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ON L-FUZZY (K, E)-SOFT NEIGHBORHOOD SPACES

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Abstaract − In this paper, we define L-fuzzy (K, E)-soft neighborhood spaces and investigated the topological properties of L-fuzzy (K, E)-soft uniformity in stsc-quantales. We obtain L-fuzzy (K, E)-soft topology and L-fuzzy (K, E)-soft neighborhood spaces induced by L-fuzzy (K, E)-soft uniformity. Moreover, we study the relations among L-fuzzy (K, E)-soft topology, L-fuzzy (K, E)-soft neighborhood system and L-fuzzy (K, E)-soft uniformity.

Keywords − *Stsc-quantales, L-fuzzy* (K, E)*-soft neighborhood space, L-fuzzy* (K, E)*-soft uniform space, L-fuzzy* (K, E)*-soft topologies.*

1 Introduction

In 1999 Molodtsov [15] initiated the theory of soft sets as a new mathematical tool to deal with uncertainties while modeling problems in engineering physics, computer science, economics, social sciences and medical sciences. In [15], Molodtsov applied successfully in directions such as, smoothness of functions, game theory, operations research, Riemann integration, Perron integration, probability and theory of measurement. Maji et al. [13,14] gave the first practical application of soft sets in decision making problems. In 2003, Maji et al. [13] defined and studied several basic notions of soft set theory. Many researchers have contributed towards the algebraic structure of soft set theory [1,3,7]. In 2011, Shabir and Naz [23] initiated the study of soft topological spaces. They defined soft topology on the collection of soft sets over X and established their several properties. Aygünoğlu et.al [4] introduced the concept of soft topology in the sense of Šostak [24].

Hájek [8] introduced a complete residuated lattice which is an algebraic structure for many valued logic and decision rules in complete residuated lattices. Höhle

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[9,10] introduced L-fuzzy topologies with algebraic structure L(cqm, quantales, MV-algebra). It has developed in many directions [11,17,18]. Lowen [12] introduced the notion of fuzzy uniformities as a view of the enourage approach. Many reachers studied the different approach as powerset or the uniform covering. Kim [11] introduced the notion of fuzzy uniformities as an extension of Lowen in a stsc-quantale L. Ramadan et al. [19,20] define the the concept , L- fuzzy (K, E)-soft uniform spaces, L-fuzzy (K, E)-soft topological spaces in strictly two sided commutative quantales and investigated the relation between them.

In this paper, we define L-fuzzy (K, E)-soft neighborhood spaces and investigated the topological properties of L-fuzzy (K, E)-soft uniformity in stsc-quantales. We obtain L-fuzzy (K, E)-soft topology and L-fuzzy (K, E)-soft neighborhood spaces induced by L-fuzzy (K, E)-soft uniformity. Moreover, we study the relations among L-fuzzy (K, E)-soft topology, L-fuzzy (K, E)-soft neighborhood system and L-fuzzy (K, E)-soft uniformity.

2 Preliminaries

Let $L = (L, \leq, \vee, \wedge, 0, 1)$ be a completely distributive lattice with the least element 0 and the greatest element 1 in L.

Definition 2.1. [8,10,22] A complete lattice $(L, \leq, \odot)$ is called a strictly two-sided commutative quantale (stsc-quantale, for short) iff it satisfies the following properties.

(L1) $(L, \odot)$ is a commutative semigroup,
(L2) $x = x \odot 1$, for each $x \in L$ and 1 is the universal upper bound,
(L3) $\odot$ is distributive over arbitrary joins, i.e. $(\bigvee_i x_i) \odot y = \bigvee_i (x_i \odot y)$.

There exists a further binary operation $\rightarrow$ (called the implication operator or residuated) satisfying the following condition

$$x \rightarrow y = \bigvee \{z \in L | x \odot z \leq y\}.$$

Then it satisfies Galois correspondence; i.e, $(x \odot z) \leq y$ iff $z \leq (x \rightarrow y)$.

In this paper, we always assume that $(L, \leq, \odot, \rightarrow, ^*)$ is a stsc-quantales with an order reversing involution * which is defined by $x^* = x \rightarrow 0$ unless otherwise specified.

Remark 2.2. Every completely distributive lattice $(L, \leq, \wedge, ^*)$ with order reversing involution * is a stsc-quantale $(L, \leq, \odot, \oplus, ^*)$ with a strong negation * where $\odot = \wedge$.

Lemma 2.3. [8,22] For each $x, y, z, x_i, y_i, w \in L$, we have the following properties.

(1) $1 \to x = x$, $0 \odot x = 0$.
(2) If $y \le z$, then $x \odot y \le x \odot z$, $x \to y \le x \to z$ and $z \to x \le y \to x$.
(3) $x \le y$ iff $x \to y = 1$.
(4) $(\bigwedge_i y_i)^* = \bigvee_i y_i^*$, $(\bigvee_i y_i)^* = \bigwedge_i y_i^*$.
(5) $x \to (\bigwedge_i y_i) = \bigwedge_i (x \to y_i)$.
(6) $(\bigvee_i x_i) \to y = \bigwedge_i (x_i \to y)$.
(7) $x \to (\bigvee_i y_i) \ge \bigvee_i (x \to y_i)$.
(8) $(\bigwedge_i x_i) \to y \ge \bigvee_i (x_i \to y)$.
(9) $(x \odot y) \to z = x \to (y \to z) = y \to (x \to z)$.
(10) $x \odot y = (x \to y^*)^*$,
(11) $(x \to y) \odot (z \to w) \le (x \odot z) \to (y \odot w)$ and $x \le y \to x \odot y$.
(12) $x \to y \le (x \odot z) \to (y \odot z)$ and $(x \to y) \odot (y \to z) \le x \to z$.
(13) $y \to z \le (x \to y) \to (x \to z)$.
(14) $x \to y = y^* \to x^*$.
(15) $\bigvee_{i\in\Gamma} x_i \to \bigvee_{i\in\Gamma} y_i \ge \bigwedge_{i\in\Gamma}(x_i \to y_i)$ and $\bigwedge_{i\in\Gamma} x_i \to \bigwedge_{i\in\Gamma} y_i \ge \bigwedge_{i\in\Gamma}(x_i \to y_i)$.

