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SOLVING ASSIGNMENT PROBLEM WITH LINGUISTIC COSTS

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Abstract – The purpose of this paper is to solve an assignment problem with linguistic costs/ times. In this paper, an assignment problem has been considered whose costs/times relating to the problem are linguistic variables and its solution methodology has been proposed. The solution method is based on fuzzy representation of linguistic costs/times and defuzzification of fuzzy costs/times. Here, a new defuzzification method based on statistical beta distribution has been used. Then the assignment problem has been converted into an assignment problem whose costs/times are crisp valued and solved by existing linear programming and/or Hungarian method. Finally, a numerical example has been considered and solved to illustrate the solution procedure.

Keywords – *Assignment problem, linguistic variables, fuzzy number, Beta distribution, Defuzzification*

1 Introduction

Assignment problem (AP) is the effective tool in solving real world decision making problem. AP plays a vital role in industrial management system. It is connected with the problem of production planning, telecommunication system, VLSI design problem etc. The main objective of the AP is how to assign/select several tasks/jobs in an effective way for which an optimal assignment can be performed in the best possible way. In an AP n resources (workers) and n activities (jobs) and effectiveness (which are generally represented in terms of cost, profit, time etc.) of each resource for each activity, the problem involves in assigning each resource to one and only one activity so that the total measure of effectiveness is optimized in an efficient way. In the existing literature, it has been noticed that several solution methods and algorithms, viz. linear programming method (Balinski [3], Barr et al. [4], Hung and Rom [10]), Hungarian algorithm (Khun [13]), Neural network method (Eberhardt et al. [9]), genetic algorithm (Avis et al. [2]) etc. were used to solve the assignment problem. But in reality the precise assumptions about

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cost/time related to assignment problem is not realistic in nature and it is observed that the decision makers may be not able to evaluate the same exactly due to lack of proper information. As a result, costs/times of the assignment problem are not measured precisely. To overcome these types of situation, the problem can be formulated using the concept of uncertainty and costs/times are treated as imprecise /uncertain in nature. In such cases fuzzy set theory plays an important role to handle such situation. In this paper, we have treated imprecise costs/times considering linguistic variables. A linguistic variable [22] is a variable whose values are represented using linguistic terms. To tackle the linguistic variables fuzzy representation is the best representation as qualitative data is converted into quantitative data using the same. Therefore, the fuzzy assignment problem provides an efficient framework which solves real-life problems with linguistic information. In the last few years, several attempts have been made in the existing literature for solving fuzzy assignment problem. Sakawa et al. [18] handled the problems on production work force assignment in a firm using interactive fuzzy programming for two level linear and linear functional programming models. Fuzzy assignment model that considers all persons to have same skill proposed by Chen [8].Based on restriction on qualification Huang and Zhang [11] presented a mathematical model for the fuzzy assignment problem and they transformed the model as certain assignment problem. Chen Liang-Hsuan and Lu Hai-Wen [12] developed a method for resolving assignments problem with multiple inadequate inputs and outputs in crisp form for each possible assignment using linear programming model. Majumdar and Bhunia [14] developed an efficient genetic algorithm to solve a generalized assignment problem with imprecise costs/times. X Ye and Jiuping Xu [20] developed a priority based genetic algorithm to a fuzzy vehicle routing assignment problem with connection network. Chen and Chen [7] well discussed fuzzy optimal solution of the assignment problem based on the ranking of generalized fuzzy numbers. Fuzzy risk analysis based on ranking fuzzy numbers using α-cuts proposed by Chen and Wang. Rommelfanger [17] proposed a ranking of fuzzy cost present in the fuzzy assignment problem, which takes more advantages over the existing fuzzy ranking methods. Kar et al/ [12] solved fuzzy generalized assignment problem with restriction on available cost. Solution of fuzzy assignment problem with ranking of generalized trapezoidal fuzzy numbers proposed by Thangavelu et al. [19]. Andal et al. [1] well discussed novel approach to minimum cost flow of fuzzy assignment problem with fuzzy membership functions. Pramanik and Biswas [15] developed multi-objective assignment problem with generalized trapezoidal fuzzy numbers.

