http://www.newtheory.org ISSN: 2149-1402

Journal of New Theory

*Received*: 10.02.2016
*Published*: 04.10.2017
*Year*: **2017**, *Number*: **16**, *Pages*: **01-09**
*Original Article*

# MULTICRITERIA DECISION MAKING BASED ON CUBIC SET

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**Abstaract** − This paper solves multicriteria decision making problems based on cubic set. The whole cubic set information given by the decision maker has been presented in a matrix form along with the weights assigned to each criteria. We have applied proposed method to select best alternative among available altermatives.

***Keywords*** − *Fuzzy sets, Cubic sets, Score function.*

# 1 Introduction

The idea of fuzzy sets (FSs) was first proposed by Zadeh and has achieved a huge success in many areas.The concept of fuzzy sets was generalized as intuitionistic fuzzy sets (IFSs) by Atanassov. In 2008, Xu proposed some geometric aggregation operators, like the intuitionistic fuzzy weighted geometric (IFWG) operator, the intuitionistic fuzzy ordered weighted geomtric (IFOWG) operator and the intuitionistic fuzzy hybrid geometric (IFHG) operator, and applied IFGH operator to multicriteria decision-making problems with intuitionistic fuzzy knowledge. Some of the arithmetic aggregation operators like intuitionistic fuzzy weighted averaging (IFWA) etc. were introduced by Xu (2000). Tursken (1986) and Gorzaleczany (1987) gave the idea of so-called interval-valued fuzzy sets (IVFSs) which was considered to be furthur general form of a fuzzy set, but really there is solid bond between IFSs and IVFSs. Both the IFSs and IVFSs were furthur generalized by Gargov (1989), named as interval-valued intuitionistic fuzzy sets (IVIFSs). For IVIFSs some aggregation operators, labelled as the interval-valued intuitionistic fuzzy weighted geometric aggregation (IIFWGA) operator and the interval-valued intuitionistic fuzzy weighted

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arithematic aggregation (IIFWAA) operator were introduced, and utilized these operators to decision making problems involving multicriteria with the help of the score function of interval-valued intuitionistic fuzzy information.

In the current article we have proposed the application of cubic set instead of IVIFS to decision-making problems having multicriteria. Our proposed score function or an accuracy function does not lead to the paradox of the difficult decision to the alternatives. The remaining article is arranged as follows. In section no. 3, we briefly introduce some aggregation operators for cubic sets. In third section, we suggest a score function, and then we provide two examples to justify that the suggested function is more suitable in the process of decision-making . In section 4, we have established a algorithm to recognize the best alternative. We make the use of cubic set weighted geometric aggregation (CSWGA) and cubic set weighted aggregation (CSWAA) operators to aggregate cubic set information corresponding to each alternative, and then give ranking to the alternatives and choose the best one(s) in view of the accuracy degrees of the aggregated cubic set information corresponding to score function. We show the worth of the adopted method by presenting illustrative examples in section 5.

# 2 Preliminaries

A fuzzy set in a set $U$ is a function defined by $\mu : U \to I$ where $I = [0, 1]$. The closed subinterval $\widetilde{c} = [c^-, c^+]$ of $I$, is called an interval number, where $0 \le c^- \le c^+ \le 1$. The interval number $\widetilde{c} = [c^-, c^+]$ with $c^- = c^+$ denoted by $c$. For the set of all interval numbers we will use the notation $[I]$ .

Let $U$ be a nonempty set. A function $B : U \to [I]$ is called an interval-valued fuzzy set (IVF) in $U$. Let $[I]^U$ denote the set of all IVF sets in $U$. For every $B \in [I]^U$ and $u \in U$, $B(u) = [B^-(u), B^+(u)]$ is called the degree of the membership of an element $u$ to $B$, where $B^- : U \to I$ and $B^+ : U \to I$ are fuzzy sets in $U$ which are termed as lower fuzzy set and upper fuzzy set in $U$ resp. For every $F, G \in [I]^U$, we define $F \subseteq G \iff F(u) \le G(u)$ for all $u \in U$, and $F = G \iff F(u) = G(u)$ for all $u \in U$.

