,B))^c) \tilde{\subseteq} (F,B)^c$, and $(F,B)^c$ is soft open set. By hypothesis, there is a soft e-open set (G,K) of Y such that $(f((F,A)^c))^c \tilde{\subseteq} (G,K)$ and $f^{1}(G,K) \tilde{\subseteq} (F,B)$, Thus $(F,B) \tilde{\subseteq} f^{1}(G,K)$. Hence $(G,K)^c \tilde{\subseteq} f(G,K) \tilde{\subseteq} f(f^{1}(G,K))^c) \tilde{\subseteq} (G,K)$ which implies $f(F,B) = (G,K)^c$. Since $(G,K)^c$ is soft e-closed set, f (F,B) is sb-closed set. So, f is a soft e-closed map.

Theorem 4.20. Let $f : X \to Y$, $g : Y \to Z$ be two maps such that $gof : X \to Z$ is sb-closed map.
(i) If f is soft continuous and surjective, then g is soft e-closed map.
(ii) If g is soft e-irresolute and injective, then f is soft e-closed map.

Proof. **(i)** Let (H,K) be a soft closed set of Y. Then, $f^{1}(H,K)$ is soft closed set in X as f is soft continuous. Since gof is soft e-closed map, $(gof)\ (f^{1}(H,K))= g(H,K)$ is soft e-closed set in Z. Hence $g : Y \to Z$ soft e-closed map.

(ii) Let (F,A) be a soft closed set in X. Then, $(gof)\ (F,A)$ is soft e-closed set in Z, and so $g^{-1}(gof)\ (F,A) = f(F,A)$ is soft e-closed set in Y. Since g is soft e-irresolute and injective. Hence, f is a soft e-closed map.

5. Applications in Separation Axioms and in Soft Group Theory

In this section e-separation axioms has been introduced and investigated with the help of soft e-open sets. Finally, we have shown that the collection Ser-h(X,τ,E) form a soft group.

Definition 5.1. A soft topological space (X,τ,E) is said to be soft e-T_1 if for each pair of distinct soft points P_λ^F and P_μ^G of X, there exists soft e-open sets (U,A) and (V,B) such that $P_\lambda^F \in (U,A)$ and $P_\mu^G \in (V,B)$, $P_\lambda^F \notin (V,B)$ and $P_\mu^G \notin (U,A)$.

Theorem 5.2. If f : X $\to$ Y is soft e-continuous injective function and Y is soft T_1, then X is soft e-T_1.

Proof. Suppose that Y is soft T_1. For any two distinct soft points P_λ^F and P_μ^G of X, there exists soft open sets (U,A) and (V,A) in Y such that $f(P_\lambda^F) \in$ (U,A), $f(P_\mu^G) \in$ (V,A), $f(P_\lambda^F) \notin$ (V,A) and $f(P_\mu^G) \notin$ (U,A). Since f is injective soft e-continuous function, we have f^{1}(U,A) and f^{1}(V,A) are soft e-open sets in X. Hence by definition X is soft e-T_1.

Definition 5.3. A soft topological space (X,τ,E) is said to be soft e-T_2 (i.e., soft e-Hausdorff) if for each pair of distinct soft points P_λ^F and P_μ^G of X, there exists disjoint soft e-open sets (U,A) and (V,B) such that $P_\lambda^F \in$ (U,A) and $P_\mu^G \in$ (V,B).

Theorem 5.4. If f : (X,τ_1, E) $\to$ (Y,τ_2, E) is soft e-continuous injective function and Y is soft T_2 then X is soft e-T_2.

Proof. Suppose that Y is soft T_2 space. For any two distinct soft points P_λ^F and P_μ^G of X, there exists disjoint soft open sets (U,A) and (V,B) in Y such that $f(P_\lambda^F) \in$ (U,A) , $f(P_\mu^G) \in$ (V,B), $f(P_\lambda^F) \notin$ (V,B) and $f(P_\mu^G) \notin$ (U,A). Since f is injective soft e-continuous function, we have f^1(U,A) and f^1(V,B) are disjoint soft e-open sets in X. Hence by definition, X is soft e-T_2.

