http://www.newtheory.org ISSN: 2149-1402

Journal of New Theory

Received: 14.03.2016 *Year*: **2017**, *Number*: **16**, *Pages*: **10-18**
Published: 04.10.2017 *Original Article*

JUST CHROMATIC EXCELLENCE IN FUZZY GRAPHS

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Abstaract − Let G be a simple fuzzy graph. A family $\Gamma^f = \{\gamma_1, \gamma_2, \ldots, \gamma_k\}$ of fuzzy sets on a set V is called k-fuzzy colouring of $V = (V, \sigma, \mu)$ if i)$\cup\Gamma^f = \sigma$, ii) $\gamma_i \cap \gamma_j = \emptyset$, iii)for every strong edge (x, y)(i.e., $\mu(xy) > 0$) of G $\min\{\gamma_i(x), \gamma_i(y)\} = 0, (1 \leq i \leq k)$. The minimum number of k for which there exists a k-fuzzy colouring is called the fuzzy chromatic number of G denoted as $\chi^f(G)$. Then Γ^f is the partition of independent sets of vertices of G in which each sets has the same colour is called the fuzzy chromatic partition. A graph G is called the just χ^f-excellent if every vertex of G appears as a singleton in exactly one χ^f-partition of G. This paper aims at the study of the new concept namely Just Chromatic excellence in fuzzy graphs. Fuzzy colourful vertex is defined and studied. We explain these new concepts through examples.

Keywords − *fuzzy chromatic excellent, fuzzy just excellent,fuzzy colourful vertex*

1 Introduction

A fuzzy graph is a symmetric binary fuzzy relation on a fuzzy subset. The concept of fuzzy sets and fuzzy relations was introduced by L.A.Zadeh in 1965[1] and further studied[2]. It was Rosenfeld[5] who considered fuzzy relations on fuzzy sets and developed the theory of fuzzy graphs in 1975. The concepts of fuzzy trees, blocks, bridges and cut nodes in fuzzy graph has been studied[3]. Computing chromatic sum of an arbitrary graph introduced by Kubica [1989] is known as NP-complete problem. Graph coloring is the most studied problem of combinatorial optimization. As an advancement fuzzy coloring of a fuzzy graph was defined by authors Eslahchi and Onagh in 2004, and later developed by them as Fuzzy vertex coloring[4] in 2006.This fuzzy vertex coloring was extended to fuzzy total coloring in terms of family of fuzzy sets by Lavanya. S and Sattanathan. R[6]. In this paper we are introducing "Just Chromatic excellence in fuzzy graphs".

2 Preliminary

Definition 2.1. A fuzzy graph $G = (\sigma, \mu)$ is a pair of functions $\sigma : V \to [0, 1]$ and $\mu : V \times V \to [0, 1]$ where for all $u, v \in V$, we have $\mu(u, v) \leq \sigma(u) \wedge \sigma(v)$.

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Definition 2.2. The order p and size q of a fuzzy graph $G = (\sigma, \mu)$ are defined to be $p = \sum\limits_{x \in V} \sigma(x)$ and $q = \sum\limits_{xy \in E} \mu(xy)$.

Definition 2.3. The degree of vertex u is defined as the sum of the weights of the edges incident at u and is denoted by $d(u)$.

Definition 2.4. The union of two fuzzy graphs G_1 and G_2 is defined as a fuzzy graph $G = G_1 \cup G_2 : ((\sigma_1 \cup \sigma_2, \mu_1 \cup \mu_2))$ defined by

$$\begin{aligned} (\sigma_1 \cup \sigma_2)(u) &= \begin{cases} \sigma_1(u), \text{ if } u \in V_1 - V_2 \text{ and} \\ \sigma_2(u) \text{ if } u \in V_2 - V_1 \end{cases} \\ (\mu_1 \cup \mu_2)(uv) &= \begin{cases} \mu_1(uv), \text{ if } uv \in E_1 - E_2 \text{ and} \\ \mu_2(uv) \text{ if } uv \in E_2 - E_1 \end{cases} \end{aligned}$$