Throughout this paper, X refers to an initial universe, E and K are the sets of all parameters for X, and L^X is the set of all L-fuzzy sets on X.

Definition 2.4. [4,7] A map f is called an L- fuzzy soft set on X, where f is a mapping from E into L^X, i.e., $f_e := f(e)$ is an L- fuzzy set on X, for each $e \in E$. The family of all L- fuzzy soft sets on X is denoted by $(L^X)^E$. Let f and g be two L- fuzzy soft sets on X.

(1) f is an L-fuzzy soft subset of g and we write $f \sqsubseteq g$ if $f_e \le g_e$, for each $e \in E$. f and g are equal if $f \sqsubseteq g$ and $g \sqsubseteq f$.

(2) The intersection of f and g is an L- fuzzy soft set $h = f \sqcap g$, where $h_e = f_e \wedge g_e$, for each $e \in E$.

(3) The union of f and g is an L- fuzzy soft set $h = f \sqcup g$, where $h_e = f_e \vee g_e$, for each $e \in E$.

(4) An L- fuzzy soft set $h = f \odot g$ is defined as $h_e = f_e \odot g_e$, for each $e \in E$.

(5) The complement of an L- fuzzy soft sets on X is denoted by f^*, where $f^* : E \to L^X$ is a mapping given by $f_e^* = (f_e)^*$, for each $e \in E$.

(6) 0_X (resp. 1_X) is an L-fuzzy soft set if $(0_X)_e(x) = 0$ (resp. $(1_X)_e(x) = 1$), for each $e \in E$, $x \in X$.

Definition 2.5. [4] Let $\varphi : X \to Y$ and $\psi : E \to F$ be two mappings, where E and K are parameters sets for the crisp sets X and Y, respectively. Then $\varphi_\psi : (X, E) \to (Y, F)$ is called a fuzzy soft mapping. Let f and g be two fuzzy soft sets over X and Y, respectively and let φ_ψ be a fuzzy soft mapping from (X, E) into (Y, F).

(1) The image of f under the fuzzy soft mapping φ_ψ, denoted by $\varphi_\psi(f)$ is the fuzzy soft set on Y defined by ,$\forall e_1 \in F, \forall y \in Y$,

$$\varphi(f)_{e_1}(y) = \begin{cases} \bigvee_{\varphi(x)=y}\Big(\bigvee_{\psi(e)=e_1} f_e(x)\Big), & \text{if } x \in \varphi^{-1}(\{y\}), e \in \psi^{-1}(\{e_1\}) \\ 0, & \text{otherwise,} \end{cases}$$

(2) The pre-image of g under the fuzzy soft mapping φ_ψ, denoted by $\varphi_\psi^{-1}(g)$ is the fuzzy soft set on X defined by

$$\varphi_\psi^{-1}(g)_e(x) = g_{\psi(e)}(\varphi(x)), \forall e \in E, \forall x \in X.$$

Definition 2.6. [4,19] A mapping $\mathcal{T} : K \to L^{(L^X)^E}$ (where $\mathcal{T}_k := \mathcal{T}(k) : (L^X)^E \to L$ is a mapping for each $k \in K$) is called an L-fuzzy (K, E)-soft topology on X if it satisfies the following conditions for each $k \in K$.

(SO1) $\mathcal{T}_k(0_X) = \mathcal{T}_k(1_X) = 1$,
(SO2) $\mathcal{T}_k(f \odot g) \geq \mathcal{T}_k(f) \odot \mathcal{T}_k(g) \;\; \forall\, f, g \in (L^X)^E$,
(SO3) $\mathcal{T}_k(\bigsqcup_i f_i) \geq \bigwedge_{i\in I} \mathcal{T}_k(f_i) \; \forall\, f_i \in (L^X)^E$, $i \in I$.

The pair $(X, \mathcal{T})$ is called an L-fuzzy (K, E)-soft topological space.

An L-fuzzy (K, E)-soft topology is called enriched if

(SR) $\mathcal{T}_k(\alpha \odot f) \geq \mathcal{T}_k(f)$ for all $f \in (L^X)^E$ and $\alpha \in L$.

Let $(X, \mathcal{T}^1)$ be an L-fuzzy (K_1, E_1)-soft topological space and $(Y, \mathcal{T}^2)$ be an L-fuzzy (K_2, E_2)-soft topological space. Let $\varphi : X \to Y$, $\psi : E_1 \to E_2$ and $\eta : K_1 \to K_2$ be mappings. Then $\varphi_{\psi,\eta}$ from $(X, \mathcal{T}^1)$ into $(Y, \mathcal{T}^2)$ is called L-fuzzy soft continuous if

$$\mathcal{T}^2_{\eta(k)}(f) \leq \mathcal{T}^1_k(\varphi_\psi^{-1}(f)) \;\; \forall\, f \in (L^Y)^{E_2}, k \in K_1.$$

Lemma 2.7. [18] Define a binary mapping $S : (L^X)^E \times (L^X)^E \to L$ by

$$S(f, g) = \bigwedge_{x\in X} \bigwedge_{e\in E} (f_e(x) \to g_e(x)) \;\; \forall\, f, g \in (L^X)^E, \;\; \forall\, e \in E.$$

Then $\forall f, g, h, m, n \in (L^X)^E$ the following statements hold.

(1) $f \sqsubseteq g$ iff $S(f, g) = 1$.
(2) If $f \sqsubseteq g$, then $S(h, f) \leq S(h, g)$ and $S(f, h) \geq S(g, h)$.
(3) $S(f, h) \odot S(h, g) \leq S(f, g)$. Moreover, $\bigvee_{h\in(L^X)^E}(S(f, h) \odot S(h, g)) = S(f, g)$
(4) $S(f, g) \odot S(m, n) \leq S(f \odot m, g \odot n)$.
(5) If $\varphi_\psi : (X, E) \to (Y, F)$ is a fuzzy soft mapping, then $S(p, q) \leq S(\varphi_\psi^{-1}(p), \varphi_\psi^{-1}(q))$, for each $p, q \in (L^Y)^F$.

Definition 2.8. [18] An L- fuzzy (K, E)-soft quasi- uniformity is a mapping $\mathcal{U} : K \to L^{(L^{X\times X})^E}$ which satisfies the following conditions .

