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ON CLASS OF FUNCTION VIA GRILL

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Abstaract − If we have (X, τ, G) be a topological space with a grill, our aim in this paper is to provide a study of new class of functions via grill called slightly G-precontinuous functions. From this new definition, we are enables some of the important characterizations and principle properties of this class of functions are obtained.

Keywords − *Grill topological spaces, G-preopen sets, Slightly G-precontinuity.*

1 Introduction

The idea of grill topological space depended on the two influential are Φ and Ψ. Choquet [4] in 1947 is the first exposure to the concept of grill and explain its conditions. There is a similarity between the notion of ideals, nets, filters and grill. Also, the theory of compactifications is similar to the investigation of many topological notions in terms of grill, proximity spaces was used (see [5], [6], [12] for details). In [10] Roy and Mukherjee introduced grill toplogical space τ_G, some of characterizations and the relationship between τ and τ_G. In 2012, Mandal and Mukherjee [9] have defined new classes of sets in grill topological space and obtained a new decoposition of continuity in terms of grills. Quite recently, Baskar and Rajesh [3] introduced new class of functions called slightly G-semi-continuous functions in grill topological spaces. Our purpose in this paper, slightly G-precontinuity is introduced and studied. Moreover, basic properties and preservation theorems of slightly G-precontinuous functions are investigated and relationships between this class of functions and graphs are investigated.

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2 Preliminaries

In what follows, by a space X we shall mean a topological space (X, τ). For a subset A of the space X, we shall adopt the usual notations $Cl(A)$ and $Int(A)$ to respectively denote the closure and interior of A in X. Also, the power set of X will be written as $P(X)$. A collection G of nonempty subsets of a space X is called a grill ([1]) if (i) $\phi \notin G$, (ii) $A \in G$ and $A \subset B \subset X$ implies that $B \in G$, (iii) $A, B \subset X$ and $A \cup B \in G$ implies that $A \in G$ or $B \in G$.

Definition 2.1 (2)**.** Let (X, τ, G) be a grill topological space. An operator $\Phi : P(X) \to P(X)$ is defined as follows: $\Phi(A) = \Phi_G(A, \tau) = \{x \in X : A \cap U \in G$ for every open set U containing x $\}$ for each $A \in P(X)$. The mapping Φ is called the operator associated with the grill G and the topology τ.

Definition 2.2 (2)**.** Let G be a grill on a topological space (X, τ). Then we define a map $\Psi : P(X) \to P(X)$ by $\Psi(A) = A \cup \Phi(A)$ for all $A \in P(X)$. The map Ψ is a Kuratowski closure axioms. Corresponding to a grill G on a topological space (X, τ), there exists a unique topology τ_G on X given by $\tau_G = \{U \subset X : \Psi(X \setminus U) = X \setminus U\}$, where for any $A \subset X$, $\Psi(A) = A \cup \Phi(A) = \tau_G Cl(A)$. For any grill G on a topological space (X, τ), $\tau \subset \tau_G$. If (X, τ) is a topological space with a grill G on X, then we call it a grill topological space and denote it by (X, τ, G).

Definition 2.3 (5)**.** Let (X, τ, G) be a grill topological space. A subset A in X is said to be G-preopen if $A \subseteq Int(\Psi(A))$. The complement of a G-preopen set is called a G-preclosed set.

Definition 2.4. The intersection of all G-preclosed sets containing $A \subset X$ is called the G-preclosure of A and is denoted by $pCl_G(A)$. The family of all G-preopen (resp. G-preclosed) sets of (X, τ, G) is denoted by $GPO(X)$ (resp. $GPC(X)$). The family of all G-preopen (resp. G-preclosed) sets of (X, τ, G) containing a point $x \in X$ is denoted by $GPO(X, x)$ (resp. $GPC(X, x)$).

Definition 2.5. A function $f : (X, \tau) \to (Y, \sigma)$ is said to be slightly continuous [8] (resp. slightly precontinuous [2]) if $f^{-1}(V)$ is open (resp. preopen) in X for every clopen set V of Y.

Remark 2.6. For several sets, we have implications as in the Figure 1. Where converses of the above implications need not be true as shown by the following examples.