## 2.1 Cubic Sets

Let $U \ne \Phi$ be a set. A cubic set in $U$ has the form, $B = \{\langle\ u, B(u), \mu(u)\rangle \mid u \in U\}$ ,where $B$ is an IVF set in $U$ and $\mu$ is a fuzzy set in $U$. A cubic set $B = \{\langle\ u, B(u), \lambda(u)\rangle \mid u \in U\}$ is denoted by $B = \langle B, \mu\rangle$ for simplicity.

### 2.1.1 Internal Cubic Set ( briefly, ICS)

Let $U \ne \Phi$ be a set. A cubic set $B = \langle B, \mu\rangle$ in $U$ is known as an internal cubic set (ICS) if $B^-(u) \le \mu(u) \le B^+(u)$ for all $u \in U$.

### 2.1.2 External Cubic Set ( briefly,ECS)

Let $U \ne \Phi$ be a set. A cubic set $B = \langle B, \mu\rangle$ in $U$ is known as an external cubic set (ECS) if $\mu(x) \notin (B^-(u), B^+(u))$ for all $u \in U$

### 2.1.3 Example

Let $B = \{\langle u, B(u), \mu(u)\rangle \mid u \in I\}$ be a cubic set in $I$. If $B(u) = [0.2, 0.5]$ and $\mu(u) = 0.4$ for all $\mu \in I$, then $B$ is an ICS. If $B(x) = [0.2, 0.5]$ and $\mu(u) = 0.7$ for all $u \in I$, then $B$ is an ECS. If $B(u) = [0.2, 0.5]$ and $\mu(u) = u$ for all $u \in I$, then $B$ does not belong to the class of ICS and ECS.

# 3 Score Function

Before defining score function, we define two weighted aggregation operators related to CSs.

**Definition 3.1.** *Let $B = \langle B, \mu\rangle$ and $C = \langle C, \nu\rangle$ be cubic sets in $U$. Then we define (i) (Equality) $B = C \iff B = C$ and $\mu = \nu$. (ii) (P-order) $B \subseteq_p C \iff B \subseteq C$ and $\mu \leq \nu$. (ii) (R-order) $B \subseteq_R C \iff B \subseteq C$ and $\mu \geq \nu$.*

From here on we will denote by CS$(U)$ the set of all cubic sets in $U$. The value of a cubic set will be conventionally denoted by $B = ([b, c], d)$.

**Definition 3.2.** *Let $B_j(1 \leq j \leq n) \in CS(U)$. The weighted arithematic average operator is defined by $F_w(\ B_1, B_2, ..., B_n) =$*

$$\sum_{j=1}^{n} w_j B_j = \left(\left[1 - \overset{n}{\underset{j=1}{\pi}} \left(1 - B_j^-(u)\right)^{w_j}, 1 - \overset{n}{\underset{j=1}{\pi}} \left(\left(1 - B_j^+(u)\right)^{w_j}\right)\right], \left[\overset{n}{\underset{j=1}{\pi}} \mu_j^{wj}(u)\right]\right) \tag{1}$$

where $w_j$ is the weight of $B_j(1 \leq j \leq n)$, $w_j \in [0, 1]$ and $\sum_{j=1}^{n} w_j = 1$. Especially assume $w_j = \frac{1}{n}$ $(\ j = 1, 2, ..., n)$ then, $F_w$ is known as an arithematic average operator for CSs.

