http://www.newtheory.org ISSN: 2149-1402

Journal of New Theory

Received: 17.09.2017 *Year*: **2017,** *Number*: **16**, *Pages*: **19-26**
Published: 05.10.2017 *Original Article*

ON SOME IDEALS OF INTUITIOISTIC FUZZY POINTS SEMIGROUPS

Essam Hamed Hamouda <ehamouda70@gmail.com>

Department of Basic Sciences, Faculty of Industrial Education, Beni-Suef University, Beni-Suef, Egypt

Abstract – In this paper, the minimal ideal A of a semigroup S is characterized by the intuitionistic characteristic function χ_A. The existence of an intuitionistic fuzzy kernel in a semigroup is explored. Finally, we consider the semigroup $\underline{S}$ of the intuitionistic fuzzy points of a semigroup S and discuss some relations between some ideals A of S and the subset $\underline{C_A}$ of the semigroup $\underline{S}$.

Keywords – *Semigroups; Intuitionistic fuzzy points; Intuitionistic fuzzy ideals.*

1 Introduction

Semigroups are important in many areas of mathematics, for example, coding and language theory, automata theory, combinatorics and mathematical analysis. Zadeh [16] introduced the concept of a fuzzy set for the first time and this concept was applied by Rosenfeld [14] to define fuzzy subgroups and fuzzy ideals. Based on this crucial work, Kuroki [9,10,11,12] defined a fuzzy semigroup and various kinds of fuzzy ideals in semigroups and characterized them. In [13], Kim considered the semigroup $\underline{S}$ of the fuzzy points of a semigroup S, and discussed the relation between the fuzzy interior ideals and the subsets of $\underline{S}$, also see [6, 7]. Atanassov [4, 5] introduced the notion of intuitionistic fuzzy sets as a generalization of fuzzy sets. Many concepts in fuzzy set theory were also extended to intuitionistic fuzzy set theory, such as intuitionistic fuzzy relations, intuitionistic L- fuzzy sets, intuitionistic fuzzy implications, intuitionistic fuzzy logics, intuitionistic fuzzy semigroups etc. Jun and Song [8] introduced the notion of intuitionistic fuzzy points. In [15] Sardar et al., defined some relations between the intuitionistic fuzzy ideals of a semigroup S and the set of all intuitionistic fuzzy points of S. In [3] Akram characterized intuitionistic fuzzy ideals in ternary semigroups by intuitionistic fuzzy points. Also in [2] he analyzed some relations between the intuitionistic fuzzy Γ-ideals and the sets of intuitionistic fuzzy points of these Γ-ideals of a Γ-semigroup. In this paper, we consider the semigroup $\underline{S}$ of the intuitionistic fuzzy points of a semigroup S, and discuss some relations between some ideals A of S and the subset $\underline{C_A}$ of the semigroup $\underline{S}$.

2 Basic Definitions and Results

Let S be a semigroup. A nonempty subset A of S is called a *left (resp., right) ideal* of S if $SA \subseteq A(resp., AS \subseteq A)$, and *a two-sided ideal* (or simply *ideal*) of S if A is both a left and a right ideal of S. A nonempty subset A of S is called an interior ideal of S if $\subseteq A$. An ideal A of S is called *minimal* ideal of S if A does not properly contains any other ideal of S. If the intersection K of all the ideals of a semigroup S is nonempty then we shall call K*the kernel* of S. A sub-semigroup A of S is called *a bi-ideal* of S if $ASA \subseteq A$. A function f from S to the closed interval [0, 1] is called a *fuzzy set* in S. The semigroup S itself is a fuzzy set in S such that $S(x) = 1$ for all $x \in S$, denoted also by S. Let A and B be two fuzzy sets in$S$. Then the inclusion relation $A \subseteq B$ is defined $A(x) \leq B(x)$ for all $x \in S$. $A \cap B$ and $A \cup B$ are fuzzy sets in S defined by

$$(A \cap B)(x) = A(x) \wedge B(x) = min\{A(x), B(x)\}$$

$$(A \cup B)(x) = A(x) \vee B(x) = max\{A(x), B(x)\}$$

for all $x \in S$.