Definition 2.5. The join of two fuzzy graphs G_1 and G_2 is defined as a fuzzy graph $G = G_1 + G_2 : ((\sigma_1 + \sigma_2, \mu_1 + \mu_2))$ defined by

$$\begin{aligned} (\sigma_1 + \sigma_2)(u) &= (\sigma_1 \cup \sigma_2)(u) \forall u \in V_1 \cup V_2 \\ (\mu_1 + \mu_2)(uv) &= \begin{cases} (\mu_1 \cup \mu_2)(uv) \text{ if } uv \in E_1 \cup E_2 \text{ and} \\ \sigma_1(u) \wedge \sigma_2(v) \text{ if } uv \in E'. \end{cases} \end{aligned}$$

where E' is the set of all edges joining the nodes of V_1 and V_2.

Definition 2.6. The cartesian product of two fuzzy graphs G_1 and G_2 is defined as a fuzzy graph $G = G_1 \times G_2 : (\sigma_1 \times \sigma_2, \mu_1 \times \mu_2)$ on $G* : (V, E)$ where $V = V_1 \times V_2$ and $E = \{((\sigma_1, \sigma_2), (\mu_1, \mu_2))/u_1 = v_1, u_2v_2 \in E_2 or u_2 = v_2, u_1v_1 \in E_1\}$ with

$$\begin{aligned} (\sigma_1 \times \sigma_2)(u_1, v_1) &= \sigma_1(u_1) \wedge \sigma_2(u_2) forall (u_1, u_2) \in V_1 \times V_2 \\ (\mu_1 \times \mu_2)((u_1, u_2)(v_1, v_2)) &= \begin{cases} \sigma_1(u_1) \wedge \mu_2(u_2, v_2), \text{ if } u_1 = v_1 \text{ and } u_2v_2 \in E_2 \\ \sigma_2(u_2) \wedge \mu_1(u_1, v_1), \text{ if } u_2 = v_2 \text{ and } u_1v_1 \in E_1 \end{cases} \end{aligned}$$

3 Main Defintions and Results

Definition 3.1. Let G be a fuzzy graph. A family $\Gamma^f = \{\gamma_1.\gamma_2, \ldots, \gamma_k\}$ of fuzzy sets on a set V is called k-fuzzy colouring of $V = (V, \sigma, \mu)$ if

(i) $\cup \Gamma^f = \sigma$,

(ii) $\gamma_i \cap \gamma_j = \emptyset$,

(iii) for every strong edge (x, y)(i.e., $\mu(xy) > 0$) of G $\min\{\gamma_i(x), \gamma_i(y)\} = 0, (1 \leq i \leq k)$.

The minimum number of k for which there exists a k-fuzzy colouring is called the fuzzy chromatic number of G denoted as $\chi^f(G)$.

Definition 3.2. Γ^f is the partition of independent sets of vertices of G in which each sets has the same colour is called the fuzzy chromatic partition.