(SU1) There exists $u \in (L^{X\times X})^E$ such that $\mathcal{U}_k(u) = 1$.
(SU2) If $v \sqsubseteq u$, then $\mathcal{U}_k(v) \leq \mathcal{U}_k(u)$.
(SU3) For every $u, v \in (L^{X\times X})^E, \mathcal{U}_k(u \odot v) \geq \mathcal{U}_k(u) \odot \mathcal{U}_k(v)$.
(SU4) If $\mathcal{U}_k(u) \neq 0$ then $\top_\triangle \sqsubseteq u$ where, for each $e \in E$,

$$(\top_\triangle)_e(x,y) = \begin{cases} 1, & \text{if } x = y, \\ 0, & \text{if } x \neq y. \end{cases}$$

(SU5) $\mathcal{U}_k(u) \leq \bigvee\{\mathcal{U}_k(v) \mid v \circ v \sqsubseteq u\}$, where

$$v_e \circ w_e(x,z) = \bigvee_{y\in X} v_e(x,y) \odot w_e(y,z),$$

The pair $(X, \mathcal{U})$ is called an L-fuzzy (K,E)-soft quasi-uniform space.

An L-fuzzy (K,E)-soft quasi-uniform space $(X, \mathcal{U})$ is said to be an L-fuzzy (K,E)-soft uniform space if

(U) $\mathcal{U}_k(u) \leq \mathcal{U}_k(u^{-1})$, where $(u^{-1})_e(x,y) = u_e(y,x)$ for each $k \in K$ and $u \in (L^{X\times X})^E$.

An L-fuzzy (K,E)-soft quasi-uniformity $\mathcal{U}$ on X is said to be stratified if

(SR) $\mathcal{U}_k(\alpha \odot u) \geq \alpha \odot \mathcal{U}_k(u), \ \ \forall\, u \in (L^{X\times X})^E, \alpha \in L$.

Let $(X, \mathcal{U}^1)$ be an L-fuzzy (K_1, E_1)-soft quasi-uniform space and $(Y, \mathcal{U}^2)$ be an L-fuzzy (K_2, E_2)-soft quasi-uniform space. Let $\varphi : X \to Y$, $\psi : E_1 \to E_2$ and $\eta : K_1 \to K_2$ be mappings. Then $\varphi_{\psi,\eta}$ from $(X, \mathcal{U}^1)$ into $(Y, \mathcal{U}^2)$ is called L-fuzzy soft uniformly continuous if

$$\mathcal{U}^2_{\eta(k)}(v) \leq \mathcal{U}^1_k((\varphi \times \varphi)^{-1}_\psi(v)) \ \ \forall\, v \in (L^{Y\times Y})^{E_2}, k \in K_1.$$

Remark 2.9. Let $(X, \mathcal{U})$ be an L-fuzzy (K,E)-soft uniform space.

(1) By (SU1) and (SU2), we have $\mathcal{U}_k(1_{X\times X}) = 1$ because $u \sqsubseteq 1_{X\times X}$ for all $u \in (L^{X\times X})^E$.
(2) Since $\mathcal{U}_k(u) \leq \mathcal{U}_k(u^{-1}) \leq \mathcal{U}_k((u^{-1})^{-1}) = \mathcal{U}_k(u)$, then $\mathcal{U}_k(u) = \mathcal{U}_k(u^{-1})$.

3 Topological Properties of L-fuzzy (K,E)-soft Uniform Spaces

Definition 3.1. An L-fuzzy (K,E)-soft neighborhood system on X is a set $N = \{N^x \mid x \in X\}$ of mappings $N^x : K \to L^{(L^X)^E}$ such that for each $k \in K$:

(SN1) $N^x_k(\top_X) = \top$ and $N^x_k(\perp_X) = \perp$,
(SN2) $N^x_k(f \odot g) \geq N^x_k(f) \odot N_x(g)$ for each $f, g \in (L^X)^E$,

(SN3) If $f \sqsubseteq g$, then $N_k^x(f) \leq N_k^x(g)$,
(SN4) $N_k^x(f) \leq \bigwedge_{e \in E} f_e(x)$ for all $f \in (L^X)^E$. (Here $N^x(k) =: N_k^x : (L^X)^E \to L$)

An L-fuzzy (K, E)-soft neighborhood system is called stratified if

(SR) $N_k^x(\alpha \odot f) \geq \alpha \odot N_k^x(f)$ for all $f \in (L^X)^E$ and $\alpha \in L$.

The pair (X, N) is called an L-fuzzy (K, E)-soft neighborhood space.

Let (X, N) be an L-fuzzy (K_1, E_1)-soft neighborhood space and (Y, M) be an L-fuzzy (K_2, E_2)-soft neighborhood space. Let $\varphi : X \to Y$, $\psi : E_1 \to E_2$ and $\eta : K_1 \to K_2$ be mappings. Then $\varphi_{\psi,\eta}$ from (X, N) into (Y, M) is called L-fuzzy soft continuous at every $x \in X$ if $M_{\eta(k)}^{\phi(x)}(f) \leq N_k^x(\varphi_\psi^{-1}(f))$ $\forall\ f \in (L^Y)^{E_2}, k \in K_1$.

Theorem 3.2. Let $(X, \mathcal{T})$ be an L-fuzzy (K, E)- soft topological space. Define a map $(N^{\mathcal{T}})^x : K \to L^{(L^X)^E}$ by

$$(N^{\mathcal{T}})_k^x(f) = \bigvee \{ \bigwedge_{e \in E} (\mathcal{T}_k(g) \odot g_e(x)) \mid g \sqsubseteq f \}.$$

Then,

(1) $N^{\mathcal{T}}$ is an L-fuzzy (K, E)-soft neighborhood on X,
(2) If $\mathcal{T}$ is enriched, then $N^{\mathcal{T}}$ is stratified.