In this paper, we have considered assignment problem with linguistic costs/times which is more realistic and has not been generally used in the existing literature. To solve the assignment problem, we have used fuzzy representation of linguistic costs/times and defuzzification of fuzzy costs/times. Here, a new defuzzification technique based on statistical beta distribution [16] has been used for defuzzification of fuzzy number. Then the assignment problem has been transformed into an assignment problem whose costs/times are crisp valued and solved by Linear programming method and/or Hungarian method [13]. Finally, a numerical example has been considered and solved in support of the solution method. The rest of this paper is organized as follows. Section 2 gives some basic definitions used in this paper. Section 3 presents the mathematical formulation of the assignment problem. Section 4 provides the solution methodology. Numerical example is presented in Section. Section 6 presents the results and discussion of a numerical example. Finally, conclusions are made in Section 7.

2 Basic Definitions

In this section, some basic concepts and methods used in this paper are briefly described.

2.1 Fuzzy Set Theory

The fuzzy set theory [21] is developed to deal with the extraction of the primary possible outcome from a set of multiple information that is presented in vague/imprecise terms. Fuzzy set theory handles imprecise data as probability distributions in terms of membership function.

Let X be a non empty set. A fuzzy set $\tilde{A}$ is defined by a membership function $\mu_{\tilde{A}}(x)$, which maps each element x in X to a real number in the interval $[0,1]$. The function $\mu_{\tilde{A}}(x)$ is called the membership function value of $x \in X$ in the fuzzy set $\tilde{A}$.

Definition 2.1 (Fuzzy number). A fuzzy number $\tilde{A}$ is a fuzzy set on the real line R, must satisfy the following conditions.

(i) There exists at least one $x_0 \in R$ for which $\mu_{\tilde{A}}(x_0) = 1$.

(ii) $\mu_{\tilde{A}}(x)$ is pair wise continuous.

(iii) $\tilde{A}$ must be convex and normal.

Definition 2.2 (Triangular fuzzy number). The triangular fuzzy number (TFN) is a normal fuzzy number denoted as $\tilde{A} = (a_1, a_2, a_3)$ where $a_1 \le a_2 \le a_3$ and its membership function $\mu_{\tilde{A}}(x) : X \to [0,1]$ is defined by

$$\mu_{\tilde{A}}(x) = \begin{cases} \dfrac{x - a_1}{a_2 - a_1} & \text{if } a_1 \le x \le a_2 \\ 1 & \text{if } x = a_2 \\ \dfrac{a_3 - x}{a_3 - a_2} & \text{if } a_2 \le x \le a_3 \end{cases}$$

2.2 Linguistic Variables

Linguistic variable is an important concept in fuzzy set theory and plays a vital role in its applications, especially in the fuzzy expert system. Linguistic variable is a variable whose values are descriptive word in natural languages. For example "cost" is a linguistic variable, which can take the values as "Extremely Low", "Very Low", "Low", "Fairly Low" "Medium", "Fairly High", "High", "Very High" and "Extremely High".

2.3 Beta distribution:

A random variable X is said to have Beta distribution if its probability density function is as follows:

$$f(x) = \begin{cases} \dfrac{x^{p-1}(1-x)^{q-1}}{\beta(p,q)} & 0 < x < 1 \\ 0 & \text{otherwise} \end{cases}$$

where $p > 0$ and $q > 0$ are parameters of Beta distribution.

The expected value/mean value $E(X)$ of Beta distribution is $\frac{p}{p+q}$ and is denoted by μ.

If, we have assumed that the density curve of Beta distribution is unimodal and $x^* \in (0,1)$ be the point at which the density function $f(x)$ has the maximum value then we have a relation as follows:

$$q-1=(p-1)\left(\frac{1-x^*}{x^*}\right)$$

Therefore, given both the values of p and x^*, the value of q can be obtained and the expected value of Beta distribution can be calculated by $\mu = \frac{p}{p+q}$.

Theorem 2.1. Let $[a,b]$ be a real interval on $\mathbb{R}$. Then every $\mu \in (a,b)$ there exists a unique $\mu' \in (a,b)$ and vice versa.

