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## COMPACTNESS IN INTUITIONISTIC FUZZY MULTISSET TOPOLOGY

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**Abstract** – In this paper, we discuss the compactness properties of Intuitionistic Fuzzy Multiset Topological spaces. Various properties of Compact and Homeomorphic Intuitionistic Fuzzy Multiset Topological spaces are discussed.

**Keywords** -- *Intuitionistic Fuzzy Multiset, Intuitionistic Fuzzy Multiset Topology, Compact spaces, Homeomorphism.*

### 1. INTRODUCTION

The theory of sets considered to have begun with Cantor (1845-1918). For considering the uncertainty factor, Zadeh [1] introduced Fuzzy sets in 1965, in which a membership function assigns to each element of the universe of discourse, a number from the unit interval  $[0,1]$  to indicate the degree of belongingness to the set under consideration.

If repeated occurrences of any object are allowed in a set, then the mathematical structure is called as multiset [11,12]. As a generalization of this concept, Yager [2] introduced fuzzy multisets. An element of a Fuzzy Multiset can occur more than once with possibly the same or different membership values.

In 1983, Atanassov [3,10] introduced the concept of Intuitionistic Fuzzy sets. An Intuitionistic Fuzzy set is characterized by two functions expressing the degree of membership and the degree of nonmembership of elements of the universe to the Intuitionistic Fuzzy set. Among the various notions of higher-order Fuzzy sets, Intuitionistic Fuzzy sets proposed by Atanassov provide a flexible framework to explain uncertainty and vagueness.

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The concept of Intuitionistic Fuzzy Multiset is introduced in [4] by combining the all the above concepts. Intuitionistic Fuzzy Multiset has applications in medical diagnosis and robotics [13,14]. In [5] Shinoj et al. introduced algebraic structures on Intuitionistic Fuzzy Multiset.

In 1968, Chang [9] introduced Fuzzy topological spaces. And as a continuation of this, in 1997, Coker [6] introduced the concept of Intuitionistic fuzzy topological spaces. In [15], Shinoj and John generalized this concept into Intuitionistic Fuzzy Multiset by introducing Intuitionistic Fuzzy Multiset Topology. In the present work we introduced the concept of Compactness, which is considered as a “global” property in general topology. The advantage of this concept is that, one can study the whole space by studying a finite number of open subsets. Also we introduced the concept of Homeomorphism which will help to compare two spaces and corresponding properties.

## 2. Preliminaries

**Definition 2.1.** [1] Let  $X$  be a nonempty set. A Fuzzy set  $A$  drawn from  $X$  is defined as

$$A = \{ \langle x : \mu_A(x) \rangle : x \in X \}.$$

Where  $\mu : X \rightarrow [0,1]$  is the membership function of the Fuzzy Set  $A$ .

**Definition 2.2.** [2] Let  $X$  be a nonempty set. A Fuzzy Multiset (FMS)  $A$  drawn from  $X$  is characterized by a function, ‘count membership’ of  $A$  denoted by  $CM_A$  such that  $CM_A : X \rightarrow Q$  where  $Q$  is the set of all crisp multisets drawn from the unit interval  $[0,1]$ . Then for any  $x \in X$ , the value  $CM_A(x)$  is a crisp multiset drawn from  $[0,1]$ . For each  $x \in X$ , the membership sequence is defined as the decreasingly ordered sequence of elements in  $CM_A(x)$ . It is denoted by  $(\mu^1_A(x), \mu^2_A(x), \dots, \mu^p_A(x))$  where  $\mu^1_A(x) \geq \mu^2_A(x) \geq \dots \geq \mu^p_A(x)$ .

A complete account of the applications of Fuzzy Multisets in various fields can be seen in [9].

**Definition 2.3.** [3] Let  $X$  be a nonempty set. An Intuitionistic Fuzzy Set (IFS)  $A$  is an object having the form  $A = \{ \langle x : \mu_A(x), \nu_A(x) \rangle : x \in X \}$ , where the functions  $\mu_A : X \rightarrow [0,1]$  and  $\nu_A : X \rightarrow [0,1]$  define respectively the degree of membership and the degree of non membership of the element  $x \in X$  to the set  $A$  with  $0 \leq \mu_A(x) + \nu_A(x) \leq 1$  for each  $x \in X$ .