For any $\alpha \in (0,1]$ and $x \in S$, a fuzzy set x_α in S is called a *fuzzy point* in S if

$$x_\alpha(y) = \begin{cases} \alpha & if\ x = y, \\ 0 & otherwise, \end{cases}$$

for all $x \in S$. The fuzzy point x_α is said to be contained in a fuzzy set A, denoted by$x_\alpha \in A$, iff $\alpha \leq A(x)$.

Definition 1. [4, 5] The intuitionistic fuzzy sets (IFS, for short) defined on a non-empty set X as objects having the form

$$A = \{< x, \mu_A(x), \gamma_A(x) >: x \in X\},$$

where the functions $\mu_A: X \to [0,1]$ and $\gamma_A: X \to [0,1]$ denote the degree of membership and the degree of non-membership of each element $x \in X$ to the set A respectively, and $0 \leq \mu_A(x) + \gamma_A(x) \leq 1$ for all $x \in X$.

For the sake of simplicity, we shall use$A = (\mu_A, \gamma_A)$for intuitionistic fuzzy set $A = \{< x, \mu_A(x), \gamma_A(x) >: x \in X\}$.

Definition 2. [15] Let $\alpha, \beta \in [0,1]$ with $\alpha + \beta \leq 1$. An intuitionistic fuzzy point, written as $x_{(\alpha,\beta)}$ is defined to be an intuitionistic fuzzy subset of S, given by

$$x_{(\alpha,\beta)}(y) = \begin{cases} (\alpha, \beta) & if\ x = y \\ (0,1) & otherwise \end{cases}$$

Definition 3. [15] A non-empty IFS $A = (\mu_A, \gamma_A)$ of a semigroup S is called an intuitionistic fuzzy subsemigroup of S if

(i) $\mu_A(xy) \geq \mu_A(x) \wedge \mu_A(y), \forall x, y \in S$,

(ii) $\gamma_A(xy) \leq \gamma_A(x) \vee \gamma_A(y), \forall x, y \in S$.

Definition 4. [15] An intuitionistic fuzzy subsemigroup $A = (\mu_A, \gamma_A)$ of a semigroup S is called an intuitionistic fuzzy interior ideal of S if

(i) $\mu_A(xay) \geq \mu_A(x) \ \forall x, a, y \in S$,

(ii) $\gamma_A(xay) \leq \gamma_A(a) \ \forall x, a, y \in S$.

Definition 5. [15] An intuitionistic fuzzy subsemigroup $A = (\mu_A, \gamma_A)$ of a semigroup S is called an intuitionistic fuzzy bi-ideal of S if

(i) $\mu_A(xwy) \geq \mu_A(x) \wedge \mu_A(y), \forall x, w, y \in S$,

(ii) $\gamma_A(xwy) \leq \gamma_A(x) \vee \gamma_A(y) \ \forall x, w, y \in S$.

Definition 6. [15] A non-empty IFS $A = (\mu_A, \gamma_A)$ of a semigroup S is called an intuitionistic fuzzy left (right) ideal of S if

(i) $\mu_A(xy) \geq \mu_A(y)$ (resp. $\mu_A(xy) \geq \mu_A(x)$) $\forall x, y \in S$,

(ii) $\gamma_A(xy) \leq \gamma_A(y)$ (resp. $\gamma_A(xy) \leq \gamma_A(y)$) $\forall x, y \in S$.

Definition 7. [15] A non-empty IFS $A = (\mu_A, \gamma_A)$ of a semigroup S is called an intuitionistic fuzzy two-sided ideal or an intuitionistic fuzzy ideal of S if it is both an intuitionistic fuzzy left and an intuitionistic fuzzy right ideal of S.

