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ON NANO $\wedge_g$-CLOSED SETS

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Abstaract − In this paper, we introduce nano $\wedge_g$-closed sets in nano topological spaces. Some properties of nano $\wedge_g$-closed sets and nano $\wedge_g$-open sets are weaker forms of nano closed sets and nano open sets.

Keywords − *Nano $\wedge$-set, nano λ-closed set, nano $\wedge_g$-closed set.*

1 Introduction

In 2017, Rajasekaran et.al [5] introduced the notion of nano $\wedge$-sets in nano topological spaces and nano $\wedge$-set is a set H which is equal to its nano kernel and we introduced the notion of nano λ-closed set and nano λ-open sets. In this paper to introduce new classes of sets called nano $\wedge_g$-closed sets and nano $\wedge_g$-open sets in nano topological spaces. We also some properties of such sets and nano $\wedge_g$-closed sets and nano $\wedge_g$-open sets are weaker forms of nano closed sets and nano open sets.

2 Preliminaries

Throughout this paper $(U, \tau_R(X))$ (or X) represent nano topological spaces on which no separation axioms are assumed unless otherwise mentioned. For a subset H of a space $(U, \tau_R(X))$, $Ncl(H)$ and $Nint(H)$ denote the nano closure of H and the nano interior of H respectively. We recall the following definitions which are useful in the sequel.

Definition 2.1. *[4] Let U be a non-empty finite set of objects called the universe and R be an equivalence relation on U named as the indiscernibility relation. Elements*

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belonging to the same equivalence class are said to be indiscernible with one another. The pair (U, R) is said to be the approximation space. Let $X \subseteq U$.

1. *The lower approximation of X with respect to R is the set of all objects, which can be for certain classified as X with respect to R and it is denoted by $L_R(X)$. That is, $L_R(X) = \bigcup_{x\in U}\{R(x) : R(x) \subseteq X\}$, where R(x) denotes the equivalence class determined by x.*

2. *The upper approximation of X with respect to R is the set of all objects, which can be possibly classified as X with respect to R and it is denoted by $U_R(X)$. That is, $U_R(X) = \bigcup_{x\in U}\{R(x) : R(x) \cap X \neq \phi\}$.*

3. *The boundary region of X with respect to R is the set of all objects, which can be classified neither as X nor as not - X with respect to R and it is denoted by $B_R(X)$. That is, $B_R(X) = U_R(X) - L_R(X)$.*

Proposition 2.2. *[2] If (U, R) is an approximation space and $X, Y \subseteq U$; then*

1. $L_R(X) \subseteq X \subseteq U_R(X)$;

2. $L_R(\phi) = U_R(\phi) = \phi$ *and* $L_R(U) = U_R(U) = U$;

3. $U_R(X \cup Y) = U_R(X) \cup U_R(Y)$;

4. $U_R(X \cap Y) \subseteq U_R(X) \cap U_R(Y)$;

5. $L_R(X \cup Y) \supseteq L_R(X) \cup L_R(Y)$;

6. $L_R(X \cap Y) \subseteq L_R(X) \cap L_R(Y)$;

7. $L_R(X) \subseteq L_R(Y)$ *and* $U_R(X) \subseteq U_R(Y)$ *whenever* $X \subseteq Y$;

8. $U_R(X^c) = [L_R(X)]^c$ *and* $L_R(X^c) = [U_R(X)]^c$;

9. $U_R U_R(X) = L_R U_R(X) = U_R(X)$;

10. $L_R L_R(X) = U_R L_R(X) = L_R(X)$.

Definition 2.3. *[2] Let U be the universe, R be an equivalence relation on U and $\tau_R(X) = \{U, \phi, L_R(X), U_R(X), B_R(X)\}$ where $X \subseteq U$. Then by the Property 2.2, R(X) satisfies the following axioms:*

1. *U and $\phi \in \tau_R(X)$,*

2. *The union of the elements of any sub collection of $\tau_R(X)$ is in $\tau_R(X)$,*

3. *The intersection of the elements of any finite subcollection of $\tau_R(X)$ is in $\tau_R(X)$.*

That is, $\tau_R(X)$ is a topology on U called the nano topology on U with respect to X. We call $(U, \tau_R(X))$ as the nano topological space. The elements of $\tau_R(X)$ are called as nano open sets and $[\tau_R(X)]^c$ is called as the dual nano topology of $[\tau_R(X)]$.