**Remark 2.4.** Every Fuzzy set  $A$  on a nonempty set  $X$  is obviously an IFS having the form

$$A = \{ \langle x : \mu_A(x), 1 - \mu_A(x) \rangle : x \in X \}$$

Using the definition of FMS and IFS, a new generalized concept can be defined as follows:

**Definition 2.5.** [4] Let  $X$  be a nonempty set. An Intuitionistic Fuzzy Multiset  $A$  denoted by IFMS drawn from  $X$  is characterized by two functions : ‘count membership’ of  $A$  ( $CM_A$ ) and ‘count non membership’ of  $A$  ( $CN_A$ ) given respectively by  $CM_A : X \rightarrow Q$  and  $CN_A : X \rightarrow Q$

where  $Q$  is the set of all crisp multisets drawn from the unit interval  $[0, 1]$  such that for each  $x \in X$ , the membership sequence is defined as a decreasingly ordered sequence of elements in  $CMA(x)$  which is denoted by  $(\mu^1_A(x), \mu^2_A(x), \dots, \mu^p_A(x))$  where  $(\mu^1_A(x) \geq \mu^2_A(x) \geq \dots \geq \mu^p_A(x))$  and the corresponding non membership sequence will be denoted by  $(v^1_A(x), v^2_A(x), \dots, v^p_A(x))$  such that  $0 \leq \mu^i_A(x) + v^i_A(x) \leq 1$  for every  $x \in X$  and  $i = 1, 2, \dots, p$ .

An IFMS  $A$  is denoted by

$$A = \{ \langle x : (\mu^1_A(x), \mu^2_A(x), \dots, \mu^p_A(x)), (v^1_A(x), v^2_A(x), \dots, v^p_A(x)) \rangle : x \in X \}$$

**Remark 2.6.** We arrange the membership sequence in decreasing order but the corresponding non membership sequence may not be in decreasing or increasing order.

**Definition 2.7. [15]** Let  $X$  and  $Y$  be two nonempty sets and  $f : X \rightarrow Y$  be a mapping. Then

a) The image of an IFMS  $A$  in  $X$  under the mapping  $f$  is denoted by  $f(A)$  is defined as

$$CM_{f[A]}(y) = \begin{cases} \vee_{f(x)=y} CM_A(x) & ; f^{-1}(y) \neq \emptyset \\ 0 & \text{otherwise} \end{cases}$$

$$CN_{f[A]}(y) = \begin{cases} \wedge_{f(x)=y} CN_A(x) & ; f^{-1}(y) \neq \emptyset \\ 1 & \text{otherwise} \end{cases}$$

b) The inverse image of the IFMS  $B$  in  $Y$  under the mapping  $f$  is denoted by  $f^{-1}(B)$  where

$$CM_{f^{-1}[B]}(x) = CM_B f[x], CN_{f^{-1}[B]}(x) = CN_B f[x]$$

### 2.1. Intuitionistic Fuzzy Multiset Topological spaces

In this section we introduced the concept of Intuitionistic Fuzzy multiset Topology (IFMT). Here we extend the concept of Intuitionistic fuzzy topological spaces introduced by Dogan Coker in [6] to the case of Intuitionistic fuzzy multisets.

For this first we introduced  $\rightarrow 0$  and  $\rightarrow 1$  in a nonempty set  $X$  as follows.

**Definition 2.8. [15]** Let

$$\rightarrow 0 = \{ \langle x : (0, 0, \dots, 0), (1, 1, \dots, 1) \rangle : x \in X \}$$

$$\rightarrow 1 = \{ \langle x : (1, 1, \dots, 1), (0, 0, \dots, 0) \rangle : x \in X \}$$

**Definition 2.9. [15]** An intuitionistic Fuzzy multiset topology (IFMT) on  $X$  is a family  $\tau$  of intuitionistic fuzzy multisets (IFMSs) such that

1.  $\rightarrow 0, \rightarrow 1 \in \tau$
2.  $G_1 \cap G_2 \in \tau$  for any  $G_1, G_2 \in \tau$
3.  $\cup G_i \in \tau$  for any arbitrary family  $\{G_i : i \in I\}$  in  $\tau$

Then the pair  $(X, \tau)$  is called **Intuitionistic Fuzzy multiset topological space** (IFMT for short) and any IFMS in  $\tau$  is known as an open intuitionistic fuzzy multiset (OIFMS in short) in  $X$ .