Let A be a subset of a semigroup S and A^c be the complement of A. $= (C_A, C_{A^c})$ is defined as:

$$C_A(x) = \begin{cases} 1 & \text{if } x \in A, \\ 0 & \text{otherwise}, \end{cases} \quad C_{A^c}(x) = \begin{cases} 0 & \text{if } x \in A, \\ 1 & \text{otherwise}, \end{cases}$$

for all $x \in S$.

Let $\mathcal{IF}(S)$ be the set of all intuitionistic fuzzy sets in a semigroup S. For each $A = (\mu_A, \gamma_A)$, $B = (\mu_B, \gamma_B) \in \mathcal{IF}(S)$, the product of A and B is an intuitionistic fuzzy set $A \circ B$ defined as follows:

$$A \circ B = \{< x, \mu_{A \circ B}(x), \gamma_{A \circ B}(x) >: x \in S\},$$

where

$$\mu_{A \circ B}(x) = \begin{cases} \bigvee_{x=uv} \mu_A(u) \wedge \mu_B(v) & \text{if } uv = x \\ 0 & \text{otherwise}. \end{cases}$$

$$\gamma_{A\circ B}(x) = \begin{cases} \bigwedge_{x=uv} \gamma_A(u) \vee \gamma_B(v) & \text{if } uv = x \\ 1 & \text{otherwise.} \end{cases}$$

Lemma 1. [1] *For any nonempty subsets A and B of a semigroup S, we have $A \subseteq B$ if and only if $\chi_A \subseteq \chi_B$.*

Lemma 2. [1] *Let A be a nonempty subset of a semigroup S, then A is an ideal of S if and only if χ_A is an intuitionistic fuzzy ideal of S.*

Theorem 1. *A nonempty subset$A$ of a semigroup S is a minimal ideal if and only if χ_A is a minimal intuitionistic fuzzy ideal of S.*

Proof .Let A be a minimal ideal of S, then by lemma 2., χ_A is an intuitionistic fuzzy ideal of S. Suppose that χ_A is not minimalintuitionistic fuzzy ideal of S, then there exists some intuitionistic fuzzy ideal χ_B of S such that $\chi_B \subseteq \chi_A$. Hence, lemma 1 implies that $B \subseteq A$, where B is an ideal of S. This is a contradiction to the fact that A is minimal ideal of S. Thus χ_A is minimalintuitionistic fuzzy ideal of S. Conversely, let χ_A be a minimalintuitionistic fuzzy ideal of S, then$A$ is an ideal of S. Suppose that A is not minimal ideal of S, then there exists some ideal B of S such that $B \subseteq A$. Now by lemma 1, $\chi_B \subseteq \chi_A$ where χ_B is an intuitionistic fuzzy ideal of S. This contradicts that χ_A is a minimalintuitionistic fuzzy ideal of S. Thus A is minimal ideal of S. ∎

Lemma 3. *If $A = (\mu_A, \gamma_A)$ is a minimal intuitionistic fuzzy ideal of a semigroup S , then A is the intuitionistic fuzzy kernel of S.*

Proof. Let $B = (\mu_B, \gamma_B)$ be any intuitionistic fuzzy ideal of S, then $B \circ A \subseteq B \cap A$. Since $B \cap A$ is an intuitionistic fuzzy ideal of S and $B \cap A \subseteq A$, it follows that $B \cap A = A$. But then$A = B \cap A \subseteq B$, so A is contained in every intuitionistic fuzzy ideal of S and hence is an intuitionistic fuzzy kernel of S. ∎

Lemma 4. *If $A = (\mu_A, \gamma_A)$ is an intuitionistic fuzzy kernel of a semigroup S , then A is a simple intuitionistic fuzzy subsemigroup of S.*