Example 2.7. *Let $X = \{a, b, c\}$ and $\tau = \{\phi, X, \{a\}, \{b, c\}\}$. If $G = \{X, \{a\}, \{b\}, \{a, b\}, \{b, c\}, \{a, c\}\}$, then G is a grill on X. If $A = \{a, b\}$. Then*
1- A is $G - \gamma -$ open but niether G-semi-open nor $G - \alpha -$open.
2- A is γ-open but not semi-open.
3- A is preopen but not $\alpha -$open.

Example 2.8. *Let $X = \{a, b, c, d\}$ and $\tau = \{\phi, X, \{d\}, \{a, c\}, \{a, c, d\}\}$. If $G = \{X, \{a\}, \{a, c\}, \{a, c, d\}, \{a, b\}, \{a, bc\}, \{a, d\}, \{a, b, d\}\}$, then G is a grill on X. Then,*
1- A is G-preopen and G-semi-open but neither Φ-open nor α-open at $A = \{d\}$.
2- A is β-open, but not $G - \beta -$open at $A = \{c\}$.

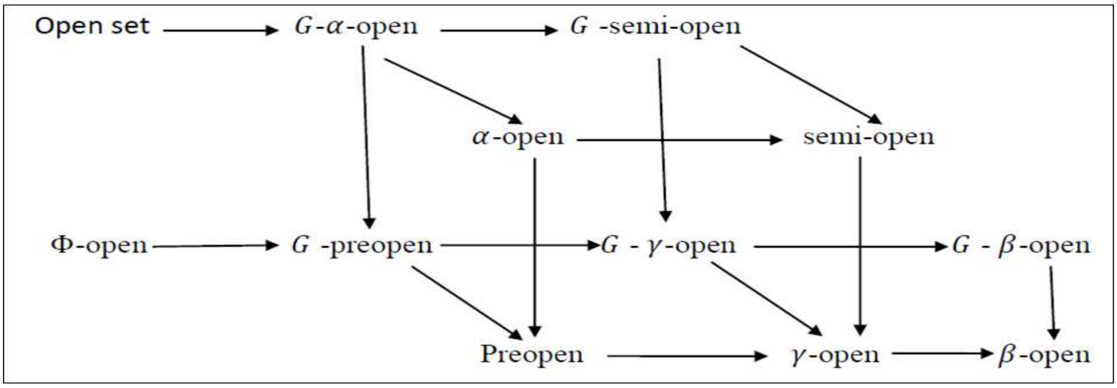

Figure 1: Relationships between some classes of open sets

3 Slightly G-precontinuous Functions

In this section, the notion of slightly G-precontinuous functions is introduced and characterizations and some relationships of precontinuous functions and basic properties of slightly G-precontinuous functions are investigated and obtained.

Definition 3.1. A function $f : (X, \tau, G) \to (Y, \sigma)$ is said to be:
1- Slightly G-precontinuous at $x \in X$ if for each clopen subset V of Y containing $f(x)$, there exists $U \in GPO(X, x)$ such that $f(U) \subset V$;
2- Slightly G-precontinuous if it is G-precontinuous at each point of X.

Theorem 3.2. *For a function $f : (X, \tau, G) \to (Y, \sigma)$, The following statements are equivalent :*
1- f is slightly G-precontinuous;
2- $f^{-1}(V)$ is G-preopen in X for each clopen subset V of Y;
3- $f^{-1}(V)$ is G-preclosed in X for each clopen subset V of Y;
4- $f^{-1}(V)$ is G-preclopen in X for each clopen subset V of Y.

Proof. The proof is clear. □

Proposition 3.3. *Every slightly G-precontinuous function is slightly precontinuous.*

Proof. It follows from proposition 3.3 of [1]. □

The converse of Proposition 3.3 is need not be true as shown by the following example.

Example 3.4. *Let $X = \{a, b, c\}, \tau = \{\phi, \{a\}, \{b, c\}, X\}$ and $G = P(X) \setminus \{\phi, \{b\}\}$. Then the function $f : (X, \tau, G) \to (X, \tau)$ is defined by $f(a) = c, f(b) = a$ and $f(c) = b$ is slightly precontinuous but not slightly G-precontinuous*

Proposition 3.5. *Every slightly continuous function is slightly G-precontinuous.*

Proof. It follows from Remark 1 of [1]. □

The converse of Proposition 3.5 is need not be true as shown by the following example.