Remark 2.4. *[2] If $[\tau_R(X)]$ is the nano topology on U with respect to X, then the set $B = \{U, \phi, L_R(X), B_R(X)\}$ is the basis for $\tau_R(X)$.*

Definition 2.5. *[2] If $(U, \tau_R(X))$ is a nano topological space with respect to X and if $H \subseteq U$, then the nano interior of H is defined as the union of all nano open subsets of H and it is denoted by $Nint(H)$.*

That is, $Nint(H)$ is the largest nano open subset of H. The nano closure of H is defined as the intersection of all nano closed sets containing H and it is denoted by $Ncl(H)$.

That is, $Ncl(H)$ is the smallest nano closed set containing H.

Definition 2.6. *[3] Let $(U, \tau_R(X))$ be a nano topological spaces and $H \subseteq U$. The nano $Ker(H) = \bigcap\{U : H \subseteq U, U \in \tau_R(X)\}$ is called the nano kernal of H and is denoted by $\mathcal{N}Ker(H)$.*

Definition 2.7. *[5] A subset H of a space $(U, \tau_R(X))$ is called*

1. *a nano $\wedge$-set if $H = NKer(H)$.*
2. *nano λ-closed if $H = L \cap F$ where L is a nano $\wedge$-set and F is nano closed.*

Definition 2.8. *A subset H of a nano topological space $(U, \tau_R(X))$ is called nano g-closed [1] if $Ncl(H) \subseteq G$, whenever $H \subseteq G$ and G is nano open.*

Remark 2.9. *[5]In a nano topological space, the concepts of nano g-closed sets and nano λ-closed sets are independent.*

3 Nano $\wedge_g$-closed Sets

Definition 3.1. *A subset H of a space $(U, \tau_R(X))$ is called nano λ-open if $H^c = U - H$ is nano λ-closed.*

Example 3.2. *Let $U = \{a, b, c, d\}$ with $U/R = \{\{a\}, \{c\}, \{b, d\}\}$ and $X = \{a, b\}$. Then the nano topology $\tau_R(X) = \{\phi, \{a\}, \{b, d\}, \{a, b, d\}, U\}$. Then $\{a\}$ is nano λ-open.*

Definition 3.3. *A subset H of a space $(U, \tau_R(X))$ is called a nano $\wedge_g$-closed set if $Ncl(H) \subseteq G$, whenever $H \subseteq G$ and G is nano λ-open.*

The complement of nano $\wedge_g$-open if $H^c = U - H$ is nano $\wedge_g$-closed.

Example 3.4. *In Example 3.2, then $\{a, c\}$ is nano $\wedge_g$-closed set.*

Lemma 3.5. *In a space $(U, \tau_R(X))$, every nano open set is nano $\wedge_g$-open but not conversely*

Remark 3.6. *The converse of statements in Lemma 3.5 are not necessarily true as seen from the following Example.*

Example 3.7. *In Example 3.2, then $\{b\}$ is nano $\wedge_g$-open but not nano open.*

Remark 3.8. *The following example shows that the concepts of nano $\wedge_g$-closed sets and nano λ-closed are independent for each other.*

Example 3.9. *In Example 3.2,*

1. *then $\{b, c\}$ is nano $\wedge_g$-closed but not nano λ-closed.*

2. *then $\{a\}$ is nano λ-closed but not nano $\wedge_g$-closed.*

Theorem 3.10. *In a space $(U, \tau_R(X))$, the union of two nano $\wedge_g$-closed sets is nano $\wedge_g$-closed.*

Proof. Let $H \cup Q \subseteq G$, then $H \subseteq G$ and $Q \subseteq G$ where G is nano λ-open. As H and Q are $\wedge_g$-closed, $Ncl(H) \subseteq G$ and $Ncl(Q) \subseteq G$. Hence $Ncl(H \cup Q) = Ncl(H) \cup Ncl(Q) \subseteq G$.