Proof. (SN1)

$$(N^{\mathcal{T}})_k^x(\top_X) \geq \bigwedge_{e \in E} (\mathcal{T}_k(\top_X) \odot (\top_X)_e(x)) = \top,$$

$$(N^{\mathcal{T}})_k^x(\bot_X) = \bigwedge_{e \in E} (\mathcal{T}_k(\bot_X) \odot (\bot_X)_e(x)) = \bot.$$

(SN2)

$$\begin{aligned} & \bigwedge_{e \in E} (\mathcal{T}_k(f \odot g) \odot (f_e \odot g_e)(x)) \\ & \geq \bigwedge_{e \in E} (\mathcal{T}_k(f) \odot \mathcal{T}_k(g) \odot (f_e \odot g_e)(x)) \\ & \geq \bigwedge_{e \in E} (\mathcal{T}_k(f) \odot f_e(x)) \odot \bigwedge_{e \in E} (\mathcal{T}_k(g) \odot g_e(x)) \end{aligned}$$

$$\begin{aligned} & (N^{\mathcal{T}})_k^x(f_1) \odot (N^{\mathcal{T}})_k^x(f_2) \\ & = \bigvee \{ \bigwedge_{e \in E} (\mathcal{T}_k(g_1) \odot (g_1)_e(x)) \mid g_1 \sqsubseteq f_1 \} \odot \bigvee \{ \bigwedge_{e \in E} (\mathcal{T}_k(g_2) \odot (g_2)_e(x)) \mid g_2 \sqsubseteq f_2 \} \end{aligned}$$

$$\leq \bigvee\{\bigwedge_{e\in E}(\mathcal{T}_k(g_1\odot g_2)\odot((g_2)_e\odot(g_2)_e)(x))\mid g_1\odot g_2\sqsubseteq f_1\odot f_2\}$$
$$\leq (N^{\mathcal{T}})^x_k(f\odot g).$$

(SN3) and (SN4) are easily proved.

(2) Let $\mathcal{T}$ be enriched. Then

$$\alpha\odot(N^{\mathcal{T}})^x_k(g)=\alpha\odot\bigvee\{\bigwedge_{e\in E}(\mathcal{T}_k(f)\odot f_e(x))\mid f\sqsubseteq g\}$$
$$\leq \bigvee\{\bigwedge_{e\in E}(\alpha\odot\mathcal{T}_k(f)\odot f_e(x))\mid f\sqsubseteq g\}$$
$$\leq \bigvee\{\bigwedge_{e\in E}(\mathcal{T}_k(\alpha\odot f)\odot(\alpha\odot f_e)(x))\mid \alpha\odot f\sqsubseteq \alpha\odot g\}$$
$$\leq (N^{\mathcal{T}})^x_k(\alpha\odot f).$$

Theorem 3.3. Let $(X,\mathcal{T}^1)$ be an L-fuzzy (K_1,E_1)-soft topological space and $(Y,\mathcal{T}^2)$ be an L-fuzzy (K_2,E_2)-soft topological space. Let $\varphi:X\to Y$, $\psi:E_1\to E_2$ and $\eta:K_1\to K_2$ be mappings. If $\varphi_{\psi,\eta}:(X,\mathcal{T}^1)\to(Y,\mathcal{T}^2)$ is L-fuzzy soft continuous, then $\varphi_{\psi,\eta}:(X,N^{\mathcal{T}^1})\to(Y,N^{\mathcal{T}^2})$ is L-fuzzy soft continuous.

Proof.

$$(N^{\mathcal{T}^2})^{\varphi(x)}_{\eta(k)}(g)\to(N^{\mathcal{T}^1})^x_k(\varphi_\psi^{-1}(g))$$
$$=\bigvee\{\bigwedge_{e_2\in E_2}(\mathcal{T}^2_{\eta(k)}(f)\odot f_{e_2}(\varphi(x)))\mid g\sqsubseteq f\}\to\bigvee\{\bigwedge_{e_1\in E_1}(\mathcal{T}^1_k(h)\odot h_{e_1}(x))\mid h\sqsubseteq\varphi_\psi^{-1}(g)\}$$
$$\geq\bigvee\{\bigwedge_{e_2\in E_2}(\mathcal{T}^2_{\eta(k)}(f)\odot f_{e_2}(\varphi(x)))\mid g\sqsubseteq f\}\to$$
$$\bigvee\{\bigwedge_{e_1\in E_1}(\mathcal{T}^1_k(\varphi_\psi^{-1}(f))\odot\varphi_\psi^{-1}(f)_{e_1}(x))\mid\varphi_\psi^{-1}(f)\sqsubseteq\varphi_\psi^{-1}(g)\}$$
$$\geq\bigwedge\{\bigwedge_{e_2\in E_2}(\mathcal{T}^2_{\eta(k)}(f)\odot f_{e_2}(\varphi(x)))\to\bigwedge_{e_1\in E_1}(\mathcal{T}^1_k(\varphi_\psi^{-1}(f))\odot\varphi_\psi^{-1}(f)_{e_1}(x))\mid f\sqsubseteq g\}$$
$$\geq\bigwedge\{\bigwedge_{e_1\in E_1}(\mathcal{T}^2_{\eta(k)}(f)\odot f_{\psi(e_1)}(\varphi(x)))\to\bigwedge_{e_1\in E_1}(\mathcal{T}^1_k(\varphi_\psi^{-1}(f))\odot f_{\psi(e_1)}(\varphi(x)))\mid f\sqsubseteq g\}$$
$$\geq\bigwedge\bigwedge_{e_1\in E_1}\{(\mathcal{T}^2_{\eta(k)}(f)\odot f_{\psi(e_1)}(\varphi(x)))\to(\mathcal{T}^1_k(\varphi_\psi^{-1}(f))\odot f_{\psi(e_1)}(\varphi(x)))\mid f\sqsubseteq g\}$$
$$\geq\bigwedge\{\mathcal{T}^2_{\eta(k)}(f)\to\mathcal{T}^1_k(\varphi_\psi^{-1}(f))\mid f\sqsubseteq g\}$$

Thus, if $\mathcal{T}_{\eta(k)}(f)\leq\mathcal{T}^{\mathcal{U}}_k(\varphi_\psi^{-1}(f))$, then $(N^{\mathcal{T}^2})^{\varphi(x)}_{\eta(k)}(g)\leq(N^{\mathcal{T}^1})^x_k(\varphi_\psi^{-1}(g))$. So, $\varphi_{\psi,\eta}$

is L-fuzzy soft continuous.

