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APPROXIMATION ON AN INTUITIONISTIC FUZZY NEAR-RINGS

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Abstaract − This paper proposes the notion of a rough near-ring with respect to an ideal of an intuitionistic fuzzy near-ring. Further the properties of rough ideals on near-rings are characterized.

Keywords − *Rough intuitionistic fuzzy ideals, Rough intuitionistic fuzzy near-ring, Rough intuitionistic fuzzy N-subgroup.*

1 Introduction

Rough set theory, proposed by Pawlak [25] is a new mathematical tool that supports uncertainty reasoning.The basic assumption of rough set theory is every knowledge in universe depends upon their capability of its classification. So that equivalennce relations are considered to define rough sets. It may be seen as an extension of classical set theory and has been successfully applied to machine learning, intelligent systems, inductive reasoning, pattern recognition, image processing, signal analysis, knowledge discovery, decision analysis, expert systems and many other fields. In 1965, Zadeh[32] initiated the novel concept of fuzzy set theory. There have been attempts to fuzzify various mathematical structures like topological spaces, groups, rings, etc., also concepts like relations measure, probability and automata etc. Biswas and Nanda[6] in 1994 introduced the concept of rough ideal in semi group. Based on an equivalence relation in 1990, Dubois and Prade[12] introduced the lower and upper approximations of fuzzy sets in Pawlak approximation space to obtain an extended notion called rough fuzzy sets. In 2008, Kazanci and Davaaz[16] introduced rough prime ideals and rough fuzzy prime ideals in commutative rings. Recently

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Jayanta Ghosh and T.K. Samantha[15] introduced rough intuitionistic fuzzy sets in semigroups.Yong Ho Yon et.al[29] introduced the concept of intuitionistic fuzzy R-subgroups of near-rings. In this paper we define rough intuitionistic fuzzy ideals of a near-ring based on its lower and upper approximation. Some interesting properties are established.

2 Preliminary

Definition 2.1. [22] By a near-ring we mean a nonempty set R with two binary operations " + " and "." satisfying the following axioms:

(i) $(R, +)$ is a group.

(ii) $(R, .)$ is a semigroup.

(iii) $x.(y + z) = x.y + x.z$ for all $x, y, z \in R$.

Definition 2.2. [22] An ideal of a near-ring R is a subset I of R such that

(i) $(I, +)$ is a normal subgroup of $(R, +)$.

(ii) $RI \subseteq I$.

(iii) $(x + i)y - xy \in I$ for all $i \in I$ and$x, y \in R$.

Definition 2.3. [3] An intuitionistic fuzzy set (IFS in short) A in X is an object having the form $A = \{\langle x, \mu_A(x), \nu_A(x)/x \in X\rangle\}$ where the function $\mu : X \to [0, 1]$ and $\nu : X \to [0, 1]$ denote the degree of membership (namely $\mu_A(x)$) and the degree of non membership (namely $\nu_A(x)$) of each element $x \in X$ to the set A, respectively and $0 \leq \mu_A(x) + \nu_A(x) \leq 1$ for each $x \in X$. Denote by IFS(X) the set of all intuitionistic fuzzy set in X.

Definition 2.4. [3] Let A and B be IFS's of the form $A = \{\langle x, \mu_A(x), \nu_A(x)/x \in X\rangle\}$ and $B = \{\langle x, \mu_B(x), \nu_B(x)/x \in X\rangle\}$. Then

1. $A \subseteq B$ if and only if $\mu_A(x) \leq \mu_B(x)$ and $\nu_A(x) \geq \nu_B(x)$ for all $x \in X$.
2. $A = B$ if and only if $A \subseteq B$ and $B \subseteq A$.
3. $\overline{A} = \{\langle x, \nu_A(x), \mu_A(x)/x \in X\rangle\}$.(Complement of A)
4. $A \cap B = \{\langle x, \mu_A(x) \wedge \mu_B(x), \nu_A(x) \vee \nu_B(x)/x \in X\rangle\}$.
5. $A \cup B = \{\langle x, \mu_A(x) \vee \mu_B(x), \nu_A(x) \wedge \nu_B(x)/x \in X\rangle\}$.

For the sake of simplicity we use the notion $A = \langle x, \mu_A, \nu_A\rangle$ instead of $A = \{\langle x, \mu_A(x), \nu_A(x)/x \in X\rangle\}$. The intuitionistic fuzzy set $0 \sim= \{\langle x, 0 \sim, 1 \sim\rangle /x \in X\}$ and $1 \sim= \{\langle x, 1 \sim, 0 \sim\rangle /x \in X\}$ are respectively the empty set and the whole set of X.

Definition 2.5. [33] An intuitionistic fuzzy set $A = (\mu_A, \nu_A)$ is called an intuitionistic fuzzy ideal of near-ring of R if for all x,y,i $\in R$.