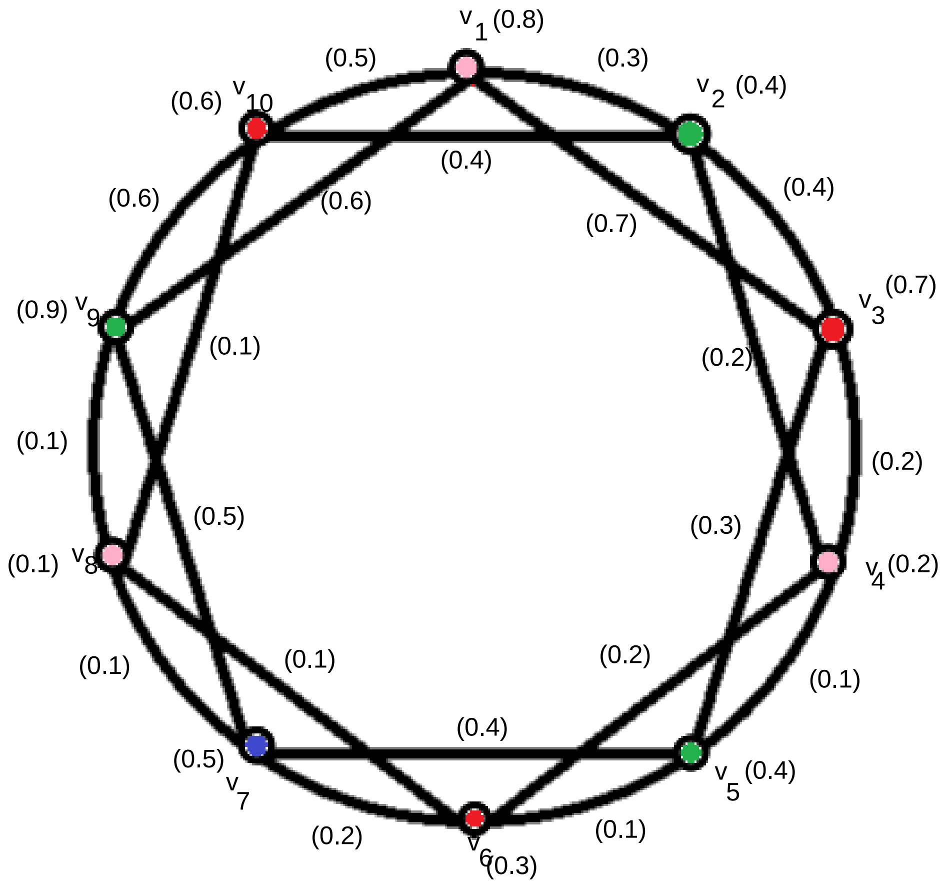

Figure 1: For Exaple 3.7

Definition 3.3. A vertex $v \in V(G)$ is called Γ^f-good if $\{v\}$ belongs to a Γ^f-partition. Otherwise v is said to be Γ^f-bad vertex.

Definition 3.4. A graph is called Γ^f-excellent fuzzy graph if every vertex of G is Γ^f-good.

Definition 3.5. A graph G is said to be Γ^f**- commendable**fuzzy graph if the number of Γ^f-good vertices is greater than the number of Γ^f-bad vertices.

A graph G is said to be Γ^f**- fair** fuzzy graph if the number of Γ^f-good vertices is equal to the number of Γ^f-bad vertices.

A graph G is said to be Γ^f**- poor** fuzzy graph if the number of Γ^f-good vertices is less than the number of Γ^f-bad vertices.

Definition 3.6. A fuzzy graph G is just χ^f-excellent if every vertex of G appears as a singleton in exactly one χ^f-partition.

Example 3.7. See Figure 1. The fuzzy colouring $\Gamma^f = \{\gamma_1, \gamma_2, \gamma_3, \gamma_4\}$
$\gamma_1(v_i) = 0.8 \quad i = 1$

$$\gamma_2(v_i) = \begin{cases} 0.4 & i = 2 \\ 0.4 & i = 5 \\ 0.1 & i = 8 \\ 0 & \text{otherwise} \end{cases}$$

$$\gamma_3(v_i) = \begin{cases} 0.7 & i = 3 \\ 0.3 & i = 6 \\ 0.9 & i = 9 \\ 0 & \text{otherwise} \end{cases}$$

$$\gamma_4(v_i) = \begin{cases} 0.2 & i = 4 \\ 0.5 & i = 7 \\ 0.6 & i = 10 \\ 0 & \text{otherwise} \end{cases}$$