**Definition 3.3.** *Let $B_j(1 \leq j \leq n) \in CS(U)$. The weighted geometric average operator is defined by*

$$G_w(\ B_1, B_2, ..., B_n) = \overset{n}{\underset{j=1}{\pi}} B_j^{w_j} = \left(\left[\overset{n}{\underset{j=1}{\pi}} B_j^{-^{w_j}}(u), \overset{n}{\underset{j=1}{\pi}} B_j^{+^{w_j}}(u)\right], \left[1 - \overset{n}{\underset{j=1}{\pi}} \left(1 - \mu_j(u)\right)^{wj}\right]\right) \tag{2}$$

where $w_j$ is the weight of $B_j(1 \leq j \leq n)$, $w_j \in [0, 1]$ and $\sum_{j=1}^{n} w_j = 1$. Especially assume $w_j = \frac{1}{n}$ $(\ j = 1, 2, ..., n)$ then, $G_w$ is known as geometric average operator for CSs.

The aggregation results $F_w$ & $G_w$ are still CS$(U)$.

Let $B = ([b, c], d)$ be a CSV, a score function $M$ of cubic set value is suggested by the formula given below

$$M(\mathbf{B}) = \frac{b + c - 1 + d}{2} \tag{3}$$

where $M(\mathbf{B}) \in [-1, +1]$. Now we consider following examples.

## 3.1 Example

If internal cubic set values for different alternatives are $B_1 = ([0.3, 0.5], 0.4)$ and $B_2 = ([0.5, 0.7], 0.6)$ the wanted alternative is selected in view of score function. After applying equation (3) we have

$$\begin{aligned} M(\mathbf{B}_1) &= \frac{0.3 + 0.5 - 1 + 0.4}{2} = 0.1 \\ M(\mathbf{B}_1) &= \frac{0.5 + 0.7 - 1 + 0.6}{2} = 0.4 \end{aligned}$$

Obviously the alternative $B_2$ has prefrence over $B_1$.

## 3.2 Example

If external cubic set values for two diffrent alternatives are $B_1 = ([0.3, 0.4], 0.5)$ and $B_2 = ([0.4, 0.5], 0.6)$ the desired alternative is choosen with the help of score function. By using equation (3) we get

$$\begin{aligned} M(\mathbf{B}_1) &= \frac{0.3 + 0.4 - 1 + 0.5}{2} = 0.10 \\ M(\mathbf{B}_2) &= \frac{0.4 + 0.5 - 1 + 0.6}{2} = 0.25 \end{aligned}$$

clearly the alternative $B_2$ has advantage over $B_1$.