Example 3.6. *Let* $X = \{a, b, c\}, \tau = \{\phi, \{a\}, \{b, c\}, X\}$ *and* $G = P(X)\backslash\{\phi\}$*. Then the function* $f : (X, \tau, G) \to (Y, \sigma)$ *is defined by* $f(a) = c, f(b) = a$ *and* $f(c) = b$ *is slightly* G*-precontinuous but not slightly continuous.*
i. e. Slightly continuiy $\Rightarrow$ *slightly* G*-precontinuity* $\Rightarrow$ *slightly precontinuity.*

Definition 3.7. A function $f : (X, \tau, G) \to (Y, \sigma, G)$ is said to be G-preirresolute if $f^{-1}(V) \in GPO(X)$ for every $V \in GPO(Y)$.

Theorem 3.8. *Let* $f : (X, \tau, G_1) \to (Y, \sigma, G_2)$ *and* $g : (Y, \sigma, G_2) \to (Z, \theta)$ *be functions, then the following properties holds:*
1- If f *is slightly* G_1*-precontinuous and* g *is slightly continuous, then* $g \circ f : (X, \tau, G_1) \to (Z, \theta)$ *is slightly* G_1*-precontinuous.*
2- If f *is* G_1*-preirresolute and* g *is slightly* G_2*-precontinuous, then* $g \circ f$ *is slightly* G_1*-precontinuous.*
3- If f *is* G_1*-preirresolute and* g *is slightly continuous, then* $g \circ f$ *is slightly* G_1*-precontinuous.*

Proof. The proof is clear. □

Definition 3.9. A function $f : (X, \tau, G_1) \to (Y, \sigma, G_2)$ is said to be strongly G-preopen if $f(U) \in G_2PO(Y)$ for every $U \in G_1PO(X)$.

Theorem 3.10. *Let* $f : (X, \tau, G_1) \to (Y, \sigma, G_2)$ *and* $g : (Y, \sigma, G_2) \to (Z, \eta)$ *be functions. Then the following properties hold:*
1- If f *is strongly* G_1*-preopen surjection and* $g \circ f$ *is slightly* G_1*-precontinuous, then* g *is slightly* G_2*-precontinuous.*
2- Let f *be strongly* G_1*-preopen and* G_1*-preirresolute surjection. then* g *is slightly* G_2*-precontinuous if and only if* $g \circ f$ *is slightly* G_1*-precontinuous*

Proof. The proof is clear. □

Remark 3.11. A subset A of a grill topological space (X, τ, G) is G-preopen if and only if for all $x \in A$, there exists $H_x \in GPO(X)$ such that $x \in H_x \subset A$.

Theorem 3.12. *A function* $f : (X, \tau, G) \to (Y, \sigma)$ *is slightly* G*-precontinuous if and only if the graph function* $g : X \to X \times Y$*, defined by* $g(x) = (x, f(x))$ *for each* $x \in X$ *is slightly* G*-precontinuous.*

Proof. Let $x \in X$ and let W be a clopen subset of $X \times Y$ containing $g(x)$. Then $W \cap (\{x\} \times Y)$ is clopen in $\{x\} \times Y$ containing $g(x)$. Also $\{x\} \times Y$ is homeomorphic to Y. Hence $\{y \in Y \mid (x, y) \in W\}$ is a clopen subset of Y. Since f is slightly G-precontinuous, $\cup\{f^{-1}(y) \mid (x, y) \in W\}$ is a G-preopen subset of (X, τ, G). Further, $x \in \cup\{f^{-1}(y) \mid (x, y) \in W\} \subset G^{-1}(W)$. Hence $g^{-1}(W)$ is G-preopen. Then g is slightly G-precontinuous. Conversely, Let F be a clopen subset of Y. Then $X \times F$ is a clopen subset of $X \times Y$. Since g is slightly G-precontinuous, $g^{-1}(X \times F)$ is a G-preopen subset of X. Also, $g^{-1}(X \times F) = f^{-1}(F)$. Hence f is slightly G-precontinuous. □

Definition 3.13. A grill topological space (X, τ, G) is said to be G-preconnected if X is not the union of two disjoint nonempty G-preopen sets of X.

