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$\Omega - \mathscr{N}$-FILTERS ON CI-ALGEBRA

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Abstaract − This paper deals the notion of $\Omega - \mathscr{N}$-structured subalgebras and $\Omega - \mathscr{N}$-structured Filters on CI-algebra. Further some of the properties and results using the idea of $\Omega - \mathscr{N}$-function on CI-algebra also established.

Keywords − *CI-algebra, Subalgebra, Filter,* $\Omega - \mathscr{N}$*-filter*

1 Introduction

After the initiation of the two classes of abstract algebras: BCK-algebras and BCI-algebras by Y. Imai and K. Iseki [2] , B. L. Meng[4][5], introduced the notion of a CI-algebra . K. H. Kim [3] also dealt about some concepts on CI-algebras. Zadeh. L. A. [9], introduced Fuzzy Sets for classifying the uncertainty. Then many researches used the notion of fuzzy in various algebraic structures. Samy. M. Mostafa [8] dealt fuzzification of ideals in CI-algebra and Intuitionistic (T, S)-fuzzy CI-algebras were discussed by A. Borumand Saeid et. al [1]. Also in [6] and [7] the authors introduced $\mathscr{N}$-ideals of a BF-algebras and $\mathscr{N}$-filters of CI-algebras. Motivated by these, this paper, intends to discuss $\Omega - \mathscr{N}$-structured filter of a CI-algebra and establish some simple, elegant and interesting results.

2 Preliminaries

This section deals with the basic definition of $\mathscr{N}$-function,$\Omega - \mathscr{N}$-function, CI-algebra, subalgebra and Filter of a CI-algebra.

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Definition.2.1. [6][7] *$\mathcal{N}$-structure and $\mathcal{N}$-function*
Consider a non-empty Set S. Denote the collection of functions from S to $[-1,0]$ by $\mathcal{F}(S,[-1,0])$. It is said that a member of $\mathcal{F}(S,[-1,0])$ is a negative valued function from S to $[-1,0]$, briefly $\mathcal{N}$-function and by an $\mathcal{N}$-structure on S, it means that an ordered pair (S,η) of S and $\mathcal{N}$-function η on S.

Definition.2.2. $\Omega-\mathcal{N}$*-function:*
A $\Omega-\mathcal{N}$-function η in a non-empty set S is a function $\eta : S\times\Omega \rightarrow [-1,0]$, where Ω is any non-empty set. The set of all $\Omega-\mathcal{N}$-functions from $S\times\Omega$ to $[-1,0]$ is denoted by $\mathcal{F}(S\times\Omega,[-1,0])$ and by the term $\Omega-\mathcal{N}$-Structure(Ω-NS) on S, it means that an ordered pair $(S\times\Omega,\eta)$ of $S\times\Omega$ and $\Omega-\mathcal{N}$-function η on $S\times\Omega$.

Definition.2.2. Consider the $\Omega-\mathcal{N}$-structure $(S\times\Omega,\eta)$ on a non-empty S. The negative Ω-Level subset η_t of η is defined as follows:
For some $t\in[-1,0]$, $\eta_t=\{x\in S \;:\; \eta(x,q)\geq t \;\forall q \in\Omega\}$.

Definition 2.3. [3][4] A CI-algebra is a non-empty set X with a consonant 1 and a single binary operation $*$ satisfying the following axioms:
(i)$x*x=1$
(ii)$1*x=x$
(iii)$x*(y*z)=y*(x*z)$ for all $x,y\in X$

Example 2.4.[3][4][5] Let $X=\{1,a,b,c\}$ and $Y=\{1,a,b,c,d\}$ be a set with the following tables

*	1	a	b	c
1	1	a	b	c
a	1	1	a	c
b	1	1	1	c
c	1	a	b	1

*	1	2	3	4	5
1	1	2	3	4	5
2	1	1	1	4	4
3	1	1	1	4	4
4	4	5	1	1	2
5	4	4	4	1	1

Then $\{X,*,1\}$ and $\{Y,*,1\}$ are CI-algebra.