(IF1) $\mu_A(x-y) \geq \mu_A(x) \wedge \mu_A(y)$ and $\nu_A(x-y) \leq \nu_A(x) \vee \nu_A(y)$

(IF2) $\mu_A(y+x-y) \geq \mu_A(x)$ and $\nu_A(y+x-y) \leq \nu_A(x)$

(IF3) $\mu_A(xy) \geq \mu_A(y)$ and $\nu_A(xy) \leq \nu_A(y)$

(IF4) $\mu_A(i(x+y)-ix) \geq \mu_A(y)$ and $\nu_A(i(x+z)-ix) \leq \nu_A(y)$

Definition 2.6. [29] An IFS $A = (\mu_A, \nu_A)$ in R is called an intuitionistic fuzzy subnear ring of R if for all $x, y \in R$

(i) $\mu_A(x-y) \geq \mu_A(x) \wedge \mu_A(y)$ and $\nu_A(x-y) \leq \nu_A(x) \vee \nu_A(y)$

(ii) $\mu_A(xy) \geq \mu_A(y)$ and $\nu_A(xy) \leq \nu_A(y)$.

Definition 2.7. [16] An equivalence relation θ on R is a reflexive, symmetric and transitive binary relation on R. If θ is an equivalence relation on R then the equivalence class of $x \in R$ is the set $\{y \in R|(x,y) \in \theta$.We write it as $[x]_\theta$.

Definition 2.8. [16] Let θ be an equivalence relation on R, then θ is called a full congruence relation if $(a,b) \in \theta$ implies $(a+x, b+x), (ax, bx)$ and $(xa, xb) \in \theta$ for all $x \in R$

Theorem 2.9. [16] Let θ be a full congruence relation on R, then $(a,b) \in \theta$ and $(c,d) \in \theta$ imply $(a+c, b+d) \in \theta, (ca, bd) \in \theta$ and $(-a, -b) \in \theta$ for all $a, b, c, d \in R$

Definition 2.10. [15] Let us consider θ to be a congruence relation of S. If X is a nonempty subset of S then the sets $\theta_*(X) = \{x \in S|[x]_\theta \subseteq X\}$ and $\theta^*(X) = \{x \in S|[x]_\theta \cap X \neq \phi\}$ are respectively called the θ-lower and θ-upper approximation of the set X and $\theta(X) = (\theta_*(X), \theta^*(X))$ is called rough set with respect to θ if $\theta_*(X) \neq \theta^*(X)$. If $A = (\mu_A, \nu_A)$ be IFS of S. Then the IFS $\theta_*(A) = (\theta_*(\mu_A, \theta_*\nu_A))$ and $\theta^*(A) = (\theta^*(\mu_A, \theta^*\nu_A))$ are respectively called $\theta-$lower and $\theta-$upper approximation of the IFS $A = (\mu_A, \nu_A)$ where for all $x \in S$

$$\theta_*(\mu_A)(x) = \wedge_{a \in [x]_\theta} \mu_A(a), \theta_*(\nu_A)(x) = \vee_{a \in [x]_\theta} \nu_A(a)$$
$$\theta^*(\mu_A)(x) = \vee_{a \in [x]_\theta} \mu_A(a), \theta_*(\nu_A)(x) = \wedge_{a \in [x]_\theta} \nu_A(a)$$

For an IFS $A = (\mu_A, \nu_A)$ of S, $\theta(A) = (\theta_*(A), \theta^*(A))$ is called rough intuitionistic fuzzy set with respect to θ if $\theta_*(A) \neq \theta^*(A)$

Definition 2.11. [29] An IFS $A = (\mu_A, \nu_A)$ in R is called an intuitionistic fuzzy N-subgroup of R if for all $x, y, n \in R$

(i) $\mu_A(x-y) \geq \mu_A(x) \wedge \mu_A(y)$ and $\nu_A(x-y) \leq \nu_A(x) \vee \nu_A(y)$.

ii $\mu_A(nx) \geq \mu_A(x)$ and $\nu_A(nx) \leq \nu_A(x)$.

iii $\mu_A(xn) \geq \mu_A(x)$ and $\nu_A(xn) \leq \nu_A(x)$.

3 ROUGH INTUITIONISTIC FUZZY IDEALS IN NEAR-RINGS

Throughout N denotes an abelian near ring and R denotes a near ring.

Lemma 3.1. If an IFS $A = (\mu_A, \nu_A)$ in R satisfies the condition (IF1) of definition [2.5], then

(i) $\mu_A(0) \geq \mu_A(x)$ and $\nu_A(0) \leq \nu_A(x)$,

(ii) $\mu_A(-x) = \mu_A(x)$and $\nu_A(-x) = \nu_A(x)$.

for all $x \in R$.

Lemma 3.2. If an IFS $A = (\mu_A, \nu_A)$ in R satisfies the condition (IF1) of definition [2.5], then

(i) $\mu_A(x-y) = \mu_A(0) \Rightarrow \mu_A(x) = \mu_A(y)$,

(ii) $\nu_A(x-y) = \nu_A(0) \Rightarrow \nu_A(x) = \nu_A(y)$.

for all $x, y \in R$.
The proof of 3.1 and 3.2 are immediate.