For the above fuzzy graph $\chi^f(G) = 4$. Similarly, the χ^f-partitions are

$$\begin{aligned}
\Gamma_1^f &= \{\{v_1\}, \{v_2, v_5, v_8\}, \{v_3, v_6, v_9\}, \{v_4, v_7, v_{10}\}\} \\
\Gamma_2^f &= \{\{v_2\}, \{v_3, v_6, v_9\}, \{v_4, v_7, v_{10}\}, \{v_5, v_8, v_1\}\} \\
\Gamma_3^f &= \{\{v_3\}, \{v_4, v_7, v_{10}\}, \{v_5, v_8, v_1\}, \{v_6, v_9, v_2\}\} \\
\Gamma_4^f &= \{\{v_4\}, \{v_5, v_8, v_1\}, \{v_6, v_9, v_2\}, \{v_7, v_{10}, v_3\}\} \\
\Gamma_5^f &= \{\{v_5\}, \{v_6, v_9, v_1\}, \{v_7, v_{10}, v_3\}, \{v_8, v_1, v_4\}\} \\
\Gamma_6^f &= \{\{v_6\}, \{v_7, v_{10}, v_3\}, \{v_8, v_1, v_4\}, \{v_9, v_2, v_5\}\} \\
\Gamma_7^f &= \{\{v_7\}, \{v_8, v_1, v_4\}, \{v_9, v_2, v_5\}, \{v_{10}, v_3, v_6\}\} \\
\Gamma_8^f &= \{\{v_8\}, \{v_9, v_2, v_5\}, \{v_{10}, v_3, v_6\}, \{v_1, v_3, v_6\}\} \\
\Gamma_9^f &= \{\{v_9\}, \{v_{10}, v_3, v_6\}, \{v_1, v_4, v_7\}, \{v_2, v_5, v_8\}\} \\
\Gamma_{10}^f &= \{\{v_{10}\}, \{v_1, v_4, v_7\}, \{v_2, v_5, v_8\}, \{v_3, v_6, v_9\}\}
\end{aligned}$$

Therefore every vertex in above figure is appears in a singleton in exactly one χ^f-partition. Hence above figure is just χ^f-excellent.

Remark 3.8. (1) Every just χ^f-excellent fuzzy graph is χ^f-excellent graph

(2) Let G be any χ^f-excellent graph. Add a vertex u to every vertex in G such that $\mu(uv) \leq \sigma(u) \wedge \sigma(v)$ for every $v \in V(G)$. Let the resulting graph be H. Then H is χ^f-excellent but not just χ^f-excellent.

For every χ^f-partition of H contains $\{u\}$. Since G is χ^f-excellent, then for any $v \in V(G)$, there exists a χ^f-partition Γ^f of G such that $\{v\} \in \Gamma^f$. Then $\Gamma^f \cup \{u\}$ is χ^f-partition of H.

(3) If G is χ^f-excellent, then G has exactly one χ^f-partition(i.e.,G is uniquely colourable) iff G is complete.

For: If G is complete, then G is χ^f-excellent and it has exactly one χ^f-partition

Conversely, if G is χ^f-excellent and it has exactly one χ^f-partition, then every vertex in G must appear as a singleton in that χ^f-partition. Therefore G is complete.

Proposition 3.9. If G is not complete fuzzy graph and G is χ^f-excellent, then G has atleast two χ^f-partitions.

Proof. Let us take Γ^f be a χ^f-patition of G. Since G is not complete, then there exists atleast non full degree vertex say u. Let $\Gamma_1^f = \{\{u\}, V_2, \ldots, V_{\chi^f}\}$ be a χ^f-partition of G. Let $v \in V(G)$ such that u and v are not adjacent(i.e.,$\mu(uv) > \sigma(u) \wedge \sigma(v)$). Let $v \in V_i, 2 \leq i \leq \chi^f$. Then $\Gamma_2^f = \{\{V_i - \{v\}, \{u, v\}, V_3, \ldots, V_{\chi^f}\}$ is also a χ^f-partition of G not containing $\{u\}$. □

Proposition 3.10. If G is not complete fuzzy graph and G is χ^f-excellent, then G has atleast three χ^f-partitions.