Theorem 3.4. Let (X, N) be an L-fuzzy (K, E)-soft neighborhood space. Define a map $\mathcal{T}^N : K \to L^{(L^X)^E}$ by

$$\mathcal{T}^N_k(f) = \bigwedge_{x\in X}\bigwedge_{e\in E}(f_e(x) \to (N^x_k)(f)).$$

Then we have the following properties.

(1) $\mathcal{T}^N$ is an L-fuzzy (K, E)-soft topology on X.
(2) If N is stratified, then $\mathcal{T}^N$ is an enriched L-fuzzy (K, E)-soft topology.
(3) $N^{\mathcal{T}^N} \leq N$.
(4) If $(X, \mathcal{T})$ is an L-fuzzy (K, E)-soft topological space with $E = \{e\}$, then $\mathcal{T}^{N^{\mathcal{T}}} \geq \mathcal{T}$.

Proof. (1) (SO1)

$$\mathcal{T}^N_k(\top_X) = \bigwedge_{x\in X}\bigwedge_{e\in E}((\top_X)_e(x) \to N^x_k(\top_X) = \top \to \top = \top,$$

$$\mathcal{T}^N_k(\bot_X) = \bigwedge_{x\in X}\bigwedge_{e\in E}((\bot_X)_e(x) \to N^x_k(\bot_X)) = \bot \to \bot = \bot.$$

(SO2)

$$\begin{aligned}\mathcal{T}^N_k(f \odot g) &= \bigwedge_{x\in X}\bigwedge_{e\in E}((f_e \odot g_e)(x) \to N^x_k(f \odot g))\\
&\geq \bigwedge_{x\in X}\bigwedge_{e\in E}((f_e \odot g_e)(x) \to (N^x_k(f) \odot N^x_k(g)))\\
&\quad\text{(by Lemma 2.3 (11))}\\
&\geq \bigwedge_{x\in X}\bigwedge_{e\in E}(f_e(x) \to N^x_k(f)) \odot \bigwedge_{x\in X}\bigwedge_{e\in E}(g_e(x) \to N^x_k(g))\\
&= \mathcal{T}^N_k(f) \odot \mathcal{T}^N_k(g).\end{aligned}$$

(SO3)

$$\begin{aligned}\mathcal{T}^N_k(\bigsqcup_i f_i) &= \bigwedge_{x\in X}\bigwedge_{e\in E}((\bigsqcup_i (f_i)_e)(x) \to N^x_k(\bigsqcup_i f_i))\\
&\geq \bigwedge_{x\in X}\bigwedge_{e\in E}((\bigsqcup_i (f_i)_e)(x) \to \bigvee_i N^x_k(f_i))(x))\\
&\quad\text{(by Lemma 2.3 (15))}\\
&\geq \bigwedge_i\bigwedge_{x\in X}\bigwedge_{e\in E}((f_i)_e)(x) \to N^x_k(f_i)) = \bigwedge_i \mathcal{T}^N_k(f_i).\end{aligned}$$

(2) (SR) By Lemma 2.3 (12), we have

$$\begin{aligned}\mathcal{T}_k^N(\alpha \odot f) &= \bigwedge_{x\in X}\bigwedge_{e\in E}((\alpha \odot f_e)(x) \to N_k^x(\alpha \odot f))\\ &\geq \bigwedge_{x\in X}\bigwedge_{e\in E}((\alpha \odot f_e)(x) \to (\alpha \odot N_k^x(f)))\\ &\geq \bigwedge_{x\in X}\bigwedge_{e\in E}(f_e(x) \to N_k^x(f)) = \mathcal{T}_k^N(f).\end{aligned}$$

(3) We have $N^{\mathcal{T}^N} \leq N$ from:

$$\begin{aligned}(N^{\mathcal{T}^N})_k^x(f) &= \bigwedge_{e\in E}(\mathcal{T}_k^N(f) \odot f_e(x))\\ &= \bigwedge_{e\in E}((\bigwedge_{z\in X}\bigwedge_{e_1\in E}(f_{e_1}(z) \to N_k^z(f)) \odot f_e(x))\\ &\leq \bigwedge_{e\in E}((f_e(x) \to N_k^x(f)) \odot f_e(x)) \leq N_k^x(f).\end{aligned}$$

(4) Let $E = \{e\}$ be given. We have $\mathcal{T}^{N^{\mathcal{T}}} \geq \mathcal{T}$ from:

$$\begin{aligned}\mathcal{T}_k^{N^{\mathcal{T}}}(f) &= \bigwedge_{x\in X}(f_e(x) \to N_k^{\mathcal{T}}(f))\\ &= \bigwedge_{x\in X}(f_e(x) \to \bigvee\{\mathcal{T}_k(g) \odot g_e(x)) \mid g \sqsubseteq f\}\\ &\geq \bigwedge_{x\in X}(f_e(x) \to \mathcal{T}_k(f) \odot f_e(x))\\ &\geq \mathcal{T}_k(f) \text{ (by Lemma 2.3 (11))}.\end{aligned}$$

Theorem 3.6. Let (X, N^1) be an L-fuzzy (K_1, E_1)-soft neighborhood space and (Y, N^2) be an L-fuzzy (K_2, E_2)-soft neighborhood space. Let $\varphi : X \to Y$, $\psi : E_1 \to E_2$ and $\eta : K_1 \to K_2$ be mappings. If $\varphi_{\psi,\eta} : (X, N^1) \to (Y, N^2)$ is L-fuzzy soft continuous, then $\varphi_{\psi,\eta} : (X, \mathcal{T}^{N^1}) \to (Y, \mathcal{T}^{N^2})$ is L-fuzzy soft continuous.