Proof. The uniqueness of μ' is proven by contradiction. Let $\mu \in (a,b)$ and $\mu' = \frac{\mu - a}{b-a}$, then $\mu' \in (0,1)$. Assuming that $\mu' \in (0,1)$ and $\mu'' \in (0,1)$ are two distinct numbers ($\mu' \neq \mu''$) corresponding to $\mu \in (a,b)$, then we have $\mu = \mu'(b-a)+a$ and $\mu = \mu''(b-a)+a$ and consequently we get $\mu' = \mu''$ which contradicts the assumption $\mu' \neq \mu''$. So every $\mu \in (a,b)$ there exists a unique $\mu' \in (a,b)$ and vice versa.

2.4 Defuzzification of Triangular Fuzzy Number Based on Statistical Beta Distribution

Let us consider a triangular fuzzy number $\tilde{A} = (a_1, a_2, a_3)$. Now, the projection of

$\tilde{A} = (a_1, a_2, a_3)$ on the interval $(0,1)$ is $\tilde{A} = (\frac{a_1 - a_1}{a_3 - a_1}, \frac{a_2 - a_1}{a_3 - a_1}, \frac{a_3 - a_1}{a_3 - a_1}) = (0, \frac{a_2 - a_1}{a_3 - a_1}, 1)$.

Now, we define the parameter p corresponding to the Beta distribution as follows:

$$p = \frac{a_2 - a_1}{a_3 - a_1} + 1 \tag{1}$$

From (1) it is clear that $m \geq 1$ and if $a_2 \neq a_1$, then the distribution curve is unimodal. If $a_3 - a_2 = -a_1$, which gives a symmetrical triangular fuzzy number, then $m = n = \frac{3}{2}$ and the distribution curve will be symmetric.

In the Beta distribution corresponding to the projection of fuzzy number $\tilde{A}=(a_1,a_2,a_3)$, we have $x^* = \dfrac{a_2-a_1}{a_3-a_1}$ and using the equation (1) we get $q=\dfrac{a_3-a_2}{a_3-a_1}+1$

Now we calculate the mean value of Beta distribution corresponding to the triangular fuzzy number $\tilde{A}=(a_1,a_2,a_3)$ is as follows:

$$\mu = \frac{p}{p+q} = \frac{a_3+a_2-2a_1}{3(a_3-a_1)} \tag{2}$$

The real number $\mu_{\tilde{A}}$ is the real number by transforming μ from the interval $(0,1)$ to the interval (a_1,a_3) is considered as the real number corresponding to the fuzzy number $\tilde{A}=(a_1,a_2,a_3)$ is as follows:

$$\mu_{\tilde{A}} = \mu(a_3-a_1)+a_1 = \frac{a_3+a_2-2a_1}{3(a_3-a_1)}(a_3-a_1)+a_1 = \frac{a_3+a_2+a_1}{3} \tag{3}$$

Remark 1. Using statistical beta distribution, the crisp real number $\mu_{\tilde{A}}$ corresponding to the triangular fuzzy number $\tilde{A}=(a_1,a_2,a_3)$ is $\mu_{\tilde{A}} = \dfrac{a_1+a_2+a_3}{3}$.

2.5 Ranking of Fuzzy Numbers

Let $\mu_{\tilde{A}}$ and $\mu_{\tilde{B}}$ are the crisp real numbers corresponding to two fuzzy numbers $\tilde{A}$ and $\tilde{B}$ respectively. Now, we have defined the ranking or order relations of two fuzzy numbers as follows:

(i) If $\mu_{\tilde{A}} > \mu_{\tilde{B}}$ then $\tilde{A} > \tilde{B}$

(ii) If $(\mu_{\tilde{A}} = \mu_{\tilde{B}}) \wedge (a_3 > \mathrm{b}_3)$ then $\tilde{A} > \tilde{B}$

(iii) If $(\mu_{\tilde{A}} = \mu_{\tilde{B}}, a_3 = \mathrm{b}_3) \wedge (a_2 > \mathrm{b}_2)$ then $\tilde{A} > \tilde{B}$

(iv) If $(\mu_{\tilde{A}} = \mu_{\tilde{B}}, a_3 = \mathrm{b}_3) \wedge (a_2 = \mathrm{b}_2)$ then $\tilde{A} = \tilde{B}$

Theorem 2.2. If a number is added to or subtracted from all of the entries of any row or column of the cost matrix, then an optimal assignment for the resulting cost matrix is also an optimal assignment for the original cost matrix.