Proof. Since A is *an intuitionistic fuzzy ideal of* S, so A is an intuitionistic fuzzy subsemigroup of S. To show that A is simple, let B be any intuitionistic fuzzy ideal of A, then $A \circ B \circ A$ is an intuitionistic fuzzy ideal of S, since

$$S \circ (A \circ B \circ A) = (S \circ A) \circ B \circ A \subseteq A \circ B \circ A$$

and

$$(A \circ B \circ A) \circ S = A \circ B \circ (A \circ S) \subseteq A \circ B \circ A$$

Also, $A \circ B \circ A \subseteq A \circ A \subseteq A$, but by lemma 3, A is minimal intuitionistic fuzzy ideal of S. Hence $A \circ B \circ A = A$. Also, $A \circ B \circ A \subseteq B \circ A \subseteq B$, which implies that $A \subseteq B$. Thus $A = B$, that is A is simple subsemigroup of S. ∎

Lemma 5. *Let $A = (\mu_A, \gamma_A)$ be an intuitionistic fuzzy left ideal of a semigroup S and $x_{(\alpha,\beta)}$ be any intuitionistic fuzzy point of S, then $A \circ x_{(\alpha,\beta)}$ is a minimal intuitionistic fuzzy left ideal of S.*

Proof. $A \circ x_{(\alpha,\beta)}$ is an intuitionistic fuzzy left ideal of S, since $S \circ (A \circ x_{(\alpha,\beta)}) = (S \circ A) \circ x_{(\alpha,\beta)} \subseteq A \circ x_{(\alpha,\beta)}$. Suppose B is an intuitionistic fuzzy left ideal of $A \circ x_{(\alpha,\beta)}$ and let $D = \{y_{(\gamma,\delta)} \in A : y_{(\gamma,\delta)} \circ x_{(\alpha,\beta)} \subseteq B\}$. Let $z_{(\sigma,\tau)}$ be in S and $y_{(\gamma,\delta)}$ be in D, then $z_{(\sigma,\tau)} \circ y_{(\gamma,\delta)} \in A$ and so $z_{(\sigma,\tau)} \circ y_{(\gamma,\delta)} \circ x_{(\alpha,\beta)} \subseteq B$. Hence $z_{(\sigma,\tau)} \circ y_{(\gamma,\delta)} \in D$, which implies that $S \circ D \subseteq D$. Thus D is an intuitionistic fuzzy left ideal of S contained in A and besause of minimality of A, we get $D = A$. Thus for all $y_{(\gamma,\delta)} \in A$, $y_{(\gamma,\delta)} \circ x_{(\alpha,\beta)} \in B$, which implies that $A \circ x_{(\alpha,\beta)} \subseteq B$. Hence $A \circ x_{(\alpha,\beta)} = B$ and therefore, $A \circ x_{(\alpha,\beta)}$is a minimal intuitionistic fuzzy left ideal of S. ∎

3 Main Results

If S is a semigroup, then $\mathcal{IF}(S)$ is a semigroup with the product "$\circ$"[15]. Let $\underline{S}$ be the set of all intuitionistic fuzzy points in a semigroup S. Then $x_{(\alpha,\beta)} \circ y_{(\gamma,\delta)} = (xy)_{(\alpha\wedge\gamma,\beta\vee\delta)} \in \underline{S}$, for $x_{(\alpha,\beta)}, y_{(\gamma,\delta)} \in \underline{S}$ and $y_{(\alpha,\beta)} \circ (x_{(\sigma,\tau)} \circ z_{(\gamma,\delta)}) = (y_{(\alpha,\beta)} \circ x_{(\sigma,\tau)}) \circ z_{(\gamma,\delta)}$. Thus $\underline{S}$ is a subsemigroup of $\mathcal{IF}(S)$[15]. For any $A \in \mathcal{IF}(S)$, $\underline{A}$ denotes the set of all intuitionistic fuzzy points contained in A, that is, $\underline{A} = \{x_{(\alpha,\beta)} \in \underline{S} : \mu_A(x) \geq \alpha,\ \gamma_A(x) \leq \beta\}$. For any $\underline{A}, \underline{B} \subseteq \underline{S}$, we define the product of $\underline{A}$ and $\underline{B}$ as $\underline{A} \circ \underline{B} = \{x_{(\alpha,\beta)} \circ y_{(\gamma,\delta)} : x_{(\alpha,\beta)} \in \underline{A}, y_{(\gamma,\delta)} \in \underline{B}\}$.