Definition 5.5. A soft topological space (X,τ,E) is said to be soft *e*-normal if for every two disjoint soft closed sets (F,A) and (H,B) of X, there exist two disjoint soft *e*-open sets (U,A) and (V,B) such that (F,A) $\tilde{\subseteq}$ (U,A) and (H,B) $\tilde{\subseteq}$ (V,B) and (U,A) $\tilde{\cap}$ (H,B) $= \tilde{\varphi}$.

Theorem 5.6. If f : (X,τ_1, E) → (Y,τ_2, E) is soft *e*-continuous closed injective function and Y is soft normal then X is soft *e*-normal.

Proof. Suppose that Y is soft normal. Let *(F,A)* and *(H,B)* be soft closed sets in X such that $(F,A) \tilde{\cap} (H,B) = \tilde{\varphi}$. Since f is soft closed injection *f(F,A)* and *f(H,B)* are soft closed in Y and $f(F,A) \tilde{\cap}\ f(H,B) = \tilde{\varphi}$. Since Y is normal, there exists soft open sets *(U,A)* and *(V,B)* in Y such that $f(F,A) \tilde{\subseteq} U$, $f(H,B) \tilde{\subseteq} V$ and $U \tilde{\cap} V = \tilde{\varphi}$. Therefore we obtain, $(F,A) \tilde{\subseteq} f^{1}(U)$ and $(H,B) \tilde{\subseteq} f^{1}(V)$ and $f^{1}(U \tilde{\cap} V) = \tilde{\varphi}$. Since f is soft *e*-continuous, $f^{1}(U)$ and $f^{1}(V)$ are soft *e*-open sets. Hence by definition X is soft *e*-normal.

Definition 5.7. A space X is said to be soft *e*-regular if for each soft closed set (F, A) of X and each soft point $P_\lambda^F \in$ X- (F,A), there exist disjoint soft *e*-open sets (U,A) and (V,B) such that $P_\lambda^F \in$ (U,A) and (F,A) $\tilde{\subseteq}$ (V,B).

Theorem 5.8. If $f:(X,\tau_1,E) \to (Y,\tau_2,E)$ is soft *e*-continuous closed injective function and Y is soft regular then X is soft *e*-regular.

Proof. Let (F,A) be soft closed set in Y with a soft point $P_\mu^G \notin$ (F,A) Take $P_\mu^G = f(P_\lambda^F)$. Since Y is soft regular, there exists disjoint soft open sets (U,A) and (V,B) such that $P_\lambda^F \in$ (U,A) and $P_\mu^G = f(P_\lambda^F) \in f$(U,A) and (F,A) $\tilde{\subseteq}$ f(V,B) such that f(U,A) and f(V,B) are disjoint soft open sets. Therefore, we obtain that,f^{1}(F,A) $\tilde{\subseteq}$ (V,B). Since f is soft *e*-continuous, f^{1}(F,A) is soft *e*-closed set in X and $P_\lambda^F \notin f^{1}$(F,A). Hence by definition X is soft *e*-regular.

Theorem 5.9. If *(F,A)* is soft *e*-closed set in X and $f: X \to Y$ is bijective, soft continuous and soft *e*-closed , then *f(F,A)* is soft *e*-closed set in Y.

Proof. Let $f(F,A) \tilde{\subseteq} (G,B)$ where *(G,B)* is a soft open set in Y. Since f is soft continuous, $f^{1}(G,B)$ is a soft open set containing *(F,A)*. Hence, S*e*-Cl$(F,A) \tilde{\subseteq} f^{1}(G,B)$ as *(F,A)* is soft *e*-closed set. Since f is soft *e*-closed, *f(Se-Cl(F,A))* is soft *e*-closed set contained in the soft open set *(G,B)*, which implies S*e*-Cl *f(Se-Cl(F,A))* $\tilde{\subseteq}$ *(G,B)* and hence S*e*-Cl *f(F,A))* $\tilde{\subseteq}$ *(G,B)*. So *f(F,A))* is soft *e*-closed set in Y.