Example 2.5.[5] Let X be the set of all positive real numbers. Then X becomes a CI-algebra by defining $x*y=\frac{y}{x}$ for all $x,y\in X$.

Definition 2.6.[3][4] A partial ordering $\leq$ on a CI-algebra $(X,*,1)$ can be defined as $x\leq y$ if, and only if, $x*y=1$.

Definition 2.7. [3][4] A non-empty subset S of a CI-algebra X is said to be a subalgebra if $x*y\in S$ for all $x,y\in S$.

Definition 2.8. [3] A non-empty subset F of a CI-algebra X is said to be a Filter of X if (i) $1 \in F$ and (ii) $x * y \in F$ and $x \in F$ then $y \in F$ for all $x, y \in X$.

Definition 2.9. An Filter F of X is called closed if $x * 1 \in F$ for all $x \in F$.

3 $\Omega - \mathscr{N}$-Subalgebra and $\Omega - \mathscr{N}$-Filter on a CI-algebra

This section introduces, the notion of $\Omega - \mathscr{N}$-subalgebra and $\Omega - \mathscr{N}$-Filter on a CI-algebra and discuss some of its results. In the rest of the paper, X represents a CI-algebra, Ω is any non-empty set and η is a $\Omega - \mathscr{N}$ function from $X \times \Omega$ to $[-1, 0]$ unless otherwise specified.

Definition 3.1. An $\Omega - \mathscr{N}$-structure $(X \times \Omega, \eta)$, on a CI-algebra X is called an $\Omega - \mathscr{N}$-subalgebra on X if $\eta((x * y), q) \leq \eta(x, q) \vee \eta(y, q)$ for all $x, y \in X$ and $q \in \Omega$.

Example 3.2. Consider the CI-algebra $X = (\{1, a, b, c, d\}, *, 1)$ given below.

*	1	a	b	c	d
1	1	a	b	c	d
a	1	1	b	b	d
b	1	a	1	a	d
c	1	1	1	1	d
d	d	d	d	d	1

The $\Omega - \mathscr{N}$-structure $(X \times \Omega, \eta)$ defined by, $\forall\ q \in \Omega$

$$\eta(x, q) = \begin{cases} -0.8 & ; \quad x = 1 \\ -0.7 & ; \quad x = a \\ -0.5 & ; \quad x = b \\ -0.3 & ; \quad x = c \\ -0.3 & ; \quad x = d \end{cases}$$

is an $\Omega - \mathscr{N}$-subalgebra on X.

Proposition 3.3. If (X, η) is an $\Omega - \mathscr{N}$-subalgebra on X then
$\eta(1, q) \leq \eta(x * 1, q) \leq \eta(x, q)$ for all $x \in X$ and $q \in \Omega$.
Proof. Let $x \in X$.
Then $\eta(1, q) = \eta((x * 1) * (x * 1), q) \leq \eta(x * 1, q) \vee \eta(x * 1, q) = \eta(x * 1, q)$ and $\eta(x * 1, q) \leq \eta(x, q) \vee \eta(1, q) = \eta(x, q) \vee \eta(x * x, q) = \eta(x, q)$. ■

Proposition 3.4. If (X, η) is an N-subalgebra of X then negative Level subset η_t of X is either empty or subalgebra of X, for all $t \in [-1, 0]$.
Proof. Let $t \in [-1, 0]$ and η_t be nonempty.