Example 3.11. *In Example 3.2, then $H = \{a, c\}$ and $Q = \{b, c\}$ is nano $\wedge_g$-closed. Clearly $H \cup Q = \{a, b, c\}$ is nano $\wedge_g$-closed.*

Theorem 3.12. *In a space $(U, \tau_R(X))$, the intersection of two nano $\wedge_g$-open sets is nano $\wedge_g$-open.*

Proof. Obvious by Theorem 3.10.

Example 3.13. *Let $U = \{a, b, c, d\}$ with $U/R = \{\{a\}, \{b\}, \{c, d\}\}$ and $X = \{b, d\}$. Then the nano topology $\tau_R(X) = \{\phi, \{b\}, \{c, d\}, \{b, c, d\}, U\}$ Then $H = \{b, c\}$ and $Q = \{b, d\}$ is nano $\wedge_g$-open. Clearly $H \cap Q = \{b\}$ is nano $\wedge_g$-open.*

Remark 3.14. *In a space $(U, \tau_R(X))$, the intersection of two nano $\wedge_g$-closed sets but not nano $\wedge_g$-closed.*

Example 3.15. *Let $U = \{1, 2, 3\}$ with $U/R = \{\{1\}, \{2, 3\}\}$ and $X = \{1\}$. Then the nano topology $\tau_R(X) = \{\phi, \{1\}, U\}$, Then $H = \{1, 21\}$ and $Q = \{1, 3\}$ is nano $\wedge_g$-closed. Clearly $H \cap Q = \{1\}$ is but not nano $\wedge_g$-closed.*

Theorem 3.16. *In a space $(U, \tau_R(X))$ is nano $\wedge_g$-closed, then $Ncl(H) - H$ contains no nonempty nano closed.*

Proof. Let P be a nano closed subset contains in $Ncl(H) - H$. Clearly $H \subseteq P^c$ where H is nano $\wedge_g$-closed and P^c is an nano open set of U. Thus $Ncl(H) \subseteq P^c$ (or) $P \subseteq (Ncl(H))^c$. Then $P \subseteq (Ncl(H))^c \cap (Ncl(H) - H) \subseteq (Ncl(H))^c \cap Ncl(H) = \phi$. This is show that $P = \phi$.

Theorem 3.17. *A subset H of a space $(U, \tau_R(X))$ is nano $\wedge_g$-closed $\iff$ $Ncl(H) - H$ contains no nonempty nano λ-closed.*

Proof. Necessity. Assume that H is nano $\wedge_g$-closed. Let K be a nano λ-closed subset of $Ncl(H) - H$. Then $H \subseteq K^c$. Since H is nano $\wedge_g$-closed, we have $Ncl(H) \subseteq K^c$. Consequently $K \subseteq (Ncl(H))^c$. Hence $K \subseteq Ncl(H) \cap (Ncl(H))^c = \phi$. Therefore K is empty.

Sufficiency. Assume that $Ncl(H) - H$ contains no nonempty nano λ-closed sets. Let $H \subseteq C$ and C be a nano λ-open. If $Ncl(H) \nsubseteq C$, then $Ncl(H) \cap C^c$ is a nonempty nano λ-closed subset of $Ncl(H) - H$. Therefore H is nano $\wedge_g$-closed.

Theorem 3.18. *In a space $(U, \tau_R(X))$, if H is a nano $\wedge_g$-closed and $H \subseteq Q \subseteq Ncl(H)$, then Q is a nano $\wedge_g$-closed.*

Proof. Let $H \subseteq Q$ and $Ncl(Q) \subseteq Ncl(H)$. Hence $(Ncl(Q) - Q) \subseteq (Ncl(H) - H)$. But by Theorem 3.17, $Ncl(H) - H$ contains no nonempty nano λ-closed subset of U and hence neither does $Ncl(B) - B$. By Theorem 3.17, Q is nano $\wedge_g$-closed.