Definition 5.10. A soft subset (F,A) of a soft topological space (X,τ,E) is soft *e*-connected iff (F,A) can't be expressed as the union of two non empty disjoint soft *e*-open sets.

Theorem 5.11. Let $f: X \to Y$ is soft *e*-continuous and surjection map. If *(H,A)* is soft *e*-connected, then *f(H,A)* is soft connected.

Proof. Suppose that *f(H,A)* is not soft connected space. Then, ∃ non empty soft open sets (F,K) and (G,K) in Y such that *f(H,A)* = (F,A) $\tilde{\cup}$ (G,A). Since f is soft *e*-continuous, $f^{1}(F,A)$ and $f^{1}(G,A)$ are soft *e*-open set in X and (H,A) $= f^{1}[(F,A) \tilde{\cup} (G,A)] = f^{1}(F,A)\ \tilde{\cup}\ f$

${}^{1}(\mu)$. It is clear that f^{-1}(F,A) and f^{1}(G,A) are soft *e*-open set in X. Therefore, (H,A) is not soft *e*-connected, which is a contradiction to the given hypothesis. Hence, *f(H,A)* is soft connected.

Definition 5.12. A function $f: (X,\tau_1, E) \to (Y,\tau_2, E)$ is called soft *e*-homeomorphism (resp. soft *e*r-homeomorphism) if f is a soft *e*-continuous bijection (resp. soft *e*-irresolute bijection) and $f^{-1}: (Y,\tau_2, E) \to (X, \tau_1, E)$ is a soft *e*-continuous (resp.soft *e*-irresolute).
Now we can give the following definition by taking the soft space (X,τ,E) instead of the soft space (Y,τ_2, E).

Definition 5.13. For a soft topological space (X,τ,E), we define the following two collections of functions:

(a) S*e*-h$(X,\tau,E) =\{f \mid f: (X,\tau,E) \to (X,\tau,E)$ is a soft *e*-continuous bijection, $f^{-1}: (X,\tau,E) \to (X,\tau,E)$ is soft *e*-continuous$\}$.

(b) S*er*-h$(X,\tau,E) =\{ f \mid f: (X,\tau,E) \to (X,\tau,E)$ is a soft *e*-irresolute bijection, $f^{-1}: (X,\tau,E) \to (X,\tau,E)$ is soft *e*-irresolute$\}$.

Theorem 5.14. For a soft topological space (X,τ,E), S-h(X,τ,E) $\tilde{\subseteq}$ S*er*-h(X,τ,E) $\tilde{\subseteq}$ S*e*-h(X,τ,E), where, S-h$(X,\tau,E) =\{ f \mid f: (X,\tau,E) \to (X,\tau,E)$ is a soft continuous bijection, $f^{-1}: (X,\tau,E) \to (X,\tau,E)$ is soft continuous i.e. f is soft homeomorphisms$\}$.

Proof. First we show that every soft-homeomorphism $f: (X,\tau_1, E) \to (Y,\tau_2, E)$ is a soft *e*-r-homeomorphism. Let (G,A)$\in$S*e*-OS(Y), then (G,A)$\tilde{\subseteq}$Int(Cl$_\delta$(G,A))$\tilde{\cup}$Cl(Int$_\delta$(G,A)). Hence, f^{-1}(G,A) $\tilde{\subseteq} f^{-1}$[Int(Cl$_\delta$(G,A))$\tilde{\cup}$Cl(Int$_\delta$(G,A))] = Int(Cl$_\delta$(f^{-1}(G,A)) $\tilde{\cup}$ Cl(Int$_\delta$(f^{-1}(G,A)) and so f^{-1}(G,A)$\in$S*e*-OS(X). Thus, f is soft *e*-irresolute. In a similar way, it can be shown that f^{-1} is soft *e*-irresolute. Hence, we have, S-h(X,τ,E) $\tilde{\subseteq}$S*er*-h(X,τ,E).
Finally, it is obvious that S*er*-h$(X,\tau,E)\tilde{\subseteq}$S*e*-h$(X,\tau,E)$, because every soft *e*-irresolute function is soft *e*-continuous.