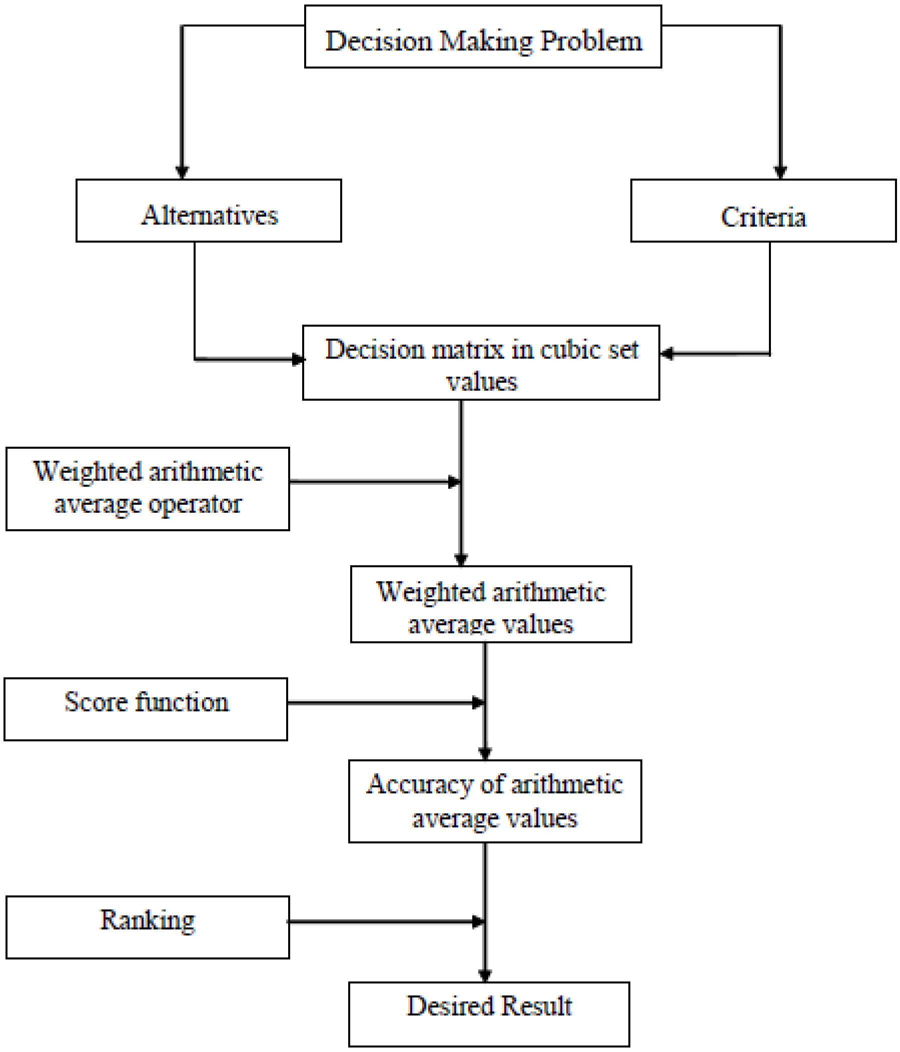
# 4 Multicriteria Cubic Set Decision Making Method Based on the Score Function

Here we are going to present a method for tackling of multicriteria cubic set decision-making problems along with weights. Suppose that $B = \{\mathbf{B}_1, \mathbf{B}_2, ..., \mathbf{B}_m\}$ is a collection of alternatives and also suppose that $C = \{C_1, C_2, ..., C_n\}$ is a set of criteria. Consider the criterion $C_j$ $(1 \leq j \leq n)$, recommended by the decision-maker, has weight $w_j, w_j \in [0, 1]$ and $\sum_{j=1}^{n} w_j = 1$. In this situation, the characterstic of the alternative $B_i$ is represented by a cubic set as

$$B_i = \left\{ \left\langle C_j, \left[B^-(C_j), B^+(C_j)\right], \left[\mu(C_j)\right] \right\rangle \mid C_j \in C \right\}.$$

The cubic set value that is the pair of IVFS and fuzzy number, i.e.

$$(\mathbf{B}_i(C_j) = [b_{ij}, c_{ij}], \mu(C_j) = d_{ij} \text{ for } C_j \in C) \text{ is denoted by } \alpha_{ij} = ([b_{ij},\ c_{ij}], d_{ij})$$




Flow chart of the proposed method.

Since $[b_{ij},\ c_{ij}] \subseteq [0,1]$ & $d_{ij} \in [0,1]$. Therefore a decision matrix of the form $D = (\alpha_{ij})$ can be formulated. The aggregating cubic set value $\alpha_i$ for $B_i\,(1 \le i \le m)$ is $\alpha_i = ([b_i,\ c_i], d_i) = F_{iw}(\alpha_{i1}, \alpha_{i2,...}, \alpha_{in})$ or $\alpha_i = ([b_i,\ c_i], d_i) = G_{iw}(\alpha_{i1}, \alpha_{i2,...}, \alpha_{in})$ which is obtained by using equation (1) or Eq. (2), in accordance with each row in the decision matrix. We will use Eq. (3) to calculate the accuracy $M(\alpha_i)$ of aggregating cubic set value $\alpha_i\,(1 \le i \le m)$ to rank the alternatives $B_i\,(1 \le i \le m)$ and then to choose the best one(s). Simply, the decision making process for the suggested technique can be described by the following steps.