Theorem 3.14. *If $f : (X, \tau, G) \to (Y, \sigma)$ is slightly G-precontinuous surjection and (X, τ, G) is G-preconnected, then Y is connected.*

Proof. Follows from the Theorem 3.2 and Definition 3.12. □

Theorem 3.15. *If f is slightly G-precontinuous function from a G-preconnected space (X, τ, G) onto space (Y, σ), then Y is not a discrete space.*

Proof. Since f is slightly G-precontinuous function, then for each clopen V of Y there exist $f^{-1}(V)$ is G-preopen in X and scince $X \neq f^{-1}(V_1) \cup f^{-1}(V_2)$, $V_1 \neq V_2$. Then Y is not discrate space. □

Theorem 3.16. *A grill topological space (X, τ, G) is G-preconnected if for every slightly G-precontinuous function from a space (X, τ, G) into any τ_0-space Y is constant.*

Proof. From Theorem 3.2 and Definition 3.12 □

Let $\{X_\alpha : \alpha \in I\}$ and $\{Y_\alpha : \alpha \in I\}$ be two families of topological spaces with the same index set I. The product space of $\{X_\alpha : \alpha \in I\}$ is denoted by $\Pi\{X_\alpha : \alpha \in I\}$(or simply ΠX_α). Let $f_\alpha : X_\alpha \to Y_\alpha$ be a function for each $\alpha \in I$. The product function $f : \Pi X_\alpha \to \Pi Y_\alpha$ is defined by $f(\{x_\alpha\}) = \{f_\alpha(x_\alpha)\}$ for each $\{x_\alpha\} \in \Pi X_\alpha$.

Theorem 3.17. *If a function $f : (X, \tau, G) \to \Pi Y_\alpha$ is slightly G-precontinuous, then $P_\alpha \circ f : (X, \tau, G) \to Y_\alpha$ is slightly G-precontinuous for each $\alpha \in I$, where P_α is the projection of ΠY_α into Y_α.*

Proof. Let V_α be any clopen set of Y_α. Then, $P_\alpha^{-1}(V_\alpha)$ is clopen in ΠY_α and hence $(P_\alpha \circ f)^{-1}(V_\alpha) = f^{-1}(P_\alpha^{-1}(V_\alpha))$ is G-preopen in X. Therefore, $P_\alpha \circ f$ is slightly G-precontinuous. □

4 Separation Axioms

In this section we begin by introducing a generalized classes of normal spaces, regular spaces, τ_1 and τ_2 in terms of grills.

Definition 4.1. A grill topological space (X, τ, G) is said to be
1- G-prei-τ_1 if for each pair of distinct points x and y of X, there exist G-preopen sets U and V of X such that $x \in U$ and $y \notin U$, and $y \in V$ and $x \notin V$.
2- G-prei-τ_2 if for each pair of distinct points x and y of X, there exist disjoint G-preopen sets U and V of X such that $x \in U$ and $y \in V$.
3- Clopen-τ_1 [11] if for each pair of distinct points x and y of X, there exist clopen sets U and V of X such that $x \in U$ and $y \notin U$, and $y \in V$ and $x \notin V$.
4- Clopen-τ_2 [11] if for each pair of distinct points x and y of X, there exist disjoint clopen sets U and V of X such that $x \in U$ and $y \in V$.

Theorem 4.2. *If $f : (X, \tau, G) \to (Y, \sigma)$ is slightly G-precontinuous injection and Y is a clopen-τ_1 space, then(X, τ, G) is a G-pre-τ_1 space.*

Proof. Suppose that Y is clopen-τ_1. For any two distinct points x and y of X, there exists clopen sets V and W of Y such that $f(x) \in V, f(y) \notin V, f(x) \notin W$ and $f(y) \in W$. Since f is slightly G-precontinuous, $f^{-1}(V)$ and $f^{-1}(W)$ are G-preopen subsets of (X, τ, G) such that $x \in f^{-1}(V), y \notin f^{-1}(V), x \notin f^{-1}(W)$ and $y \in f^{-1}(W)$. This shows that (X, τ, G) is a G-pre-τ_1 space.