Take $x, y \in \eta_t \Rightarrow \eta(x,q) \leq t$ and $\eta(y,q) \leq t$.
Then $\eta(x*y,q) \leq \eta(x,q) \vee \eta(y,q) \leq t \vee t = t \Rightarrow x*y \in \eta_t$. ■

Definition.3.5. An Ω-NS on a CI-algebra X is said to be $\Omega-\mathscr{N}$-structured filter ($\Omega-\mathscr{N}$*-filter*) on X if (i) $\eta(1,q) \leq \eta(x,q)$ and
(ii) $\eta(y,q) \leq \eta(x*y,q) \vee \eta(x,q)$ for all $x, y \in X$ and $q \in \Omega$

Definition.3.6. An Ω-NS on a CI-algebra X is said to be $\Omega-\mathscr{N}$-structured closed filter ($\Omega-\mathscr{N}c$*-filter*) on X if (i) $\eta(y,q) \leq \eta(x*y,q) \vee \eta(x,q)$ and (ii) $\eta(x*1,q) \leq \eta(1,q)$ for all $x, y \in X$ and $q \in \Omega$.

Example.3.7. The $\Omega-\mathscr{N}$-structure (X,η) on the CI-algebra in Example.2.5 defined by, $\forall\ q \in \Omega$

$$\eta(x,q) = \begin{cases} -0.8 & ; \quad x = 1 \\ -0.7 & ; \quad x = 2^n \quad ; n \in \mathrm{N} \\ -0.5 & ; \quad \text{otherwise} \end{cases}$$

is an $\Omega-\mathscr{N}$-filter but not $\Omega-\mathscr{N}c$-filter on X.

Example.3.8. The $\Omega-\mathscr{N}$-structure (X,η) on the CI-algebra in Example.2.5 defined by $\forall\ q \in \Omega$

$$\eta(x,q) = \begin{cases} -0.8 & ; \quad x = 1 \\ -0.7 & ; \quad x = 2^n \quad ; n \in Z^+ \\ -0.5 & ; \quad \text{otherwise} \end{cases}$$

is an $\Omega-\mathscr{N}c$-filter on X.

Proposition.3.9. If (X,η) is an $\Omega-\mathscr{N}$-filter on X with $x \leq y$ for all $x, y \in X$, and $q \in \Omega$ then $\eta(x,q) \geq \eta(y,q)$ that is η is order-reversing.

Proof. Let $x, y \in X$ and $q \in \Omega$ such that $x \leq y$.
Then by the partial ordering $\leq$ defined in X, we have $x*y = 1$. Thus $\eta(y,q) \leq \eta(x*y,q) \vee \eta(x,q) = \eta(1,q) \vee \eta(x,q) \leq \eta(x,q)$. This completes the proof. ■

Proposition.3.10. If (X,η) is an $\Omega-\mathscr{N}$-filter on X with $x \leq y*z$ for all $x, y, z \in X$, and $q \in \Omega$ then $\eta(z,q) \leq \eta(x,q) \vee \eta(y,q)$.

Proof. Let $x, y, z \in X$ such that $x \leq y*z$.
Then by the partial ordering $\leq$ defined in X, we have $x*(y*z) = 1$.
Then $\eta(z,q) \leq \eta(y*z,q) \vee \eta(y,q)$
$\leq (\eta((x*(y*z),q)) \vee \eta(x,q)) \vee \eta(y,q)$
$= (\eta(1,q) \vee \eta(x,q)) \vee \eta(y,q)$
$= \eta(x,q) \vee \eta(y,q)$. ■

Remark.3.11. The terms $\Omega-\mathscr{N}$-subalgebra and $\Omega-\mathscr{N}$-filter on X are independent to each other. The following examples give the illustration.

Example.3.12. Consider the $\Omega-\mathscr{N}$-filter in Example 3.7.
Here
$$\eta\left(\left(2^4 * 2^2\right), q\right) = \eta\left(\frac{1}{4}, q\right) = -0.5 > -0.7 = \eta\left(2^4, q\right) \vee \eta\left(2^2, q\right)$$
, which is not an $\Omega-\mathscr{N}$-subalgebra.

Example.3.13. Consider the $\Omega-\mathscr{N}$-subalgebra in Example 3.2.
Here
$$\eta(c, q) = -0.3 > -0.5 = -0.7 \vee -0.5 = \eta(b * c, q) \vee \eta(b, q),$$
which is not an $\Omega-\mathscr{N}$-filter.