Theorem 3.19. *In a space $(U, \tau_R(X))$, if H is nano λ-open and nano $\wedge_g$-closed, then hence H is nano closed.*

Proof. Since H is nano λ-open and nano λ-closed, $Ncl(H) \subseteq H$ and hence H is nano closed.

Theorem 3.20. *For each $x \in U$, either $\{x\}$ is nano λ-closed (or) $\{x\}^c$ is nano $\wedge_g$-closed.*

Proof. Assume $\{x\}$ is not nano λ-closed. Then $\{x\}^c$ is not nano λ-open and the only nano λ-open set containing $\{x\}^c$ is the space of U itself. Therefore $Ncl(\{x\}^c) \subseteq U$ and so $\{x\}^c$ is nano $\wedge_g$-closed.

Theorem 3.21. *In a space $U, \tau_R(X)$, H is nano $\wedge_g$-open $\iff$ $P \subseteq Nint(H)$ whenever P is nano λ-closed and $P \subseteq H$.*

Proof. Assume that $P \subseteq Nint(H)$ whenever P is nano λ-closed and $P \subseteq H$. Let $H^c \subseteq C$, where C is nano λ-open. Hence $C^c \subseteq H$. By assumption $C^c \subseteq Nint(H)$ which implies that $(Nint(H))^c \subseteq C$, so $Ncl(H^c) \subseteq C$. Hence H^c is nano $\wedge_g$-closed that is, H is nano $\wedge_g$-open.

Conversely, let H be nano $\wedge_g$-open. Then H^c is nano $\wedge_g$-closed. Also let P be a nano λ-closed set contained in H. Then P^c is nano λ-open. Therefore whenever $H^c \subseteq P^c$, $Ncl(H^c) \subseteq P^c$. This implies that $P \subseteq (Ncl(H^c))^c = Nint(H)$. Thus $H \subseteq Nint(H)$.

Theorem 3.22. *In a space $(U, \tau_R(X))$, H is $\wedge_g$-open $\iff$ $C = U$ whenever C is nano λ-open and $Nint(H) \cup H^c \subseteq C$.*

Proof. Let H be a nano $\wedge_g$-open, C be a nano λ-open and $Nint(H) \cup H^c \subseteq C$. Then $C^c \subseteq (Nint(H))^c \cap (H^c)^c = (Nint(H))^c - H^c) = Ncl(H^c) - H^c$. Since H^c is nano $\wedge_g$-closed and C^c is nano λ-closed, by Theorem 3.17 it follows that $C^c = \phi$. Therefore $U = C$. Conversely, suppose that P is nano λ-closed and $P \subseteq H$. Then $Nint(H) \cup H^c \subseteq Nint(H) \cup P^c$. It follows that $Nint(H) \cup P^c = U$ and hence $P \subseteq Nint(H)$. Therefore H is nano $\wedge_g$-open.

Theorem 3.23. *In a space $(U, \tau_R(X))$, if $Nint(H) \subseteq Q \subseteq H$ and H is nano $\wedge_g$-open, then Q is nano $\wedge_g$-open.*

Proof. Assume $Nint(H) \subseteq Q \subseteq H$ and H is nano $\wedge_g$-open. Then $H^c \subseteq Q^c \subseteq Ncl(H^c)$ and H^c is nano $\wedge_g$-closed. By Theorem 3.18, Q is nano $\wedge_g$-open.

Theorem 3.24. *In a space $(U, \tau_R(X))$, H is nano $\wedge_g$-closed $\iff$ $Ncl(H) - H$ is nano $\wedge_g$-open.*

Proof. Necessity. Assume that H is nano $\wedge_g$-closed. Let $P \subseteq Ncl(H) - H$, where P is nano λ-closed. By Theorem 3.17, $P = \phi$, Therfore $P \subseteq Nint(Ncl(H) - H)$ and by Theorem 3.21, $Ncl(H) - H$ is nano $\wedge_g$-open.