2.6 The Hungarian Method

The following algorithm applies the Theorem 2 to a given $n \times n$ cost matrix to find an optimal assignment.

Step-1 Subtract the smallest entry in each row from all the entries of its row.

Step-2 Subtract the smallest entry in each column from all the entries of its column.

Step-3 Draw lines through appropriate rows and columns so that all the zero entries of the cost matrix are covered and the minimum number of such lines is used.

Step-4 Test for optimality: (i) If the minimum number of covering lines is n, an optimal assignment of zeros is possible and we are finished. (ii) If the minimum number of covering lines is less than n, an optimal assignment of zero is not yet possible. In that case proceed to next step.

Step-5 Determine the smallest entry not covered by any line, subtract this entry from each uncovered row, and then add it to each covered column and return to **Step-3**.

3 Mathematical Formulation of Assignment Problem

Let us consider a problem of assignment of n resources (workers) to n activities (jobs) so as to minimize the total cost or time in such a way that each resource can associate with one and only one job. The assignment problem is a special case of transportation problem. The cost or time of the assignment problem is same as that of a transportation problem except that availability at each of the resource and the requirement at each of the destination is unity. The cost matrix $(c_{ij})_{n \times n}$ is given as follows:

		Activities				Availability
		J_1	J_2	$\dots$	J_n	
	W_1	c_{11}	c_{12}	$\dots$	c_{1n}	1
Resource	W_2	c_{21}	c_{22}	$\dots$	c_{2n}	1
	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
	W_n	c_{n1}	c_{n2}	$\dots$	c_{nn}	1
	Requirement	1	1	$\dots$	1	

Here c_{ij} denotes the cost associated with assigning i-th resource to j-th activity.

Let x_{ij} denotes the assignment of i-th resource to j-th activity such that

$$x_{ij} = \begin{cases} 1, \text{ if resource } i \text{ is assigned to activity } j \\ 0, \text{otherwise} \end{cases}$$

Mathematically assignment problem can be formulated as follows:

$$\text{Minimize } z = \sum_{i=1}^{n} \sum_{j=1}^{n} c_{ij} x_{ij}$$

$$\text{subject to } \sum_{i=1}^{n} x_{ij} = 1, \ j = 1, 2, \dots, n$$

$$\sum_{j=1}^{n} x_{ij} = 1,\ i = 1,2,...,n$$
$$x_{ij} \in \{0,1\}$$

In most of the cases, it is seen that the decision maker decision is not a precise type due to lack of information between workers and assigned costs of the job. For this reason, let us consider that $\tilde{c}_{ij}$ be the imprecise cost. Here, we have considered imprecise cost $\tilde{c}_{ij}$ as a linguistic variable and it is represented by descriptive word in natural language. When the costs $\tilde{c}_{ij}$ are linguistic variables, then the assignment problem becomes

$$\text{Minimize } z = \sum_{i=1}^{n}\sum_{j=1}^{n} \tilde{c}_{ij} x_{ij}$$

$$\text{subject to } \sum_{i=1}^{n} x_{ij} = 1,\ j = 1,2,...,n$$
$$\sum_{j=1}^{n} x_{ij} = 1,\ i = 1,2,...,n$$
$$x_{ij} \in \{0,1\}$$

4 Solution Methodology

Proposed approach of solution methodology of linguistic assignment problem is as follows:

Step-1 In linguistic cost matrix, replace all the cost with linguistic variables by triangular fuzzy numbers. The resulting cost matrix is fuzzy cost matrix.

Step-2 Find the crisp number $\mu_{\tilde{c}_{ij}}$ of each $\tilde{c}_{ij}$ using statistical beta distribution and re-constructed crisp cost matrix.