Theorem 3.3. Let θ be a full congruence relation on R. If $A = (\mu_A, \nu_A)$ is an intuitionistic fuzzy subnear-ring of R then so is $\theta^*(A) = (\theta^*(\mu_A), \theta^*(\nu_A))$.
Proof: Since θ is a congruence relation of R. Then for all $x, y \in R$,

(i) a) $\theta^*(\mu_A)(x-y) = \bigvee_{z \in [x-y]_\theta} \mu_A(z)$

$$= \bigvee_{z \in [x]_\theta - [y]_\theta} \mu_A(z)$$
$$= \bigvee_{a-b \in [x]_\theta - [y]_\theta} \mu_A(a-b)$$
$$\geq \bigvee_{a \in [x]_\theta, b \in [y]_\theta} [\mu_A(a) \wedge \mu_A(b)]$$
$$= [\bigvee_{a \in [x]_\theta} \mu_A(a)] \vee [\bigvee_{b \in [y]_\theta} \mu_A(b)] = \theta^*(\mu_A)(x) \wedge \theta^*(\mu_A)(y)$$

b) $\theta^*(\nu_A)(x-y) = \bigwedge_{z \in [x-y]_\theta} \nu_A(z)$

$$= \bigwedge_{z \in [x]_\theta - [y]_\theta} \nu_A(z)$$
$$= \bigwedge_{a-b \in [x]_\theta - [y]_\theta} \nu_A(a-b)$$
$$\leq \bigwedge_{a \in [x]_\theta, b \in [y]_\theta} [\nu_A(a) \vee \nu_A(b)]$$
$$= [\bigwedge_{a \in [x]_\theta} \nu_A(a)] \vee [\bigwedge_{b \in [y]_\theta} \nu_A(b)] = \theta^*(\nu_A)(x) \vee \theta^*(\nu_A)(y)$$

(ii) a) $\theta^*(\mu_A)(xy) = \bigvee_{z\in[xy]_\theta} \mu_A(z)$
$\geq \bigvee_{z\in[x]_\theta[y]_\theta} \mu_A(z)$
$= \bigvee_{ab\in[x]_\theta[y]_\theta} \mu_A(ab)$
$\geq \bigvee_{a\in[x]_\theta b\in[y]_\theta} \mu_A(ab)$
$\geq \bigvee_{b\in[y]_\theta} \mu_A(b) = \theta^*(\mu_A)(y)$
b) $\theta^*(\nu_A)(xy) = \bigwedge_{z\in[yx]_\theta} \nu_A(z)$
$\leq \bigwedge_{z\in[x]_\theta[y]_\theta} \nu_A(z)$
$= \bigwedge_{ab\in[x]_\theta[y]_\theta} \nu_A(ab)$
$\leq \bigwedge_{a\in[x]_\theta b\in[y]_\theta} \nu_A(ab)$
$\leq \bigwedge_{b\in[y]_\theta} \nu_A(b) = \theta^*(\nu_A)(y)$ This shows that $\theta^*(A)$ is an intuitionistic fuzzy subnear-ring of R. Therefore A is an upper rough intuitonistic fuzzy subnear-ring of R.

Theorem 3.4. Let θ be a full congruence relation on R. If $A = (\mu_A, \nu_A)$ is an intuitionistic fuzzy subnear-ring of R then so is $(\theta)_*(A) = (\theta_*(\mu_A), \theta_*(\nu_A))$.

Proof. Since θ is a congruence relation of R. Then for all $x, y \in R$

(i) a) $\theta_*(\mu_A)(x-y) = \bigwedge_{z\in[x-y]_\theta} \mu_A(z)$
$= \bigwedge_{z\in[x]_\theta-[y]_\theta} \mu_A(z)$
$\geq \bigwedge_{z\in[x]_\theta[y]_\theta} \mu_A(z)$
$= \bigwedge_{a-b\in[x]_\theta-[y]_\theta} \mu_A(a-b)$
$\geq \bigwedge_{a\in[x]_\theta, b\in[y]_\theta} [\mu_A(a) \wedge \mu_A(b)]$
$= [\bigwedge_{a\in[x]_\theta} \mu_A(a)] \wedge [\bigwedge_{b\in[y]_\theta} \mu_A(b)] = \theta_*(\mu_A)(x) \wedge \theta_*(\mu_A)(y)$
b) $\theta_*(\nu_A)(x-y) = \bigvee_{z\in[x-y]_\theta} \nu_A(z)$
$= \bigvee_{z\in[x]_\theta-[y]_\theta} \nu_A(z)$
$= \bigvee_{a-b\in[x]_\theta-[y]_\theta} \nu_A(a-b)$
$\leq \bigvee_{a\in[x]_\theta, b\in[y]_\theta} [\nu_A(a) \vee \nu_A(b)]$
$= [\bigvee_{a\in[x]_\theta} \nu_A(a)] \vee [\bigvee_{b\in[y]_\theta} \nu_A(b)] = \theta_*(\nu_A)(x) \vee \theta_*(\nu_A)(y)$