Sufficiency. Let $H \subseteq C$ where C is a nano λ-open set. Then $Ncl(H) \cap C^c \subseteq Ncl(H) \cap H^c = Ncl(H) - H$. Since $Ncl(H) \cap C^c$ is nano λ-closed and $Ncl(H) - H$ is nano $\wedge_g$-open, by Theorem 3.21, we have $Ncl(H) \cap C^c \subseteq Nint(Ncl(H) - H) = \phi$. Hence H is nano $\wedge_g$-closed.

Theorem 3.25. *In a nano topological space $(U, \tau_R(X))$, the following properties are equivalent:*

1. *H is nano $\wedge_g$-closed.*
2. *$Ncl(H) - H$ contains no nonempty nano λ-closed set.*
3. *$Ncl(H) - H$ is nano $\wedge_g$-open.*

Proof. This follows from by Theorems 3.17 and 3.24.

Definition 3.26. *A subset H of a space $(U, \tau_R(X))$ is called*

1. *a nano ${}_g\wedge$-closed set if $N\lambda cl(H) \subseteq G$, whenever $H \subseteq G$ and G is nano open.*
2. *a nano $\wedge$-g-closed set if $N\lambda cl(H) \subseteq G$, whenever $H \subseteq G$ and G is nano λ-open.*

The complement of the above mentioned sets are called their respective open sets.

Example 3.27. *In Example 3.2, then $\wp(U)$ is nano ${}_g\wedge$-closed and nano $\wedge$-g-closed.*

Remark 3.28. *For a subset of a space $(U, \tau_R(X))$, we have the following implications:*

$$\begin{array}{ccc}
\textbf{nano closed} & \rightarrow & \textbf{nano } \lambda\textbf{-closed} \\
\downarrow & & \downarrow \\
\textbf{nano } \wedge_g\textbf{-closed} & \rightarrow & \textbf{nano } \wedge\textbf{-}g\textbf{-closed} \\
\downarrow & & \downarrow \\
\textbf{nano } g\textbf{-closed} & \rightarrow & \textbf{nano } {}_g\wedge\textbf{-closed}
\end{array}$$

None of the above implications is reversible.

Theorem 3.29. *In a space $(U, \tau_R(X))$, H is nano $\wedge_g$-closed $\iff$ $N\lambda cl(\{x\}) \cap H \neq \phi$ for every $x \in Ncl(H)$.*

Proof. Necessity. Suppose that $N\lambda cl(\{x\}) \cap H = \phi$ for some $x \in Ncl(H)$. Then $U - N\lambda cl(\{x\})$ is a nano λ-open set containing H. Furthermore, $x \in Ncl(H) - (U - N\lambda cl(\{x\}))$ and hence $Ncl(H) \nsubseteq U - N\lambda cl(\{x\})$. This shows that H is not nano $\wedge_g$-closed.

Sufficiency. Suppose that H is not nano $\wedge_g$-closed. There exist a nano λ-open set G containing H such that $Ncl(H) - G \neq \phi$. There exist $x \in Ncl(H)$ such that $x \notin G$, hence $N\lambda cl(\{x\}) \cap G = \phi$. Therefore, $N\lambda cl(\{x\}) \cap H = \phi$ for some $x \in Ncl(H)$.

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References

[1] K. Bhuvaneshwari and K. Mythili Gnanapriya, *Nano Generalizesd closed sets*, International Journal of Scientific and Research Publications, 4(5)2014,1-3.

[2] M. Lellis Thivagar and Carmel Richard, *On Nano forms of weakly open sets*, International Journal of Mathematics and Statistics Invention,1(1) 2013, 31-37.

[3] M. Lellis Thivagar, Saeid Jafari and V. Sutha Devi, *On new class of contra continuity in nano topology*, Italian Journal of Pure and Applaied Mathematics, 2017, 1-10. .

[4] Z. Pawlak, *Rough sets*, International journal of computer and Information Sciences, 11(5)(1982), 341-356.

[5] I. Rajasekaran and O. Nethaji, *On some new subsets of nano topological spaces*, Journal of New Theory, 16(2017), 52-58.