Step-3 Test whether the given assignment problem is balanced or not.

(i) If it is a balanced then go to **Step-5.**
(ii) If it is unbalanced then go to **Step-4**.

Step-4 Introduce dummy rows and/or columns with zero cost so as to form a balanced assignment problem.

Step-5 Solved by linear programming problem method and/or by the Hungarian method to obtain the optimal assignments.

(a) In Case of linear programming method, solve

$$\text{Minimize } z = \sum_{i=1}^{n}\sum_{j=1}^{n} \mu_{\tilde{c}_{ij}} x_{ij}$$

subject to $\sum_{i=1}^{n} x_{ij} = 1,\ j = 1, 2, ..., n$

$$\sum_{j=1}^{n} x_{ij} = 1,\ i = 1, 2, ..., n$$

$$x_{ij} \in \{0,1\}$$

(b) In case of using Hungarian method to a given $n \times n$ cost matrix to find an optimal assignment follow the Hungarian Algorithm.

Step-6 Print the total minimum cost/time and optimal assignment for the assignment problem.

5 Numerical Example

Consider an assignment problem with four resources W_1, W_2, W_3, W_4 and fours jobs J_1, J_2, J_3, J_4 with assignment cost varying between 0\$ to 50\$. The cost matrix $(c_{ij})_{4\times4}$ is given whose elements are linguistic variables which are replaced by triangular fuzzy numbers. The linguistic cost matrix is given below:

Activities

Resource		J_1	J_2	J_3	J_4
	W_1	Extremely Low	Low	Fairly High	Extremely High
	W_2	Low	Very Low	High	Very High
	W_3	Medium	Extremely High	Very Low	Extremely Low
	W_4	Very High	Low	Fairly High	Fairly Low

Table 1. Linguistic term and their corresponding fuzzy data

Linguistic variable (qualitative data)	**Fuzzy data (quantitative data)**
Extremely Low	(0,2,5)
Very Low	(1,2,4)
Low	(4,8,12)
Fairly Low	(15,18,20)
Medium	(23,25,27)
Fairly High	(28,30,32)
High	(33,36,38)
Very High	(37,40,42)
Extremely High	(44,48,50)

Solution. The linguistic variables showing the qualitative data is converted into quantitative data using the Table 1. As the assignment cost varies between Rs.0 to Rs. 50.00 the minimum possible value is taken as 0 and the maximum possible value is taken as 50. Using Table 1, the fuzzy cost matrix is given below:

Activities

		J_1	J_2	J_3	J_4
	W_1	(0,2,5)	(4,8,12)	(28,30,32)	(44,48,50)
Resource	W_2	(4,8,12)	(1,2,4)	(33,36,38)	(37,40,42)
	W_3	(23,25,27)	(44,48,50)	(1,2,4)	(0,2,5)
	W_4	(37,40,42)	(4,8,12)	(28,30,32)	(15,18,20)

Here it is seen that the fuzzy assignment problem is balanced one. Now, we have obtained crisp data $\mu_{\tilde{c}_{ij}}$ of each $\tilde{c}_{ij}$ using statistical beta distribution and presented in Table 2.

Table 2. Crisp data $\mu_{\tilde{c}_{ij}}$ of each $\tilde{c}_{ij}$ using statistical beta distribution

Fuzzy data $\tilde{c}_{ij}$ **(quantitative data)**	**Crisp Data** $\mu_{\tilde{c}_{ij}}$
(0,2,5)	2.33
(1,2,4)	2.33
(4,8,12)	8.00
(15,18,20)	17.67
(23,25,27)	25.00
(28,30,32)	30.00
(33,36,38)	35.67
(37,40,42)	39.67
(44,48,50)	47.33

Now we calculate crisp real number $\mu_{\tilde{A}}$ corresponding to the fuzzy costs $\tilde{c}_{ij}$ and the reduced crisp cost matrix is as follows:

Activities

		J_1	J_2	J_3	J_4
	W_1	$\mu_{(0,2,5)}$	$\mu_{(4,8,12)}$	$\mu_{(28,30,32)}$	$\mu_{(44,48,50)}$
Resource	W_2	$\mu_{(4,8,12)}$	$\mu_{(1,2,4)}$	$\mu_{(33,36,38)}$	$\mu_{(37,40,42)}$
	W_3	$\mu_{(23,25,27)}$	$\mu_{(44,48,50)}$	$\mu_{(1,2,4)}$	$\mu_{(0,2,5)}$
	W_4	$\mu_{(37,40,42)}$	$\mu_{(4,8,12)}$	$\mu_{(15,18,20)}$	$\mu_{(15,18,20)}$

Then the assignment problem can be formulated in the following mathematical programming:

Minimize $z = z_1 + z_2 + z_3 + z_4$ (3)

Subject to

$x_{11} + x_{21} + x_{31} + x_{41} = 1$; $x_{12} + x_{22} + x_{32} + x_{42} = 1$; $x_{13} + x_{23} + x_{33} + x_{43} = 1$; $x_{14} + x_{24} + x_{34} + x_{44} = 1$

$x_{11} + x_{12} + x_{13} + x_{14} = 1$; $x_{21} + x_{22} + x_{23} + x_{24} = 1$; $x_{31} + x_{32} + x_{33} + x_{34} = 1$; $x_{41} + x_{42} + x_{43} + x_{44} = 1$

where

$z_1 = \mu_{(0,2,5)}x_{11} + \mu_{(4,8,12)}x_{21} + \mu_{(23,25,27)}x_{31} + \mu_{(37,40,42)}x_{41}$
$z_2 = \mu_{(4,8,12)}x_{12} + \mu_{(1,2,4)}x_{22} + \mu_{(44,48,50)}x_{32} + \mu_{(4,8,12)}x_{42}$
$z_3 = \mu_{(28,30,32)}x_{13} + \mu_{(33,36,38)}x_{23} + \mu_{(1,2,4)}x_{33} + \mu_{(15,18,20)}x_{43}$
$z_4 = \mu_{(44,48,50)}x_{14} + \mu_{(37,40,42)}x_{24} + \mu_{(0,2,5)}x_{34} + \mu_{(15,18,20)}x_{44}$
$x_{ij} \in \{0,1\}, i = 1,2,3,4$ and $j = 1,2,3,4$

Solving the problem (3) we get the solution as $x_{11} = 1$, $x_{22} = 1$, $x_{34} = 1$ and $x_{43} = 1$

Therefore the optimal assignment is $W_1 \to J_1, W_2 \to J_2, W_3 \to J_4, W_4 \to J_3$

Total optimal cost is $(0,2,5) + (1,2,4) + (0,2,5) + (15,18,20) = (16,24,34)$ and corresponding $\mu_{(16,24,34)} = 24.66$.

Also proceeding by Hungarian method, the optimal assignments are $W_1 \to J_1, W_2 \to J_2, W_3 \to J_4, W_4 \to J_3$ and $W_1 \to J_1, W_2 \to J_2, W_3 \to J_3, W_4 \to J_4$. For the optimal assignment $W_1 \to J_1, W_2 \to J_2, W_3 \to J_4, W_4 \to J_3$ the minimum cost is $(0,2,5) + (1,2,4) + (1,2,4) + (15,18,20) = (17,24,33)$ and corresponding $\mu_{(17,24,33)} = 24.66$, where as for optimal assignment $W_1 \to J_1, W_2 \to J_2, W_3 \to J_4, W_4 \to J_3$ the minimum cost is $(0,2,5) + (1,2,4) + (0,2,5) + (15,18,20) = (16,24,34)$ and corresponding $\mu_{(16,24,34)} = 24.66$.