**Remark 2.10.** [15] The complement of an OIFMS is called closed intuitionistic Fuzzy multiset (CIFMS in short)

## 2.2. Construction of IFMTs [15]

Here we construct Intuitionistic fuzzy multiset topology from a given IFMT. Consider a nonempty set  $X$ . Let  $A = \{ \langle x : (\mu^1_A(x), \mu^2_A(x), \dots, \mu^p_A(x)), (v^1_A(x), v^2_A(x), \dots, v^p_A(x)) \rangle : x \in X \}$  be an IFMS. Define

$$\tau_A = \{ \langle x : (\mu^1_A(x), \mu^2_A(x), \dots, \mu^p_A(x)), (1-\mu^1_A(x), 1-\mu^2_A(x), \dots, 1-\mu^p_A(x)) \rangle : x \in X \}$$

**Proposition 2.11.** Let  $(X, \tau)$  be an IFMT on  $X$ . Then  $\tau_{0,1} = \{ \tau_A : A \in \tau \}$  is an IFMS.

## 2.3. Closure and Interior

**Definition 2.12.** [15] Let  $(X, \tau)$  be an IFMT and  $A$  be an IFMS in  $X$ . Then **closure** of  $A$  denoted by  $\text{cl}(A)$  is defined as  $\text{cl}(A) = \cap \{ M : M \text{ is closed in } X \text{ and } A \subseteq M \}$ .

**Definition 2.13.** [15] Let  $(X, \tau)$  be an IFMT and  $B$  be an IFMS in  $X$ . Then **interior** of  $B$  is denoted by

$$\text{int}(B) \text{ is defined as } \text{int}(B) = \cup \{ N : N \text{ is open in } X \text{ and } N \subseteq B \}.$$

**Proposition 2.14.** [15] Let  $(X, \tau)$  be an IFMT and  $A$  be an IFMS in  $X$ . Then  $\text{cl}(A)$  is a CIFMS.

**Proposition 2.15.** [15] Let  $(X, \tau)$  be an IFMT and  $A$  be an IFMS in  $X$ . Then  $\text{int}(A)$  is an OIFMS.

**Proposition 2.16.** [15] Let  $(X, \tau)$  be an IFMT and  $A$  be an IFMS. Then  $\text{cl}(\bigvee A) = \bigvee(\text{int}(A))$

**Proposition 2.17.** [15] Let  $(X, \tau)$  be an IFMT and  $A$  be an IFMS in  $X$ . Then  $A$  is a CIFMS if and only if  $\text{cl}(A) = A$ .

**Proposition 2.18.** [15] Let  $(X, \tau)$  be an IFMT and  $A$  be an IFMS in  $X$ . Then  $A$  is an OIFMS if and only if  $\text{int}(A) = A$ .

## 2.4. Continuous Functions

**Definition 2.19.** [15] Let  $(X, \tau)$  and  $(Y, \phi)$  be two IFMTs. A function  $f : X \rightarrow Y$  is said to be **Continuous** if and only if inverse image of each OIFMS in  $\phi$  is an OIFMS in  $\tau$ .

**Theorem 2.20.** [15] Let  $(X, \tau)$  and  $(Y, \phi)$  be two IFMTs. Then the function  $f : X \rightarrow Y$  is Continuous if and only if inverse image of each CIFMS in  $\phi$  is a CIFMS in  $\tau$ .

**Theorem 2.21.** [15] Let  $(X, \tau)$  and  $(Y, \phi)$  be two IFMTs. Then the function  $f : X \rightarrow Y$  is Continuous if and only if for each IFMT  $A$  in  $X$ ,  $f[\text{cl}(A)] \subseteq \text{cl}[f(A)]$

**Theorem 2.22.** [15] Let  $(X, \tau)$  and  $(Y, \phi)$  be two IFMTs. Then the function  $f : X \rightarrow Y$  is Continuous if and only if for each IFMT  $B$  in  $Y$ ,  $\text{cl}[f^{-1}(B)] \subseteq f^{-1}[\text{cl}(B)]$

**Theorem 2.23.** [15] Let  $(X, \tau)$  and  $(Y, \phi)$  be two IFMTs. Then the function  $f : X \rightarrow Y$  is Continuous if and only if for each IFMT  $A$  in  $X$ ,  $\text{int}[f(A)] \subseteq f[\text{int}(A)]$ .