The following gives a sufficient condition for an $\Omega-\mathscr{N}$-subalgebra to be an $\Omega-\mathscr{N}$-filter.

Theorem.3.14. In a $\Omega-\mathscr{N}$-subalgebra (X,η), If $\eta(x*y,q) \leq \eta(y*x,q)$ $\forall\ x,y \in X$ and $q \in \Omega$ then (X,η) is an $\Omega-\mathscr{N}$-filter of X.

Proof. Let (X,η) be a $\Omega-\mathscr{N}$-subalgebra of X with
$\eta(x*y,q) \leq \eta(y*x,q)\ \forall\ x,y \in X$ and $q \in \Omega$.
Then $\eta(y,q) = \eta(1*y,q) \leq \eta(y*1,q)$
$= \eta((y*(x*x),q))$
$\leq \eta((x*(y*x),q))$
$\leq \eta(x,q) \vee \eta(y*x,q)$.
Hence (X,η) is an $\Omega-\mathscr{N}$-filter of X. ■

Theorem.3.15. If the $\Omega-\mathscr{N}$-structure (X,η) of X is a $\Omega-\mathscr{N}c$-filter of X, then the set $K = \{x \in X; \eta(x,q) = \eta(1,q) \forall\ q \in \Omega\}$ is a filter of X.

Proof. Clearly, K is nonempty (since $1 \in K$). Let $x, x*y \in K$.
Then $\eta(x*y,q) = \eta(x,q) = \eta(1,q)$
$\Rightarrow \eta(y,q) \leq \eta(x*y,q) \vee \eta(x,q)$
$= \eta(1,q) \vee \eta(1,q)$
$= \eta(1,q)$.
But $\eta(1,q) \leq \eta(y,q) \Rightarrow \eta(y,q) = \eta(1,q)$.
Thus $y \in K$. Hence K is a filter of X. ■

The following theorem shows the arbitrary union of family of $\Omega-\mathscr{N}c$-filters of X is also an $\Omega-\mathscr{N}c$-filter of X.

Theorem.3.16. Let $\{\eta_i : i \in I\}$ be the family of $\Omega-\mathscr{N}c$-filter of X. Then $\bigcup_i \eta_i$ is also $\Omega-\mathscr{N}c$-filter of X.

Proof. Let $x * y \in X$.
Since $\{\eta_i : i \in I\}$ is the family of $\Omega - \mathscr{N}c$-filter of X, for any $i \in I$ we have,

$$(i)\ \eta_i(y,q) \leq \eta_i(x*y,q)\ \vee\ \eta_i(x,q)\ and\ (ii)\ \eta_i(x*1,q) \leq \eta_i(x,q).$$

Now $\bigcup_i \eta_i(y,q) = sup\{\eta_i : i \in I\}$
$\leq sup\{\eta_i(x*y,q) \vee \eta_i(x,q) : i \in I\}$
$= sup\{\eta_i(x*y,q) : i \in I\} \vee sup\{\eta_i(x,q) : i \in I\}$
$= \bigcup_i \eta_i(x*y,q) \vee \bigcup_i \eta_i(x,q)$
and $\bigcup_i \eta_i(x*1,q) = sup\{\eta_i(x*1,q) : i \in I\} \leq sup\{\eta_i(x,q) : i \in I\} = \bigcup_i \eta_i(x,q)$
Hence $\bigcup_i \eta_i$ is an $\Omega - \mathscr{N}c$-filter of X. ■

Conclusion

In this paper, the notion of $\Omega - \mathscr{N}$-subalgebra and $\Omega - \mathscr{N}$-filter on a CI-algebra are introduced and some of the results have been discussed. In future it is planned to extend these ideas to homomorphism on $\Omega - \mathscr{N}$-filters,Cartesian products on $\Omega - \mathscr{N}$-filters and translation on $\Omega - \mathscr{N}$-filters.

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