(ii) a) $\theta_*(\mu_A)(xy) = \bigwedge_{z\in[xy]_\theta} \mu_A(z)$

$= \bigwedge_{z\in[x]_\theta[y]_\theta} \mu_A(z)$
$= \bigwedge_{ab\in[x]_\theta[y]_\theta} \mu_A(ab)$
$\geq \bigwedge_{a\in[x]_\theta b\in[y]_\theta} \mu_A(ab)$
$= \bigvee_{b\in[y]_\theta} \mu_A(b) = \theta_*(\mu_A)(y)$
b) $\theta_*(\nu_A)(xy) = \bigvee_{z\in[yx]_\theta} \nu_A(z)$
$= \bigvee_{z\in[x]_\theta[y]_\theta} \nu_A(z)$
$= \bigvee_{ab\in[x]_\theta[y]_\theta} \nu_A(ab)$
$\leq \bigvee_{a\in[x]_\theta b\in[y]_\theta} \nu_A(ab)$
$\leq \bigvee_{b\in[y]_\theta} \nu_A(b) = \theta_*(\nu_A)(y)$ This shows that $\theta_*(A)$ is an intuitionistic fuzzy subnear-ring of R. Therefore A is an lower rough intuitonistic fuzzy subnear-ring of R.

Corollary 3.5. If $A = (\mu_A, \nu_A)$ is an intuitionistic fuzzy subnear-ring of R then $\theta(A) = (\theta_*(A), \theta^*(A))$ is a rough intuitionistic fuzzy subnear-ring of R.

Example 3.6. Let $R = \{0, a, b, c\}$ be a set with two binary operations as follows

+	0	a	b	c
0	0	a	b	c
a	a	0	c	b
b	b	c	0	a
c	c	b	a	0

.	0	a	b	c
0	0	a	b	c
a	a	0	c	b
b	b	c	0	a
c	c	b	a	0

Then $(R, +, .)$ is a near-ring. Let θ be a congruence relation on R such that the θ-congruence classes are the subsets $\{0\}, \{c\}, \{a, b\}$. Let $A = \{\langle x, \mu_A(x), \nu_A(x)\rangle | x \in R\}$ be an intuitionistic fuzzy subset of R defined by

$$A = \{\langle 0, 0.4, 0.4\rangle, \langle a, 0.1, 0.5\rangle, \langle b, 0.4, 0.6\rangle, \langle c, 0.4, 0.4\rangle\}$$

Since for every $x \in R, \theta^*(\mu_A)(x) = \bigvee_{\alpha\in[x]_\theta} \mu_A(\alpha)$ and $\theta^*(\nu_A)(x) = \bigwedge_{\alpha\in[x]_\theta} \nu_A(\alpha)$, so the upper approximation $\theta^*(A) = \{\langle x, \theta^*(\mu_A(x)), \theta^*(\nu_A(x))\rangle | x \in T\}$ is given by

$$\theta^*(A) = \{\langle 0, 0.4, 0.4\rangle, \langle a, 0.4, 0.5\rangle, \langle b, 0.4, 0.5\rangle, \langle c, 0.4, 0.4\rangle\}$$

then it can be easily verified that

$$\theta^*(\mu_A)(x - y) \geq \theta^*(\mu_A)(x) \wedge \theta^*(\mu_A)(y)$$
$$\theta^*(\nu_A)(x - y) \leq \theta^*(\nu_A)(x) \vee \theta^*(\nu_A)(y)$$

and

$$\theta^*(\mu_A)(x.y) \geq \theta^*(\mu_A)(y)$$
$$\theta^*(\nu_A)(x.y) \leq \theta^*(\nu_A)(y)$$

for all $x, y \in R$. Therefore $\theta^*(A)$ is an intuitionistic fuzzy subnear-ring of R. Hence A is an upper rough intuitionistic fuzzy subnear-ring of R.

Theorem 3.7. Let θ be a congruence relation on R then, if A is an intuitionistic fuzzy N-subgroup of R, then A is an upper rough intuitionistic fuzzy N-subgroup of R.

Proof. Since θ is a congruence relation on R $[a]_\theta[b]_\theta \subseteq [ab]_\theta \forall a, b \in R$.
Let $A = (\mu_A, \nu_A)$ be an intuitionistic fuzzy N-subgroup of N and $x \in N$. Now $\theta^*(A) = (\theta^*(\mu_A), \theta^*(\nu_A))$. Thus

(i) a) $\theta^*(\mu_A)(x-y) \geq \theta^*(\mu_A)(x) \wedge \theta^*(\mu_A)(y)$
b) $\theta^*(\nu_A)(x-y) \leq \theta^*(\nu_A)(x) \vee \theta^*(\nu_A)(y)$

(ii) a) $\theta^*(\mu_A)(nx) = \bigvee_{z \in [nx]_\theta} \mu_A(z)$
$\geq \bigvee_{z \in [n]_\theta[x]_\theta} \mu_A(z)$
$= \bigvee_{ab \in [n]_\theta[x]_\theta} \mu_A(ab)$
$\geq \bigvee_{a \in [n]_\theta b \in [x]_\theta} \mu_A(ab)$
$\geq \bigvee_{b \in [x]_\theta} \mu_A(b) = \theta^*(\mu_A)(x)$
b) $\theta^*(\nu_A)(nx) = \bigwedge_{z \in [nx]_\theta} \nu_A(z)$
$\leq \bigwedge_{z \in [n]_\theta[x]_\theta} \nu_A(z)$
$= \bigwedge_{ab \in [n]_\theta[x]_\theta} \nu_A(ab)$
$\leq \bigwedge_{a \in [n]_\theta b \in [x]_\theta} \nu_A(ab)$
$\leq \bigwedge_{b \in [x]_\theta} \nu_A(b) = \theta^*(\nu_A)(x)$

(iii) a) $\theta^*(\mu_A)(xn) \geq \theta^*(\nu_A)(x)$
b) $\theta^*(\nu_A)(xn) \leq \theta^*(\nu_A)(x)$
Proof is analogs to (ii).