Step( a). Obtain the weighted arithmetic average values by applying Eq. (1) if we prefer the influence of group, otherwise get the weighted geometric values with the help of Eq. (2).

Step( b). Obtain the accuracy $M(\alpha_i)$ of cubic set value $\alpha_i\,(1 \le i \le m)$ by the application of Eq. (3).

Step (c). Rank the alternatives $B_i\,(1 \le i \le m)$ and choose the best one(s) in comparison with $M(\alpha_i)\,(1 \le i \le m)$.

# 5 Illustrative Examples

This section is consisting of two examples. First example adapted from Herrera and Herrera -Viedma (2000) for a decision-making problem of alternatives along with multicriteria is used to potray the suggested fuzzy decision making method in the spectrum of reallity, as well as the validity of the effectiveness of the suggested algorithm.

Here is a set of people provided with four options to invest the money: (1) $B_1$ is a company of car; (2) $B_2$ is a company of food; (3) $B_3$ is a company of computer; (4) $B_4$ is a company of arms. The investor must have to decide by keeping in mind these three criteria: (1) $C_1$ is the analysis of risk; (2) $C_2$ is the analysis of growth; (3) $C_3$ is the analysis of enviromental impact. Now decider will evaluate the four possible alternatives under the above mentioned criteria, as provided in the following matrices. First we consider the matrix $D_1$ consisting of internal cubic set values.

$$D_1 = \begin{bmatrix} ([0.1, 0.3], 0.2) & ([0.2, 0.4], 0.3) & ([0.3, 0.6], 0.4) \\ ([0.5, 0.7], 0.5) & ([0.3, 0.4], 0.3) & ([0.7, 0.8], 0.7) \\ ([0.3, 0.5], 0.4) & ([0.7, 0.9], 0.8) & ([0.6, 0.8], 0.7) \\ ([0.4, 0.6], 0.4) & ([0.1, 0.2], 0.2) & ([0.6, 0.8], 0.7) \end{bmatrix}$$

Now assume that the weights of $C_1$, $C_2$ & $C_3$ are $0.35, 0.25$ and $0.40$ resp. Then we use the following algorithm.

Step 1. Eq. (1) provides us the weighted arithmetic average value $\alpha_i$ for $B_i\,(i = 1, 2, ..., 4)$.
$\alpha_1 = ([0.2097, 0.4615], 0.2921)$
$\alpha_2 = ([0.5566, 0.6967], 0.5035)$
$\alpha_3 = ([0.5472, 0.7682], 0.5950)$
$\alpha_4 = ([0.3827, 0.5243], 0.3678)$

Step 2. By applying Eq. (3) we can compute $M(\alpha_i)$ where $i = 1, 2, 3, 4$ as $M(\alpha_1) = 0.4817$, $M(\alpha_2) = 0.3784$, $M(\alpha_3) = 0.4552$, $M(\alpha_4) = 0.1374$.

Step 3. Awarding ranks to all alternatives in view of the accuracy degree of $M(\alpha_i)\ (i = 1, 2, 3, 4) : B_1 \succ B_3 \succ B_2 \succ B_4$, and thus the best alternative is $B_1$.

Now we consider the matrix $D_2$ consisting of external cubic set values.

$$D_2 = \begin{bmatrix} ([0.4, 0.5], 0.3) & ([0.4, 0.6], 0.2) & ([0.1, 0.3], 0.5) \\ ([0.6, 0.7], 0.2) & ([0.5, 0.7], 0.2) & ([0.4, 0.7], 0.1) \\ ([0.3, 0.6], 0.1) & ([0.5, 0.6], 0.4) & ([0.5, 0.6], 0.3) \\ ([0.7, 0.8], 0.1) & ([0.6, 0.7], 0.3) & ([0.3, 0.4], 0.2) \end{bmatrix}$$

Consider the same weights for $C_1$, $C_2$ & $C_3$ as mentioned above and use the following algorithm.