Proof. We know that any χ^f-excellent non complete fuzzy graph has atleast two χ^f-partitions(from the above proposition). Suppose that G has exactly two χ^f-partitions Γ_1^f and Γ_2^f. Let $\Gamma_1^f = \{V_1, V_1, \ldots, V_{\chi^f}\}$ and $\Gamma_2^f = \{W_1, W_2, \ldots, W_{\chi^f}\}$ be the two partitions of G. Since G is χ^f-excellent and not complete, Γ_1^f has r singletons and Γ_2^f contains atleast $n - r$ singletons. Let Γ_1^f contains $\{u_1\}, \{u_2\}, \ldots, \{u_n\}$ and let $\{u_{r+1}\}, \ldots, \{u_n\}$ be the elements of Γ_2^f. Then $\langle u_1, u_2, \ldots, u_r\rangle$ is complete and also $\langle u_{r+1}, u_{r+2}, \ldots, u_n\rangle$ is complete.

Therefore in Γ_1^f there will be $\{u_1\}, \{u_2\}, \ldots, \{u_r\}, \{u_{r+1}\}, \ldots, \{u_n\}$ elements, a contradiction. Hence there are atleast three χ^f-partitions. □

Remark 3.11. A similar arguement as in the above proposition shows that there are atleast four χ^f-partitions

Remark 3.12. There exists fuzzy graphs having not full degree vertex and not just χ^f-excellent but χ^f-excellent.

Proposition 3.13. If G is just χ^f-excellent fuzzy graph and $G \neq K_n$ then $\chi^f = \left\lfloor \frac{n+1}{2} \right\rfloor$. The converse is not true.

Remark 3.14. P_n is not just χ^f-excellent fuzzy graph but is an induced subgraph of a just χ^f-excellent fuzzy graph.(If n is odd say $n = 2k + 1$, then P_n is an induced subgraph of C_{2k+3}. If n is even say $n = 2k$,then P_n is an induced subgraph of C_{2k+1}).

Remark 3.15. Let $G \neq K_n$, be a χ^f-excellent fuzzy graph with a full degree vertex. Then G is not just χ^f-excellent.

Proof. Since $G \neq K_n$, then $\chi^f(G) < n$. Let $\{u\}$ be a full degree vertex of G. Then clearly, G has atleast two χ^f-partitions. Then $\{u\}$ appears in all χ^f-partitions of G. Therefore G is not just χ^f-excellent. □

Proposition 3.16. If G is a just χ^f-excellent fuzzy graph and $G \neq K_n$,then any χ^f-partition of G can contain exactly one singleton.

Proof. Let us assume that there exists a χ^f-partition Γ^f of G containing more than one singleton. Let $\Gamma_1^f = \{\{u_1\}, \{u_2\}, V_3, \ldots, V_{\chi^f}\}$ be a partition of G. Since G is just χ^f-excellent and $G \neq K_n$, no vertex of $V(G)$ is a full degree vertex. Therefore there exists $v_1 \in V(G)$ such that u_1 and v_1 are not adjacent such that $\mu(u_1v_1) > \sigma(u_1) \wedge \sigma(v_1)$. Let $v_1 \in V_i, 3 \leq i \leq \chi^f$. Clearly, $|V_i| \geq 2$, for if $V_i = \{v_1\}$,then u_1 and v_1 are adjacent. Let $\Gamma_2^f = \{\{u_1, v_1\}, \{u_2\}, V_3, \ldots, V_i - \{v_1\}, \ldots, V_{\chi^f}\}$. Then Γ_2^f is a χ^f-partition containing $\{u_2\}$,which is a contradiction to G is just χ^f-excellent. □

Corollary 3.17. If G is just χ^f-excellent fuzzy graph and $G \neq K_n$, then $\chi^f \leq \left\lfloor \frac{n+1}{2} \right\rfloor$.