6 Results and Discussion

Generally, in real life situation the assignment cost cannot be taken as precise one. So, in this paper we have solved an assignment problem considering each cost of the assignment is linguistic variable. For this purpose, to solve the assignment problem with linguistic cost we have used fuzzy representation of linguistic variables and defuzzification of fuzzy cost. Here we have considered triangular fuzzy number to represent linguistic variable and for defuzzification we have used newly defuzzification technique based on statistical beta distribution. For optimal solution we have used linear programming method and Hungarian method. By using linear programming method we have found optimal assignment as $x_{11} = 1$, $x_{22} = 1$, $x_{34} = 1$ and $x_{43} = 1$, where as using Hungarian method we have found multiple assignment either as $x_{11} = 1$, $x_{22} = 1$, $x_{34} = 1$ and $x_{43} = 1$ or $x_{11} = 1$, $x_{22} = 1$, $x_{33} = 1$ and $x_{44} = 1$. But it is seen that optimal assignment cost is 24.66 for all solution sets. Hence, the optimal assignments are $W_1 \to J_1, W_2 \to J_2, W_3 \to J_4, W_4 \to J_3$ and $W_1 \to J_1, W_2 \to J_2, W_3 \to J_3, W_4 \to J_4$. For the fuzzy optimal costs (17, 24, 33) and (16, 24, 34), we have obtained $\mu_{(17,24,33)} = 24.66$ and $\mu_{(16,24,34)} = 24.66$. Here, we have seen that $(\mu_{(17,24,33)} = \mu_{(16,24,34)}) \wedge (34 > 33)$ so (16, 24, 34) is greater than (17, 24, 33). Also from Figure 1, it is seen that intersection of (16, 24, 34) and (17, 24, 33) is (17, 24, 33). Therefore we have taken (17, 24, 33) is the optimal fuzzy assignment cost and its linguistic value has considered as Medium cost.

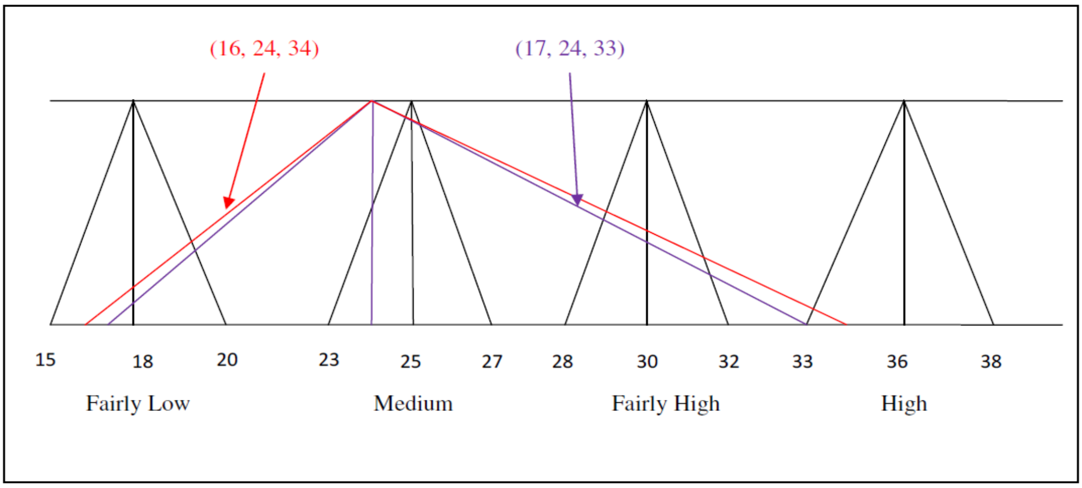

Figure 1. Pictorial representation of linguistic variables and fuzzy data

7 Conclusions

Here assignment problem with linguistic cost/time is presented and its solution methodology has been discussed. Assignment problem is one of the most important problems in decision making. In many real life situations, costs/times of the assignment problem are not precise. The assignment problem with linguistic costs is more realistic than the assignment problem with precise costs because most of the real life situations are uncertain. The proposed method described here is very simple and easy to implement. This method is based on fuzzy representation of linguistic costs/times and defuzzification of fuzzy costs/times. Here, a new defuzzification technique based on the statistical beta distribution has been used for defuzzification of fuzzy number. Then the assignment problem has been converted into an assignment problem whose costs/times are crisp /precise valued and solved by existing Hungarian method/ linear programming method. Finally, an assignment problem with linguistic cost has been solved and computed results has been presented and compared. It may be claimed that the proposed method attempted in this paper can be applied to solve realistic decision making problems in the near future involving linguistic parameters.