Step 1. Applying Eq. (1) we obtain the weighted arithmetic average value $\alpha_i$ for $B_i\,(i = 1, 2, ..., 4)$.
$\alpha_1 = ([0.2944, 0.4590], 0.3325)$
$\alpha_2 = ([0.5026, 0.7000], 0.1516)$
$\alpha_3 = ([0.4375, 0.6000], 0.2195)$
$\alpha_4 = ([0.5476, 0.6565], 0.1737)$

Step 2. By applying Eq. (3) we can compute $M(\alpha_i)$ where $i = 1, 2, 3, 4$ as $M(\alpha_1) = 0.0430$, $M(\alpha_2) = 0.1771$, $M(\alpha_3) = 0.1285$, $M(\alpha_4) = 0.1889$.

Step 3. By ranking all alternatives in view of the accuracy degree of $M(\alpha_i)$ $(i = 1, 2, 3, 4) : B_4 \succ B_2 \succ B_3 \succ B_1$, and thus the alternative $B_4$ is the best one.

Finally we consider the matrix $D_3$ consisting of cubic set values which are neither internal cubic set values nor external cubic set values.

$$D_3 = \begin{bmatrix} ([0.3, 0.7], 0.1) & ([0.3, 0.7], 0.2) & ([0.3, 0.7], 0.4) \\ ([0.3, 0.7], 0.4) & ([0.3, 0.7], 0.5) & ([0.4, 0.7], 0.1) \\ ([0.3, 0.7], 0.7) & ([0.3, 0.7], 0.8) & ([0.3, 0.7], 0.6) \\ ([0.2, 0.5], 1) & ([0.2, 0.5], 0.3) & ([0.2, 0.5], 0.6) \end{bmatrix}$$

Again using the similar procedure as stated above with similar weights we have $M(\alpha_1) = 0.1036$, $M(\alpha_2) = 0.1215$, $M(\alpha_3) = 0.3403$, $M(\alpha_4) = 0.3017$ so $B_3 \succ B_4 \succ B_2 \succ B_1$ and thus the alternative $B_3$ is the most wishful one.

Now we present another example in this section in which we want to investigate the suitability of an S-box to image encryption applications. We have been provided with nine different alternatives of S-boxes: (1) $B_1$ is Plain Image; (2) $B_2$ is Advanced Encryption Standard; (3) $B_3$ is Affine Power Affine; (4) $B_4$ is Gray; (5) $B_5$ is $\mathrm{S}_8$; (6) $B_6$ is Liu; (7) $B_7$ is Prime; (8) $B_8$ is Xyi; (9) $B_9$ is Skipjack. We have to make the decision according to the following criterion: (1) $C_1$ is the entropy analysis; (2) $C_2$ is the contrast analysis; (3) $C_3$ is the average correlation analysis; (4) $C_4$ is the energy analysis; (5) $C_5$ is the homogeneity analysis; (6) $C_6$ is the mean of absolute deviation analysis. The nine possible alternatives are to be sorted out using the cubic set information by the decider from the given criterion as presented in the following matrix.