Theorem 5.15. For a soft topological space (X,τ,E), the collection S*er*-h(X,τ,E) forms a group under the composition of functions.

Proof. If $f: (X,\tau_1,E) \to (Y,\tau_2,E)$ and $g: (Y,\tau_2,E) \to (Z,\tau_3,E)$ are soft *e*r-homeomorphism, then their composition $gof: (X,\tau_1,E) \to (Z,\tau_3,E)$ is a soft *e*r-homeomorphism.It is obvious that for a bijective soft *e*r-homeomorphism $f: (X,\tau_1,E) \to (Y,\tau_2,E)$, $f^{-1}: (Y,\tau_2,E) \to (X,\tau_1,E)$ is also a soft *e*r-homeomorphism and the identity function $I: (X,\tau_1,E) \to (X,\tau_1,E)$ is a soft *e*r-homeomorphism. A binary operation α : S*er*-h$(X,\tau,E)\times$S*er*-h$(X,\tau,E) \to$ S*er*-h(X,τ,E) is well defined by $\alpha(a,b) = boa$, where a,b$\in$S*er*-h(X,τ,E) and boa is the composition of a and b. By using the above properties, the set S*er*-h(X,τ,E) forms a group under composition of function.

Theorem 5.16. The group S-h(X,τ,E) of all soft homeomorphisms on (X,τ,E) is a subgroup of S*er*-h(X,τ,E).

Proof. For any a,b$\in$S-h(X,τ,E), we have, $\alpha(a,b^{-1}) = b^{-1}oa\in$S-h$(X,\tau,E)$ and $I_X \in$S-h $(X,\tau,E) \neq \varphi$. Thus, using (Theorem 4.14) and (Theorem 4.15), it is obvious that the group S-h(X,τ,E) is a subgroup of S*er*-h(X,τ,E).

For a soft topological space (X,τ,E), we can construct a new group S*e*r-h(X,τ,E) satisfying the property: If there exists a homeomorphism (X,τ,E) ≅ (Y,τ,E), then there exists a group isomorphism S*e*r-h(X,τ,E) ≅ S*e*r-h(X,τ,E).

Corollary 5.17. Let f : $(X,\tau_1,E) \rightarrow (Y,\tau_2,E)$ and g : $(Y,\tau_2,E) \rightarrow (Z,\tau_3,E)$ be two functions between soft topological spaces.
(i) For a soft *e*r-homeomorphism f : $(X,\tau_1,E) \rightarrow (Y,\tau_2,E)$, there exists an isomorphism, say, f_* : S*e*r-h(X,τ,E) → S*e*r-h(X,τ,E), defined f_* (a) $= f \circ a \circ f^{-1}$, for any element a ∈ S*e*r-h(X,τ,E).
(ii) For two soft *e*r-homeomorphisms f : $(X,\tau_1,E) \rightarrow (Y,\tau_2,E)$ and g : $(Y,\tau_2,E) \rightarrow (Z,\tau_3,E)$, $(gof)_* = g_* o\ f_*$: S*e*r-h(X,τ_1,E) → S*e*r-h(Z,τ_3,E) holds.
(iii) For the identity function I_X : (X,τ,E) → (X,τ,E), $(I_X)_* = I$: S*e*r-h(X,τ,E) → S*e*r-h(X,τ,E) holds where I denotes the identity isomorphism.

Proof. Straightforward.

6. Conclusion

In this work we introduced the concept of soft *e-open* set and investigated some properties of them. Then focused the relationships among soft δ-pre open sets, soft δ-semi open sets, soft pre-open sets and soft *e-open* sets. We also investigated the concepts of soft *e-open* functions, soft *e*-continuous, soft *e*-irresolute and soft *e*-homomorphism on soft topological space and discussed their relations with existing soft continuous and other weaker forms of soft continuous functions. Further soft *e*-separation axioms have been introduced and investigated with the help of soft *e*-open sets. Finally, we observed that the collection S*e*r-h(X,τ,E) form a soft group. We hope that the findings in this work will help researcher enhance and promote the further study on soft topological spaces to carry out a general framework for their applications in separation axioms, connectedness, compactness etc. and also in practical life.

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