Proof. Since G is just χ^f-excellent,then any χ^f-partition contains exactly one singleton. Therefore $n \geq 1 + 2(\chi^f - 1)$. That is $n \geq 2(\chi^f - 1)$. Hence $\chi^f \leq \left\lfloor \frac{n+1}{2} \right\rfloor$. □

Remark 3.18. (1) W_6 has chromatic number $4 > \left\lfloor \frac{n+1}{2} \right\rfloor$ and W_6 is χ^f-excellent. Clearly, W_6 is not just χ^f-excellent.

(2) The bound is sharp as seen in C_5 ($\chi^f(C_5) = 3 = \frac{5+1}{2}$) and C_5 is just χ^f-excellent.

Remark 3.19. The sum of two just χ^f-excellent graphs need not be just χ^f-excellent.

For: C_5 is just χ^f-excellent but $C_5 + C_5$ is not just χ^f-excellent.

Remark 3.20. If $G + H$ is just χ^f-excellent fuzzy graph then G and H are just χ^f-excellent graph. Proof: Any chromatic partition of $G + H$ is a union of a chromatic partition of G and H.Then $G + H$ is just χ^f-excellent,then G and H are just χ^f-excellent.

Proposition 3.21. If G and H are just χ^f-excellent fuzzy graph and one of them is not complete if other is K_1 then $G + H$ is not just χ^f-excellent. Proof: Let $G = K_1$. Then H is not not complete fuzzy graph.Then $G + H$ is not complete but it has a full degree vertex. Therefore $G + H$ is not just χ^f-excellent graph.

Let $G \neq K_1$ and $h \neq K_1$. Since G and H are just χ^f-excellent, $G, H \neq \bar{K_n}$ for $n \geq 2$. Then any χ^f-partition of G and H contains atleast two elements. Then for any χ^f-partition of G with a singleton element, we can associate several χ^f-partitions of H, giving a χ^f-partition of $G + H$. Therefore $G + H$ is not just χ^f-excellent.

Proposition 3.22. Let G and H be two fuzzy graphs. $G + H$ is just χ^f-excellent if and only if both of them are complete graphs. Proof: Let us assume that G and H are complete fuzzy graph. Then $G + H$ is complete fuzzy graph and hence just χ^f-excellent.

Conversely, assume that $G + H$ are just χ^f-excellent. Therefore both G and H are just χ^f-excellent.If G or H is not complete, then using above remark, $G + H$ is not just χ^f-excellent, a contradiction. Therefore G and H are complete. Hence $G + H$ is complete fuzzy graph.

Proposition 3.23. Let $G \neq K_n$ be just χ^f-excellent graph. Let $u \in V(G)$. Let $\Gamma^f = \{\{u\}, V_2, \ldots, V_{\chi^f}$ be a χ^f-partition of G. Then for every vertex in $V_i, 2 \leq i \leq \chi^f$ is adjacent with atleast one vertex in V_j, for all j,$j \neq i, 2 \leq j \leq \chi^f$. Proof: Since G is just χ^f-excellent, $|V_i| \geq 2$ for all $i, 2 \leq i \leq \chi^f$. Let $v \in V_i$. suppose v is not adjacent to any vertex of some V_j such that $\mu(uv) > \sigma(u) \wedge \sigma(v)$,for $u \in V_j$,$j \neq i, 2 \leq j \leq \chi^f$. Then $\Gamma_1^f = \{\{u\}, V_2, \ldots, V_i - \{v\}, \ldots, V_j \cup \{v\}, \ldots, V_{\chi^f}\}$ is a χ^f-partition of G(note that $V_i - \{v\} \neq \emptyset$) different from Γ^f a contradiction.

Definition 3.24. A vertex of a fuzzy graph G with respect to a χ^f-partition Γ^f of G is called a fuzzy colourful vertex if it is adjacent to every colour class other than the one to which it belongs.