Lemma 6. [15] *Let* $A = (\mu_A, \gamma_A)$ *and* $B = (\mu_B, \gamma_B)$ *be two intuitionistic fuzzy subsets of a semigroup* S, *then*

1) $\underline{A \cup B} = \underline{A} \cup \underline{B}$
2) $\underline{A \cap B} = \underline{A} \cap \underline{B}$.
3) $\underline{A} \circ \underline{B} \subseteq \underline{A \circ B}$.

Lemma 7. *Let* A *be nonempty subset of a semigroup* S, *we have* $x_{(\alpha,\beta)} \in \underline{\chi_A}$ *if and only if* $x \in A$.

Proof. Suppose that $x_{(\alpha,\beta)} \in \underline{\chi_A}$ for any $x \in S$, then $C_A(x) \geq \alpha$. Hence $C_A(x) = 1$ for any $\alpha > 0$, which implies that $x \in A$.Conversely, Let $x \in A$, then $C_A(x) = 1 \geq \alpha$ and $C_A(x) = 0 < \beta$ for any $\alpha, \beta > 0$. This means that $x_{(\alpha,\beta)} \in \underline{\chi_A}$. ∎

Lemma 8. *For any nonempty subsets* A *and* B *of a semigroup* S, *we have*

1) $A \subseteq B$ *if and only if* $\underline{\chi_A} \subseteq \underline{\chi_B}$.
2) $\chi_A \subseteq \chi_B$ *if and only if* $\underline{\chi_A} \subseteq \underline{\chi_B}$.

Proof. (1)Assume that $A \subseteq B$, and let $x_{(\alpha,\beta)} \in \underline{\chi_A}$. By lemma 7, $x \in A \subseteq B$ and $x_{(\alpha,\beta)} \in \underline{\chi_B}$, this implies that $\underline{\chi_A} \subseteq \underline{\chi_B}$. Conversely, suppose that $\underline{\chi_A} \subseteq \underline{\chi_B}$. Let $x \in A$, then by lemma 7, for any $\alpha, \beta > 0$ $x_{(\alpha,\beta)} \in \underline{\chi_A}andx_{(\alpha,\beta)} \in \underline{\chi_B}$ which implies that $x \in B$. (2) it is obvious that if $\chi_A \subseteq \chi_B$ then $\underline{\chi_A} \subseteq \underline{\chi_B}$. Now assume that $\underline{\chi_A} \subseteq \underline{\chi_B}$ and let $x_{(\alpha,\beta)} \in \underline{\chi_A} \subseteq \underline{\chi_B}$, then $A \subseteq B$and consequently, we have $\chi_A \subseteq \chi_B$. This completes the proof. ∎

Lemma 9. *Let* A *be a nonempty subset of a semigroup* S. *Then* A *is an ideal of* S *if and only if* $\underline{\chi_A}$ *is an ideal of* $\underline{S}$.

Proof. By lemma 2, A is an ideal of S if and only if χ_A is a fuzzy ideal of S. And from theorem 3.5[13], χ_A is a fuzzy ideal of S if and only if $\underline{\chi_A}$ is an ideal of $\underline{S}$. ∎

Theorem 2. *A nonempty subset A of a semigroup S is minimal ideal if and only if χ_A is a minimal intuitionistic fuzzy ideal of S.*