$$D = \begin{bmatrix} ([0.1, 0.2], 0.3) & ([0.1, 0.3], 0.2) & ([0.3, 0.4], 0.1) & ([0.4, 0.5], 0.6) & ([0.3, 0.6], 0.5) & ([0.5, 0.6], 0.4) \\ ([0.5, 0.7], 0.4) & ([0.3, 0.4], 0.2) & ([0.7, 0.8], 0.6) & ([0.4, 0.5], 0.3) & ([0.6, 0.7], 0.2) & ([0.4, 0.7], 0.1) \\ ([0.3, 0.5], 0.4) & ([0.7, 0.9], 0.8) & ([0.6, 0.8], 0.7) & ([0.5, 0.6], 0.3) & ([0.7, 0.8], 0.1) & ([0.1, 0.3], 0.5) \\ ([0.4, 0.6], 0.4) & ([0.1, 0.2], 0.2) & ([0.3, 0.6], 0.4) & ([0.3, 0.4], 0.1) & ([0.3, 0.4], 0.2) & ([0.6, 0.7], 0.3) \\ ([0.1, 0.3], 0.3) & ([0.5, 0.6], 0.7) & ([0.2, 0.4], 0.3) & ([0.6, 0.8], 0.7) & ([0.1, 0.2], 0.2) & ([0.3, 0.5], 0.1) \\ ([0.5, 0.6], 0.2) & ([0.4, 0.7], 0.6) & ([0.5, 0.7], 0.9) & ([0.8, 0.9], 0.8) & ([0.4, 0.6], 0.3) & ([0.7, 0.8], 0.2) \\ ([0.7, 0.8], 0.9) & ([0.4, 0.7], 0.5) & ([0.4, 0.6], 0.2) & ([0.7, 0.9], 0.2) & ([0.8, 0.9], 0.7) & ([0.2, 0.5], 0.4) \\ ([0.8, 0.9], 0.7) & ([0.7, 0.9], 0.8) & ([0.1, 0.2], 0.1) & ([0.3, 0.2], 0.1) & ([0.5, 0.6], 0.1) & ([0.4, 0.8], 0.6) \\ ([0.8, 0.9], 0.6) & ([0.6, 0.9], 0.7) & ([0.3, 0.5], 0.6) & ([0.4, 0.7], 0.3) & ([0.4, 0.6], 0.5) & ([0.1, 0.2], 0.3) \end{bmatrix}$$

Now we assume the same weight for each of $C_1$, $C_2$ ,..., $C_6$, that is 0.167 and use the following algorithm.

Step 1. We calculate the weighted arithmetic average value $\alpha_i$ for $B_i$ $(i = 1, 2, ..., 9)$ with the aid of Eq. (1).

$\alpha_1 = ([0.3035, 0.4592], 0.2922)$
$\alpha_2 = ([0.5096, 0.6646], 0.2501)$
$\alpha_3 = ([0.5330, 0.7200], 0.3797)$
$\alpha_4 = ([0.3575, 0.5170], 0.2334)$
$\alpha_5 = ([0.3350, 0.5194], 0.3025)$
$\alpha_6 = ([0.5884, 0.7499], 0.4088)$
$\alpha_7 = ([0.5912, 0.7845], 0.4068)$
$\alpha_8 = ([0.5330, 0.7242], 0.2567)$
$\alpha_9 = ([0.4942, 0.7272], 0.4670)$

Step 2. By applying Eq. (3) we can compute $M(\alpha_i)$ where $i = 1, 2, ..., 9$ as $M(\alpha_1) = 0.0275$, $M(\alpha_2) = 0.2122$, $M(\alpha_3) = 0.3164$, $M(\alpha_4) = 0.0540$, $M(\alpha_5) = 0.0785$, $M(\alpha_6) = 0.3736$, $M(\alpha_7) = 0.8913$, $M(\alpha_8) = 0.7570$, $M(\alpha_9) = 0.3342$.

Step 3. After awarding ranks to all alternatives in view of the accuracy degree of $M(\alpha_i)$ $(i = 1, 2\ , ..., 9.)$ : $B_7 \succ B_8 \succ B_6 \succ B_9 \succ B_3 \succ B_2 \succ B_5 \succ B_4 \succ B_1$ and thus the alternative $B_7$ is the most desired one.

## References

[1] K. Atanassov, Intuitionistic fuzzy sets, VII ITKR's session, Sofia, June (1983).

[2] K. Atanassov, Intuitionistic fuzzy sets, Fuzzy Sets and Systems 20 (1986) 87–96.

[3] K. Atanassov, Operators over interval-valued intuitionistic fuzzy sets. Fuzzy Sets and Systems 64(2) (1994) 159–174.