Theorem 3.8. Let θ be a complete congruence relation on R then, if A is an intuitionistic fuzzy N-subgroup of R, then A is an lower rough intuitionistic fuzzy N-subgroup of R.

Proof. Since θ is a congruence relation on R $[a]_\theta[b]_\theta = [ab]_\theta \forall a, b \in R$.
Let $A = (\mu_A, \nu_A)$ be an intuitionistic fuzzy N-subgroup of N and $x, \in N$. Now $\theta_*(A) = (\theta_*(\mu_A), \theta_*(\nu_A))$. Thus

(i) a) $\theta_*(\mu_A)(x-y) \geq \theta_*(\mu_A)(x) \vee \theta_*(\mu_A)(y)$
b) $\theta_*(\nu_A)(x-y) \leq \theta_*(\nu_A)(x) \wedge \theta_*(\nu_A)(y)$
Proof of (i) is similar to subnear-rings.

(ii) a) $\theta_*(\mu_A)(nx) = \bigwedge_{z\in[nx]_\theta} \mu_A(z)$
$= \bigwedge_{z\in[n]_\theta[x]_\theta} \mu_A(z)$
$= \bigwedge_{ab\in[n]_\theta[x]_\theta} \mu_A(ab)$
$\geq \bigwedge_{a\in[n]_\theta b\in[x]_\theta} \mu_A(ab)$
$= \bigwedge_{b\in[x]_\theta} \mu_A(b) = \theta_*(\mu_A)(x)$
b) $\theta_*(\nu_A)(nx) = \bigvee_{z\in[nx]_\theta} \nu_A(z)$
$= \bigvee_{z\in[n]_\theta[x]_\theta} \nu_A(z)$
$= \bigvee_{ab\in[n]_\theta[x]_\theta} \nu_A(ab)$
$\leq \bigvee_{a\in[n]_\theta b\in[x]_\theta} \nu_A(ab)$
$= \bigvee_{b\in[x]_\theta} \nu_A(b) = \theta_*(\nu_A)(x)$

(iii) a) $\theta_*(\mu_A)(xn) \geq \theta_*(\nu_A)(x)$
b) $\theta_*(\nu_A)(xn) \leq \theta_*(\nu_A)(x)$
Proof is analogous to (ii).

Theorem 3.9. Let θ be a congruence relation of R, then if A is an intuitionistic fuzzy ideal of R then A is an upper rough intuitionistic fuzzy ideal of R.

Proof. (i) a. $\theta^*(\mu_A(x-y)) \geq \theta^*(\mu_A)(x) \wedge \theta^*(\mu_A)(y)$.
b. $\theta^*(\nu_A(x-y)) \leq \theta^*(\nu_A)(x) \vee \theta^*(\nu_A)(y)$.

(ii) a. $\theta^*(\mu_A(xn)) \geq \theta^*(\mu_A)(x)$
b. $\theta^*(\nu_A(xn)) \leq \theta^*(\nu_A)(x)$

(iii) a. $\theta^*(\mu_A)(y+x-y) = \bigvee_{z\in[y+x-y]_\theta} \mu_A(z)$
$= \bigvee_{z\in[y+x]_\theta-[y]_\theta} \mu_A(z)$
$= \bigvee_{a-d\in[y+x]_\theta-[y]_\theta} \mu_A(a-d)$
$= \bigvee_{a\in[y+x]_\theta, d\in[y]_\theta} \mu_A(a-d)$
$= \bigvee_{d+c\in[y+x]_\theta, d\in[y]_\theta} \mu_A(d+c-d)$
$= \bigvee_{c\in[x]_\theta, d\in[y]_\theta} \mu_A(d+c-d)$
$= \bigvee_{c\in[x]_\theta, d\in[y]_\theta} \mu_A(c)$

$= \bigvee_{c\in[x]_\theta} \mu_A(c) = \theta^*(\mu_A)(x).$

b. $\theta^*(\nu_A)(y+x-y) = \bigwedge_{z\in[y+x-y]_\theta} \mu_A(z)$

$= \bigwedge_{z\in[y+x]_\theta-[y]_\theta} \nu_A(z)$

$= \bigwedge_{a-d\in[y+x]_\theta-[y]_\theta} \nu_A(a-d)$

$= \bigwedge_{a\in[y+x]_\theta, d\in[y]_\theta} \nu_A(a-d)$

$= \bigwedge_{d+c\in[y+x]_\theta, d\in[y]_\theta} \nu_A(d+c-d)$

$= \bigwedge_{c\in[x]_\theta, d\in[y]_\theta} \nu_A(d+c-d)$

$= \bigwedge_{c\in[x]_\theta, d\in[y]_\theta} \nu_A(c)$

$= \bigwedge_{c\in[x]_\theta} \nu_A(c) = \theta^*(\nu_A)(x).$

Theorem 3.10. Let θ be a complete congruence relation of R, then if A is an intuitionistic fuzzy ideal of R then A is an lower rough intuitionistic fuzzy ideal of R.