Proof.

$$\begin{aligned}&\mathcal{T}_{\eta(k)}^{N^2}(f) \to \mathcal{T}_k^{N^1}(\varphi_\psi^{-1}(f))\\ &= \bigwedge_{y\in Y}\bigwedge_{e_2\in E_2}(f_{e_2}(y) \to (N^2)_{\eta(k)}^y(f)) \to \bigwedge_{x\in X}\bigwedge_{e_1\in E_1}(\varphi_\psi^{-1}(f)_{e_1}(x) \to (N^1)_k^x(\varphi_\psi^{-1}(f)))\\ &\geq \bigwedge_{x\in X}\bigwedge_{e_1\in E_1}(\varphi_\psi^{-1}(f)_{e_1}(x) \to (N^2)_{\eta(k)}^{\varphi_\psi(x)}(f)) \to \bigwedge_{x\in X}\bigwedge_{e_1\in E_1}(\varphi_\psi^{-1}(f)_{e_1}(x) \to (N^1)_k^x(\varphi_\psi^{-1}(f)))\end{aligned}$$

$$\geq \bigwedge_{x\in X}\bigwedge_{e_1\in E_1}\Big((\varphi_\psi^{-1}(f)_{e_1}(x)\to (N^2)^{\varphi_\psi(x)}_{\eta(k)}(f))\to(\varphi_\psi^{-1}(f)_{e_1}(x)\to (N^1)^x_k(\varphi_\psi^{-1}(f)))\Big)$$

(by Lemma 2.3 (13))

$$\geq \bigwedge_{x\in X}\Big((N^2)^{\varphi_\psi(x)}_{\eta(k)}(f)\to (N^1)^x_k(\varphi_\psi^{-1}(f))\Big).$$

Thus, if $(N^2)^{\varphi_\psi(x)}_{\eta(k)})(f)\leq (N^1)^x_k(\varphi_\psi^{-1}(f))$, then $\mathcal{T}^2_{\eta(k)}(f)\leq \mathcal{T}^1_k(\varphi_\psi^{-1}(f))$. So, $\varphi_{\psi,\eta}$ is L-fuzzy soft continuous.

Theorem 3.7. Let $(X,\mathcal{U})$ be an L-fuzzy (K,E)-soft uniform space. Define a map $N^{\mathcal{U}}: K\to L^{(L^X)^E}$ by

$$(N^{\mathcal{U}})^x_k(f)=\bigvee_u \mathcal{U}_k(u)\odot S(u[x],f),\quad \forall\ f\in (L^X)^E,\ x\in X,$$

where $u_e[x](y)=u_e(y,x)$. Then the following properties hold.

(1) $(X,N^{\mathcal{U}})$ is an L-fuzzy (K,E)-soft neighborhood space.
(2) If $\mathcal{U}$ is stratified, then $N^{\mathcal{U}}$ is also stratified.

Proof. (1) (SN1) For $\mathcal{U}_k(u)\neq\bot,\ \top_\triangle\sqsubseteq u$. Then

$$\begin{aligned}(N^{\mathcal{U}})^x_k(\bot_X)&=\bigvee_u \mathcal{U}_k(u)\odot S(u[x],\bot_X)\\ &\leq \bigvee_u\big(\mathcal{U}_k(u)\odot(u_e(x,x)\to\bot)\big)\\ &\leq \bigvee_u\big(\mathcal{U}_k(u)\odot((\top_\triangle)_e(x,x)\to\bot)\big)=\bot.\end{aligned}$$

Hence $(N^{\mathcal{U}})^x_k(\bot_X)=\bot$. Also, $(N^{\mathcal{U}})^x_k(\top_X)=\top$, because

$$(N^{\mathcal{U}})^x_k(\top_X)\geq \mathcal{U}_k(\top_{X\times X})\odot\bigwedge_{e\in E}\bigwedge_{y\in X}((\top_{X\times X})_e(x,y)\to(\top_X)_e(y))=\top.$$

(SN2) By Lemma 2.7 (4) , we have

$$\begin{aligned}&(N^{\mathcal{U}})^x_k(f)\odot(N^{\mathcal{U}})^x_k(g)\\ &=\big(\bigvee_u \mathcal{U}_k(u)\odot S(u[x],f)\big)\odot\big(\bigvee_u \mathcal{U}_k(v)\odot S(v[x],g)\big)\\ &=\bigvee_{u,v}\mathcal{U}_k(u)\odot\mathcal{U}_k(v)\odot S(u[x],f)\odot S(v[x],g)\\ &\leq\bigvee_{u,v}\mathcal{U}_k(u\odot v)\odot S((u\odot v)[x],f\odot g)\\ &\leq\bigvee_{w}\mathcal{U}_k(w)\odot S(w[x],f\odot g)=(N^{\mathcal{U}})^x_k(f\odot g).\end{aligned}$$

(SN3) By Lemma 2.7 (2), we have

$$(N^{\mathcal{U}})^x_k(f) = \bigvee_u \mathcal{U}_k(u) \odot S(u[x], f)$$
$$\leq \bigvee_u \mathcal{U}_k(u) \odot S(u[x], g) = (N^{\mathcal{U}})^x_k(g).$$

(SN4) For $\mathcal{U}_k(u) \neq \perp$, $\top_\triangle \sqsubseteq u$.

$$(N^{\mathcal{U}})^x_k(f) = \bigvee_u \mathcal{U}_k(u) \odot \bigwedge_{e \in E} \bigwedge_{y \in X} (u_e(y, x) \to f_e(y))$$
$$\leq \bigvee_u \big\{\mathcal{U}_k(u) \odot (u_e(x, x) \to f_e(x))\big\} \leq f_e(x).$$

This implies that $(X, N^{\mathcal{U}}_x)$ is an L-fuzzy soft neighborhood space.

(2)

$$\alpha \odot (N^{\mathcal{U}})^x_k(f) = \alpha \odot \bigvee_u \mathcal{U}_k(u) \odot S(u[x], f)$$
$$= \bigvee_u \alpha \odot \mathcal{U}_k(u) \odot S(\alpha, \alpha) \odot S(u[x], f)$$
$$\leq \bigvee_u \mathcal{U}_k(\alpha \odot u) \odot S(\alpha \odot u[x], \alpha \odot f)$$
$$\leq (N^{\mathcal{U}})^x_k(\alpha \odot f).$$

Theorem 3.8. Let $(X, \mathcal{U})$ be an L-fuzzy (K_1, E_1)-soft uniform space and $(Y, \mathcal{V})$ be an L-fuzzy (K_2, E_2)-soft uniform space. Let $\varphi : X \to Y$, $\psi : E_1 \to E_2$ and $\eta : K_1 \to K_2$ be mappings. If $\varphi_{\psi,\eta} : (X, \mathcal{U}) \to (Y, \mathcal{V})$ is L-fuzzy soft uniformly continuous, then $\varphi_{\psi,\eta} : (X, N^{\mathcal{U}}) \to (Y, N^{\mathcal{V}})$ is L-fuzzy soft continuous.