□

Theorem 4.3. *If $f : (X, \tau, G) \to (Y, \sigma)$ is slightly G-precontinuous injection and Y is a clopen-τ_2 space, then (X, τ, G) is a G-pre-τ_2 space.*

Proof. For any pair of distinct points x and y of X, there exists clopen sets V and U of Y such that $f(x) \in U, f(y) \in V$ Since f is slightly G-precontinuous, $f^{-1}(U)$ and $f^{-1}(V)$ are G-preopen sets in (X, τ, G) containing x and y, respectively. Therefore, $f^{-1}(U) \cap f^{-1}(V) = \phi$ because $U \cap V = \phi$. This shows that the space (X, τ, G) is a G-pre-τ_2 space. □

Definition 4.4. A grill topological space (X, τ, G) is said to be G-preregular if for each closed set F and each point $x \notin F$, there exist disjoint G-preopen sets U and V of X such that $F \subset U$ and $x \in V$.

Example 4.5. *Let $X = \{a, b, c\}$, $\tau = \{X, \phi, \{a\}, \{b, c\}\}$ and $G = \{X, \{a\}, \{a, b\}\}$. Then a grill topological space (X, τ, G) is G-preregular space.*

Definition 4.6. A grill topological space (X, τ, G) is said to be G-prenormal if for any pair of disjoint closed subsets F_1 and F_2 of X, there exist disjoint G-preopen sets U and V of X such that $F_1 \subset U$ and $F_2 \subset V$.

Definition 4.7. A topological space (X, τ) is said to be:
1-Ultra Hausdroff [11] if every two distinct points of X can be separated by disjoint clopoen sets.
2- Ultra regular [11] if each pair of a point and a closed set not containing the point can be separated by disjoint clopen sets.
3- Ultra normal [11] if every two disjoint closed sets of X can be separated by clopen sets.

Theorem 4.8. *Let $f : (X, \tau, G) \to (Y, \sigma)$ be a slightly G-precontinuous injection then the following properties are hold:*
1-If (Y, σ) is ultra Hausdroff, then (X, τ, G) is G-pre-τ_2,
2- If (Y, σ) is ultra regular and f is open or closed, then (X, τ, G) is G-preregular,
3- If (Y, σ) is ultra normal and f is closed, then (X, τ, G) is G-prenormal.

Proof. (1) Let x_1, x_2 be two distinct points of X. Then since f is injective and Y is ultra Hausdroff, there exists clopen sets V_1 and V_2 of Y such that $f(x_1) \in V_1$, $f(x_2) \in V_2$, and $V_1 \cap V_2 = \phi$. By Theorem 3.2, $x_i \in f^{-1}(V_i) \in GPO(X)$ for $i = 1, 2$ and $f^{-1}(V_1) \cap f^{-1}(V_2) = \phi$. Thus, (X, τ, G) is G-pre-τ_2.

(2) Case-1: Suppose that f is open. Let $x \in X$ and U be an open set containing x. Then $f(U)$ is an open set of Y containing $f(x)$. Since Y is ultra regular, there exists a clopen set V such that $f(x) \in V \subset f(U)$. Since f is a slightly G-precontinuous injection, by Definition 3.1 $x \in f^{-1}(V) \subset U$ and $f^{-1}(V)$ is G-preclopen in X. Therefore, (X, τ, G) is G-preregular. Case-2: Suppose that f is closed. Let $x \in X$ and F be any closed se of X not containing x. Since f is injective and closed, $f(x) \notin f(F)$ and $f(F)$ is closed in Y. By the ultra regularity of Y, there exists a clopen set V such that $f(x) \in V \subset Y \setminus f(F)$. Therefore, $x \in f^{-1}(V)$ and $F \subset X \setminus f^{-1}(V)$. By Theorem 3.2, $f^{-1}(V)$ is an G preclopen set in (X, τ, G). Thus, (X, τ, G) is G preregular.
3- Similar to the proof of Statment(2). □

5 Conclusion

The study of grill topological spaces generalized most of near open sets, near continuous function, a new classes of functions, slightly G- semi-continuous functions and obtained a new decomposition of continuity in terms of grills. So we introduced a new class of function called slightly G- precontinuous functions and some basic properties, beside theorems of slightly G- precontinuous functions are investigated and relationships between this classes of functions.

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