Proof. (i) a. $\theta_*(\mu_A(x-y)) \geq \theta_*(\mu_A)(x) \vee \theta_*(\mu_A)(y).$

b. $\theta_*(\nu_A(x-y)) \leq \theta_*(\nu_A)(x) \wedge \theta_*(\nu_A)(y.)$

(ii) a. $\theta_*(\mu_A(xn)) \geq \theta_*(\mu_A)(x)$

b. $\theta_*(\nu_A(xn)) \leq \theta_*(\nu_A)(x).$

(iii) a. $\theta_*(\mu_A)(y+x-y) = \bigwedge_{z\in[y+x-y]_\theta} \mu_A(z)$

$= \bigwedge_{z\in[y+x]_\theta-[y]_\theta} \mu_A(z)$

$= \bigwedge_{a-d\in[y+x]_\theta-[y]_\theta} \mu_A(a-d)$

$= \bigwedge_{a\in[y+x]_\theta, d\in[y]_\theta} \mu_A(a-d)$

$= \bigwedge_{d+c\in[y+x]_\theta, d\in[y]_\theta} \mu_A(d+c-d)$

$= \bigwedge_{c\in[x]_\theta, d\in[y]_\theta} \mu_A(d+c-d)$

$= \bigwedge_{c\in[x]_\theta, d\in[y]_\theta} \mu_A(c)$

$= \bigwedge_{c\in[x]_\theta} \mu_A(c) = \theta_*(\mu_A)(x).$ b. $\theta_*(\nu_A)(y+x-y) = \bigvee_{z\in[y+x-y]_\theta} \mu_A(z)$

$= \bigvee_{z\in[y+x]_\theta-[y]_\theta} \nu_A(z)$

$= \bigvee_{a-d\in[y+x]_\theta-[y]_\theta} \nu_A(a-d)$

$= \bigvee_{a\in[y+x]_\theta, d\in[y]_\theta} \nu_A(a-d)$

$= \bigvee_{d+c\in[y+x]_\theta, d\in[y]_\theta} \nu_A(d+c-d)$

$$= \bigvee_{c\in[x]_\theta, d\in[y]_\theta} \nu_A(d+c-d)$$
$$= \bigvee_{c\in[x]_\theta, d\in[y]_\theta} \nu_A(c)$$
$$= \bigvee_{c\in[x]_\theta} \nu_A(c) = \theta_*(\nu_A)(x).$$

Let N denote an abelian near-ring.

Definition 3.11. Let A be an intuitionistic fuzzy ideal of R. For each $\alpha, \beta \in [0,1]$ with $\alpha+\beta \leq 1$, the set $A_\alpha^\beta = \{(a,b) \in R\times R | \mu_A(a-b) \leq \alpha, \nu_A(a-b) \geq \beta\}$ is called a (α,β)level relation of A.

Proposition 3.12. If A and B be two intuitionistic fuzzy sets of an universal set X, then the following holds

(i) $A_\alpha^\beta \subseteq A_\gamma^\delta$ if $\alpha \geq \gamma$ and $\beta \leq \delta$.

(ii) $A_{1-\beta}^\beta \subseteq A_\alpha^\beta \subseteq A_\alpha^{1-\alpha}$.

(iii) $A \subseteq B \Rightarrow A_\alpha^\beta \subseteq B_\alpha^\beta$.

(iv) $(A\cap B)_\alpha^\beta = A_\alpha^\beta \cap B_\alpha^\beta$.

(v) $(A\cup B)_\alpha^\beta \supseteq A_\alpha^\beta \cup B_\alpha^\beta$.

Proof. (i) Let $(x,y) \in A_\alpha^\beta \Rightarrow \mu_A(x-y) \geq \alpha$ and $\nu_A(x-y) \leq \beta$. Since $\gamma \leq \alpha$ and $\delta \geq \beta \Rightarrow \mu_A(x-y) \geq \alpha \geq \gamma$ and $\nu_A(x-y) \leq \beta \leq \delta \Rightarrow \mu_A(x-y) \geq \gamma$ and $\nu_A(x-y) \leq \delta$. Hence $(x,y) \in A_\gamma^\delta$.

(ii) Since $\alpha+\beta \leq 1 \Rightarrow 1-\beta \geq \alpha$ and $\beta \leq \beta$. Thus $A_{1-\beta}^\beta \subseteq A_\alpha^\beta$. Also $\alpha \geq \alpha$ and $\beta \leq 1-\alpha$. Thus $A_\alpha^\beta \subseteq A_\alpha^{1-\alpha}$.

(iii) Let $(x,y) \in A_\alpha^\beta \Rightarrow \mu_A(x-y) \geq \alpha$ and $\nu_A(x-y) \leq \beta$. As $A \subseteq B \Rightarrow \mu_B(x-y) \geq \alpha$ and $\nu_B(x-y) \leq \beta$ and so $(x,y) \in B_\alpha^\beta$.