Proof. First we show that $\varphi^{-1}_\psi(v_e[\varphi_\psi(x)]) = (\varphi_\psi \times \varphi_\psi)^{-1}(v_e)[x]$ from

$$\varphi^{-1}_\psi(v_e[\varphi_\psi(x)])(z) = v_e[\varphi_\psi(x)](\varphi_\psi(z)) = v_e(\varphi_\psi(z), \varphi_\psi(x))$$
$$= (\varphi_\psi \times \varphi_\psi)^{-1}(v_e)(z, x) = (\varphi_\psi \times \varphi_\psi)^{-1}(v_e)[x](z).$$

Thus, by Lemma 2.7(4), we have

$$S(v[\varphi_\psi(x)], f) \leq S(\varphi^{-1}_\psi(v[\varphi_\psi(x)]), \varphi^{-1}_\psi(f))$$
$$= S((\varphi_\psi \times \varphi_\psi)^{-1}(v)[x], \varphi^{-1}_\psi(f)).$$

$$(N^{\mathcal{V}})^{\varphi_\psi(x)}_{\eta(k)}(f) = \bigvee_v \mathcal{V}_{\eta(k)}(v) \odot S(v[\varphi_\psi(x)], f)$$
$$\leq \bigvee_v \mathcal{V}_{\eta(k)}(v) \odot S((\varphi_\psi \times \varphi_\psi)^{-1}(v)[x], \varphi_\psi^{-1}(f))$$
$$\leq \bigvee_u \mathcal{U}_k((\varphi_\psi \times \varphi_\psi)^{-1}(v)) \odot S((\varphi_\psi \times \varphi_\psi)^{-1}(v)[x], \varphi_\psi^{-1}(f))$$
$$\leq (N^{\mathcal{U}})^x_k(\varphi_\psi^{-1}(f)).$$

From Theorems 3.4 and 3.7, we obtain the following corollary.

Corollary 3.9. Let $(X,\mathcal{U})$ be an L-fuzzy (K,E) soft uniform space and $N^{\mathcal{U}} = \{N^{\mathcal{U}}_x \mid x \in X\}$ be an L-fuzzy (K,E) soft neighborhood system on X. Define a map $\mathcal{T}^{\mathcal{U}} : K \to L^{(L^X)^E}$ by

$$\mathcal{T}^{\mathcal{U}}(f) = \bigwedge_{x\in X} (f_e(x) \to (N^{\mathcal{U}})^x_k(f)).$$

Then,

(1) $\mathcal{T}^{\mathcal{U}}$ is an L-fuzzy (K,E) soft topology on X,
(2) If $N^{\mathcal{U}}$ is stratified, then $\mathcal{T}^{\mathcal{U}}$ is an enriched L-fuzzy topology.

From Theorems 3.6 and 3.8, we obtain the following corollary.

Corollary 3.10. Let $(X,\mathcal{U})$ be an L-fuzzy (K_1,E_1)-soft uniform space and $(Y,\mathcal{V})$ be an L-fuzzy (K_2,E_2)-soft uniform space. Let $\varphi : X \to Y$, $\psi : E_1 \to E_2$ and $\eta : K_1 \to K_2$ be mappings. If $\varphi_{\psi,\eta} : (X,\mathcal{U}) \to (Y,\mathcal{V})$ is L-fuzzy soft uniformly continuous, then $\varphi_{\psi,\eta} : (X,\mathcal{T}^{\mathcal{U}}) \to (Y,\mathcal{T}^{\mathcal{V}})$ is L-fuzzy soft continuous.

Example 3.11. Let $X = \{h_i \mid i = \{1,2,3\}\}$ with h_i=house and $E = \{e,b\}$ with e=expensive, b= beautiful. Define a binary operation $\odot$ on $[0,1]$ by

$$x \odot y = \max\{0, x+y-1\},\ x \to y = \min\{1-x+y, 1\}$$

Then $([0,1], \wedge, \to, 0, 1)$ is a stsc-quantle (ref [8-10]). Put $f, g \in (L^X)^E$ such that

$$f_e(h_1) = 0.5, f_e(h_2) = 0.5, f_e(h_3) = 0.6$$
$$f_b(h_1) = 0.6, f_b(h_2) = 0.3, f_b(h_3) = 0.6$$
$$g_e(h_1) = 0.8, g_e(h_2) = 0.7, g_e(h_3) = 0.5$$
$$g_b(h_1) = 0.9, g_b(h_2) = 0.6, g_b(h_3) = 0.6$$

(1) Put $K = \{k_1, K_2\}$. We define $\mathcal{T} : K \to [0,1]^{([0,1]^X)^E}$ as follows:

$$\mathcal{T}_{k_1}(h) = \begin{cases} 1, & \text{if } h = 1_X \text{or } h = 0_X \\ 0.6, & \text{if } h = f, \\ 0.3, & \text{if } h = f \odot f, \\ 0, & \text{otherwise.} \end{cases}$$

$$\mathcal{T}_{k_2}(h) = \begin{cases} 1, & \text{if } h = 1_X \text{or } h = 0_X \\ 0.5, & \text{if } h = g, \\ 0, & \text{otherwise.} \end{cases}$$