Proof. Let A be a minimal ideal of S, then by lemma 2, χ_A is an intuitionistic fuzzy ideal of S. Suppose that χ_A is not minimalintuitionistic fuzzy ideal of S, then there exists some intuitionistic fuzzy ideal χ_B of S such that $\chi_B \subseteq \chi_A$. Hence, lemma 1 implies that $B \subseteq A$, where B is an ideal of S. This is a contradiction to the fact that A is minimal ideal of S. Thus χ_A is minimalintuitionistic fuzzy ideal of S. Conversely, let χ_A be a minimalintuitionistic fuzzy ideal of S, then by lemma, A is an ideal of S. Suppose that A is not minimal ideal of S, then there exists some ideal B of S such that $B \subseteq A$. Now by lemma, $\chi_B \subseteq \chi_A$ where χ_B is an intuitionistic fuzzy ideal of S. This contradicts that χ_A is a minimal intuitionistic fuzzy ideal of S. Thus A is minimal ideal of S. ∎

Theorem 3. *Let A be a nonempty subset of a semigroup S. Then A is a minimal ideal of S if and only if $\underline{\chi_A}$ is a minimal ideal of $\underline{S}$.*

Proof. By theorem 1, A is a minimal ideal of S if and only if χ_A is an intuitionistic fuzzy minimal ideal of S. We only need to prove that, χ_A is a minimal intuitionistic fuzzy ideal of S if and only if $\underline{\chi_A}$ is a minimal ideal of $\underline{S}$. Let χ_A be a minimal intuitionistic fuzzy ideal of S, then $\underline{\chi_A}$ is an ideal of $\underline{S}$. Suppose that $\underline{\chi_A}$ is not minimal, then there exists some ideals $\underline{\chi_B}$ of $\underline{S}$ such that $\underline{\chi_B} \subseteq \underline{\chi_A}$ which implies that $\chi_B \subseteq \chi_A$, where χ_B is an intuitionistic fuzzy ideal of S. This is a contradiction to χ_A is a minimal intuitionistic fuzzy ideal of S. Thus $\underline{\chi_A}$ is a minimal ideal of $\underline{S}$. Conversely, assume that $\underline{\chi_A}$ is a minimal ideal of $\underline{S}$ and that χ_A is not a minimal intuitionistic fuzzy ideal of S. Then there exists an intuitionistic fuzzy ideal χ_B of S such that $\chi_B \subseteq \chi_A$. Hence $\underline{\chi_B} \subseteq \underline{\chi_A}$, where $\underline{\chi_B}$ is an ideal of $\underline{S}$. This contradicts that $\underline{\chi_A}$ is a minimal ideal of $\underline{S}$. This completes the proof of the theorem. ∎

Theorem 4. *Let A be a nonempty subset of a semigroup S. Then A is the kernel of S if and only if $\underline{\chi_A}$ is the kernel of $\underline{S}$.*

Proof. Suppose that A is the kernel of S, then $A = \cap_i I_i$, where I_i is an ideal of S. Let $\underline{\chi_B}$ be an ideal of $\underline{S}$, then B is an ideal of S. Now we need to show that, $\underline{\chi_A} \subseteq \underline{\chi_B}$. Let $x_{(\alpha,\beta)} \in \underline{\chi_A}$, by lemma 7, $x \in A$ and also $x \in B$, since A is the kernel of S. This implies that $x_{(\alpha,\beta)} \in \underline{\chi_B}$ and hence, $\underline{\chi_A}$ is the kernel of $\underline{S}$. Conversely, Let $\underline{\chi_A}$ be the kernel of $\underline{S}$, then $\underline{\chi_A} \subseteq \underline{\chi_B}$, for every ideal $\underline{\chi_B}$ of $\underline{S}$. Thus $A \subseteq B$ and therefore, A is the kernel of S. ∎

The following lemma weakens the condition of theorem 4.