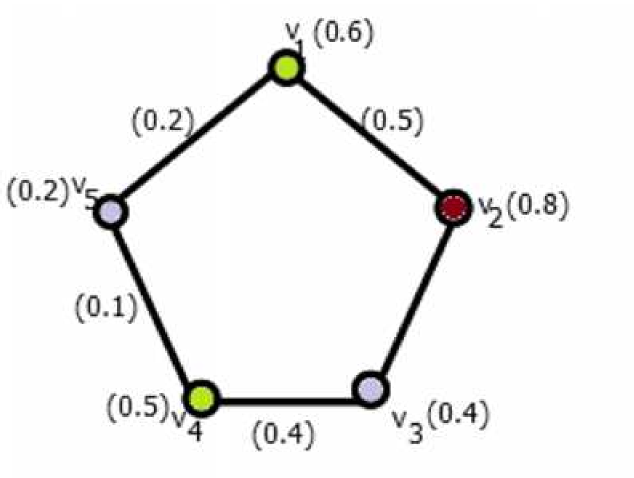

Figure 2: For Exaple 3.25

Let $\Gamma^f = \{V_1, V_2, \ldots, V_{\chi^f}\}$ be a χ^f-partition of G.Let $u \in V_i$ is said to be fuzzy colourful vertex if u is adjacent to every colour class in Γ^f-partition but not adjacent to V_i such that $\mu(uv_i) \leq \sigma(u) \wedge \sigma(v_i)$ for some vertex $v_i \in V_1, \ldots, V_{i-1}, V_{i+1}, \ldots, V_{\chi^f}$ and $\mu(uv_j) > \sigma(u) \wedge \sigma(v_j)$ for every $v_j \in V_i$

Example 3.25. See Figure 2. For the above figure $\chi^f(G) = 3$. Let

$$\Gamma^f = \{V_1 = \{v_1\}, V_2 = \{v_2, v_4\}, V_3 = \{v_3, v_5\}\}$$

be a χ^f-partition. From this partition $\{v_1\}$ is adjacent to some vertex in V_2 and V_3, $\{v_2\}$ is adjacent to V_1 and V_3, $\{v_4\}$ is adjacent to V_3 but not $\{v_1\}$, $\{v_3\}$ is adjacent to V_2 but not V_1, $\{v_5\}$ is adjacent to V_1 and V_2. Hence $\{v_1, v_2, v_5\}$ are colourful vertices with respect to the Γ^f-partition.

Corollary 3.26. (1) If G is just χ^f-excellent then every vertex in $N^f[u], u \in V(G)$ is a fuzzy colourful vertex in the χ^f-partition in which $\{u\}$ is an element. Then the number of colourful vertices is $deg^f(u) + 1$.

(2) There exists a χ^f-partition in which the number of fuzzy colourful vertices is equal to $\Delta^f(G) + 1$ which is greater than or equal to $\chi^f(G)$.

Theorem 3.27. Let G be a just χ^f-excellent fuzzy graph which is not complete.Let $u \in V(G)$ and let $\Gamma^f = \{\{u\}, V_2, \ldots, V_{\chi^f}\}$ be a χ^f-partition of G. If $|V_i| \geq 3|$ for some $2 \leq i \leq \chi^f$ then there exists a atleast some V_j with $|V_j| \geq 3$ containing a vertex not adjacent to u. Proof: Suppose let u is adjacent to every vertex in V_i with $|V_i| \geq 3(2 \leq i \leq \chi^f)$.

Case(1): $|V_i| \geq 3$ for all i, $2 \leq i \leq \chi^f$. Then u is a full degree vertex and it appears singleton in every χ^f-partition of G, which is a contradiction to G is just χ^f-excellent and $G \neq K_n$.