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References

[1] S. Andal, S. Murgesan, B.K. Ramesh, *A novel approach to minimum cost flow of fuzzy assignment problem with fuzzy membership functions*, Engineering, science and Technology: an International Journal 6(2) (2016) 85-88.

[2] D. Avis, L. Devroye, *An analysis of decomposition heuristic for assignment problem*, Operations Research Letter 3(6) (1985) 279-283.

[3] M. L. Balinski, *A competitive (dual) Simplex method for the assignment problem*, Math Program 34(2) (1986) 125-141.

[4] R. S. Barr, F. Glover, D. Klingman, *The alternating basis algorithm for assignment problems*, Math Program 13(1) (1977) 1-13.

[5] C. Liang-Hsuan, L. Hai-wen, *An extended assignment problem considering multiple inputs and outputs*, Applied mathematical modeling 31 (2007) 2239-2248.

[6] S. M. Chen, C. H. Wang, *Fuzzy risk analysis based on ranking fuzzy numbers using* α*-cuts*, Expert System Applications 36 (2009) 5576-5581.

[7] M. S. Chen, J.H. Chen, *Fuzzy risk analysis based on the ranking generalized fuzzy numbers with different heights and different spreads*, Expert System Applications 36 (2009) 6833-6842.

[8] M.S. Chen, *On a fuzzy assignment problem*, Tamkang Journal 22 (1985) 407-411.

[9] S.P. Eberhardt, T. Daud, A. Kerns, T. X. Brown, A. P. Thakoor, *Competitive neural architecture for hardware solution to the assignment problem*, Neural Networks 4(4) (1991) 431-442.

[10] M. S. Huang, W. O. Rom, *Solving the assignment problem by relaxation*, Operations Research 28(4) (1980) 969-982.

[11] L. S. Huang, Li-pu Zhang, *Solution method for fuzzy assignment problem with restriction on Qualification*, In: Proceeding of Sixth International Conference on Intelligent Systems Design and Applications (ISDA06) 2006.

[12] S. Kar, K. Basu, S. Mukherjee, *Solution of generalized fuzzy assignment problem with restriction on costs under fuzzy environment*, International journal of fuzzy mathematics and systems 4(2) (2014) 169-180.

[13] H. W. Kuhn, *The Hungarian method for assignment problem*, Naval Research Logistic Quarterly 2, (1955) 83-97.

[14] J. Majumdar, A. K. Bhunia, *Elitist Genetic algorithm for assignment problem with imprecise goa*l, European Journal of Operational Research 177 (2007) 684-692.

[15] S. Pramanik, P. Biswas, *Multi-objective assignment problem with generalized trapezoidal fuzzy numbers*, International journal of applied information systems 2(6) (2012) 13-20.

[16] A. Rahmani, Lotfi, F.Hosseinzadeh, M. Rostamy-Malkhalifeh , T. Allahviranloo , *A new method for defuzzification and ranking of fuzzy numbers based on the statistical beta distribution*, Advances in fuzzy systems 2016 (2016) 1-8.

[17] H. J. Rommelfanger, *The advantages of fuzzy optimization models in practical use*, Fuzzy optimization and decision making 3(4) (2004) 295-309.

[18] M. Sakawa, I. Nishizaki, Y. Uemura, *Interactive fuzzy programming for two-level linear and linear fractional production and assignment problems: a case study*, European Journal of Operational Research 135 (2001) 142-157.

[19] K. Thangavelu, G. Uthra, R. M. Umamageswari, *Solution of fuzzy assignment problem with ranking of generalized trapezoidal fuzzy numbers*, International journal of pure and applied mathematics 106(6) (2016) 9-16.

[20] X. Ye, J. Xu, *A fuzzy vehicle routing assignment model with connection network based on priority-based genetic algorithm*, World journal of modeling and simulation 4 (2008) 257-268.

[21] L. A. Zadeh, *Fuzzy sets,* Information and Control 8(3) (1965) 338-352.

[22] L. A. Zadeh, *The concept of linguistic variable and its application to approximate reasoning-I,* Information Science 8 (1975) 199-249.