[4] K. Atanassov, G. Gargov, Interval-valued intuitionistic fuzzy sets, Fuzzy Sets and Systems 31(3) (1989) 343–349.

[5] H. Bustine, P. Burillo, Vague sets are intuitionistic fuzzy sets, Fuzzy Sets and Systems 79 403–405.

[6] S. M. Chen, J. M. Tan, Handling multicriteria fuzzy decision-making problems based on vague set theory, Fuzzy Sets and Systems 67 (1994) 163–172.

[7] W. L. Gau, D. J. Buehrer, Vague sets, IEEE Transactions on Systems, Man and Cybernetics 23(2) (1993) 610–614.

[8] M. B. Gorzaleczany, A method of inference in approximate reasoning based on interval-valued fuzzy sets, Fuzzy Sets and Systems 21(1987) 1–17.

[9] F. Herrera, Herrera - E. Viedma, Linguistic decision analysis: Steps for solving decision problems under linguistic information, Fuzzy Sets and Systems 115 (2000) 67–82.

[10] D. H. Hong, C.-H. Choi, Multicriteria fuzzy decision-making problems based on vague set theory, Fuzzy Sets and Systems 114 (2000) 103–113.

[11] D. F. Li, Multiattribute decision making models and methods using intuitionistic fuzzy sets, Journal of Computer and System Sciences 70 (2005) 73–85.

[12] L. Lin, X. H.Yuan, Z. Q. Xia, Multicriteria decision-making methods based on intuitionistic fuzzy sets, Journal of Computer and System Sciences 73 (2007) 84–88.

[13] H. W. Liu, G. J. Wang, Multicriteria decision-making methods based on intuitionistic fuzzy sets, European Journal of Operational Research 179 (2007) 220–233.

[14] H. B. Mitchell, Ranking intuitionistic fuzzy numbers, International Journal of Uncertainty, Fuzziness and Knowledge Based Systems 12(3) (2004) 377–386.

[15] B. Turksen, Interval valued fuzzy sets based on normal forms, Fuzzy Sets and Systems 20 (1986) 191–210.

[16] Z. S. Xu, Intuitionistic preference relations and their application in group decision making, Information Science 177(11) (2007) 2363–2379.

[17] Z. S. Xu, Intuitionistic fuzzy aggregation operators, IEEE Transactions on Fuzzy Systems 15(6) (2007) 1179–1187.

[18] Z. S. Xu, Methods for aggregating interval-valued intuitionistic fuzzy information and their application to decision making, Control and Decision 22(2) (2007) 215–219.

[19] Z. S. Xu, J. Chen, An approach to group decision making based on interval-valued intuitionistic fuzzy judgment matrices, System Engineer – Theory and Practice 27(4) (2007) 126–133.

[20] Z. S. Xu, J. Chen, On geometric aggregation over interval-valued intuitionistic fuzzy information. In FSKD, 4th international conference on fuzzy systems and knowledge discovery (FSKD 2007) (Vol. 2, pp. 466–471).

[21] Z. S. Xu, Q. L. Da, An overview of operators for aggregating information, International Journal of Intelligent Systems 18 (2003) 953–969.

[22] Z. S. Xu, R. R. Yager, Some geometric aggregation operators based on intuitionistic fuzzy sets, International Journal of General System 35 (2006) 417–433.

[23] J. Ye, Multicriteria fuzzy decision-making based on a novel accuracy function under interval-valued intuitionistic fuzzy environment, Fuzzy Systems with Application 36 (2009) 6899-6902.

[24] Young Bae Jun, Chang Su Kim, Kim Oong Yang, Cubic sets, Annals of Fuzzy Mathematics and Informatics (2011) Vol. 4, No, 1, pp. 83-98

[25] L. A. Zadeh, Fuzzy sets. Information and Control 8(3) (1965) 338–356.