Lemma 10. *Let A be a minimal ideal of a semigroup S, then $\underline{\chi_A}$ is the kernel of $\underline{S}$.*

Proof. Since A is a minimal ideal of S, then χ_A is a minimal intuitionistic fuzzy ideal of S. Also lemma 3 implies that χ_A is the fuzzy kernel of S. Now, let χ_B be an intuitionistic fuzzy ideal of S, then we have $\chi_A \subseteq \chi_B$ and hence $\underline{\chi_A} \subseteq \underline{\chi_B}$. So $\underline{\chi_A}$ is a minimal ideal contained in every ideal of $\underline{S}$. Thus $\underline{\chi_A}$ is the kernel of $\underline{S}$. ■

Theorem 5. *Let A be a nonempty subset of a semigroup S. Then A is an interior ideal of S if and only if $\underline{\chi_A}$ is an interior ideal of $\underline{S}$.*

Proof. Let A be an interior ideal of S, and let $y_{(\alpha,\beta)}, z_{(\gamma,\delta)} \in \underline{S}$ and $x_{(\sigma,\tau)} \in \underline{\chi_A}$. Since $x \in A$, then $y_{(\alpha,\beta)} \circ x_{(\sigma,\tau)} \circ z_{(\gamma,\delta)} = (yxz)_{(\alpha\wedge\sigma\wedge\gamma,\ \beta\vee\tau\vee\delta)} \in \underline{\chi_A}$. This implies that $\underline{S} \circ \underline{\chi_A} \circ \underline{S} \subseteq \underline{\chi_A}$, thus $\underline{\chi_A}$ is an interior ideal of $\underline{S}$. Conversely, suppose that $\underline{\chi_A}$ is an interior ideal of $\underline{S}$. Let $y, z \in S$ and $x \in A$, then $x_{(\sigma,\tau)} \in \underline{\chi_A}$. Assume that, $y_{(\alpha,\beta)} \circ x_{(\sigma,\tau)} \circ z_{(\gamma,\delta)} = (yxz)_{(\alpha\wedge\sigma\wedge\gamma,\ \beta\vee\tau\vee\delta)} \in \underline{S} \circ \underline{\chi_A} \circ \underline{S} \subseteq \underline{\chi_A}$, then $yxz \in A$. This implies that $SAS \subseteq A$, and hence A is an interior ideal of S. ■

Theorem 6. *Let A be a nonempty subset of a semigroup S. Then A is a bi- ideal of S if and only if $\underline{\chi_A}$ is a bi- ideal of $\underline{S}$.*

Proof. Let A be a bi- ideal of S, and let $y_{(\alpha,\beta)}, z_{(\gamma,\delta)} \in \underline{\chi_A}$ and $x_{(\sigma,\tau)} \in \underline{S}$. Since $y, z \in A$ and $yxz \in A$ then

$$y_{(\alpha,\beta)} \circ x_{(\sigma,\tau)} \circ z_{(\gamma,\delta)} = (yxz)_{(\alpha\wedge\sigma\wedge\gamma,\ \beta\vee\tau\vee\delta)} \in \underline{\chi_A}.$$

This implies that $\underline{\chi_A} \circ \underline{S} \circ \underline{\chi_A} \subseteq \underline{\chi_A}$, thus $\underline{\chi_A}$ is a bi-ideal of $\underline{S}$. Conversely, suppose that $\underline{\chi_A}$ is a bi-ideal of $\underline{S}$. Let $y, z \in A$ and $x \in S$, then $y_{(\alpha,\beta)}, z_{(\gamma,\delta)} \in \underline{\chi_A}$ by lemma 7. Assume that, $y_{(\alpha,\beta)} \circ x_{(\sigma,\tau)} \circ z_{(\gamma,\delta)} = (yxz)_{(\alpha\wedge\sigma\wedge\gamma,\ \beta\vee\tau\vee\delta)} \in \underline{\chi_A} \circ \underline{S} \circ \underline{\chi_A} \subseteq \underline{\chi_A}$, then $yxz \in A$. This implies that $ASA \subseteq A$, and hence A is a bi- ideal of S. ■

Acknowledgements

The author is grateful for Editor-in-Chief and the referee for their valuable efforts.

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