(iv) Since $A\cap B \subseteq A$ and $A\cap B \subseteq B$. Thus $(A\cap B)_\alpha^\beta \subseteq A_\alpha^\beta$ and $(A\cap B)_\alpha^\beta \subseteq B_\alpha^\beta$. Thus $(A\cap B)_\alpha^\beta \subseteq A_\alpha^\beta \cap B_\alpha^\beta$. Let $(x,y) \in A_\alpha^\beta \cap B_\alpha^\beta \Rightarrow (x,y) \in A_\alpha^\beta$ and $(x,y) \in B_\alpha^\beta$
$\Rightarrow \mu_A(x-y) \geq \alpha \; and \; \nu_A(x-y) \leq \beta \; and \; \mu_B(x-y) \geq \alpha \; and \; \nu_B(x-y) \leq \beta$
$\Rightarrow \mu_A(x-y) \geq \alpha \; and \; \mu_B(x-y) \geq \alpha \; and \; \nu_A(x-y) \leq \beta \; and \; \nu_B(x-y) \leq \beta$
$\Rightarrow \mu_A(x-y) \wedge \mu_B(x-y) \geq \alpha \; and \; \nu_A(x-y) \vee \nu_B(x-y) \leq \beta$,
$\Rightarrow (\mu_A \cap \mu_B)(x-y) \geq \alpha$ and $(\nu_A \cup \nu_B)(x-y) \leq \beta$,
$\Rightarrow (x,y) \in (A\cap B)_\alpha^\beta$.

(iv) Since $A \subseteq A\cup B$ and $B \subseteq A\cup B \Rightarrow A_\alpha^\beta \subseteq (A\cup B)_\alpha^\beta$ and $B_\alpha^\beta \subseteq (A\cup B)_\alpha^\beta$. Thus $A_\alpha^\beta \cup B_\alpha^\beta \subseteq (A\cup B)_\alpha^\beta$ Now if $\alpha+\beta = 1$ then $(A\cup B)_\alpha^\beta \subseteq A_\alpha^\beta \cup B_\alpha^\beta$. Let $(x,y) \in (A\cup B)_\alpha^\beta \Rightarrow (\mu_A \cup \mu_B)(x-y) \geq \alpha$ and $(\nu_A \cup \nu_B)(x-y) \leq \beta \Rightarrow \mu_A(x-y) \vee \mu_B(x-y) \geq \alpha$ and $\nu_A(x-y) \vee \nu_B(x-y)) \leq \beta$. If $\mu_A(x-y) \geq \alpha$, then $\nu_A(x-y) \leq 1-\mu_A(x-y) \leq 1-\alpha = \beta \Rightarrow (x,y) \in A_\alpha\beta \subseteq A_\alpha^\beta \cup B_\alpha^\beta$. Similarly if $\mu_B(x-y) \geq \alpha$, then $\nu_B(x-y) \leq 1-\mu_B(x-y) \leq 1-\alpha = \beta \Rightarrow (x,y) \in B_\alpha\beta \subseteq A_\alpha^\beta \cup B_\alpha^\beta$. Thus $(A\cup B) = A_\alpha^\beta \cup B_\alpha^\beta$.

Remark 3.13. Every rough ring is a rough near-ring.

Lemma 3.14. Let A be an intuitionistic fuzzy ideal of an abelian near-ring N, and let $\alpha, \beta \in [0,1]$ with $\alpha + \beta \leq 1$. Then A_α^β is a congruence relation on N.

Proof. For any element $a \in N$, $\mu_A(a-a) = \mu_A(0) \geq \alpha$ and so $(a,a) \in A_\alpha^\beta$. If $(a,b) \in A_\alpha^\beta$, then $\mu_A(a-b) \geq \alpha$ and $\nu_A(a-b) \leq \beta$. Since A is an ideal of abelian near-ring N, $\mu_A(b-a) = \mu_A(-(a-b)) = \mu_A(a-b) \geq \alpha, \nu_A(b-a) = \nu_A(-(a-b)) = \nu_A(a-b) \leq \beta \Rightarrow (b,a) \in A_\alpha^\beta$. If $(a,b) \in A_\alpha^\beta$ and $(b,c) \in A_\alpha^\beta$, then since A is a fuzzy ideal of N, $\mu_A(a-c) = \mu_A((a-b)+(b-c)) \geq min\{\mu_A(a-b), \mu_A(b-c)\} \geq min\{\alpha, \alpha\} = \alpha$, and so $(a,c) \in A_\alpha^\beta$. Therefore A_α^β is an equivalence relation on N. Now, let $(a,b) \in A_\alpha^\beta$ and x be any element of N. Then since $\mu_A(a-b) \geq \alpha$ and $\nu_A(a-b) \leq \beta$ $\mu_A((a+x)-(b+x)) = \mu_A((a+x)+(-x-b)) = \mu_A(a+(x-x)-b) = \mu_A(a+0-b), \mu_A(a-b) \geq \alpha, \nu_A((A+x)-(b+x)) = \nu_A((a+x)+(-x-b)) = \nu_A(a+(x-x)-b) = \nu_A(a+0-b)\nu_A(a-b) \leq \beta \Rightarrow (a+x, b+x) \in A_{\alpha,\beta}$. Since (N,+) is an abelian group, we have $(x+a, x+b) \in A_{\alpha,\beta}$. Therefore $A_{\alpha,\beta}$ is a congruence relation on R.