**Theorem 2.24.**[15] Let  $(X, \tau)$  and  $(Y, \phi)$  be two IFMTs. Then the function  $f : X \rightarrow Y$  is Continuous if and only if for each IFMT  $B$  in  $Y$ ,  $f^{-1}[\text{int}(B)] \subseteq \text{int}[f^{-1}(B)]$

### 2.5. Subspace Topology

**Definition 2.25.** [15] Let  $(X, \tau)$  and  $(Y, \phi)$  be two IFMTs. The topological space  $Y$  is called a **subspace** of the topological space  $X$  if  $Y \subseteq X$  and if the open subsets of  $Y$  are precisely the subsets  $O'$  of the form

$$O' = O \cap Y$$

for some open subsets  $O$  of  $X$ . Here we may say that each open subset  $O'$  of  $Y$  is the *restriction* to  $Y$  of an open subset  $O$  of  $X$ .  $O'$  is also called **relative open** in  $Y$ .

### 3. Compactness on Intuitionistic Fuzzy Multisets

**Definition 3.1.** Let  $(X, \tau)$  be an IFMT. Let  $\{G_i : i \in I\}$  be a family of OIFMSs in  $X$  such that  $\bigcup \{G_i : i \in I\} = \rightarrow 1$ , then it is called an **open cover** of  $X$ . A finite subfamily of  $\{G_i : i \in I\}$  is an open cover of  $X$ , then it is called a *finite subcover* of  $X$ .

**Definition 3.2.** A family  $\{H_i : i \in I\}$  of CIFMSs in  $X$  satisfies the **finite intersection property** iff every finite subfamily  $\{H_i : i=1,2,\dots,n\}$  of the family satisfies the condition

$$\bigcap_{i=1}^n H_i \neq \rightarrow 0.$$

**Definition 3.3.** Let  $(X, \tau)$  be an IFMT. Then  $X$  is **compact** iff every open cover of  $X$  has a finite subcover.

**Example 3.4.** Let  $X = \{1, 2\}$  and define the IFMSs in  $X$  as follows. For  $n \in \mathbb{N}^+, p \in \mathbb{N}$

$$G_n = \{ \langle 1: (n/n+1, n+1/n+2, \dots, n+p/n+p+1), (1/n+2, 1/n+3, \dots, 1/n+p+2) \rangle, \langle 2: (n+1/n+2, n+2/n+3, \dots, n+p+1/n+p+2), (1/n+3, 1/n+4, \dots, 1/n+p+3) \rangle \}$$



Similarly we can prove the converse part.

The well known theorems in the modern Topology are also holds good for IFMTs. Some of them are given below.

**Theorem 3.6.** Any closed subspace of a compact IFMT is compact.

*Proof.* Let  $(X, \tau)$  be an IFMT on  $X$ . Also assume that  $(X, \tau)$  is compact. Let  $(Y, \phi)$  be a closed subspace of  $X$ . Let  $\{A_i : i \in I\}$  be an open cover of  $Y$ , where

$$A_i = \{ \langle x : (\mu^1_{A_i}(x), \mu^2_{A_i}(x), \dots, \mu^p_{A_i}(x)), ((v^1_{A_i}(x), v^2_{A_i}(x), \dots, v^p_{A_i}(x)) \rangle) \rangle : x \in X \}$$

ie,

$$\cup A_i = \rightarrow 1 \tag{1}$$

By Definition 4.16,  $\exists$  open sets  $B_i$  in  $X \ni$

$$A_i = B_i \cap Y \tag{2}$$

Since  $Y$  is closed,  $\forall Y \cup \{B_i\}$  forms an open cover of  $X$ .

Since  $X$  is compact, this open cover has a finite subcover. Discard  $\forall Y$  if it occurs in this subcover. Let  $\{B_1, B_2, \dots, B_n\}$  be the finite subcover of  $X$ . Then from (2), the corresponding  $\{A_1, A_2, \dots, A_n\}$  forms a finite subcover of  $Y$ . ie.

$$\cup_{i=1}^n A_i = \rightarrow 1 \tag{3}$$

Then by definition 4.20,  $Y$  is compact. Hence the proof.

**Theorem 3.7.** Continuous image of a compact IFMT is compact.

*Proof.* Let  $(X, \tau)$  be an IFMT on  $X$  and assume that  $(X, \tau)$  is compact. Let  $f : X \rightarrow Y$  be continuous. To prove  $f(X)$  is a compact subspace of  $X$ .