Case(2): Let $|V_i| \geq 3$ for all i, $2 \leq i \leq t$ and $|V_{t+1}| = 2$. Let $|V_{t+1} = \{v_1, v_2\}|$. Suppose there exists $V_{t+1}, V_{t+2}, \ldots, V_{\chi^f}$ such that $|V_{t+j}| = 2, 2 \leq j \leq \chi^f - t$(Note that no V_i,$(2 \leq i \leq \chi^f)$ is a singleton since G is just χ^f-excellent). Since Γ^f is a χ^f-partition, u is adjacent with atleast one vertex in each of $V_{t+1}, \ldots, V_{\chi^f}$. Suppose u

is adjacent with v_1 and not adjacent with v_2 in V_{t+1} such that $\mu(uv_1) \leq \sigma(u) \wedge \sigma(v_1)$ and $\mu(uv_2) > \sigma(u) \wedge \sigma(v_2)$for $v_1, v_2 \in V_{t+1}$.Then u is adjacent with every vertex $V_{t+j}, 2 \leq j \leq \chi^t - 1$ such that $\mu(uv_i) \leq \sigma(u) \wedge \sigma(v_i)$ for every $v_i \in V_{t+j}, 2 \leq j \leq \chi^f - 1$. For: otherwise there exists some vertex $w \in V_{t+j}$ not adjacent with u. Therefore $\Gamma_1^f = \{\{u, v_2, w\}, V_2, \ldots, V_t, \{v_1\}, \ldots, V_{t+j} - \{w\}, \ldots, V_{\chi^f}\}$ which is a contradiction to G is just χ^f-excellent. Hence u is adjacent with every vertex in $V - \{v_1\}$.(Note that if $V_{t+1} = V_{\chi^f}$ then also u is adjacent with every vertex in $V - \{v_2\}$). Since G is just χ^f-excellent there exists a χ^f-excellent $\Gamma_2^f = \{\{v_2\}, V_2', \ldots, V_{\chi^f}'\}$. Therefore $u \in V_i'$, a contradiction since u is adjacent with every vertex in $V - \{v_2\}$ such that $\mu(uv_i) \leq \sigma(u) \wedge \sigma(v_i)$ for every vertex $v_i \in V - \{v_2\}$. Hence the theorem.

Remark 3.28. Let G be a graph which is just χ^f-excellent. If there exists a χ^f-partition in which one of the element is a singleton $\{u\}$ and some other element with cardinality greater than or equal to 3, then there exists a χ^f-partition in which none of the elements is singleton. Proof: Let G be a just χ^f-excellent fuzzy graph satisfying the hypothesis. Then there exists a χ^f-partition $\Gamma^f = \{\{u\}, V_2, \ldots, V_{\chi^f}\}$ in which $|V_i| \geq 3$ for some i,$2 \leq i \leq \chi^f$ and V_i contains a non-neighbour,say, v and u. Then $\Gamma_1^f = \{\{u, v\}, V_2, \ldots, V_i - \{v\}, \ldots, V_{\chi^f}\}$ is a χ^f-partition of G in which each class contains atleast 2 vertices of G.

Remark 3.29. If G is just χ^f-excellent and $G \neq K_n$ and $\beta_0^f(G) = 2$, then the number of χ^f-partitions of G is exactly 'n'. For:

Let $V(G) = \{u_1, u_2, \ldots, u_k$, by the hypothesis there exists a χ^f-partitions $\{\{u_i, V_2, \ldots, V_k\}$ and $|V_i| = 2$ for all $2 \leq i \leq k$. Therefore $|V(G)| = 2k + 1$. Hence there can not exists a χ^f-partitions in which one of the element is a singleton.

Remark 3.30. If G is just χ^f-excellent and $G \neq K_n$, then G has exactly 'n' χ^f-partitions if and only if in those χ^f-partitions in which one element is a singleton, the cardinality of any other element of the partition is 2.

Remark 3.31. If G is just χ^f-excellent fuzzy graph, then $deg^f(u) \leq n - 3$ for any vertex $u \in V(G)$

4 Application

Fuzzy graph coloring has extensive applications in the following fields and solving different problems as follows: In Human Resource management such as assignment, job allocation, scheduling,In telecommunication process,In Bioinformatics, In traffic light problem.

5 Conclusion

In this paper we define new parameter called just chromatic excellence in fuzzy graphs. We can extend this concept to new type of fuzzy chromatic excellence and study the characteristics of this new parameter.

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