Definition 3.15. Let A be an intuitionisic fuzzy ideal of an abelian near-ring N and $\alpha, \beta \in [0,1]$ with $\alpha + \beta \leq 1$. We know that A_α^β is an equivalence relation(congruence relation) on N. Therefore when the universe is an abelian near-ring then (N, A_α^β) can be used instead of the approximation space (U, θ).

Let A be an intuitionistic fuzzy ideal of N and A_α^β be an (α,β)-level congruence relation of A on N. Let X be a non-empty subset of N. Then the sets

$$\underline{R}(A_\alpha^\beta, X) = \{x \in N | [x]_{A_\alpha^\beta} \subseteq X\}$$
$$\overline{R}(A_\alpha^\beta, X) = \{x \in N | [x]_{A_\alpha^\beta} \cap \neq \phi\}.$$

Proposition 3.16. Let A be an intuitionistic fuzzy ideal of an abelian near-ring N. Then for any $\alpha, \beta \in [0,1]$ with $\alpha + \beta \leq 1$. If B is an ideal of an abelian near-ring R, then A is an upper rough left ideal of N.

Proof. **Proof:** Let $a, b \in \underline{R}(A_\alpha^\beta, B)$. Then $[a]_{A_\alpha^\beta} \subseteq B$ and $[b]_{A_\alpha^\beta} \subseteq B$. Since A_α^β is a congruence relation

$$\begin{aligned} [a+b]_{A_\alpha^\beta} &= [a]_{A_\alpha^\beta} + [b]_{A_\alpha^\beta} \\ &\subseteq B + B \subseteq B \\ &\Rightarrow a + b \in \underline{R}(A_\alpha^\beta, B). \end{aligned}$$

Let a be any element of $\underline{R}(A_\alpha^\beta, B)$. Then $[a]_{A_\alpha^\beta} \subseteq B$. Let x be any element of $[-a]_{A_\alpha^\beta}$. Then $(x, -a) \in A_\alpha^\beta$ and so $(-x, a) \in A_\alpha^\beta$. Thus $-x \in [a]_{A_\alpha^\beta} \subseteq B$.. Since A is an intuitionistic fuzzy ideal of N it is a subgroup of N, thus $x \in A$ and so $[-a]_{A_\alpha^\beta} \subseteq A$. Thus $-a \in \underline{R}(A_\alpha^\beta, B)$. Let a and x be any element of $\underline{R}(A_\alpha^\beta, B)$. Then $[a]_{A_\alpha^\beta} \subseteq B$. Let z be any element of $[x+a-x]_{A_\alpha^\beta}$. Then $(z, (x+a-x)) \in A_\alpha^\beta$. Since A_α^β is a congruence relation on N, $(-x+z+x, a) \in A_\alpha^\beta$ and so $-x+z+x \in [a]_{A_\alpha^\beta} \subseteq B$.Thus

$-x+z+x=b$ for some $b \in B$. Since B is normal $z=x+b-x \in x+B-x \in B$ and so we have $[x+b-x]_{A_\alpha^\beta} \in B$. Therefore $x+b-x \in \underline{R}(A_\alpha^\beta, B)$, which means $\underline{R}(A_\alpha^\beta, B)$ is normal subgroup of N.
Let $r \in R$ and $a \in \underline{R}(A_\alpha^\beta, B)$, then $[a]_{A_\alpha^\beta} \subseteq B$. Let a be any element of $[x]_{A_\alpha^\beta}$. Then $(x,a) \in A_\alpha^\beta$

$$\begin{aligned} &\Rightarrow (rx, ra) \in A_\alpha^\beta \\ &\Rightarrow rx \in [ra]_{A_\alpha^\beta} \subseteq B \\ &\Rightarrow ra \in \underline{R}(A_\alpha^\beta, B). \end{aligned}$$

Lemma 3.17. Let A be an intuitionistic fuzzy ideal of an abelian near-ring N and $\alpha, \beta \in [0,1]$ where $\alpha+\beta \leq 1$. If $\underline{R}(A_\alpha^\beta, X)$ is a non-empty set then $[0]_{A_\alpha^\beta} \subseteq A$.

Proposition 3.18. Let A be an intuitionistic fuzzy ideal of an abelian near-ring N and $\alpha, \beta \in [0,1]$ where $\alpha+\beta \leq 1$. Let B be an ideal of N. If $\underline{R}(A_\alpha^\beta, X)$ is a non-empty set then it is equal to A.

Corollary 3.19. If A is an intuitionistic fuzzy ideal of an abelian near-ring then $(\underline{R}(A_\alpha^\beta, X), \overline{R}(A_\alpha^\beta, X))$ is a rough left ideal of N.

4 Conclusion

The idea of rough intuitionistic fuzzy ideals of a near-ring based on its lower and upper approximation is discussed. Some interesting properties are established which hold directly for a ring.We hope that by the extension of this work further we would get new results satisfing more properties for a ring.

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