We obtain a $[0,1]$-fuzzy (K,E)-soft neighborhood system on X as:

$$(N^{\mathcal{T}})^{h_1}_{k_1}(h) = \begin{cases} 1, & \text{if } h = 1_X \\ 0.1, & \text{if } f \sqsubseteq h \neq 1_X, \\ 0, & \text{otherwise.} \end{cases} \quad (N^{\mathcal{T}})^{h_1}_{k_2}(h) = \begin{cases} 1, & \text{if } h = 1_X \\ 0.2, & \text{if } g \sqsubseteq h \neq 1_X, \\ 0, & \text{otherwise.} \end{cases}$$

$$(N^{\mathcal{T}})^{h_2}_{k_1}(h) = \begin{cases} 1, & \text{if } h = 1_X \\ 0, & \text{otherwise.} \end{cases} \quad (N^{\mathcal{T}})^{h_2}_{k_2}(h) = \begin{cases} 1, & \text{if } h = 1_X \\ 0.1, & \text{if } g \sqsubseteq h \neq 1_X, \\ 0, & \text{otherwise.} \end{cases}$$

$$(N^{\mathcal{T}})^{h_3}_{k_1}(h) = \begin{cases} 1, & \text{if } h = 1_X \\ 0.2, & \text{if } f \sqsubseteq h \neq 1_X, \\ 0, & \text{otherwise.} \end{cases} \quad (N^{\mathcal{T}})^{h_3}_{k_2}(h) = \begin{cases} 1, & \text{if } h = 1_X \\ 0.1, & \text{if } g \sqsubseteq h \neq 1_X, \\ 0, & \text{otherwise.} \end{cases}$$

Then

$$\mathcal{T}^{N^{\mathcal{T}}}_{k_1}(f) = \bigwedge_{x\in X}(\bigvee_{e\in E} f_e(x) \to N^x_{k_1}(f)) = (0.6 \to 0.1) \wedge (0.5 \to 0) \wedge (0.6 \to 0.2) = 0.5$$

$$\mathcal{T}^{N^{\mathcal{T}}}_{k_1}(f \odot f) = (0.2 \to 0) \wedge (0 \to 0) \wedge (0.2 \to 0) = 0.8,$$
$$\mathcal{T}^{N^{\mathcal{T}}}_{k_2}(g) = (0.9 \to 0.2) \wedge (0.7 \to 0.1) \wedge (0.6 \to 0.1) = 0.3.$$

Since $E = \{e, b\}$, by Theorem 3.4(4), in general, $\mathcal{T}^{N^{\mathcal{T}}} \not\geq \mathcal{T}$.

(2) Put $v, v \odot v, w \in ([0,1]^{X\times X})^E$ as

$$v_e = \begin{pmatrix} 1 & 0.6 & 0.5 \\ 0.3 & 1 & 0.5 \\ 0.4 & 0.6 & 1 \end{pmatrix} v_b = \begin{pmatrix} 1 & 0.5 & 0.3 \\ 0.7 & 1 & 0.5 \\ 0.6 & 0.6 & 1 \end{pmatrix}$$

$$(v \odot v)_e = \begin{pmatrix} 1 & 0.2 & 0 \\ 0 & 1 & 0 \\ 0 & 0.2 & 1 \end{pmatrix} (v \odot v)_b = \begin{pmatrix} 1 & 0 & 0 \\ 0.4 & 1 & 0 \\ 0.2 & 0.2 & 1 \end{pmatrix}$$

$$w_e = \begin{pmatrix} 1 & 0.4 & 0.5 \\ 0.4 & 1 & 0.5 \\ 0.4 & 0.6 & 1 \end{pmatrix} v_b = \begin{pmatrix} 1 & 0.5 & 0.3 \\ 0.3 & 1 & 0.5 \\ 0.2 & 0.3 & 1 \end{pmatrix}$$

We define $\mathcal{U} : K \to [0,1]^{([0,1]^{X\times X})^E}$ as follows:

$$\mathcal{U}_{k_1}(u) = \begin{cases} 1, & \text{if } u = 1_{Y\times Y} \\ 0.6, & \text{if } v \sqsubseteq u \neq 1_{Y\times Y}, \\ 0.3, & \text{if } v \odot v \sqsubseteq u \not\sqsupseteq v, \\ 0, & \text{otherwise.} \end{cases}$$

$$\mathcal{U}_{k_2}(u) = \begin{cases} 1, & \text{if } u = 1_{Y\times Y} \\ 0.5, & \text{if } w \sqsubseteq u \neq 1_{Y\times Y}, \\ 0, & \text{otherwise.} \end{cases}$$

Since $(N^{\mathcal{U}})^x_k(f) = \bigvee_u \mathcal{U}_k(u) \odot S(u[x], f)$, we have

$$(N^{\mathcal{U}})^{h_1}_{k_1}(f) = 0.1, (N^{\mathcal{U}})^{h_2}_{k_1}(f) = 0.1, (N^{\mathcal{U}})^{h_3}_{k_1}(f) = 0.2$$
$$(N^{\mathcal{U}})^{h_1}_{k_2}(f) = 0, (N^{\mathcal{U}})^{h_2}_{k_2}(f) = 0, (N^{\mathcal{U}})^{h_3}_{k_2}(f) = 0.1$$

Since $\mathcal{T}^{\mathcal{U}}_k(f) = \bigwedge_{x\in X}(\bigvee_{e\in E} f_e(x) \to (N^{\mathcal{U}})^x_k(f))$,

$$\mathcal{T}^{\mathcal{U}}_{k_1}(f) = (0.6 \to 0.1) \wedge (0.5 \to 0.1) \wedge (0.6 \to 0.2) = 0.5,$$

$$\mathcal{T}^{\mathcal{U}}_{k_2}(f) = (0.6 \to 0) \wedge (0.5 \to 0) \wedge (0.6 \to 0.1) = 0.4.$$

$$(N^{\mathcal{U}})^{h_1}_{k_1}(g) = 0.4, (N^{\mathcal{U}})^{h_2}_{k_1}(g) = 0.2, (N^{\mathcal{U}})^{h_3}_{k_1}(g) = 0.1,$$
$$(N^{\mathcal{U}})^{h_1}_{k_2}(g) = 0.3, (N^{\mathcal{U}})^{h_2}_{k_2}(g) = 0.1, (N^{\mathcal{U}})^{h_3}_{k_2}(g) = 0.1,$$
$$\mathcal{T}^{\mathcal{U}}_{k_1}(g) = (0.9 \to 0.4) \wedge (0.7 \to 0.2) \wedge (0.6 \to 0.1) = 0.5,$$
$$\mathcal{T}^{\mathcal{U}}_{k_2}(g) = (0.9 \to 0.3) \wedge (0.7 \to 0.1) \wedge (0.6 \to 0.1) = 0.4.$$

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