Let  $\{A_i : i \in I\}$  be an open cover of  $f(X)$ , where

$$A_i = \{ \langle x : (\mu^1_{A_i}(x), \mu^2_{A_i}(x), \dots, \mu^p_{A_i}(x)), ((v^1_{A_i}(x), v^2_{A_i}(x), \dots, v^p_{A_i}(x)) \rangle) \rangle : x \in X \}$$

ie,

$$\cup A_i = \rightarrow 1 \tag{1}$$

Since  $f$  is continuous,  $\{f^{-1}(A_i)\}_{i \in I}$  is an open cover of  $X$ . Since  $X$  is compact  $\exists$  a finite subcover  $\{f^{-1}(A_i) : i = 1, 2, \dots, n\}$  which covers  $X$ .

$$\Rightarrow \bigcup_{i=1}^n A_i = \rightarrow 1 \tag{2}$$

$\Rightarrow f(X)$  is compact. Hence the proof.

**Theorem 3.8.** An IFMT is compact if and only if every class of CIFMSs with empty intersection has a finite subclass with empty intersection.

*Proof:* Let  $(X, \tau)$  be a compact IFMT.

Let  $\{C_i : i \in I\}$  be a family of closed sets  $\ni$

$$\bigcap C_i = \rightarrow 0 \tag{1}$$

where  $C_i = \{ \langle x : (\mu^1_{C_i}(x), \mu^2_{C_i}(x), \dots, \mu^p_{C_i}(x)), (v^1_{C_i}(x), v^2_{C_i}(x), \dots, v^p_{C_i}(x)) \rangle : x \in X \}$

$$\begin{aligned} (1) \quad &\Rightarrow \bigcup (\nabla C_i) = \rightarrow 1 \\ &\Rightarrow \{ \nabla C_i : i \in I \} \text{ be an open cover of } X. \end{aligned}$$

Since  $X$  is compact,  $\exists \{ \nabla C_1, \nabla C_2, \dots, \nabla C_n \} \ni \bigcup_{i=1}^n (\nabla C_i) = \rightarrow 1$   
 $\Rightarrow \bigcap_{i=1}^n C_i = \rightarrow 0$

Conversely assume that every class of CIFMSs with empty intersection has a finite subclass with empty intersection.

To prove  $X$  is compact. Let  $\{A_i : i \in I\}$  be an open cover of  $X$ .

$$\begin{aligned} &\Rightarrow \bigcup A_i = \rightarrow 1 \\ &\Rightarrow \bigcap (\nabla A_i) = \rightarrow 0 \end{aligned}$$

Hence by assumption  $\exists \{ \nabla A_1, \nabla A_2, \dots, \nabla A_n \} \ni \bigcap_{i=1}^n (\nabla A_i) = \rightarrow 0$

$$\Rightarrow \bigcup_{i=1}^n A_i = \rightarrow 1$$

Hence the proof.

### 3.1. Homeomorphism on Intuitionistic Fuzzy Multisets

**Definition 3.9.** A homeomorphism is a one-to-one continuous mapping of one topological space onto another. The IFMTs  $(X, \tau)$  and  $(Y, \phi)$  are said to be **homeomorphic** if there exist functions  $f : X \rightarrow Y$  and  $g : Y \rightarrow X$  such that  $f$  and  $g$  are continuous. If  $X$  and  $Y$  are homeomorphic, then their points can be put into one-to-one correspondence in such a way that their open sets also correspond to one another. The two spaces differ only in the nature of their points, so it can be considered that they are identical.



**Theorem 3.10.** Let  $(X, \tau)$  and  $(Y, \phi)$  are homeomorphic. Then  $X$  is compact if and only if  $Y$  is compact.

*Proof.* Let  $f : X \rightarrow Y$  be a homeomorphism. Let  $(X, \tau)$  be a compact IFMT. To prove  $Y$  is compact. Let  $\{A_i : i \in I\}$  be an open cover of  $Y$ . ie

$$\bigcup A_i = \rightarrow 1 \text{ in } Y.$$

Then  $\{f^{-1}(A_i)\}_{i \in I}$  be an open cover of  $X$ . Since  $X$  is compact there exist  $\{f^{-1}(A_i) : i=1,2,\dots,n\} \ni$

$$\bigcup_{i=1}^n f^{-1}(A_i) = \rightarrow 1 \text{ in } X.$$

Since  $f$  is onto

$$\bigcup_{i=1}^n A_i = \rightarrow 1 \text{ in } Y.$$

Hence  $Y$  is compact. Similarly we can prove the converse.

#### 4. Conclusion

In this work we extended the concept topological structures of Intuitionistic Fuzzy Multisets. We introduced the concept of Intuitionistic Fuzzy Multiset Topology in our previous work. In the current work we introduced the concept of compactness on Intuitionistic Fuzzy Multisets. The homeomorphism between two Intuitionistic Fuzzy Multisets are defined. Characterization of compactness is discussed in detail.

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